Stability boundaries for fast particle driven TAE in stellarators

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1. Introduction
2. numerical 3D kinetic MHD model
3. stability boundaries for TAE in W7-AS #39042 and W7-X
4. contribution of thermal ions

Acknowledgments: S. Zegenhagen, J. Nührenberg, A. Weller

Introduction

kinetic effects may interact with ideal MHD modes:

- **destabilization of MHD gap modes by resonant interaction of fast particles**
- **source of free energy: gradient of fast particles**
- **experimental observations:**
  - W7-AS: Weller et al. (1998, 2000, 2003)
  - CHS/LHD: Toi et al. (2000, 2004)
- **theoretical approaches for three dimensions:**
  - analytical approach for passing particles: Kolesnichenko et al. (2001, . . .)
  - gyrofluid model 2D (Spong) - M3D nonlinear two fluid - kinetic hybrid model (Strauss, Park et al., 2002)

Kinetic energy integral

there is an energy integral considering kinetic effects

$$\delta W_{\mathrm{kin}} = \omega^2 \frac{1}{2} \int d^3\vec{x} \left|\vec{\xi}_\perp\right|^2 \rho_M = \delta W_{\mathrm{mag}} + \sum_{s=\mathrm{i,e,fast}} \delta W_s(\omega)$$

(Kruskal/Oberman 1958 ... Antonsen/Lee 1984)

$$\delta W_{\mathrm{mag}} = \frac{1}{2} \int d^3\vec{x} \left\{ \left|B_\perp^{(1)}\right|^2 + \left|B_\parallel^{(1)}\right|^2 + \vec{j}_\parallel \cdot \left(\vec{\xi}_\perp \times \vec{B}_\perp^{(1)}\right) - \frac{B_\parallel^{(1)}}{B} \vec{\xi}_\perp^* \cdot \vec{\nabla} p + \left(\vec{\nabla} \cdot \vec{\xi}_\perp^*\right) \left(\vec{\xi}_\perp \cdot \vec{\nabla} p\right) \right\}$$

the non-adiabatic contributions from the hot and thermal component replace the MHD fluid compression term

the contributions from the thermal plasma ($\delta W_{\mathrm{i,e}}$) and the fast particles δW_{fast}) depend on the perturbed particle Lagrangian $L^{(1)}$

Kinetic contribution

particle- wave- energy- exchange by resonant interaction

$$\delta W_s = \frac{\pi}{M_s^2}\left\{\sum_\sigma\right\}\int ds\int d\varphi\int d\mu\, d\epsilon\left(-\int\frac{d\vartheta}{|v_{||}|}\sqrt{g}B\right)\sum_{\substack{n,m\\ n',m'}}\sum_{p=-\infty}^{\infty} e^{-i\frac{2\pi}{N_p}(n'-n)\varphi}\times$$

$$\times\left(\frac{\partial F_s}{\partial\epsilon}\right)_\mu \frac{\omega-2\pi(\frac{n}{N_p}J-mI)\omega^*}{m\left\langle\omega_d^\vartheta\right\rangle+\frac{n}{N_p}\left\langle\omega_d^\varphi\right\rangle+\left\{\begin{matrix}\sigma(p+nq)\\ p\end{matrix}\right\}\omega_{\left\{\begin{smallmatrix}t\\ b\end{smallmatrix}\right\}}-\omega} L^{(1)*}_{m'n'}\mathcal{M}^{m'n'*}_{pn}\,L^{(1)}_{mn}\mathcal{M}^{mn}_{pn}$$

definition of $\mathcal{M}^{m'n'}_{pn}$:

for passing particles:

$$\mathcal{M}^{m'n'}_{pn} = \left\langle e^{i[2\pi(m'+n'q)\vartheta''-(p+nq)\omega_t t'']}\right\rangle_{\vartheta''}$$

for reflected particles:

$$\mathcal{M}^{m'n'}_{pn} = \left\langle e^{2\pi i(m'+n'q)\vartheta''}[\cos^2(\frac{\pi}{2}p)\cos(p\omega_b t'')-i\sin^2(\frac{\pi}{2}p)\sin(p\omega_b t'')]\right\rangle_{\vartheta''}$$

perturbed particle Lagrangian:

$$L^{(1)} = -(Mv_{||}^2-\mu B)\vec{\xi}_\perp\cdot\vec{\kappa}+\mu B\vec{\nabla}\cdot\vec{\xi}_\perp$$

$\langle\ldots\rangle$ denotes the transit or bounce average

Approximations in CAS3D-K

CAS3D-K: perturbative stability code based on a hybrid MHD-drift kinetic model

- 3-dimensional
- general mode structure and equilibrium
- particle drifts are approximated as bounce averaged drifts
- zero radial orbit width and passing particles (at the moment)
- perturbative growth/damping rates from:

$$\Delta\omega_s + i\gamma_s \approx \frac{1}{2}\frac{\delta W_{\mathrm{s}}(\omega_0)}{\delta W_{\mathrm{mag}}}\omega_0$$

  using the MHD eigenfunctions and the MHD frequency ω_0

- δW_{mag} from the ideal MHD stability code $CAS3D$
  (C. Nührenberg, 1996, 1998, 2000, . . .)

3D analytical theory

valid in the limit of very localized modes and for an isotropic distribution of the hot particles (Kolesnichenko et al. 2001)

hot particle growth rate:

$$\gamma = \frac{3\pi\beta_\alpha}{64k^2r^2}\sum_{\nu,\mu,j}\left|\epsilon^{(\mu\nu)}\right|^2 \frac{w\int_w^{w/\sqrt{\epsilon_{eff}}} du\,u(u^2+w^2)^2(\omega\partial/\partial u^2+\omega_d)f_0}{\int_0^\infty du\,u^4 f_0}$$

with

$$w = \left| v_{A*}\left(1+2j\frac{\iota_*-\nu N}{\mu_0\iota_*-\nu_0 N}\right)\right| / v_0 \qquad u = v/v_0$$

$$\iota_* = (2n+\nu N)/(2m+\mu_0) \qquad k = [(m+p)\iota - n + s]R_0^{-1}$$

3D analytical theory - What can we learn?

