Effects on Ion Cyclotron Emission of the Orbit Topology Changes from the Wave-Particle Interactions

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Abstract. It is known that non-relaxed distribution functions can give rise to excitation of magnetosonic waves by ion cyclotron interactions when the distribution function increases with respect to the perpendicular velocity. We have found that in a toroidal plasma also collisional relaxed distribution functions of central peaked high-energy ions can destabilise magnetosonic eigenmodes by ion cyclotron interactions, due to the change in localisation of the orbits establishing inverted distribution functions with respect to energy along the characteristics describing the cyclotron interactions. This can take place by interactions with barely co-passing and marginally trapped high-energy ions at the plasma boundary. The interactions are enhanced by tangential interactions, which can also prevent the interactions to reach the stable part of the characteristics where they interact with more deeply trapped orbits.

1. Introduction

Emission of waves in the ion cyclotron range of frequencies has received considerable attention because of the possibility to yield information on fast particle distributions, in particular on thermonuclear alpha particles. The emission spectra are characterised by series of narrow peaks corresponding to multiple harmonics of the cyclotron frequencies of suprathermal ions at the low field side of the plasma edge. In JET, emission related to fusion products has been found to correlate with the fusion reactivity over six orders in magnitude; in high current sawtoothing discharges (6MA discharges) the emission displayed inverted sawteeth [1], and vanished after large ELMs, but was less affected by small ELMs. This behaviour is consistent with excitation of waves at the outer part of the plasma by high-energy ions [2]. In plasmas heated with ion cyclotron resonance heating emission peaks appear when the power exceeds a threshold, and is delayed with a slowing down time after the application of RF power [3]. In TFTR supershot experiments with NBI a sudden change of the spectrum occurred during the discharge. In the early part of the heating phase, high amplitude peaks were seen corresponding to unshifted ion cyclotron resonances located just outside the plasma. At a later time the amplitude of the peaks were lower and the frequency higher, the corresponding cyclotron resonances were located just inside the plasma [2].

The high intensity of the emission is consistent with magnetosonic ion cyclotron instabilities driven by suprathermal high-energy ions, originating either from thermonuclear reactions, neutral beam- or ion cyclotron heating. Such instabilities can occur when the distribution function is inverted along the characteristics of the quasi-linear diffusion operator describing ion cyclotron interactions. It has been suggested that the anisotropy caused by high-energy ions with trapped drift orbits extending out to the plasma edge on the low-field side gives rise to magnetosonic instabilities causing the emission [4, 5]. Analysis by Dendy *et al* [4], in a uniform approximation of the plasma with a mono energetic distribution function, showed that the magnetosonic wave could be destabilised by obliquely propagating waves avoiding interactions with the thermal part due to the finite Doppler shift. A comprehensive analysis of the emission taking into account the two dimensional structure of the magnetsonic eigenmode and the finite banana width effects was made by Gorelenkov *et al* [6]; for instability it was necessary to have a collisional un-relaxed energy distribution. Such distribution function could exist provided the fast ions are lost before they are slowed down. However, emission during ICRH appears only after times comparable to a slowing down time, when collisonal relaxed, steady state distribution functions, with sufficiently many high-energy ions have been established; such distribution functions would not satisfy the above mentioned condition for instability.

2. Wave-particle interactions in a toroidal geometry.

Whether an ion takes or delivers energy to the wave depends, for ion cyclotron interactions, on the difference between the phase of the wave oscillation and the gyro phase of the ion. On averaged, ions will give energy to the wave when the distribution function increases in energy along the characteristics for the cyclotron interactions. The variation of the distribution function along the characteristics depends on how the orbit changes as the invariants vary due to the interactions, in particular the innermost and the outermost positions of the orbit. The characteristics can be obtained by calculating the changes in the orbit invariants due to ion cyclotron interactions by integrating the equation of motion along the orbit. Here we use the orbit invariants W, P_ϕ and Λ in a toroidal geometry, where W is the energy, P_ϕ the canonical momentum and Λ an adiabatic invariant; defined by $P_\phi = mR\upsilon_\phi + eZ\Psi$ and $\Lambda = \mu B_0/W$, where $2\pi\Psi$ is the poloidal magnetic flux, μ is the magnetic moment and B_0 the magnetic field on the magnetic axis. The change in energy, ΔW, is given by

$$\Delta W = eZ\int_0^{\tau_B} \underline{\upsilon}\cdot\underline{E}\exp(-i\vartheta)dt \tag{1}$$

where ϑ is the phase difference between the wave oscillation and the gyro phase of the ion defined by

$$\vartheta = \int_0^t (\omega - n\omega_c - \underline{k}\cdot\underline{\upsilon})dt\,. \tag{2}$$

