Self-consistent Study of Fast Particle Redistribution by Alfvén Eigenmodes During Ion Cyclotron Resonance Heating

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Abstract. Alfvén eigenmodes (AEs) excited by fusion born α particles can degrade the heating efficiency of a burning plasma and throw out αs. To experimentally study the effects of excitation of AEs and the redistribution of the fast ions, ion cyclotron resonance heating (ICRH) is often used. The distribution function of thermonuclear αs in a reactor is expected to be isotropic and constantly renewed through DT reactions. The distribution function of cyclotron heated ions is strongly anisotropic, and the ICRH do not only renew the distribution function but also provide a strong decorrelation mechanism between the fast ions and the AE. Because of the sensitivity of the AE dynamics on the details of the distribution function, the location of the resonance surfaces in phase space and the extent of the overlapping resonant regions for different AEs, a self-consistent treatment of the AE excitation and the ICRH is necessary. Interactions of fast ions with AEs during ICRH has been implemented in the SELFO code. Simulations are in good agreement with the experimentally observer pitch-fork splitting and rapid damping of the AE as ICRH is turned off. The redistribution of fast ions have been studied in the presence of several driven AEs.

1. Introduction

Alfvén eigenmodes (AEs are used here as a common name for various types of Alfvén eigenmodes) are often seen in experiments during ion-cyclotron resonance heating (ICRH). The ICRH produces peaked density profiles of anisotropic high-energy ions with wide trapped or non-standard drift orbits which can resonate with AEs. The AEs redistribute the high-energy resonant particles and can affect the performance of ICRH. The resonance is defined by two-dimensional surfaces in the three dimensional phase space of drift orbit invariants. Interactions with AEs displace the particle along a one-dimensional characteristic in phase space. In absence of decorrelation of the phase between the particle and AEs, the particle will undergo a superadiabatic oscillation in phase space without a net exchange of energy with the mode. The decorrelation caused by Coulomb collisions will be weak for high-energy ions while ICRH provides a strong decorrelation mechanism for these particles. In some parts of phase space the distribution function of resonant ions is increasing with energy along the characteristics of an AE, and hence drive the mode while the distribution function is flattened in the resonant regions. In other parts of phase space the distribution function decreases with energy along the characteristics, which will damp the mode as the distribution function is flattened. In absence of mechanisms restoring the distribution function an unstable mode will grow while flattening the distribution function in the most unstable parts of phase space. As the mode grows and the unstable parts of the distribution function is flattened the stable parts in phase space will become more important and damp the mode.

Coulomb collisions and ICRH will displace ions in and out of the AE resonant region with an energy above or below the mean energy along the AE characteristic. An ion entering at the high-energy part of the characteristic will drive the mode while if it enters at the low-energy part it will damp the mode and vice versa for an ion leaving the resonant region. The effect of decorrelation by Coulomb collisions and ICRH is important since it leads to an effective broadening of the resonant regions in phase space, and hence increases the energy transport, the saturation level of the AE and the number of regions with overlapping modes.

In experiments it has been observed that when the ICRH is turned off the ICRH-excited AEs are damped much faster than the time scales of both slowing down and resistive damping by the background plasma [1]. Another observation is the pitch fork splitting, appearing as symmetric side bands centred at the mode frequency in the Fourier decomposed time evolution of the measured magnetic signal [1, 2]. The splitting is interpreted as an oscillation of the mode amplitude [3, 4], and are related to an effective collision frequency [2]. The observed separation of the side bands is larger than can be explained by Coulomb collisions [2]. It has been proposed that the renewal of the distribution function by ICRH can explain this difference [1].

Interactions with several AEs lead to a decorrelation of the 1D superadiabatic oscillation resulting in a two dimensional redistribution. It also leads to several resonant regions and if these regions overlap in phase space the distribution function is flattened over a larger region.

The SELFO code [5] used for calculating the distribution functions and the fast magnetosonic wave field self-consistently during ICRH has been upgraded to include the effects of interactions with AEs [6]. The SELFO code consists of the FIDO code for calculating the distribution functions during ICRH, including effects of finite drift orbit widths, and the LION code for calculating the electric field.

