# Neoclassical Momentum Transport in an Impure Rotating Tokamak Plasma

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**Abstract.** Radial electric field shear is believed to create transport barriers in tokamak plasmas, giving improved confinement. As turbulence is suppressed in transport barriers and ion thermal transport is comparable to the neoclassical prediction, neoclassical momentum transport may govern the radial electric field. We calculate the collisional transport matrix for a low collisionality plasma with collisional impurity ions. The bulk plasma toroidal rotation velocity is taken to be subsonic, but the heavy impurities undergo poloidal redistribution due to the centrifugal force. Previously only rotation shear was considered to drive radial angular momentum transport, with a small momentum diffusivity typical of the Pfirsch-Schlüter regime. The impurities now give rise to off-diagonal terms in the transport matrix, causing the plasma to rotate spontaneously, even in the absence of momentum sources. At conventional aspect ratio, poloidal impurity redistribution is seen to increase the angular momentum flux by a factor up to $\varepsilon^{-3/2}$, making it comparable to banana regime heat transport. Radial pressure and temperature gradients are the primary driving forces of the flux.

## 1. Introduction

It is widely believed that internal transport barriers (ITBs) form in regions where the radial electric field is strongly sheared. As the radial electric field profile is controlled by angular momentum transport, if turbulence is suppressed in an ITB, neoclassical angular momentum transport is likely to play a key role in the formation and sustainment of the ITB. Ion thermal transport is observed in ITB plasmas at the neoclassical level[1,2] predicted assuming the bulk ions are in a low collisionality regime. However, experimentally the radial angular momentum transport has remained anomalous, being typically an order of magnitude larger than the neoclassical prediction.[2]

Angular momentum is carried predominantly by the bulk ions, so angular momentum transport is determined by the bulk ion viscosity. This neoclassical viscosity was first calculated for the case of slow toroidal rotation of a pure plasma by Rosenbluth *et al.*[3] Hinton and Wong[4] and Catto *et al.*[5] provided the extension to toroidal rotation at a speed comparable to the ion thermal speed. Despite taking the ion collisionality to be low, the viscosity scaling with aspect ratio was characteristic of the bulk ions being in the collisional Pfirsch-Schlüter regime, thus was substantially smaller than the heat diffusivity.

Heavy impurities, such as carbon, are usually present in a plasma. Wong[6] determined the viscosity for an arbitrarily rotating plasma, in which both the bulk ions and a single impurity species were in the low collisionality banana regime. He found no significant enhancement over the viscosity in a pure plasma. However, impurities in the region of an ITB may be expected to be in a high charge state and will thus be collisional. Hirshman[7] calculated the particle and heat transport in a mixed collisionality plasma, in which the bulk ions were in the banana regime and the single impurity species was collisional. He found that the heat flux was as expected for transport dominated by trapped banana particles, but the particle flux was characteristic of transport by collisional particles and thus smaller than the heat flux. In a rotating plasma, particles are subject to a centrifugal force, when viewed in a co-moving frame, which pushes particles to the outside of the torus. As cross-field transport acts slowly, the particles accumulate on the outboard side of each flux surface. Due to their large mass, impurities feel a substantial centrifugal force, thus their distribution around a flux surface is expected to be significantly non-uniform (for example, see Wesson[8]), which is seen experimentally.[9] Fülöp and Helander[9] took this redistribution into account in the mixed collisionality plasma of Hirshman and found that both the particle and heat fluxes then displayed the scaling with aspect ratio characteristic of the banana regime. However, the viscosity has not previously been determined for a mixed collisionality rotating plasma with impurity redistribution.