- **proportionality to equilibrium quantities**

$$\frac{\gamma}{\omega_0} \propto A^2 \sum_{\mu\nu} |\epsilon^{\kappa}_{\mu\nu}|^2 \approx A^2 \sum_{\mu\nu} |\epsilon^{B}_{\mu\nu}|^2$$

- **coupling is approximately given by the structure of B**
  $\Rightarrow$ **investigate spectrum of B**

- **note, that for a TAE in a large aspect ratio tokamak: $\frac{\gamma}{\omega_0}$ is independent of the equilibrium**

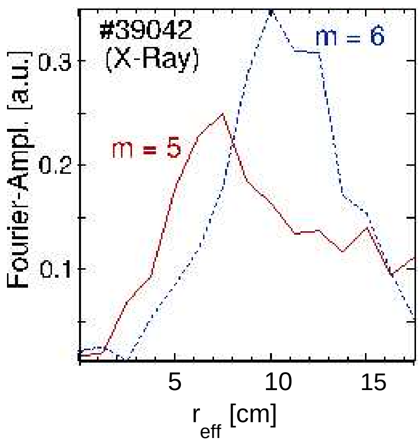
- **the resonance condition $\omega - k_{||} v_{th} = 0$ determines**

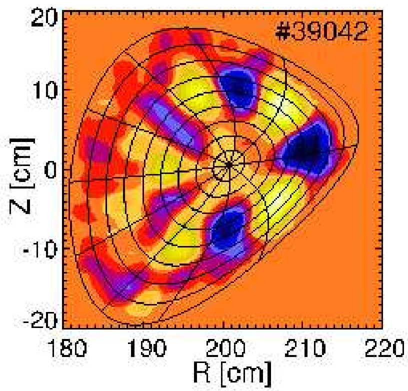
$$v^{\mathrm{res}}_{m'n'} = v_A \left| 1 \pm \frac{m'\iota^* + n' N_p}{m\iota^* + n} \right|^{-1}$$

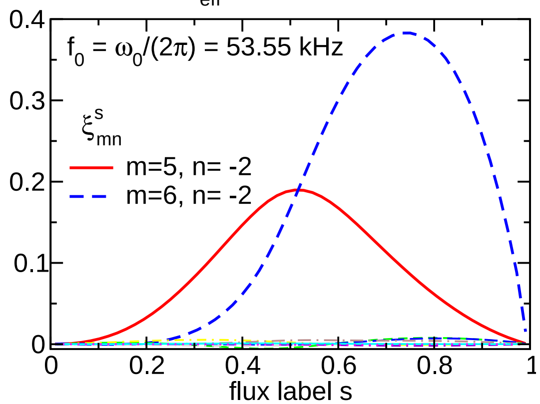
i.e. well known resonances at $v_0 = v_A$ and $v_0 = v_A/3$ for a Tokamak

TAE in W7-AS #39042

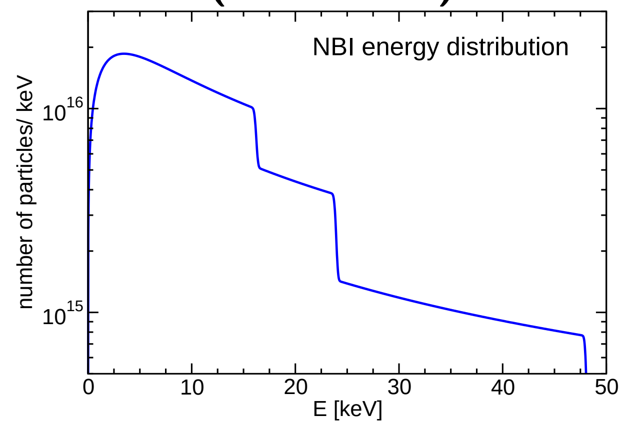
A. Weller et al., Phys. Plasmas, **8**, 931 (2001):







fast ions (deuterium):



electrons: Maxwellian $T_e = 518\text{eV}$

ions (deuterium): Maxwellian $T_e = 400\text{eV}$

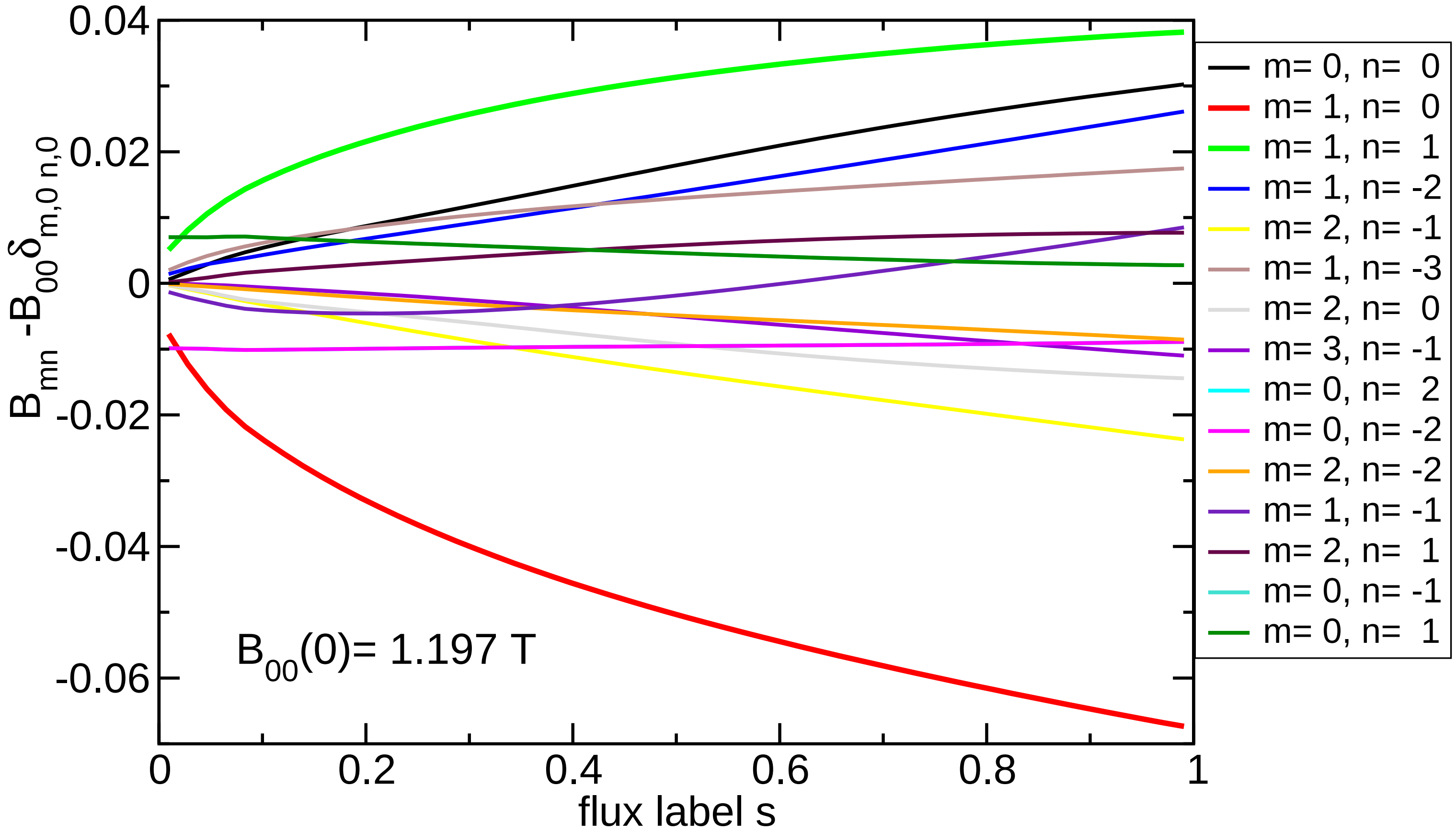
Alfvén velocity: $v_A = 4.65 \cdot 10^6\,\text{ms}^{-1}$