In general, the invariants of motion in a toroidal geometry, where the cyclotron frequency varies along the orbit, experience significant net changes only near the Doppler shifted cyclotron resonances, $\omega - n\omega_c - \underline{k}\cdot\underline{\upsilon} = 0$, due to the rapid variation of ϑ. Using the stationary phase method to integrate Eq. (1) one obtains

$$\Delta W = eZ\upsilon_\perp^{res}\,\mathrm{Re}\left\{\left(e^{-i\zeta}E_+\,\mathrm{J}_{n-1}(k_\perp\rho) + e^{i\zeta}E_-\,\mathrm{J}_{n+1}(k_\perp\rho)\right)\sqrt{\frac{i\pi}{\ddot{\vartheta}}}\exp i\vartheta_0\right\}, \tag{3}$$

where ζ defines the direction of the wave through $k_y/k_x = \tan\zeta$ in a local (x, y)-coordinate system with the x–direction perpendicular to the magnetic flux surface and the y–direction parallel to the magnetic flux surface and perpendicular to the magnetic field. ϑ_0 is the phase difference at the resonance, *i.e*. where the phase is stationary, ρ is the gyro radius, $k_\perp$ the perpendicular wave number. E_+ and E_-, given by $E_\pm = \frac{1}{2}(E_x \pm iE_y)$, are the perpendicular electric field components rotating in the direction of the ions (+) and counter to them (–).

Considerable enhancement of the wave-particle interactions takes place at tangential resonances where $\ddot{\vartheta}\to 0$ because $\ddot{\vartheta}\propto\sin\theta$. In this case a more accurate expression is required than that given by Eq. (3), which can be obtained by expanding the change in gyro phase around the Doppler shifted resonance yielding [7-9]

$$\Delta W = eZ\upsilon_\perp^{res}\,\mathrm{Re}\left\{\left(e^{-i\varsigma}E_+J_{n-1} + e^{i\varsigma}E_-J_{n+1}e^{i\Psi_R}\right)2\pi\left(\frac{2}{\dddot{\vartheta}}\right)^{1/3}\mathrm{Ai}\left[-\frac{1}{2^{2/3}}\frac{\ddot{\vartheta}^2}{\dddot{\vartheta}^{4/3}}\right]\right\}, \tag{4}$$

where Ai is the Airy function. The above expression defines the change in energy when two resonances are close together, merge into a tangential one and afterwards when they just disappear. In the latter case the argument in the Airy function becomes imaginary, with Ai decreasing exponentially.

The changes in P_ϕ and $\Lambda = \mu B_0/W$ are given by:

$$\Delta P_\phi = \frac{n_\phi}{\omega}\Delta W \qquad \Delta\Lambda = \left(\Lambda_r - \Lambda\right)\frac{\Delta W}{W}, \tag{5}$$

where $\Lambda_r = n\omega_{c0}/\omega$, ω_{c0} is the cyclotron frequency at the magnetic axis and n the harmonics of the cyclotron frequency. In absence of decorrelation of the wave-particle interactions, a superadiabatic oscillation of the orbit invariants along the characteristics takes place. Decorrelated interactions with a single mode near one harmonic will describe a 1D-diffusion process of the orbit invariants along the characteristics [10, 11].