Radial transport may be characterised by the transport matrix, $L$, which relates radial gradients of the three plasma parameters ion pressure, $p_i$, ion temperature, $T_i$ and toroidal angular velocity, $\omega$, to the radial ion flux of particles, $\Gamma$, heat, $q$ and angular momentum, $\Pi$. Here we discuss the form of the three transport coefficients $L_{31}$, $L_{32}$ and $L_{33}$ which characterise the radial angular momentum transport, for a mixed collisonality plasma, in which the bulk deuterium ions (mass $m_i$, density $n_i$) are collisionless and rotating subsonically and the impurities (mass $m_z$, density $n_z$, charge $z$) are collisional and undergo poloidal redistribution. Full details of the calculation, as well as the remaining elements of the transport matrix, will be provided in a forthcoming paper.

## 2. Formulation

Hinton and Wong[4] established a systematic way of expanding the ion kinetic equation to analyse the neoclassical transport in a plasma with arbitrary toroidal rotation speed, which we have used as a basis for this work. They transformed the bulk ion kinetic equation to a frame moving with the local toroidal rotation velocity: $\mathbf{u}_0 = \omega\left(\Psi, t\right) R\hat{e}_\phi$. $\hat{e}_\phi$ is the unit vector in the toroidal direction, $R$ the major radius, $\Psi$ the poloidal flux function and $\omega$ is the angular rotation frequency, which is constant on each flux surface. The basic expansion parameter, $\delta$, was taken to be the ratio of the ion gyroradius, $\rho_i$, to the typical macroscopic radial scale length, $L_r$, so the bulk ion distribution function is written as $f = f_0 + f_1 + ...$, where $f_1 \sim \delta f_0$. The leading term in the expansion of the electric field, $E = E_{-1} + E_0 + E_1 + ...$, was taken to be of order $T_i/eL_r\delta$, where $T_i$ is the ion temperature, which allows the bulk flow velocity to be comparable to the ion thermal speed. If the magnetic field varies slowly, the electric field is given by an electrostatic potential, $\Phi$. Hinton and Wong showed that the rotation velocity is related to this potential via: $\omega = -\partial\Phi_{-1}/\partial\Psi$. This gives the connection between the evolution of toroidal rotation via angular momentum transport and the development of the radial electric field in neoclassical regions. The variation in the zeroth-order density, $n_0 = \int f_0 d^3v$, where $v$ is the ion velocity, around a magnetic flux surface, due to the rotation, is expected to have a significant effect on the analysis. It may be obtained from the fluid momentum equation in the lab frame. Wesson[8] determined that the impurity density, for the case of a trace, heavy impurity present in a hydrogenic plasma, is given by:

$$n_z = n_{z0} \exp\left[\left(1 - \frac{T_e}{T_i + T_e} z \frac{m_i}{m_z}\right) \frac{m_z \omega^2 \left(R^2 - R_0^2\right)}{2T_z}\right], \tag{1}$$

where $n_{z0}$ is the density at the reference radius $R_0$ and a subscript $e$ indicates electrons. The impurities are therefore expected to be moved outward in major radius by the centrifugal force. The effect experienced by the bulk ions is much weaker due to their lower mass, so poloidal redistribution of the bulk ions is neglected here, except in Section 3.1.

Radial transport is second order with respect to $\delta$, so is most conveniently evaluated using flux-friction relations, which require $f$ only to first order in $\delta$. The derivation of these relations is discussed by Hinton and Wong.[4] They show that the leading order term in the radial angular momentum flux is given by:

$$\Pi = -\frac{m_i}{e}\left\langle \int d^3v \frac{1}{2} m_i R^2 v_\phi^2 C_i^l\left(f_1\right)\right\rangle, \tag{2}$$

where $C_i^l$ is the linearised ion collision operator and $v_\phi$ is the toroidal ion velocity.