$< \beta >= 2.5 \cdot 10^{-3}$

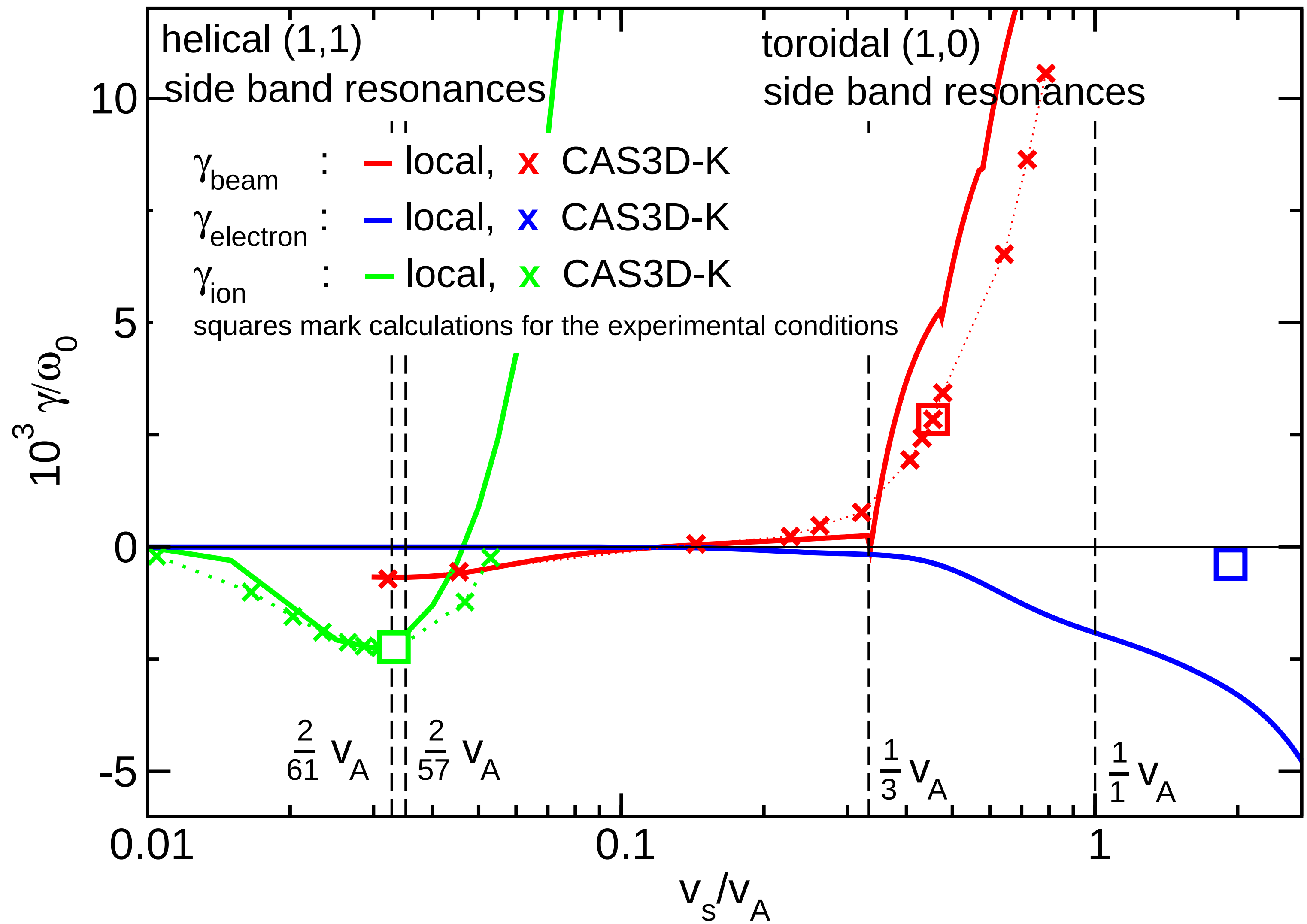
$< \beta_{\text{fast}} >= 1.2315 \cdot 10^{-4}$

$\frac{v_{\text{fast}}}{v_{\text{A}}} = 0.239$

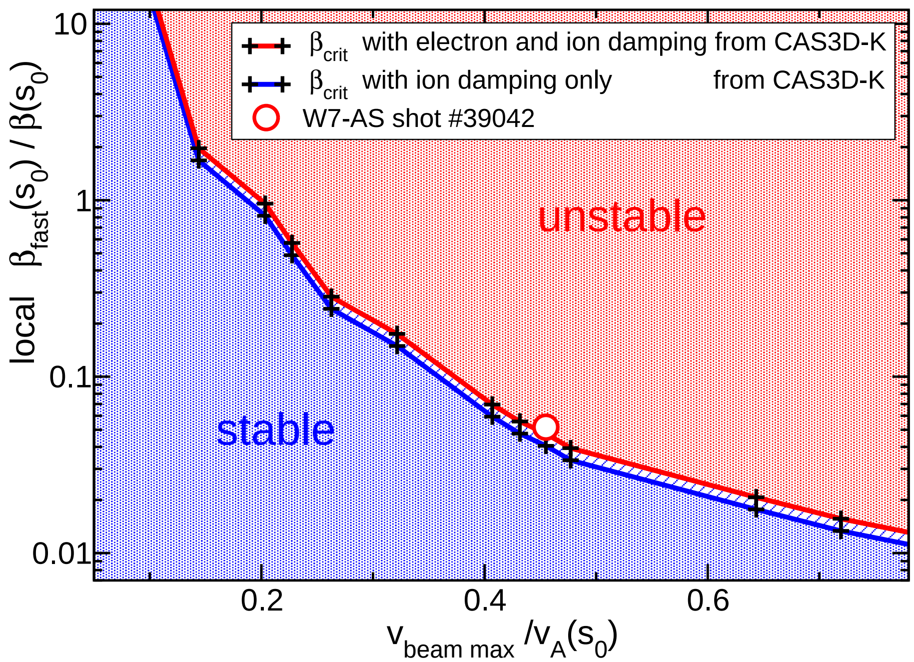
extract possible coupling from B spectrum