When an ion gains energy from the wave, the major radius of the Doppler shifted resonance increases and vice versa when it loses energy [11]. The energy an ion can reach due to the cyclotron interactions alone is limited either by the ion orbit intersecting the wall or by the Doppler shifted resonances merging into a tangential one at the low-field side for a passing orbit or at the low-field side of the inner or the outer leg for a trapped orbit [9, 11]. If the Doppler shifted resonances merge into a tangential one at the high-field side of the orbit, the minimum energy the ion can reach becomes also limited. In general, the distribution function increases only in a finite interval along the characteristics, the tangential resonances can prevent the interactions to continue into an upper stable part of the characteristics and at the same time enhance the interactions at the most unstable part by suitable choice of the frequency.

Unstable interactions appear when barely co-passing ions interact near the plasma boundary with co- or counter-propagating waves, for which the distribution function will be decreasing in energy below the trapped-passing boundary; fusion reactions and cyclotron heating produce only few high-energy co-passing ions there. Fig.1 illustrates the de-trapping of an orbit into a co-passing one as the wave takes energy from the ion. In the case the unshifted cyclotron resonance is located outside on the low-field side of the orbit the upper energy limit prevents interaction with the stable part of the distribution function consisting of deeply trapped ions.

For instability, the sum of the background damping and the interactions with all resonant ions has to result in a net increase of the mode energy. This can be achieved, if the interactions with the most unstable part of the distribution function takes place at tangential resonances and the mode is sufficiently localised at the plasma edge [5].

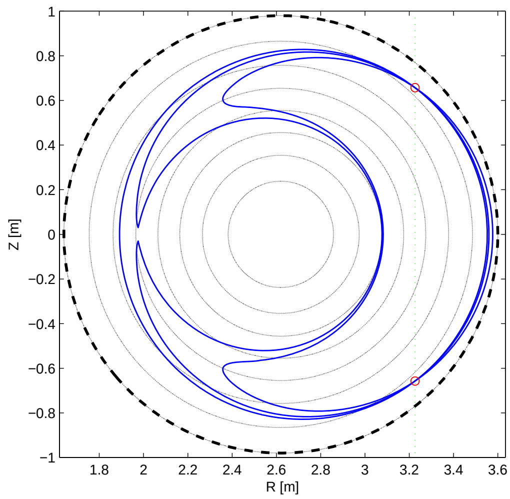

Fig. 1. Detrapping of a trapped orbit into a co-passing orbit as the energy is reduced due to cyclotron interaction, W = 1.0MeV, Λ= 0.9, (deeply trapped) W = 0.69MeV, Λ= 0.75 (marginal trapped) and W = 0.48MeV, Λ= 0.55 (co-passing) for n_ϕ= 0. The Cyclotron resonance is located at R=3.2m.

3. Numerical simulation.

The power absorbed in a volume element centred at (r, θ) is given by

$$P = \mathrm{Re}\left\{\frac{-i\omega}{8\pi}\iint \underline{E}^{*}\,\underline{\underline{\chi}}^{A}\,\underline{E}\; rdrd\theta\right\} \tag{6}$$

with

$$\underline{\underline{\chi}}^{A} = \frac{\omega_p^2}{\omega\omega_c}\int_0^\infty 2\pi\upsilon_\perp d\upsilon_\perp \int d\upsilon_{//}\,\frac{\omega_c}{\omega-\underline{k}\cdot\underline{\upsilon}-n\omega_c}\underline{\underline{S}}_n\,. \tag{7}$$

When neglecting the parallel electric field component $\underline{\underline{S}}_n$ becomes

$$\underline{\underline{S}}_n = \upsilon_\perp\begin{pmatrix}\left(\dfrac{n\omega_c J_n}{k_\perp\upsilon_\perp}\right)^2 & \dfrac{in\omega_c J_n J'_n}{k_\perp\upsilon_\perp}\\ -\dfrac{in\omega_c J_n J'_n}{k_\perp\upsilon_\perp} & \left(J'_n\right)^2\end{pmatrix}\left\{\frac{\partial f}{\partial\upsilon_\perp}\left(1-\frac{k_{//}\upsilon_{//}}{\omega}\right)+\frac{k_{//}\upsilon_\perp}{\omega}\frac{\partial f}{\partial\upsilon_{//}}\right\}. \tag{8}$$