The first-order correction to the distribution function is the sum of two parts: a classical piece, $\tilde{f}_1$, dependent on gyrophase and a neoclassical piece, $\bar{f}_1$, obtained from a drift kinetic equation. The form of $\tilde{f}_1$, valid for any species and independent of the collision operator, was determined by Hinton and Wong.[4] From this point on, we take $\mathbf{v}$ to be the ion velocity in the rotating frame. Expressing the density as $n_0 = N\left(\Psi\right)\exp\left(-\frac{e}{T_i}\tilde{\Phi}_0 + \frac{m_i\omega^2R^2}{2T_i}\right)$ and with the magnetic moment and energy of a particle in the rotating frame given by: $\mu = m_i v_\perp^2/2B$ and $H = \left(v_\parallel^2 + v_\perp^2\right)m_i/2 + e\tilde{\Phi}_0 - m_i\omega^2R^2/2$ the ion drift kinetic equation derived by Hinton and Wong[4] is:

$$v_\parallel \hat{b}\cdot\nabla\bar{f}_1 - C_i^l\left(\bar{f}_1\right) = -\frac{e}{T_i}v_\parallel\hat{b}\cdot\nabla\Phi_1 f_0 - v_\parallel f_0\sum_{j=1}^{3}A_j\left(\Psi\right)\hat{b}\cdot\nabla\alpha_j. \tag{3}$$

$\hat{b} = \mathbf{B}/B$ is the unit vector in the direction of the magnetic field, parallel and perpendicular are with respect to this direction and $\tilde{\Phi}_0$ is the leading order, poloidally varying part of the electrostatic potential. The driving terms, $A_j$, are the following radial gradients, where $' \equiv \partial/\partial\Psi$:

$$A_1 = \frac{N_i'}{N_i} + \frac{T_i'}{T_i}, \qquad A_2 = \frac{T_i'}{T_i}, \qquad A_3 = \frac{\omega'}{\omega},$$

and with $I = RB_\phi$, the functions, $\alpha_j$, are given by: $\alpha_1 = \frac{m_i}{e}\left[\frac{Iv_\parallel}{B} + \omega R^2\right]$, $\alpha_2 = \left(\frac{H}{T_i} - \frac{5}{2}\right)\alpha_1$ and $\alpha_3 = \frac{m_i^2\omega}{2eT_i}\left[\left(\frac{Iv_\parallel}{B} + \omega R^2\right)^2 + \frac{\mu|\nabla\Psi|^2}{m_iB}\right]$.

By using a suitable form of the ion collision operator, we may obtain the form of $\bar{f}_1$ from Eq. (3). Due to the small electron-ion mass ratio, ion-electron collisions can be neglected. The impurity density in a tokamak is typically such that the frequency of bulk ion self-collisons, $\nu_{ii}$, is comparable to that of bulk ion-impurity collisions, $\nu_{iz}$, so $n_z z^2/n_i \sim O(1)$. Thus we assume the linearised collision operator is a sum of two parts: $C_i^l = C_{ii} + C_{iz}$. $C_{ii}$ describes the collisions between bulk ions and is taken to be the model self-collision operator introduced by Kovrizhnykh.[10] $C_{iz}$ describes the bulk ion-impurity interactions. Due to the disparate masses, the interaction may be described exactly and is analogous to that between electrons and ions:

$$C_{iz}^l\left(f\right) = \nu_{iz}\left(v,\Psi,\theta\right)\left(\mathcal{L}\left(f\right) + \frac{m_i\mathbf{v}\cdot\mathbf{V}_z}{T_i}f_0\right). \tag{4}$$

The first term represents pitch-angle scattering and the second term takes into account the motion of the impurities, so the impurity flow velocity measured in the rotating frame, $\mathbf{V}_z$, enters. $x = v/v_{Ti}$ where $v_{Ti} = (2T_i/m_i)^{1/2}$ is the ion thermal velocity, $\nu_{iz} = \left(3\pi^{1/2}/4\tau_{iz}\right)/x^3$ and the ion-impurity collision time $\tau_{iz} = 3(2\pi)^{3/2} m_i^{1/2} T_i^{3/2} \varepsilon_0^2 / n_z z^2 e^4 \ln\Lambda$, where $\ln\Lambda$ is the Coulomb logarithm. $\mathcal{L}$ is the Lorentz operator and the angle $\theta$ measures poloidal position on the flux surface, with $\theta = 0$ at the outboard side. The $\theta$ dependence of $C$ results from the non-uniform poloidal impurity distribution.