Detailed studies of the dynamics of toroidicity-induced Alfvén eigenmode (TAE) excitation during ICRH is in good agreement with experiments on the separation of the side bands and the fast damping of the TAE when the ICRH is turned off [6]. The effects on the distribution function of a single unstable TAE is, in general, small, since the resonant regions in phase space are rather narrow. The effects of several modes significantly increases the transport of resonant particles in phase space. The change in energy of the particle distribution is still rather small, but the relatively large redistribution in space caused by the change in toroidal angular momentum can affect the heating profile. Whereas the redistribution in toroidal angular momentum takes place on a time scale of an orbit time, the effect of the redistribution on the fast particle energy content takes place on a slowing down time.

2. Theory

Three invariants of the equation of motion (E, P_ϕ, μ) are used to describe the guiding centre orbit of a charged particle. E is the energy, $P_\phi = mRv_\phi + eZ\Psi$ is the canonical toroidal angular momentum where Ψ is the poloidal flux and μ is the magnetic momentum. There is also a label, σ, to distinguish between different types of orbits with the same invariants, e.g. co- and counter-passing orbits. The change in invariants due to interactions with AEs are obtained by integrating the equation of motion along the drift orbit

$$dE/dt = eZ\mathbf{E}_1 \cdot \mathbf{v}_{d0} + \mu \partial B_{1\|}/\partial t \tag{1}$$

where index 0 indicates unperturbed quantities and 1 first order perturbations. The zeroth order drift velocity, $\mathbf{v}_{d0}$, is given by gradients and curvature of $\mathbf{B}_0$. The first order perturbed electric and magnetic field can be written on the form $\Phi(r,\theta)A(t)e^{i(n\phi-\omega t-\alpha(t))}$, where $\Phi(r,\theta)$ is the poloidal structure of the eigenmode, $A(t)$ is the mode amplitude, n is the toroidal mode number, ω is the mode frequency and α the phase of the mode. The resonance condition is given by

$$n\frac{\Delta\phi}{\tau_b} - \omega \pm j\frac{2\pi}{\tau_b} = 0 \tag{2}$$

where $\Delta\phi$ is the precessional drift during a bounce time τ_b, and j is an integer representing higher harmonics which lead to a set of resonance surfaces in phase space. In the space defined by (E, P_ϕ, μ) for an axisymmetric plasma, the superadiabatic oscillation of an orbit takes place near its resonance along a characteristic defined by $\Delta P_\phi = \frac{n}{\omega}\Delta E$ and $\Delta\mu = 0$ [7].

Phase decorrelations of the interaction between a particle and an AE occur due to changes in the invariants by collisions or interactions with other waves, such as the magnetosonic waves used for cyclotron heating. The changes in invariants due to these interactions move the particle to a neighbouring AE characteristic where the guiding centre orbit has a different orbit time.

As time passes, the phase between the particle and the AE starts to differ from what it would have had if it had not been subjected to ion-cyclotron interactions or Coulomb collisions. After several such interactions the phase between the guiding centre orbit and AE will change but the displacement from the original AE characteristic will be small due to the diffusive nature of the interactions. The decorrelation time is given by [6]

$$\tau_d^3 = \frac{3 \cdot 2\pi}{\dot{\sigma}_{IC}^{EE} G_{IC}^2 + \dot{\sigma}_C^{EE} G_E^2 + \dot{\sigma}_C^{\Lambda\Lambda} G_\Lambda^2} \tag{3}$$

where $\sigma_L^{I_i I_j}$ is the covariance of the invariants I_i and I_j, where $\Lambda = \mu B_0 / E$, from the interaction with operator L, where C denotes collisions and IC ion-cyclotron interactions. The rate of change in phase of the particle along the invariant I is denoted by G_I, where the invariant direction IC denotes the characteristic of the ion-cyclotron interactions. By limiting the allowed deviation of phase between AE and particle to 2π during a decorrelation time, the resonance surfaces are expanded into volumes and the resonance condition can be written as

$$\left| n\frac{\Delta\phi}{\tau_b} - \omega \pm j\frac{2\pi}{\tau_b} \right| \tau_d \leq 2\pi \tag{4}$$

Outside the resonance region the phase between the particle and AE varies rapidly, giving only a small contribution, which has been neglected here.

AEs which overlap along a characteristic in phase space will increase the extent of the region in which the modes redistribute the ions. An unstable distribution function in such a continous region in phase space will be flattened over a larger region and thus decrease its energy more than if the resonant regions were barely overlapping. A distribution function which is unstable along some part of the characteristic will during mode excitation be flattened and possibly build up an unstable distribution function in a neighbouring overlapping AE region, which in its turn grows up and flattens the distribution function and further redistribute the ions, leading to a cascade of AEs.