Assuming the magnetic field to vary according to the toroidal equilibrium, expanding the nominator, $\omega - n\omega_c - \underline{k}\cdot\underline{\upsilon}$, around the resonance and integrating with respect to θ we obtain after some algebra

$$P \approx \frac{\omega}{4}\int rdr\int_{-\infty}^{\infty} d\upsilon_{//}\,\frac{\omega_p^2}{\omega\omega_c}\int_0^\infty 2\pi\upsilon_\perp d\upsilon_\perp\,\frac{\omega_c\upsilon_{//}}{|\ddot{\vartheta}|qR}\underline{E}^{*}\underline{\underline{S}}_n\underline{E}\;\Big|_{\theta 0} \tag{9}$$

When applying the quasi-homogenous dielectric tensor to a toroidal geometry there is no enhancement of the anti-Hermitian part with a factor $1/|\sin\theta|$ due to tangential interactions as $\ddot{\vartheta}\to 0$. The enhancement instead appears as an increase of the volume elements, where the anti-Herimtian part is significant, with the same factor. The integration in velocity space in Eq. (9) should only include particles that are resonant in the considered volume element. The ions give on averaged energy to the wave when

$$\frac{\partial f}{\partial\upsilon_\perp}\left(1-\frac{k_{//}\upsilon_{//}}{\omega}\right)+\frac{k_{//}\upsilon_\perp}{\omega}\frac{\partial f}{\partial\upsilon_{//}} = \frac{n\omega_c}{\omega}\frac{\partial f}{\partial\upsilon_\perp}+\frac{k_{//}\upsilon_\perp}{\omega}\frac{\partial f}{\partial\upsilon_{//}} < 0 \tag{10}$$

To illustrate how thermonuclear alpha-particles can destabilize edge localised modes we calculate the susceptibility tensor with the SELFO code [12, 13] for a steady state alpha-particle distribution function, arising from slowing down of thermonuclear particles including finite orbit width effects in a circular tokamak with parameters similar to those of TFTR: R_0 =2.52m, a =0.9m, I_p =1.6MA, n_T =2.5 $\times 10^{19}$m^{-3}, n_D =2.5 $\times 10^{19}$m^{-3}, n_C =2.0 $\times 10^{18}$ m^{-3}, T_e =5keV , T_i =20keV, B_0 =5.0T, $n(\mathrm{r})=n_0(1-0.99(\mathrm{r/a})^2)^{0.2}$ and $T(\mathrm{r})= T_0(1-0.2(\mathrm{r/a})^2)^{10}$. For simplicity we have assumed that the resonances are located where $k_{||} = n_\phi/R$. Because of the relatively small value of $\upsilon_\perp k_\perp/\omega_c$ the anti-Hermitian part of the susceptibility tensor elements χ_{xx}, χ_{xy} and χ_{yy} are nearly similar. Some differences occur because the largest contribution to the susceptibility comes from the high-energy ions, for which $(n\omega_c J_n/\, k_\perp\upsilon_\perp)$ and J'_n starts to deviate. The anti-Hermitian part of the susceptibility tensor elements χ_{xy}, which in general has the largest negative anti-Hermitian part, is shown in Fig. 2 for some frequencies around $\omega \approx 4\omega_{c\alpha}$ at the low-field side edge, for waves with n_ϕ = 25 and n_ϕ = –25. Large regions with positive values of the anti-Herimitian part of the susceptibility tensor elements χ_{xy} can be seen in the central region giving rise to damping; because of the weaker spatial gradients of the alpha particle density, the distribution function is decreasing in energy along the characteristics. For n_ϕ = 25 with f = 110MHz, Fig. 2f, when the 4th harmonic resonance is just outside the plasma at the low field side a large negative region at the low-field side appears corresponding to interactions with co-current passing ions and with marginally trapped ions at the outer leg. The unstable contributions from trapped and co-passing orbits were confirmed by separating the anti-Hermitian part of the susceptibility element into contributions from co-, counter-passing and trapped alpha particles. As the frequency is increased and the 4th harmonic resonance is displaced into the plasma at the low-field side, as for f = 116MHz shown in Fig. 2f, the unstable region is shifted into the plasma followed by a stable region. The stable region consists of deeply trapped high-energy ions and lower energy ions. This is consistent with as one follows the characteristics in the region of trapped orbits towards higher energy, the Doppler shifted resonances are shifted closer to the unshifted resonance as the orbits become more deeply trapped. Since the number of ions become fewer as they become more deeply trapped, the distribution function will decrease with energy along these parts of the characteristics, giving rise to a stable region. As the 4th harmonic resonance moves into the plasma on the low-field side also a large stable region caused by interactions with counter-passing ions at the high-field side at the low-field side of the 2nd harmonic resonance that will damp edge localised magnetosonic waves propagating poloidaly around the magnetic axis at the plasma edge.