We determine the impurity flow velocity by considering the parallel impurity momentum equation. Taking the cross product of this equation with $\hat{b}$ gives $V_{z\perp}$ and as cross-field transport occurs on the slow second order timescale, the flow velocity must be divergence free on a flux surface to first order. Hence we find $V_{z\parallel} = -I|\mathbf{V}_{z\perp}|/|\nabla\Psi| + K(\Psi)\,B/n_z$. The constant of integration, $K(\Psi)$, is determined by multiplying the momentum equation by $B/n_z$, then taking the flux surface average. This gives: $\left\langle BR_{z\parallel}/n_z\right\rangle = 0$, where the friction felt by the impurities, $\mathbf{R}_z$, is due to collisions with bulk ions: $\mathbf{R}_z = -\int m_i \mathbf{v} C_{iz}(f)\, d^3v$.

## 3. Results

As $f_1 = \tilde{f}_1 + \bar{f}_1$, Eq. (2) shows that we may write the angular momentum flux as the sum of a classical part, $\tilde{\Pi}$, dependent on $\tilde{f}_1$ and a neoclassical part, $\bar{\Pi}$, dependent on the gyroaveraged distribution function, $\bar{f}_1$. Each element of the transport matrix, $L$, is thus the sum of a classical, $\tilde{L}$ and a neoclassical, $\bar{L}$, term.

### 3.1. Classical Flux

The form of $\tilde{f}_1$ derived by Hinton and Wong[4] may be used directly to evaluate the classical contribution to the angular momentum flux for arbitrary toroidal rotation speed. As the collision operator is linear, $C_i^l = C_{ii} + C_{iz}$, we split the flux into two parts, $\tilde{\Pi}^{ii}$ and $\tilde{\Pi}^{iz}$, driven by bulk ion self-collisions and by bulk ion-impurity collisions respectively. $\tilde{\Pi}^{ii}$ is the classical angular momentum flux expected in a pure plasma.

Following the procedure introduced by Braginskii, $\tilde{\Pi}^{ii}$ may be evaluated and the result of Hinton and Wong[4] recovered:

$$\tilde{\Pi}^{ii} = -\left\langle \frac{p_i|\nabla\Psi|^2}{m_i\Omega_i^2}\frac{3}{10\tau_i}\left(3\frac{I^2}{B^2} + R^2\right)\right\rangle \omega', \tag{5}$$

where $\tau_i$ is the ion collision time defined by Braginskii and $\Omega_i = eB/m_i$.

Using the form of $C_{iz}$ given in Eq. (4) we obtain:

$$\tilde{\Pi}^{iz} = -\left\langle \frac{p_i|\nabla\Psi|^2}{m_i\Omega_i^2} m_i\omega R^2 \frac{G}{\tau_{iz}}\right\rangle - \left\langle \frac{p_i|\nabla\Psi|^2}{m_i\Omega_i^2}\frac{3}{10\tau_{iz}}\left[3\frac{I^2}{B^2} + R^2\right]\right\rangle \omega', \tag{6}$$

Assuming $T_z = T_i$, $G$ defines the following combination of driving terms:

$$G = \frac{N_i'}{N} - \frac{1}{2}\frac{T_i'}{T_i} - \left(m_i - \frac{m_z}{z}\right)\frac{\omega^2 R^2}{2T_i}\left\{\frac{T_i'}{T_i} - 2\frac{\omega'}{\omega}\right\}.$$

$\tau_{iz}$ is inversely proportional to the impurity density, which, by Eq. (1), will be pushed towards the outboard side of a flux surface. Thus the flux surface averages in $\tilde{\Pi}^{iz}$ will be primarily determined by conditions at the outboard side. With $B_\phi$ and $B_p$ the toroidal and poloidal components of the magnetic field respectively, as $|\nabla\Psi| = RB_p$ we may expect the classical angular momentum transport, $\tilde{\Pi}/|\nabla\Psi|$ to be enhanced by the ratio of $R^2/B^2$ at the outboard side to its flux surface average. This enhancement can be understood as follows. The classical transport is collisional diffusion with a step size of order the ion gyroradius. The angular momentum, $m_i \omega R^2$, carried by a particle and the step size are larger at the low field outboard side, which is where most of the collisions causing the radial transport occur. The enhancement will be most significant in a spherical tokamak, where $R^2/B^2$ varies substantially over a flux surface.