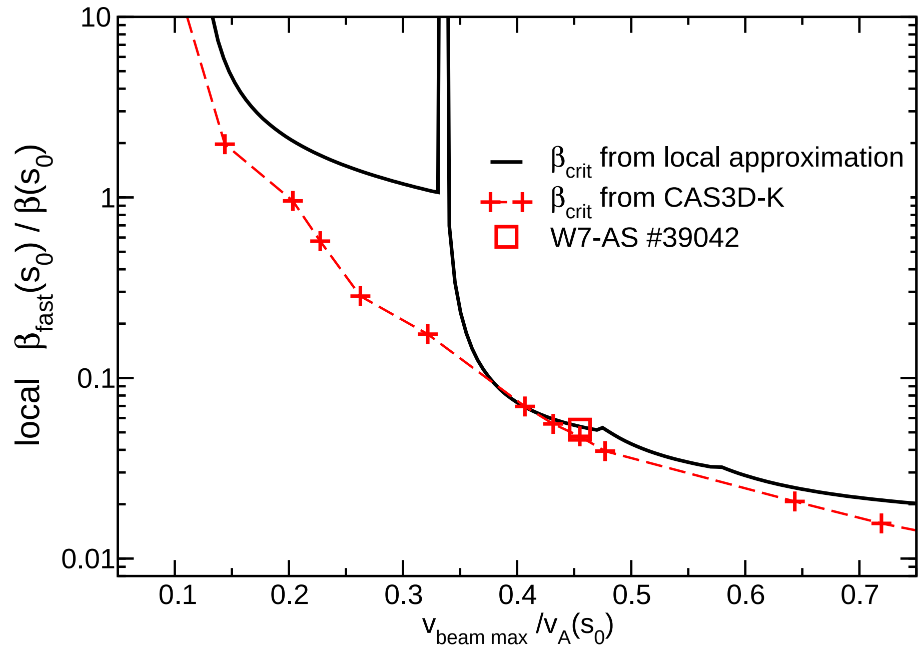


growth and damping rates for TAE in #39042



stability diagram for (5,-2)/(6,-2) TAE

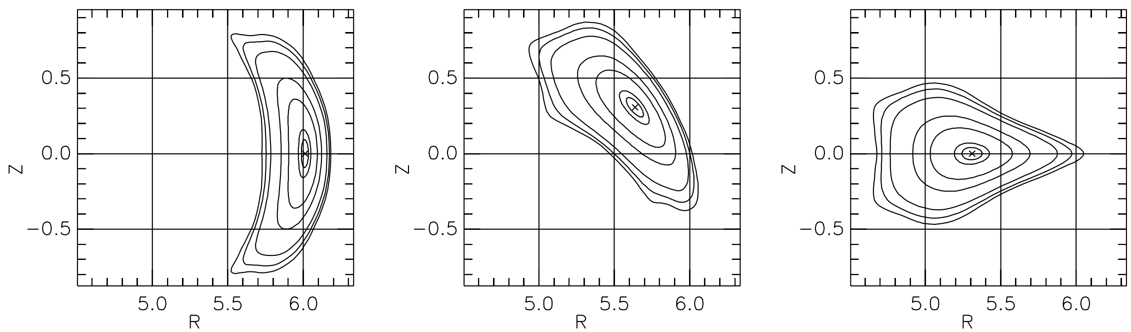




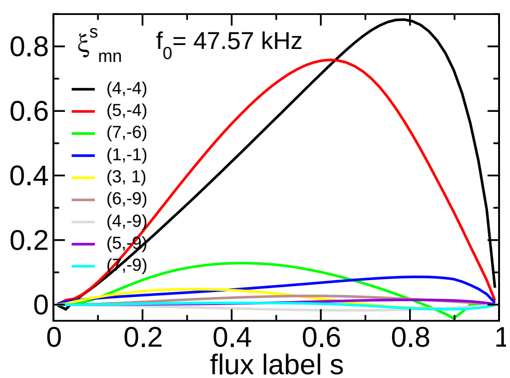
(4,-4),(5,-4) TAE in W7-X

equilibrium:
M. Drevlak et al., Phys. Plasmas, 8, 931 (2001):
from PIES calculation: practically island free

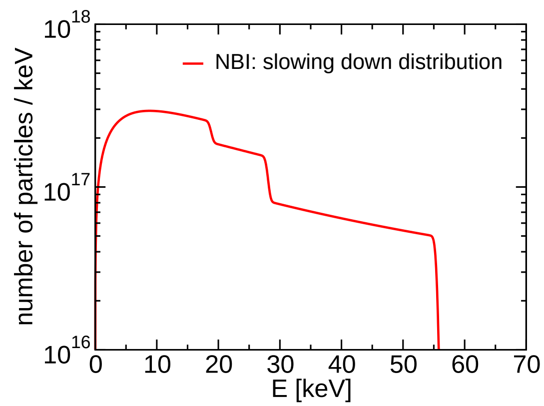
electrons: Maxwellian $T_e = 3.8\text{keV}$
ions (hyrogen): Maxwellian $T_e = 3.8\text{keV}$
$<\beta> = 4.2 \cdot 10^{-2}$