Regions with negative anti-Hermitian part of the susceptibility element χ_{xy} appear also for waves propagating counter to the plasma current as for n_ϕ = 25 at f = 124MHz, Fig. 2d. An unstable region similar to that in Fig. 2g appears at the low-field side edge, but on the low-field side of the 4th harmonic resonance, which in this case also corresponds to interactions with co-current passing ions and with trapped ions on the outer leg. However, as the energy increases due to cyclotron interactions and the trapped orbits become more deeply trapped the Doppler shifted resonances are shifted towards the unshifted resonance thus into the plasma resulting in a stable region close to the unshifted resonance, which also include interactions with less energetic ions for which the distribution function is not inverted with respect to energy. Because of the presence of this stable region close to the unstable region unstable excitation of edge localised modes requires stronger localisation of the mode to the edge, which may be difficult to achieve.

By performing a frequency scan to identify the range, in which the instability at the edge, illustrated in Fig. 2f, can appear, we find it to be limited within the range of 104MHz to 113MHz, thus less than 10% of the frequency.

The behaviour of the anti-Hermitian part of the susceptibility tensor is similar at higher frequencies when the mode numbers are scaled up accordingly, but the regions, where the anti-Hermitian part of the susceptibility tensor are significant, start to overlap.

In most of the plasma the anti-Hermitian parts of the susceptibility tensor components are positive, in particular, near the centre, where they are large and will damp the waves. Thus only waves propagating near the plasma edge avoiding the stable regions can become unstable [5]. The typical width of edge localised modes, Δr, varies between 0.12 and 0.18r_0, when m_θ varies between 50 and 25, using the formula in Ref. [14]. Thus the width of the edge localised mode is conceivable with unstable excitation of a mode corresponding to the case outlined in Fig. 2f.