### 3.2. Neoclassical Flux in a Plasma with Subsonic Rotation

At this point we restrict the analysis to the case of subsonic rotation of the bulk plasma. Specifically we assume the bulk ion Mach number, $M_i$, satisifes: $M_i^2 = m_i \omega^2 R^2 / 2T_i \sim 1/z \ll 1$. This ordering allows the impurity Mach number, $M_z^2 = m_z \omega^2 R^2 / 2T_z$, to be of order 1 and strong poloidal redistribution of impurities will be expected on the basis of Eq. (1), but $n_i$ will be approximately constant around a flux surface. The velocity is now an approximate constant of the motion, so the independent variables are taken to be $(v, \lambda, \sigma, \Psi)$, where $\lambda = v_\perp^2 / \left(v^2 B\right)$ and $\sigma = v_\| / |v_\||$. The Lorentz operator may now be written as $\mathcal{L} = \frac{2\xi}{B}\frac{\partial}{\partial\lambda}\lambda\xi\frac{\partial}{\partial\lambda}$, where the pitch angle cosine, $\xi = v_\|/v$. The gyroaveraged form of the collision operator is used and in the case of subsonic rotation, the parallel impurity velocity becomes:

$$V_{z\|} = -\frac{I}{B}\frac{\partial\langle\Phi_0\rangle}{\partial\Psi} + \frac{K\left(\Psi\right)B}{n_z}. \tag{7}$$

To determine $\bar{f}_1$, we perform a subsidary expansion of Eq. (3) in the small ratio of the bulk ion collision frequency to their typical bounce frequency. A bracketed superscript denotes the order of expansion. The two lowest order equations give:

$$f_1^{(0)} = F + g \qquad \text{and} \qquad v_\| \nabla_\| f_1^{(1)} = C_i^l\left(f_1^{(0)}\right),$$

where $F = \left(-\frac{e}{T_i}\Phi_1 - \sum_{j=1}^{3} A_j\left(\Psi\right)\alpha_j\right) f_0$ and $g$ is a constant along a field line so is a function only of $v$, $\lambda$, $\sigma$ and $\Psi$.

The neoclassical contribution to the angular momentum flux, $\bar{\Pi}$, may now be evaluated for arbitrary flux surface geometry and poloidal impurity distribution. We define the following normalised quantities: $n = n_z / \langle n_z\rangle$, $r^2 = R^2/R_0^2$, $R_0^2 = \langle R^2\rangle$, $b^2 = B^2/B_0^2$, $B_0^2 = \langle B^2\rangle$, $\hat{\Omega} = \Omega B_0^2/B^2$, $\zeta = Z_{\text{eff}}/\langle Z_{\text{eff}}\rangle$, $B_{min} = B\left(\theta = 0\right)$ and write:

$$\bar{\Pi} = -\frac{p_i I^2 \omega R_0^2}{\hat{\Omega}_i^2}\left\langle \tau_{iz}^{-1}\right\rangle\left(l_{31}A_1 + l_{32}A_2 + l_{33}A_3\right), \tag{8}$$

so each transport coefficient is $\bar{L}_{jk} = -l_{jk} p_i I^2 \omega R_0^2 \left\langle\tau_{iz}^{-1}\right\rangle / \hat{\Omega}_i^2$ and $l_{33} = l_{33}^{(1)} + l_{33}^{(2)}$. With $\eta_1 = 2^{1/2} - \ln\left(1 + 2^{1/2}\right)$ and $\eta_2 = \left(5/2\right)\eta_1 - 2^{-1/2}$, after lengthy algebra we find:

$$l_{31} = \left\langle \frac{nr^2}{b^2}\right\rangle - f_c\left\langle nr^2\right\rangle - f_t \frac{1 - \langle nr^2\rangle\left\langle\frac{b^2}{n}\zeta\right\rangle f_c}{\left\langle\left(1 - f_c\zeta\right)\frac{b^2}{n}\right\rangle}$$

$$l_{32} = -\frac{3}{2}l_{31} + \left(\frac{3}{2} - \frac{\eta_2}{\eta_1}\right)\frac{f_c f_t}{\langle Z_{\text{eff}}\rangle}\frac{\left\langle\frac{b^2}{n}\right\rangle\langle nr^2\rangle - 1}{\left\langle\left(1 - f_c\zeta\right)\frac{b^2}{n}\right\rangle}$$

$$l_{33}^{(1)} = 2\left\langle M_i^2\right\rangle\left[\left\langle\frac{nr^4}{b^2}\right\rangle - f_c\left\langle nr^2\right\rangle\left\langle\zeta r^2\right\rangle - \left(1 - f_c\left\langle\zeta r^2\right\rangle\right)\frac{1 - f_c\langle nr^2\rangle\left\langle\frac{b^2}{n}\zeta\right\rangle}{\left\langle\left(1 - f_c\zeta\right)\frac{b^2}{n}\right\rangle}\right]$$

$$l_{33}^{(2)} = -\frac{2}{5}\frac{R_0^2 B_0^2}{\left\langle \tau_{iz}^{-1} \right\rangle I^2}\left[\frac{15}{4}B_0^2\int_0^{B_{min}^{-1}}\frac{\left\langle \frac{r^2}{b\tau'}\left(\frac{B_\phi^2 - B_p^2/2}{B_0^2}\right)\sqrt{1-\lambda B}\right\rangle^2}{\left\langle \frac{1}{\tau'}\sqrt{1-\lambda B}\right\rangle}\lambda d\lambda - \left\langle \frac{r^4}{\tau'}\left(\frac{B_\phi^2 - B_p^2/2}{B^2}\right)^2\right\rangle\right]$$

where $\tau'^{-1} = \tau_i^{-1} + \tau_{iz}^{-1}$, $f_t = 1 - f_c$ and the 'effective fraction' of circulating particles, $f_c$, is a generalisation of that defined in Ref. [11]:

$$f_c = \frac{3}{4}\left\langle B^2 \right\rangle \langle Z_{\rm eff} \rangle \int_0^{\lambda_c} \frac{\lambda d\lambda}{\left\langle Z_{\rm eff}\sqrt{1-\lambda B}\right\rangle}. \tag{9}$$

In the limit of very high impurity content, $Z_{\rm eff} \gg 1$, these coefficients reduce to those presented previously in Ref. [12]. There we considered the coefficients in the limit of large aspect ratio, but not restricted to circular geometry as the inverse aspect ratio, $\varepsilon$, was a function of $\theta$, so $\varepsilon(\theta) \ll 1$ and strong impurity redistribution, $\langle n\cos\theta\rangle \sim O(1)$. As the bulk ions are not significantly redistributed in the case we are considering here, the forms presented in Ref. [12] actually give the behaviour of the above coefficients, in the same limit. So for $z^2 n_z \sim n_i$, that is for $Z_{\rm eff} - 1 \sim O(1)$, with the classical diffusion coefficient $D_{cl} = \rho_i^2/\tau_{iz}$ and $q$ the safety factor, the neoclassical transport coefficients take the form:

$$\bar{L}_{31} \sim \bar{L}_{32} \sim \frac{q^2 D_{cl}}{\epsilon^{3/2}} \qquad \text{and} \qquad \bar{L}_{33} \sim M_i^2 \frac{q^2 D_{cl}}{\epsilon^{3/2}}.$$

Overall, the angular momentum transport shows an enhancement of $\epsilon^{-3/2}$ over previous predictions[4] and a scaling with aspect ratio now typical of the banana regime. Radial pressure and temperature gradients are seen to be the primary driving forces, whereas previously only rotation shear was effective. Thus, if an ITB forms in a region of radial electric field shear and the turbulence is suppressed, the strong density and temperature gradients may sustain the shear by driving a neoclassical angular momentum flux.