The transport of resonant particles caused by AE interactions inside the resonant regions lead to an energy transfer, ΔE_{AE} , to the mode according to

$$\Delta E_{AE} = \int\int \Delta E_r \mathbf{\Gamma}_{AE} \cdot \mathbf{ds} dt \propto \int A^2 dt \tag{5}$$

where $\Delta E_r(E, P_\phi, \Lambda, \sigma)$ is the extension in energy of the resonant region along a characteristic, $\mathbf{\Gamma}_{AE}$ is the flux of particles inside the resonant region caused by AE interactions and A is the mode amplitude. Since the change in particle energy is related to a change in toroidal angular momentum according to $\Delta P_\phi = \frac{n}{\omega}\Delta E$ the change in mode energy is thus proportional to $\int\int \Delta P_\phi \mathbf{\Gamma}_{AE} \cdot \mathbf{ds} dt$. For trapped particles, which in general are those responsible for destabilization of AEs, the change in P_ϕ is related to a radial displacement of the turning points $\Delta r = (\partial\Psi/\partial r)^{-1} \Delta P_\phi / eZ$. The radial redistribution of trapped particles depends on the width of the resonant region, which in its turn depends on the phase decorrelation caused by ICRH and Coulomb collisions and the flux of particles across the resonant region, which depends on the renewal rate of the distribution function caused by ICRH. As a trapped ion heated by ICRH enters the resonant region of the AE with an energy higher than the mean energy of the resonant part of the AE characteristic it will transfer energy to the mode and at the same time be displaced outwards, where it will leave the AE resonant region by ion-cyclotron interactions.

The evolution of the distribution function is calculated with a Monte Carlo method. The diffusion coefficients of AE interaction is obtained by integrating Eq. (1) for a set of mode numbers during a decorrelation time assuming the phase to be randomly distributed in the interval $[0, 2\pi]$. The resonance condition, Eq. (4), yields a set of resonant regions in phase space in which the variation of the changes in energy, ΔE, due to interactions with the AE have a rather complex structure. The structure of the diffusion coefficient is determined by the structure, $\Phi(r,\theta)$, of the eigenmode.

3. Results

In the first scenario the dynamics of an unstable TAE during ICRH is studied for a JET-like H-minority heating scenario with $5\ \mathrm{MW}$ of ICRH power at $51\ \mathrm{MHz}$ with $+90^0$ phasing between the currents in the antenna straps in a plasma with circular cross-section, $r_0 = 0.9\ \mathrm{m}$, $R_0 = 2.97\ \mathrm{m}$, $n_H/n_D = 0.04$, $n_D = 2 \times 10^{19}\ \mathrm{m}^{-3}$, $Z_{eff} = 2.2$, $T_e(0) = T_D(0) = 10\ \mathrm{keV}$, $B_0 = 3.45\ \mathrm{T}$ and $I_p = 2.6\ \mathrm{MA}$. The steady state distribution function of H ions in the absence of TAEs is first computed. The effect of the renewal of the distribution function on the evolution of the mode amplitude of an unstable TAE, which is described by a simplified model [3], is shown in Fig. 1 and a close up of the initial stage in Fig. 2. The effect of the renewal of the distribution function by collisions and ICRH have been studied for three cases: in the absence of collisions and ICRH; in the absence of ICRH and including collisions; and including collisions and ICRH. To isolate the effect of the renewal rate from the effect of the change of the width of the resonant regions, which is determined by the diffusive contribution from collisions and ICRH according to Eqs. (3) and (4), the decorrelation time is determined by both collisions and ICRH in all three cases. The initial growth rates for the three cases are almost identical with $\gamma/\omega = 2.6\ \%$ where the mode frequency is $\omega = 1.45 \times 10^6\ \mathrm{s}^{-1}$. When the mode grows the distribution function in the high-energy regions in phase space is flattened along the TAE characteristics and at the same time reducing the drive. The interactions in the regions in phase space where the resonant ions have lower energy then become more important. If the local distribution function decreases with energy along the TAE characteristics, these interactions will then damp the mode on longer time scales. Whereas the calculation of the linear growth rate is rather straightforward, the damping of the mode is more difficult to estimate since it is subjected to both fluctuations, caused by statistical noise and chaotic behaviour of the non-linearity of the system. The intrinsic damping rate coming from ion Landau damping, essentially of high-energy ions, in the absence of collisions and ion-cyclotron interactions is estimated to be $\gamma_d/\omega \approx 1\ \%$. In the absence of mechanisms restoring the distribution function, such as collisions or ion-cyclotron interactions, the intrinsic damping will damp out the mode even in the absence of a background damping. The Coulomb collisions will still cause a flow of particles through the resonant regions, as discussed earlier, giving rise to new bursts as the distribution function is partially restored. The simulations demonstrate that the renewal rate of the distribution function and the start of the new burst are strongly affected by ICRH, but the initial growth rate is not significantly affected by Coulomb collisions or by ion-cyclotron interactions. When collisions are included we estimate the damping rate to be $\gamma_d/\omega = 1.4\ \%$ while collisions and ion-cyclotron interactions together yield an estimate of $\gamma_d/\omega = 0.5\ \%$.

