Non-Scaling Fixed Field Gradient Accelerator (FFAG) Design for the Proton and Carbon Therapy*

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Abstract. The *non-scaling* Fixed Field Alternating Gradient (FFAG-from now on) accelerator provides few advantages with respect to the other fixed field accelerators like CYCLOTRONS or *scaling*-FFAG's. One of the advantages is smaller required aperture due to small orbit offsets during acceleration. The large and heavy magnets are avoided. The beam is very well controlled in a strong focusing regime. This concept has been extensively investigated during the last eight FFAG workshops in Japan, USA, Canada, and CERN in Europe.

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1. Introduction

The Fixed Field Alternating Gradient (FFAG) accelerators became again a subject of great interest[1] in many accelerator physics applications, after more than fifty years of their first appearance [2,3,4]. The original FFAG's are the "*scaling*" designs, where particle orbits during acceleration scale with momentum. In Japan a number of *scaling* FFAG's have been built, or are under construction. The *scaling* and *non-scaling* designs are proposed for many applications: proton acceleration in medical field for cancer therapy, electron acceleration for the low (food radiation, electron demonstration ring) and high energies (future e-RHIC 10 GeV), acceleration of muons (the "PRISM"-project in Japan), proton acceleration for the AGS upgrade at Brookhaven National Laboratory, etc. There are many advantages of the *scaling* FFAG with respect to the today common used synchrotrons, cyclotrons, or linear accelerators-linacs like: the magnetic field is fixed, possibility of high repetition rate, savings on the RF requirements. Disadvantages of the *scaling* FFAG are the large required aperture and circumference. This is due to the *scaling* law between the orbit and momentum and a relatively large opposite bend magnet requirement. The *non-scaling* design had been extensively investigated in many respects. A proposal to build a *non-scaling* FFAG electron demonstration ring in United Kingdom has recently being applied to the European funding agency. A program of the recent international Cyclotron conference in Tokyo, Japan included a special section for the FFAG acceleration.

The *non-scaling* FFAG's acceleration reduces the aperture and circumference [5]. If the fixed magnetic field produces the linear gradient, there is a tune variation during fast acceleration and resonances are a crossed. The small dispersion function and strong focusing reduces the aperture size for almost an order of magnitude with respect to standard scaling FFAG design. We present one of the possible applications of the non-scaling proton and carbon cancer therapy FFAG accelerator.

The cancer proton therapy exists today in many medical facilities and more are being built. These centers treat mostly prostate or eye cancers like in: Loma Linda University Hospital in

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* This manuscript has been authored by Brookhaven Science Associates, LLC under contract No. DE-AC02-98CH1-886 with the U.S. Department of Energy, and supported by the U.S. Department of Energy under contract No. DE-AC03-76SF0009.

California, Therapy Center at Massachusetts General Hospital in Boston in U.S.A., Paul Scherer Institute in Villigen – Switzerland, the Heavy-Ion Research at GSI in Germany, the Shizouka , HYOGO Medical Center, Kashiwa Center, Wakaza Bay project, the KEK in Tsukuba and other medical centers in Japan, the Faure hospital in South Africa, Uppsala in Sweden and many other facilities are either being built or have very strong support and proposals.

Advantages of the *non-scaling* FFAG with respect to the synchrotrons are the fixed magnetic field and possibilities of higher repetition rates for spot scanning. The cyclotrons are made of very large magnets (weight is more than a ton) and could not easily change the final energy. The exact energy sweep is necessary due to the ion energy deposition inside of the patient's body precisely localized by the "Bragg peak" localized at the tumor location. The *non-scaling* FFAG adjusts the final energy by adjusting the number of turns during acceleration. The very strong focusing provides not only a smaller aperture but also a better orbit control than in cyclotrons, thus leading to lower losses.

2. Basic Lattice Cell

The *non-scaling* FFAG basic cell, as presented in Figure 1, is composed of only two kinds of combined focusing magnets: the main bend with the defocusing gradient located at the center of a symmetrically arranged cell.