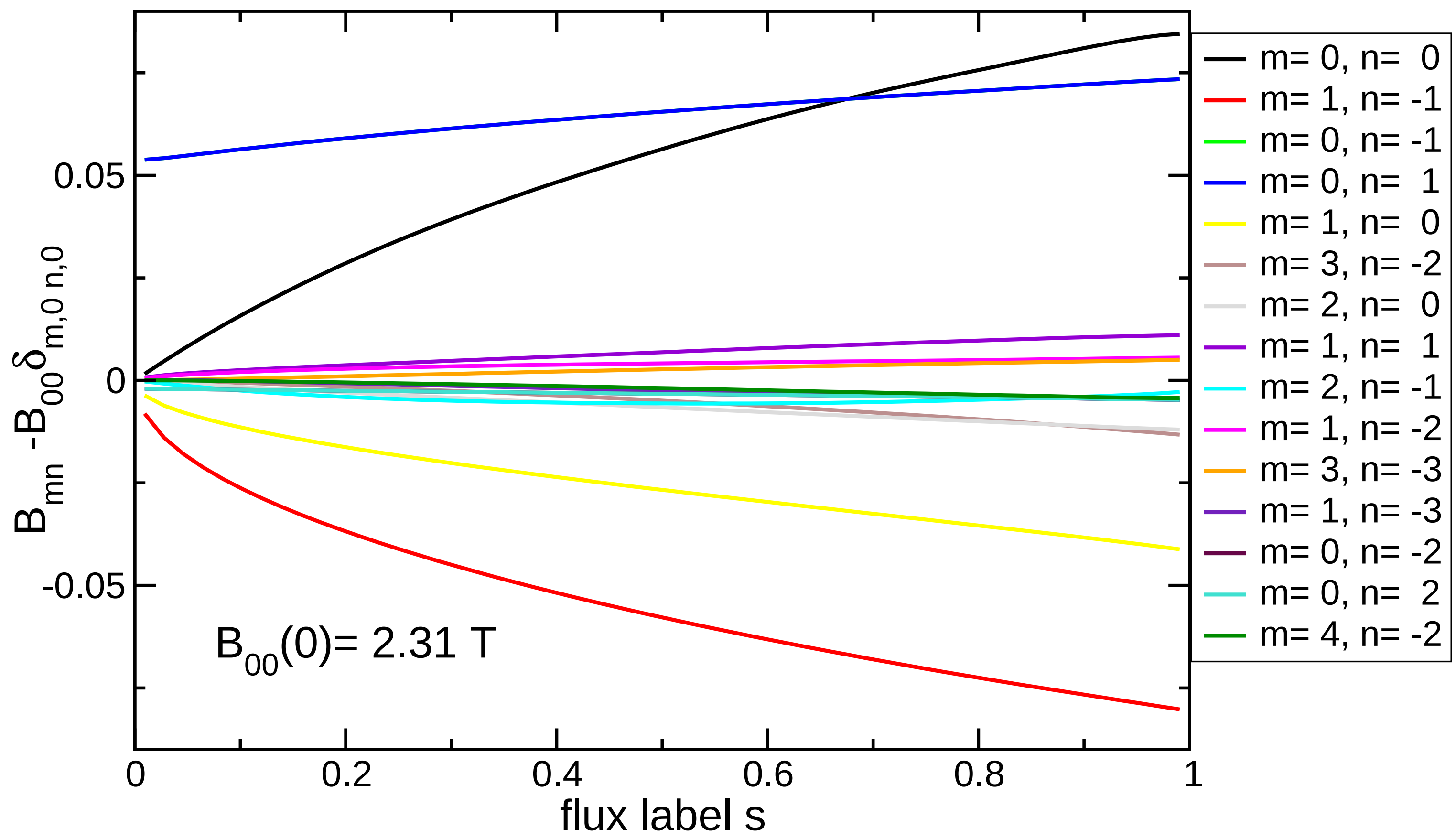
mode:



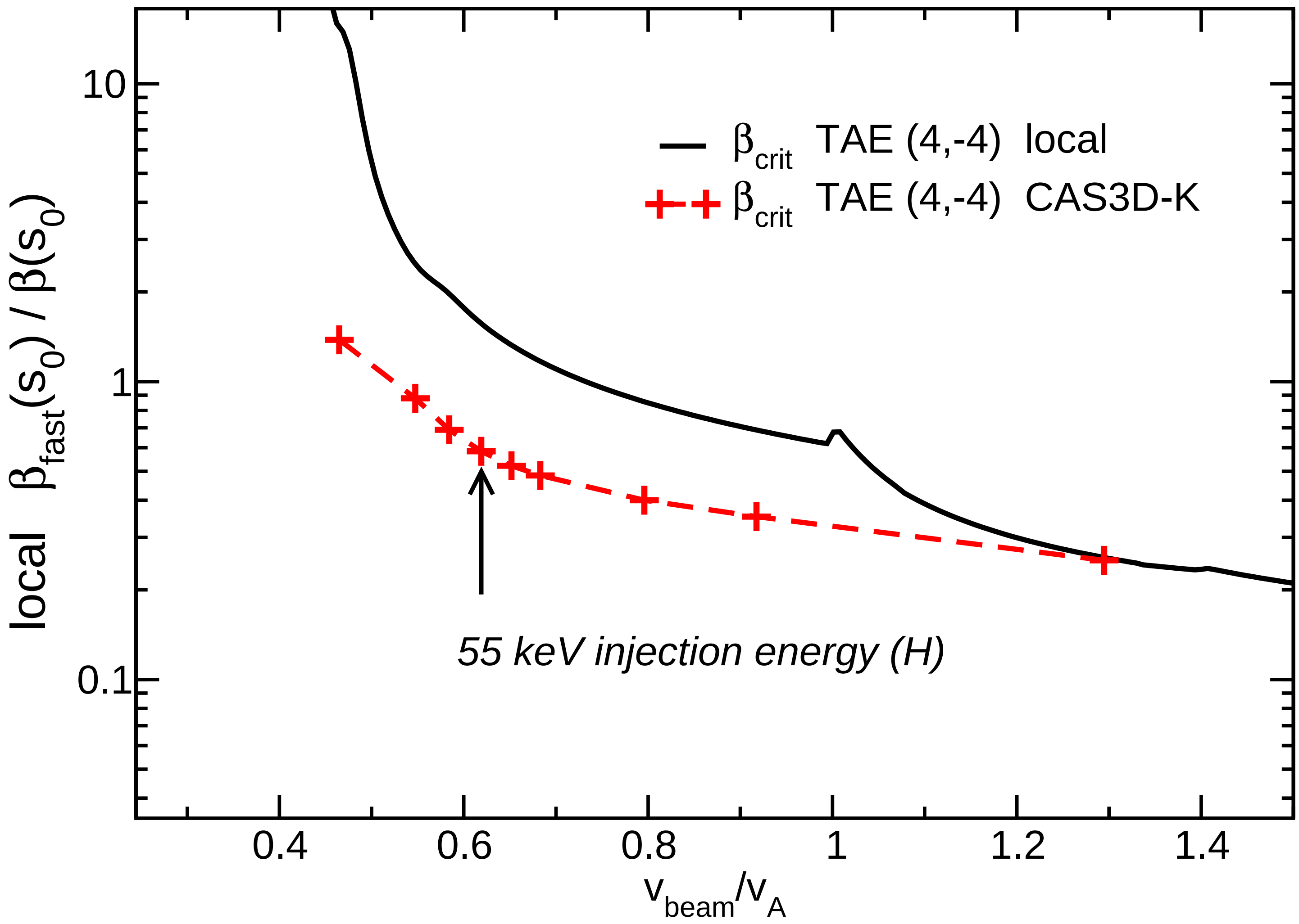
fast ions (hydrogen):