As the particles are heated by ICRH and enter the resonant region they are, in general, displaced outwards with respect to the minor radius while transferring energy to the mode. This may lead to an oscillation of the mode amplitude; or rather bursts of TAE activity, since the flattening and renewal of the distribution function take place on different time scales. The initial condition for starting the TAE simulation with a preheated distribution function affects essentially only the first TAE burst. In the following bursts the distribution function is partially restored by collisions and ion-cyclotron interactions. The fact that the growth and damping of the TAE take place on the same time scale suggests that the unstable TAE takes off when the linear growth rate just exceeds the damping as assumed in the model by Berk *et al* [3].

The frequent bursts give rise to a frequency splitting of the Fourier decomposed time-dependent wave field as illustrated in Fig. 3. In the absence of ion-cyclotron interactions the

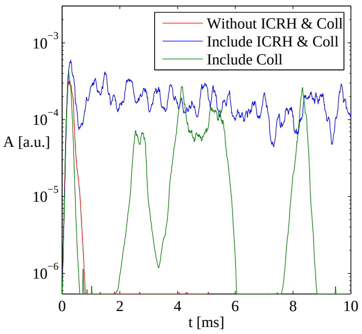

Fig. 1: *TAE amplitude with and without ICRH and collisional operators.*

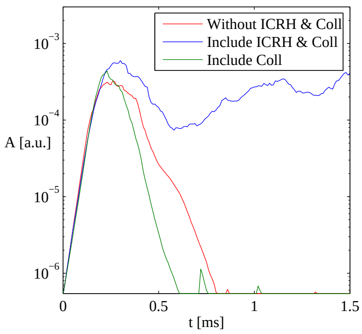

Fig. 2: *Initial stage of TAE amplitude with and without ICRH and collisional operators.*

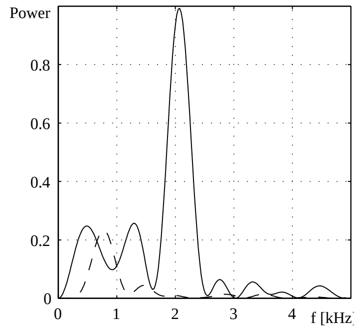

Fig. 3: *Fourier decomposition of mode amplitude. (—) ICRH and collisions, (- - -) collisions.*

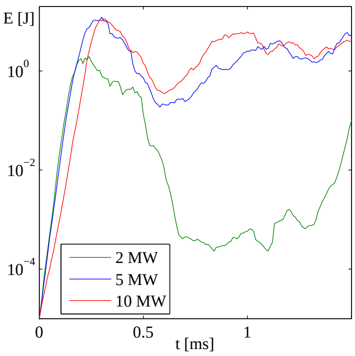

Fig. 4: *TAE amplitude for different ICRH powers.*

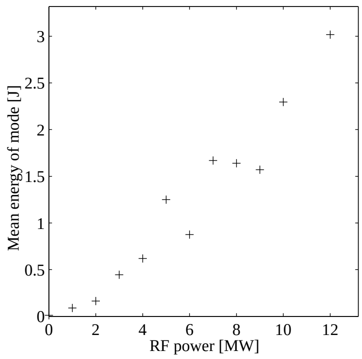

Fig. 5: *Time averaged mode energy versus ICRH power.*

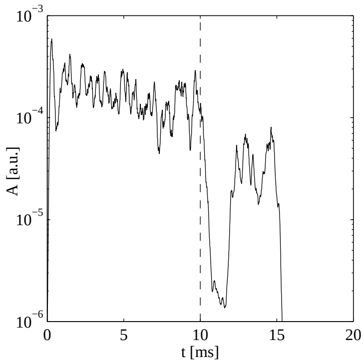

Fig. 6: *TAE amplitude when ICRH is turned off at* 10 ms.

typical period of the fluctuations of the mode amplitude is 1.5 ms, corresponding to $\Delta\omega = 2\pi \times 6.7 \times 10^2\ \mathrm{s}^{-1}$. When both ion-cyclotron interactions and Coulomb collisions are included the resulting period between the bursts decreases to 0.5 ms, corresponding to $\Delta\omega = 2\pi \times 2.0 \times 10^3\ \mathrm{s}^{-1}$.

