The case of the trapped singularities

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In this work two different views of the physics behind plasma confinement transitions are smoothly reconciled in a unified model for the coupled dynamics of potential energy, turbulent kinetic energy, and shear flow kinetic energy subsystems.

Magnetized fusion plasmas are strongly driven nonequilibrium systems in which the kinetic energy of small-scale turbulence can drive the formation of large-scale coherent structures such as shear and zonal flows. This inherent tendency to self-organise is a striking characteristic of flows where Lagrangean fluid elements see a strongly two-dimensional velocity field, and is a consequence of the inverse energy cascade [1]. The distinctive properties of quasi 2-d fluid motion are the basis of natural phenomena such as zonal structuring of planetary flows, but are generally under-exploited in technology.

In plasmas the most potentially useful effect of 2-d fluid motion is suppression of high wavenumber turbulence that degrades confinement, which can manifest as a dramatic enhancement of sheared poloidal or zonal flows and reduction in cross-field turbulent transport. These low- to high-confinement (L–H) transitions have been the subject of intensive experimental, *in numero*, and theoretical investigations since the 1980s. Two major strands in the literature emerged early and have persisted: (1) They are an internal phenomenon that occurs spontaneously when the rate of upscale kinetic energy transfer exceeds the nonlinear dissipation rate; (2) They are due to ion orbit losses near the plasma edge or induced biasing, the resulting electric field providing a torque which drives shear flows nonlinearly.

These two different views of the physics of confinement transitions are assimilated in this work in a dynamical model developed by a systematic iterative method: 1. Interrogate degenerate singularities in the simplest model; 2. Unfold the singularities in physically meaningful ways, 3. Interrogate any new singularities that appear in enhanced model; 4. Repeat steps 2 and 3 until the model is free of pathological or persistent degenerate singularities, is self-consistent, reflects observations in experiments, and is therefore predictive.

The work is presented as a case study in bifurcation and stability analysis. A systematic methodology for characterizing the equilibria of dynamical systems involves finding and classifying high-order singularities then perturbing around them to explore and map the bifurcation landscape [2]. In a broad sense, this paper is about applying singularity theory as a diagnostic tool while an impasto picture of confinement transition dynamics is compounded [3, 4, 5].

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