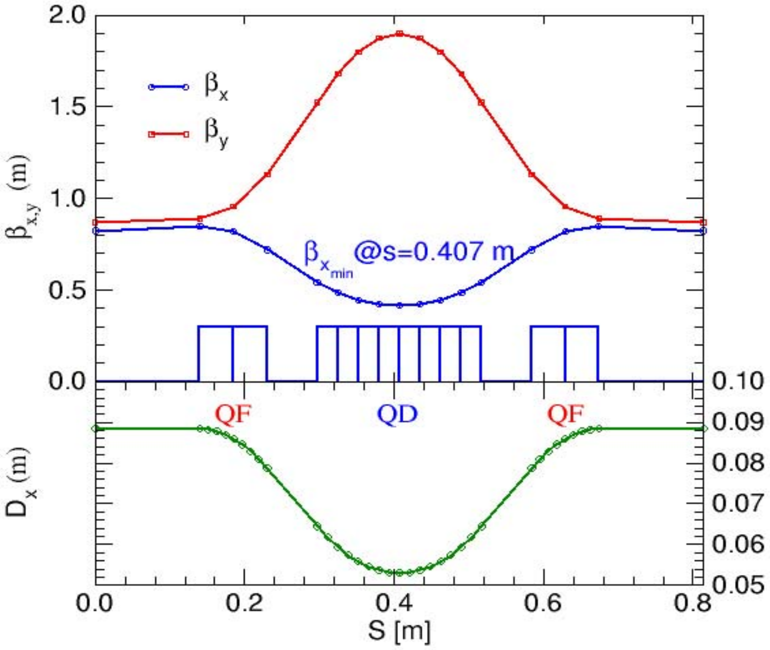

FIG. 1. Properties the non-scaling FFAG basic cell. Amplitude and dispersion functions are shown.

This magnet is surrounded with two opposite combined function bends with a focusing gradients. The magnets are powered with the fixed field and gradients and allow acceleration within $\delta p/p=+/-$ 50 %. The picture shows magnet sizes and Courant-Snyder amplitude and dispersion functions (β_x, β_y, *and* D_x) for the central momentum of $p_c=1/2*(p_{max}+p_o)=0.48415$ GeV/c [$\Delta p=0$]. The injection kinetic energy is $E_{k(o)}=30$ MeV, while the maximum kinetic energy is $E_{k(max)}$=250 MeV. This corresponds to the initial and maximum momentum, respectively, of p_o=0.2392 GeV/c and p_{max}=0.7291 GeV/c, while corresponding kinetic energy at the central momentum p_c is $E_{k(center)}$=117.5459 MeV. Parameters of the proton therapy requirements are presented in TABLE 1.

TABLE 1. PROTON THERAPY REQUIREMENTS:

	Injection ($\delta p/p$=–50%)	Cent. momentum ($\delta p/p$=0)	Top energy $\delta p/p$=50%
p (GeV/c)	0.243044580	0.486089161687	0.72913374
E_k (MeV)	30.9673989490	117.545851	250.000000
E (GeV)	0.969239427944	1.0558178797	1.18827203
γ	1.03300471291	1.125279074	1.26644725
$\beta\gamma$	0.25903423884	0.515997087	0.77710271
$B\rho$ (Tm)		1.621418914	

The bending magnetic fields, gradients, bending angles, and lengths of elements and drifts are provided in TABLE 2.

TABLE 2. MAGNET PROPERTIES FOR THE FFAG LATTICE FOR PROTON ACCELERATION FROM 30 TO 250 GeV KINETIC ENERGY

Combined function magnet	QF focusing gradient	QD defocusing gradient
Length	0.09	0.22
Bending Field (T)	1.563239217	0.30
Bending angle (rad)	0.212957438	0.016718929
Gradient (T/m)	41.227598	28.954765
Length of cavity drift (cm)	28.077650	
Drift between magnets (cm)	6.6675464	

4. *Non-scaling* FFAG's for accelerating the fully stripped carbon ions

If the magnet rigidity used for the proton therapy is kept constant and the fully stripped carbon ions $^{12}C^{6+}$ are pre-accelerated the maximum achievable kinetic energy/per nucleon for the carbon treatment is presented in TABLE 3.

TABLE 3. ACCELERATION OF FULLY STRIPPED CARBON IONS WITH THE 250 MeV PROTON NON-SCALING FFAG. THE MAXIMUM ACHIEVABLE KINETIC ENERGY PER NUCLEON IS EQUAL TO E_{kmax} = 68.271 MeV/n.