Using a set of typical equilibrium flux surfaces from the MAST tokamak, we evaluated the transport coefficients numerically, assuming a range of toroidal rotation speeds and impurity content, as defined by the flux surface averaged value of $Z_{\rm eff}$. The values of the coefficients at various aspect ratio were determined by evaluating them on different flux surfaces. In Figure 1, $\bar{L}_{31}$, evaluated on a flux surface with an inverse aspect ratio of 0.14, is plotted as a function of bulk ion Mach number. The full line is the case for $\langle Z_{\rm eff}\rangle = 1.5$, the dashed line for $\langle Z_{\rm eff}\rangle = 2$. $\bar{L}_{31}$ is exactly zero for a pure plasma and rises with impurity content. It increases with Mach number, as the impurity redistribution increases, so the flux surface averages in $l_{31}$ are increasingly determined by the outboard values of $r$ and $b$. The dotted line shows the value of $\bar{L}_{33}$ given in Ref. [4] for a pure plasma, which has previously defined the expected level of transport for all values of $Z_{\rm eff}$. Thus we see the expected level of transport is around 10 times higher than previously predicted, for $A_1 \sim A_3$.

Finally we note that, in the absence of sources of angular momentum, the angular momentum flux must be zero, so there is the following balance between the driving terms:

$$\frac{\omega'}{\omega} = -\frac{1}{L_{33}}\left(L_{31}\frac{p_i'}{p_i} + L_{32}\frac{T_i'}{T_i}\right). \tag{10}$$

The large values of $\bar{L}_{31}$ and $\bar{L}_{32}$ may thus lead to spontaneous rotation, as is sometimes observed experimentally.[2] The rotation direction will be set by the edge boundary condition.

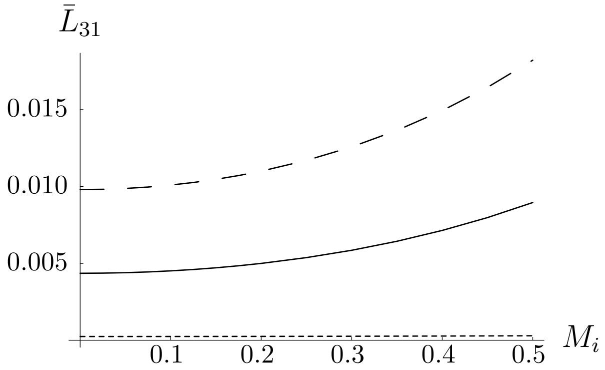


*Figure 1: $\bar{L}_{31}$ as a function of ion Mach number, $M_i$.*

## 4. Conclusions

Experimentally, angular momentum transport in regions of neoclassical ion thermal transport has remained anomalous. In a rotating plasma heavy impurities will undergo poloidal redistribution. Including this effect, a general form for the angular momentum flux has been derived for a mixed collisionality plasma typical of experiment, with collisionless bulk ions and a highly charged, collisional, impurity species. In the experimentally relevant limit of conventional aspect ratio, with impurities pushed towards the outboard side, the flux is seen to increase by a factor of $\varepsilon^{-3/2}$, making it comparable to ion energy transport by particles in the banana regime.

Radial gradients of the bulk ion pressure and temperature are now seen to be the primary driving forces of the flux, not rotation shear. Strong density and temperature gradients can thus drive angular momentum transport to sustain a sheared electric field, which may help to make transport barriers self-sustaining. Also, this may allow spontaneous toroidal rotation to arise in a plasma with no external angular momentum source, as is seen experimentally. The magnitude and direction of this rotation will depend on the edge boundary condition.

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