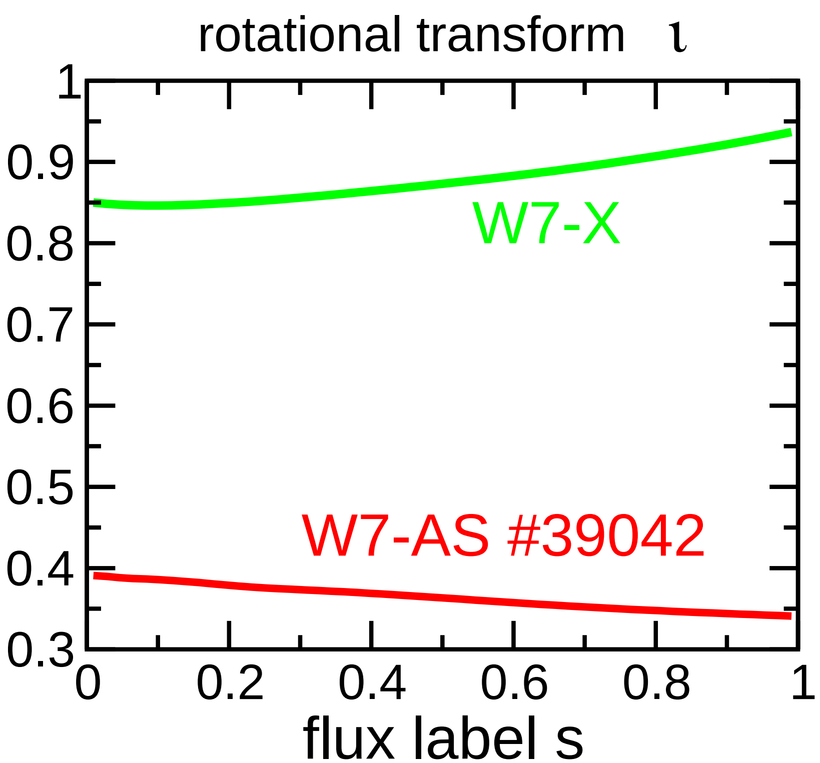
again: extract possible coupling from B spectrum

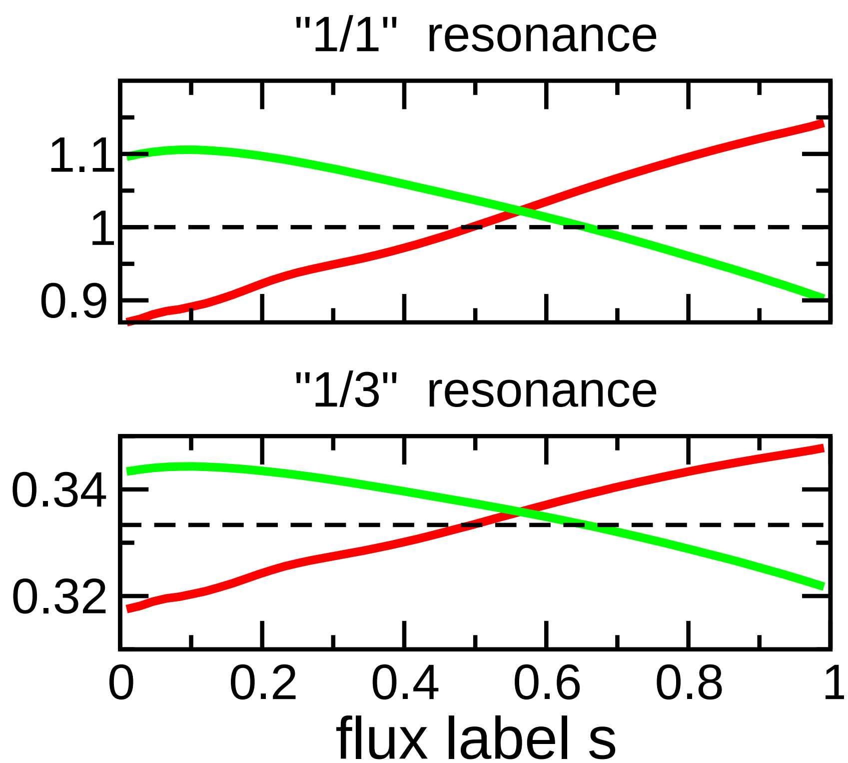


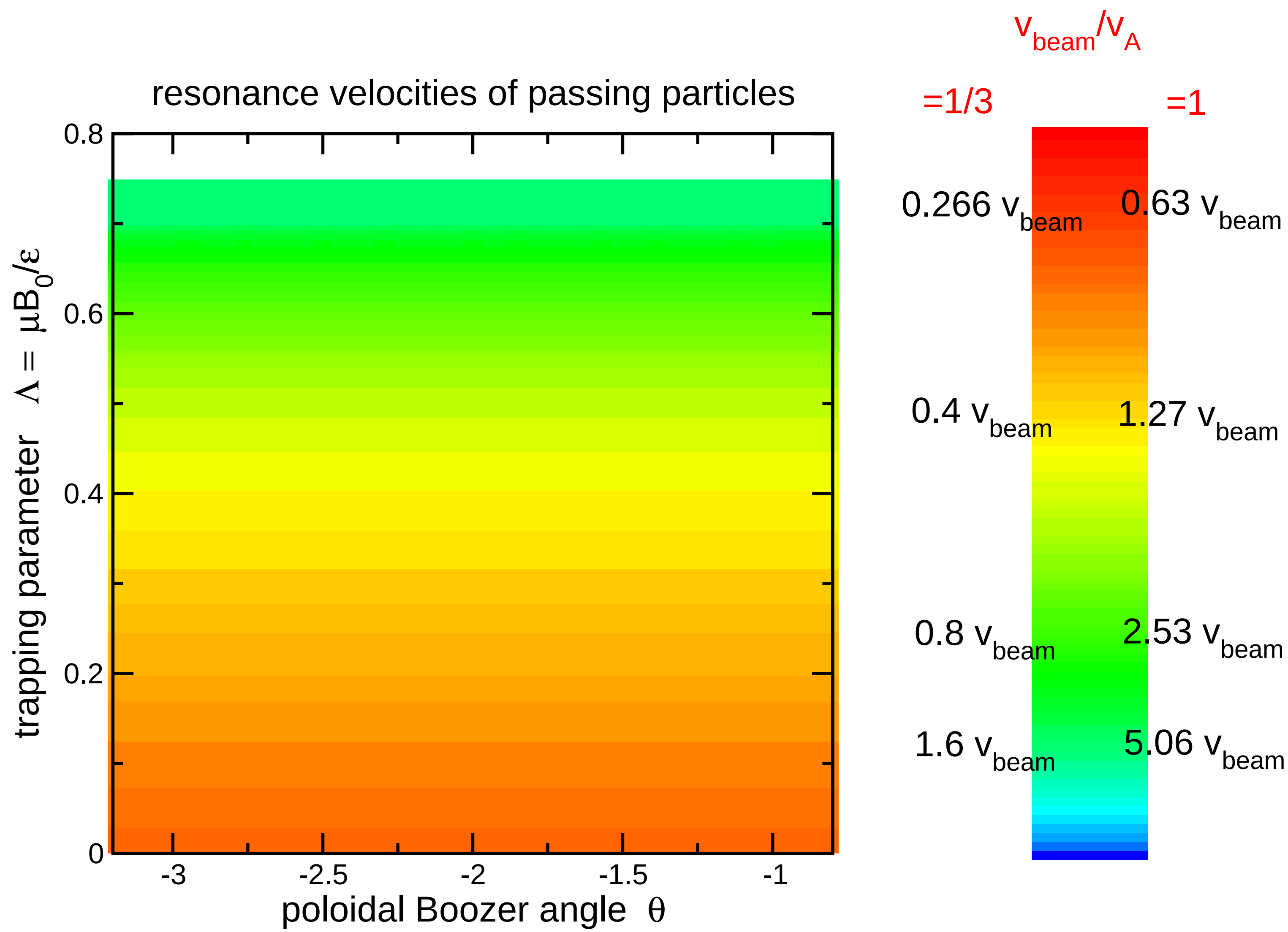
Comparison of critical β for mode in W7-X