	Injection ($\delta p/p$=–50%)	Cent. momentum ($\delta p/p$=0)	Top energy $\delta p/p$=50%
p (GeV/c)	0.1210364084	0.24207281687	0.363109225
E(GeV/n)	0.9393250132	0.96243488967	0.999764961
E_k(MeV/u)	7.8306931643	30.940569673	68.27064122
γ	1.0084065925	1.0332160583	1.073291527
$\beta\gamma$	0.1299378921	0.2598757842	0.389813676
$B\rho$ (Tm)		1.1614936003	

If the proton ring 250 MeV is used to accelerate fully stripped carbon ions with the same magnetic rigidity then the maximum carbon kinetic energy per nucleon is E_k =68.27 MeV/n, assuming the same momentum range $\delta p/p$=+-50 %. The maximum kinetic energy per nucleon in this case is an input for the next ring to be designed. This new ring with the same momentum range of $\delta p/p$=+-50 % could provide the carbon maximum kinetic energy per nucleon of more than necessary for the carbon treatment E_{kmax} =501.79434 MeV/n. The required magnetic rigidity for this machine is $B\rho$=4.8448 Tm.

TABLE 4. THE PARAMETERS OF THE SUPER-CONDUCTING NON-SCALING FFAG FOR ACCELERATION OF THE FULLY STRIPPED CARBON IONS. THE CENTRAL MOMENTUM OF p_c=0.72621845 MeV/nucl. DEFINES THE REQUIRED MAGNETIC RIGIDITY OF: $ze\ B\rho = A$ ATMU $\beta\gamma/c$ = 4.8448 Tm

	Injection ($\delta p/p$=–50%)	Cent. momentum ($\delta p/p$=0)	Top energy $\delta p/p$=50%
p (GeV/c)$_n$	0.363109225	0.726218450	1.089327675
E(GeV/n)	0.999764961	1.181132891	1.433288684
E_k(MeV/u)	68.27064122	249.6385706	501.7943640
γ	1.073291527	1.267997953	1.538698254
$\beta\gamma$	0.389813676	0.779627352	1.1694410278
$B\rho$ (Tm)		4.844808004	

The acceleration of fully stripped carbon ions for the same size of the ring as for the proton requires super-conducting dipoles. The lengths of elements and circumference of the accelerator are kept the same but the field and gradients are adjusted for carbon ions. Values of the lengths, strengths of magnetic fields and gradients, bending angles are shown in Table 5.

TABLE 5. MAGNETIC PROPERTIES FOR THE *NON-SCALING* FFAG LATTICE FOR CARBON ACCELERATION FROM 68.27 MeV/nuc. TO 501.79 MeV/nuc. KINETIC ENERGY

Combined function magnet	QF focusing gradient	QD defocusing gradient
Length	0.09	0.22
Bending Field (T)	4.689711518	0.900000000
Bending angle (rad)	0.212957490	0.016718955
Gradient (T/m)	123.6833	-86.864685
Length of cavity drift (cm)	28.077920	
Drift between magnets (cm)	6.6753299	

5. Layout of the ring

The ring is composed of 35 cells with injection and extraction designed with a single turn kickers using a drift space in the existing cell. The circumference of the ring is C_o=28.5 m and each cell is L_{cell}=0.8143 m long. A drift for the cavity is 28.08 cm while the distance between the magnets is 6.67 cm. The major bend the combined function magnet with an angle of θ_1=0.21296 rad and the opposite bend with the defocusing gradient of θ_2=0.01671 rad. A drift for the cavity or injection/extraction kicker is ~28 cm. The ring is presented partially showing only three cells in Fig. 3 and the whole layout in Fig. 3. It could fit within a square room ten meters wide.