The dynamics of the TAE activity is strongly affected by the strength of the ion-cyclotron interactions. This is clearly seen in Fig. 4, which illustrates the time variation of the mode amplitude for different ICRH powers. When the power rises from 2 to 5 MW there is a significant change in the repetition rate of the TAE bursts. As the power increases further from 5 to 10 MW it is not so much the frequency of the bursts that increases but rather the mode amplitude that stays at a higher level for a longer time.

To analyse how the spatial redistribution of the fast ions by a single TAE is affected by cyclotron interactions, we use the fact that the time-averaged redistribution of resonant ions by a TAE is in steady state related to the time-averaged mode energy, assuming that the damping of the mode by resonant ions and background damping is constant in time. To reduce the effect of the preheating and the associated growth of an initially unstable mode we average the mode energy from 1.5 to 10 ms. In Fig. 5 we have plotted the time-averaged mode energy versus ICRH power for simulations using the same initial distribution function obtained through 5 MW of heating.

To study the intrinsic damping by the resonant ions in the simulation with 5 MW of ICRH power the ICRH is turned off after 10 ms of TAE activity. The rapid decay of the mode amplitude is illustrated in Fig. 6 and the damping rate is estimated to $\gamma_d/\omega = 0.6\ \%$.

Interactions with several modes can be added straightforward in the Monte Carlo operators. However, analysis of the AE dynamics in presence of several modes become very difficult since the structure of the diffusion coefficient in phase space is very complicated and the location of

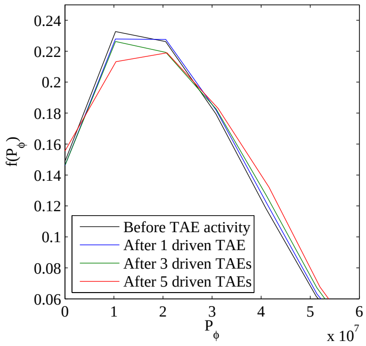

Fig. 7: *Change in* P_ϕ*-distribution of trapped particles above* 100 keV.

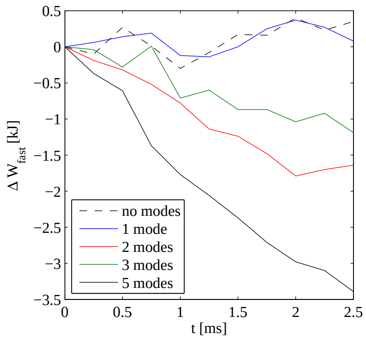

Fig. 8: *Change in fast particle energy content during TAE activity.*

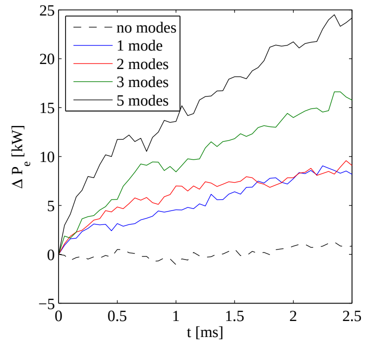

Fig. 9: *Change in power transer from ions to electrons during TAE activity.*

the resonant regions and their overlap is not known. To isolate the effect of the redistribution of resonant particles and how it effects the heating profiles the power transfer from ions to electrons has been studied using constant mode amplitudes. Several modes significantly increases the redistribution of fast particles towards higher P_ϕ. As P_ϕ increases the turning points of trapped particles are displaced outwards where the electron temperature is lower. The collision frequency, and hence the power transfer, between the high-energy ions and electrons depends on the electron temperature and density according to $\gamma_s \propto nT_e^{-3/2}$. The redistribution in P_ϕ of trapped particles above 100 keV is shown in Fig. 7 after 2.5 ms of mode activity with both one single mode, 3 modes and 5 modes. The change in fast particle energy content for different number of driven modes is shown in Fig. 8 and the case with no modes is simulated to determine noise level. As the orbits are displaced outwards by AE interactions the power transfer to the electrons increases as shown in Fig. 9. In the simulation with one single mode it is seen that even though the change in fast particle energy content and the redistribution in P_ϕ is small, the effect on the power transfer from ions to electrons is noticeable. As more modes are added it is clear that the effect on both fast particle energy content and redistribution in P_ϕ is increasing. In Fig. 10, the sum of the fraction of resonant particles in each separate mode and the total fraction of resonant particles is plotted. The fraction of particles resonating with more than one mode increases as more modes are added indicating an increased overlap in phase space.