ι profile and resonance velocities



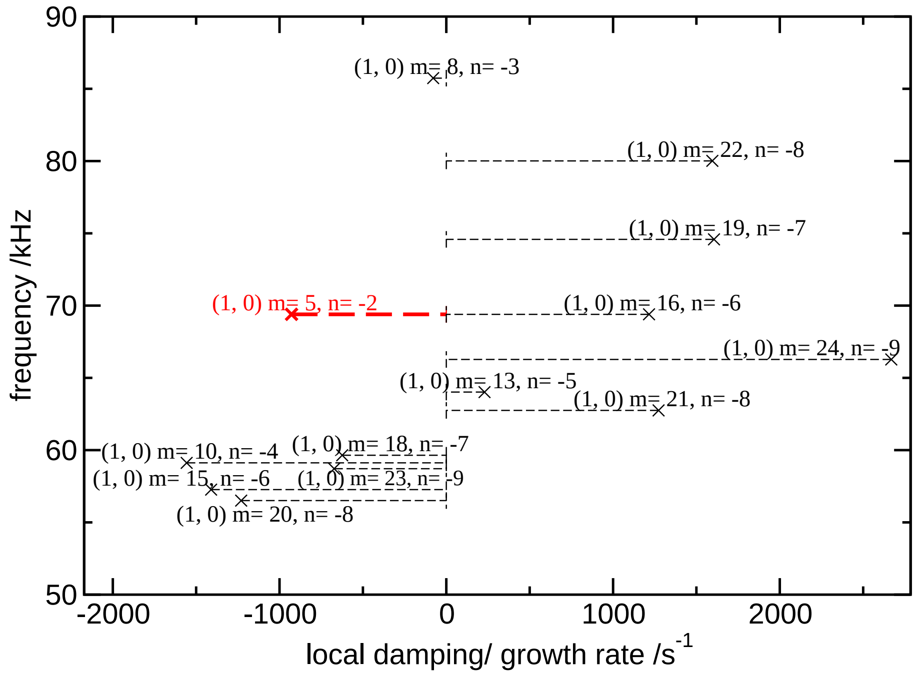



resonance velocities of passing particles
trapping parameter Λ = μB0/ε
poloidal Boozer angle θ
0.8
0.6
0.4
0.2
0
-3
-2.5
-2
-1.5
-1
vbeam/vA
=1/3
=1
0.266 vbeam
0.63 vbeam
0.4 vbeam
1.27 vbeam
0.8 vbeam
2.53 vbeam
1.6 vbeam
5.06 vbeam

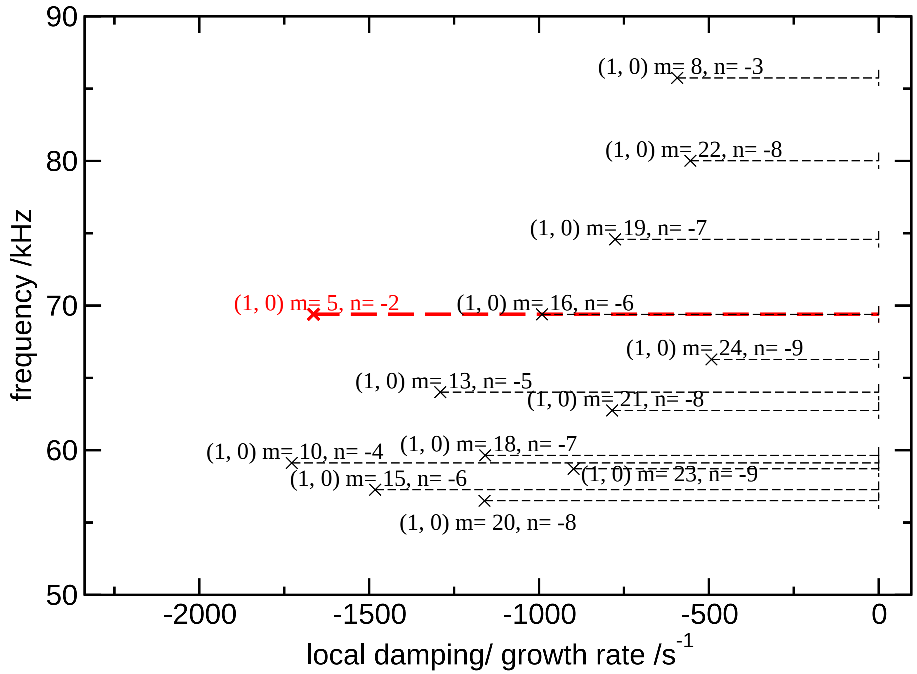
destabilization by temperature gradients