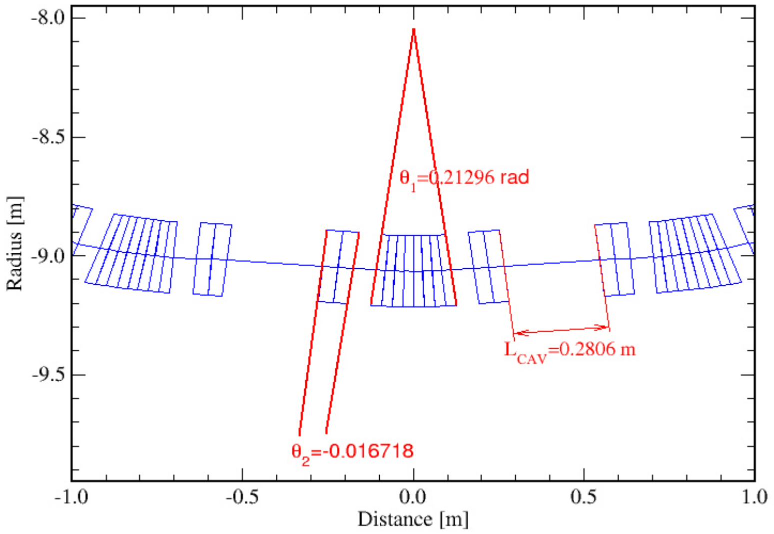

FIG. 2. A part to the ring includes almost three complete cells. The combined function magnets with linear gradients and positive and a small opposite bend are 22 cm and 9 cm effective length, respectively.

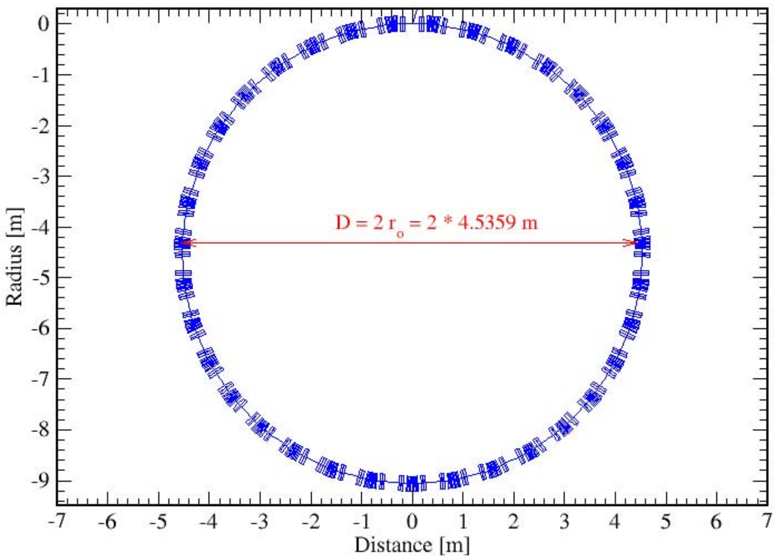

FIG. 3. Ring layout. The circumference is C_o*=28.5 meters with the average radius of* R_{avg} *= 4.5359 m. The ring occupies space of ten meters wide square size room. The major bending field is 1.56 T.*

5. Orbits and lattice properties during acceleration

5.1 Orbit Offsets

During acceleration particles with the lowest energy follow orbits closer to the center of bending. As protons or carbon ions get accelerated their orbits move toward outside of the center of curvature. We have arbitrarily chosen the range in momentum to be between δp/p=+/- 50 % of the central momentum. A combination of the edge focusing effect and strong gradients with bending produces very densely populated orbits at negative δp/p<0. Orbit offsets are within an aperture of 9 cm.

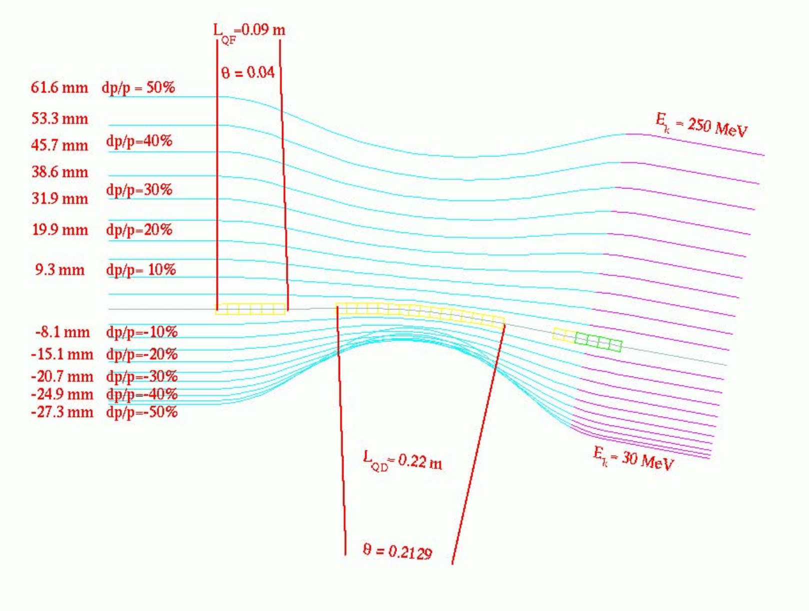

FIG. 4. The orbits during acceleration calculated by the PTC code of Etienne Forest[8]. A required horizontal aperture for the major bending combined function dipole is of the order of 65 mm, while the opposite defocusing bend requires larger aperture 90 mm wide.