From Fig. 9 it is not possible to determine if the increased power transfer is a sum of the contributions from each mode or if overlap between modes has a stronger effect. To study the effect of the increased overlap caused by an increase in ICRH power the case with three driven modes is studied. An increase in the overlapping regions can be seen in Fig. 11 where the fraction of particles resonating with more than one mode increases as the ICRH power is increased. In Fig. 12 it is shown how the power transfer from ions to electrons is altered by an increase in ICRH power. As the ICRH power is increased the overlap increases, the redistribution takes place over a larger region in phase space and the restoration of the distribution function is faster. With driven modes with constant amplitude it is seen in Fig. 12 that the redistribution of the distribution function reaches a steady state after about 3 ms and that the increased restoration rate has a stronger effect than the increased overlap and increased redistribution.

4. Conclusions

A model allowing self-consistent studies of the effects of decorrelations by ICRH and Coulomb collisions of ions interacting with AEs has been developed and implemented in the SELFO code taking into account the complex structure of the resonant regions in phase space [8]. The variation of the distribution function in phase space produces regions with destabilizing and

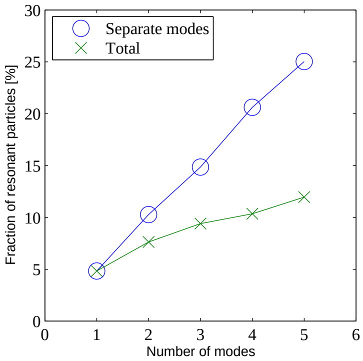

Fig. 10: *Fraction of resonant particles as a function of number of modes with 5MW ICRH power.*

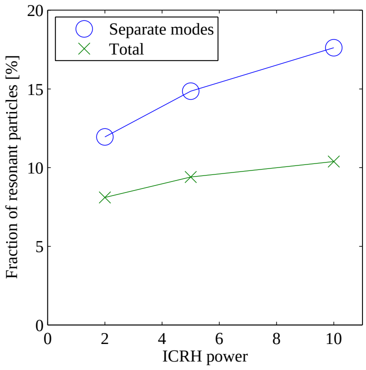

Fig. 11: *Fraction of resonant particles for 3 TAEs as a function of ICRH power.*

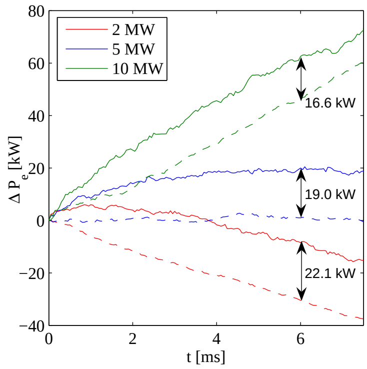

Fig. 12: *Change in power transer from ions to electrons. (—) with 3 TAEs, (- - -) without TAEs.*

stabilizing AEs. A typical intrinsic damping rate of about 1 ms is found, comparable with the damping by resistivity and ELD, and the growth rate of the AEs. The decorrelation of fast ions and the renewal of the distribution function by ion-cyclotron interactions have a strong effect on the dynamics of the modes and the redistribution of resonant ions. The typical oscillation period, which is of the order 1 ms, of the mode amplitude is seen to decrease with increasing decorrelation by ion-cyclotron interactions, which is in agreement with experimental observations [2, 9, 10] and numerical simulations. The fast decay of the TAE-mode, which is observed in experiments when the ICRH is turned off, is also reproduced in the simulations.

One single driven TAE with constant amplitude have a small effect on the fast particle energy content and the redistribution in toroidal angular momentum. Several TAEs however have a significant effect on the radial redistribution and the associated loss in power from high-energy ions to electrons. There are three effects associated with an increased ICRH power: increase of resonant regions in phase space; increased overlap of resonant regions; faster restoration of the distribution function. Which of these effects is the dominant depends on where in phase space the resonant regions are located and the distribution function within these regions.

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