TAE mode frequencies and growth/ damping rates from a local computation

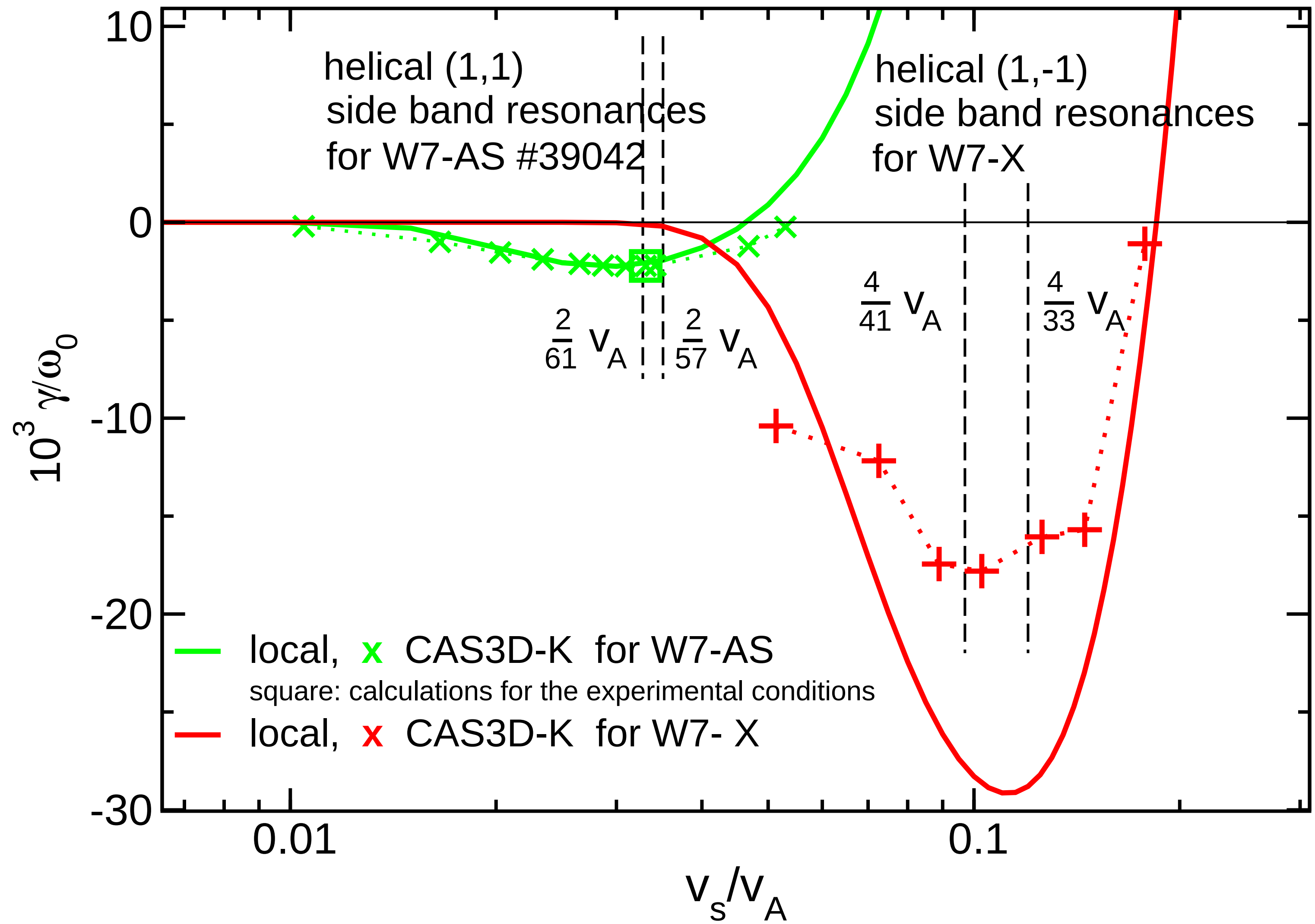
with a temperature gradient:



without a temperature gradient:



damping by thermal ions



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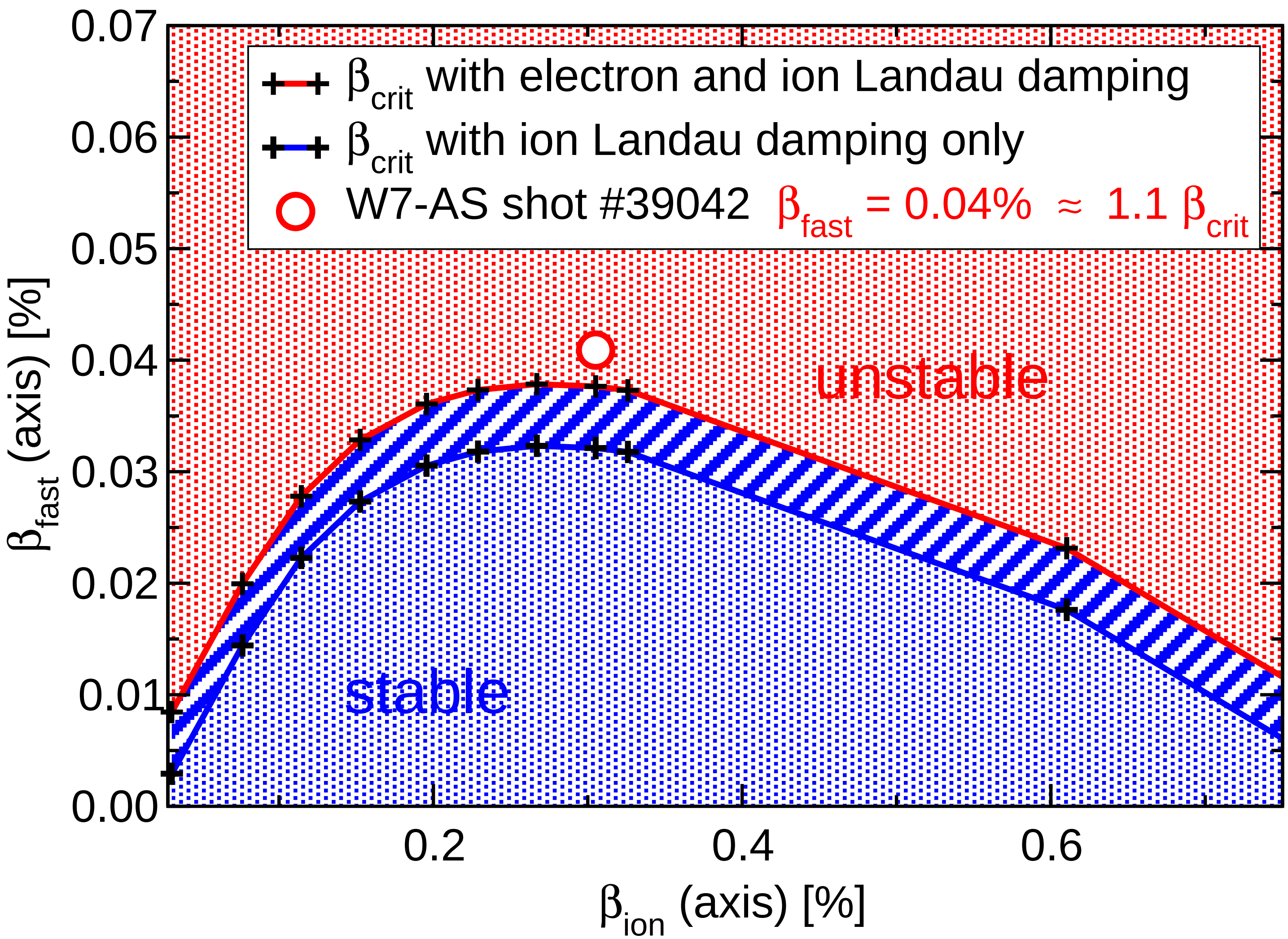

Summary

- the stability boundaries obtained predict a weak instability of a TAE in agreement with the experiment (#39042) in spite of a very small β_{fast}
- surprisingly good agreement between local and global approaches (presumably because of low shear of the equilibria)
- stability diagrams shown allow a direct comparison with the experiment
- damping is mainly due to ions and caused by the helical resonances (genuine 3D effect)
- for high plasma beta the TAE may be driven unstable by thermal ions
- thermal ion destabilization due ion temperature gradients (extension of the theory is necessary: $E_{||}$, ρ_i, . . .)