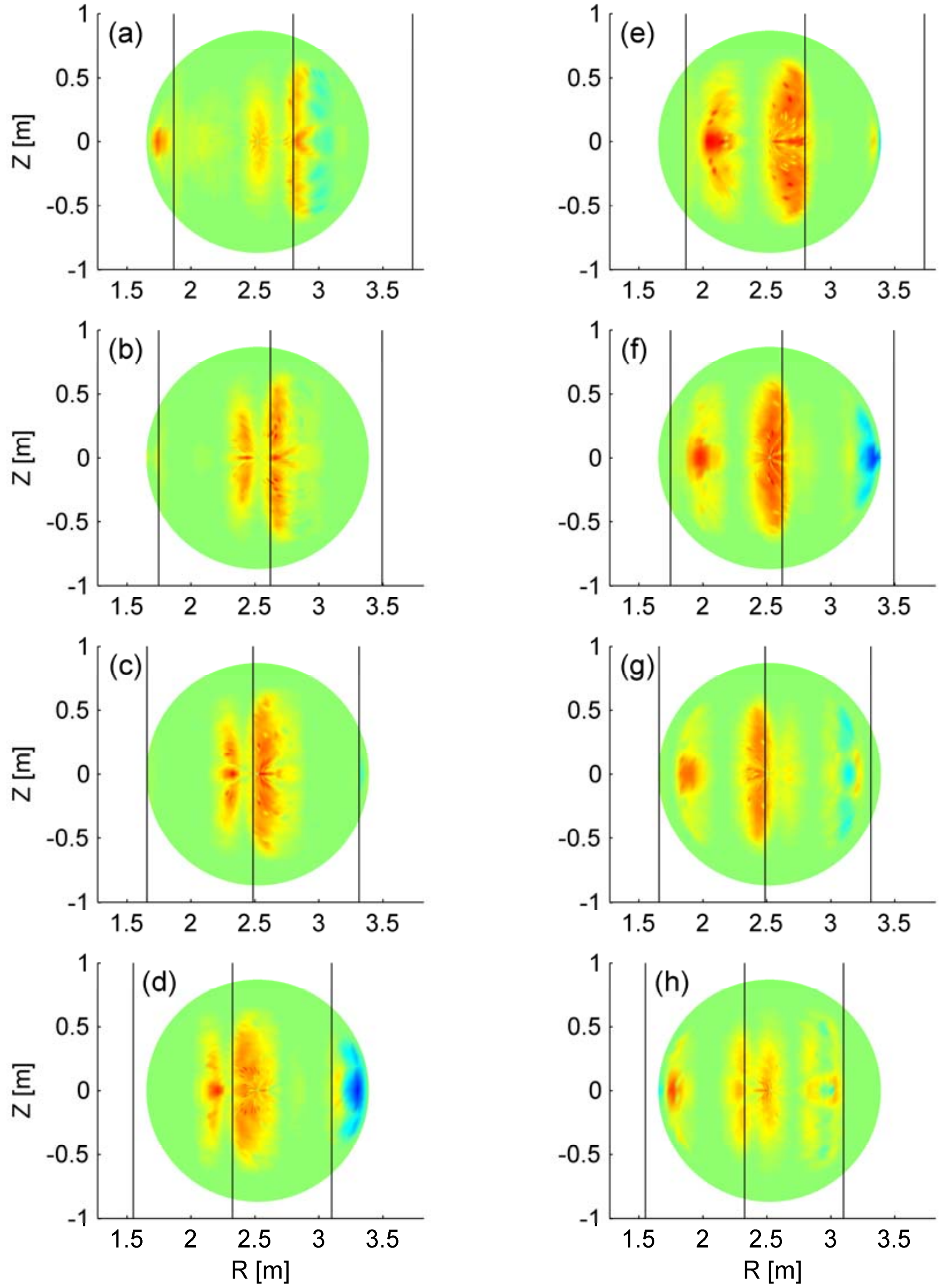

Fig. 2. The anti-Hermitian susceptibility tensor element χ_{xy} for alpha particles (blue is negative, green zero and yellow-red positive values) a-d) $n_\phi = -25$ a) f = 103MHz, b) f = 110MHz, c) f = 115MHz, d) f = 124MHz, e-h) $n_\phi = 25$, e) f = 103MHz f) f = 110MHz, g) f = 116MHz, h) f = 124MHz. The vertical lines indicates the harmonic cyclotron resonances $\omega = n\omega_c$ for n = 2, 3, 4.

4. Conclusions and Discussions

We have found that in a toroidal plasma, with a centrally peaked, collisional relaxed, steady state distribution function inverted distribution functions along the characteristics describing the quasi-linear diffusion by ion cyclotron interactions can be obtained because of the toroidal geometry. Interactions with marginally trapped and barely co-passing ions with co- and counter propagating waves are possible. Interactions at or near tangential resonances in the outer midplane can enhance the drive by the unstable part of the distribution function and limit it from interacting with the stable part.

The pattern with positive and negative regions of the anti-Hermitian part of the susceptibility tensor of a steady state thermonuclear alpha-particle distribution function is conceivable with excitation of edge localised magnetosonic waves with $m_\theta \approx -n_\phi$; even though the edge localised magnetosonic eigenmodes are not localised in major radius and that the distribution function has reached steady state by collisions.

The emission spectrum at the early phase in the TFTR supershots with emission peaks corresponding to harmonic cyclotron resonances just outside the plasma at the low-field side [2] is in good agreement with the case illustrated in Fig. 2f. The change in the spectrum at a later time in presence of NBI can be caused with the same mechanism, but since the ions have less energy the unshifted resonance has to be displaced closer or into the plasma.

5. References

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