4.2 Amplitude Functions and tunes during acceleration

The major constraints in the *non-scaling* FFAG lattice design are requests for the tunes within a cell to be within a range between $0.4<\nu_x ,\nu_y<0.1$. This is to allow to avoid half and full integer resonances repeating. At the same time a large momentum variation is required.

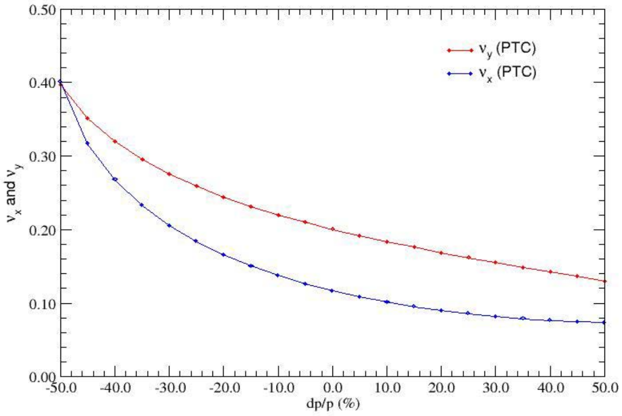

FIG. 5. The horizontal and vertical tune dependence on momentum from 30 MeV to 250 MeV kinetic energy or δp/p=+/- 50 % momentum variations.

The presented range in momentum of δp/p<+/- 50% is not as large as in the case of cyclotrons and *scaling* FFAG's, but the major advantage is the significantly smaller aperture size. The lattice solution presented in this report produced the tune variations with momentum within the required range as it is shown in Fig. 5. The tune variations are sharper and faster at the first part of the acceleration at very low momentum. The amplitude of the Courant-Snyder amplitude function dependence on momentum is shown in Fig. 6. The large variation in tunes at the beginning is reflected also on the amplitude dependence on momentum as shown in Fig. 6.

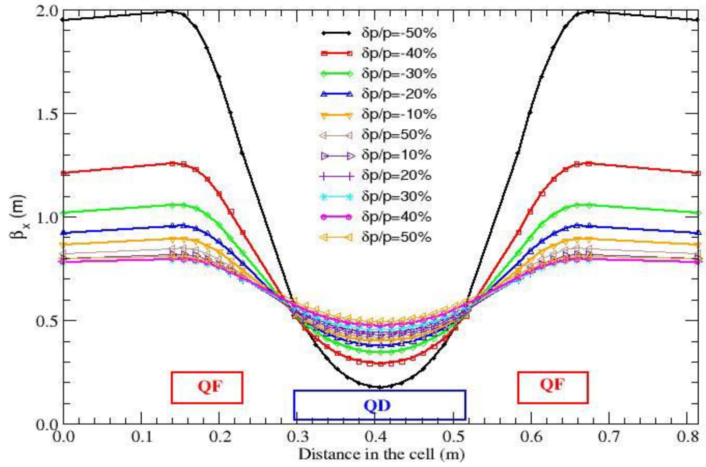

FIG. 6. The horizontal amplitude functions β_x dependence on momentum variations $\delta p/p$=+/- 50 % or kinetic energy variations from 30 MeV to 250 MeV.

6. Acceleration

The acceleration within the *non-scaling* FFAG is occurring very fast similar to the other fixed field accelerators: cyclotrons and *scaling* FFAG. The frequency variation requires a special attention. The harmonic number is selected as h=8. The path lengths at injection and at the top energy 'measured' by the PTC Etienne Forest code [8] are C_{min}=28.559 m and C_{max}=28.827m, respectively. The time of flight at the injection and the top energy are calculated to be τ=0.38 and τ=0.157 μs. The required frequencies, for the h=8, for the injection and the top energy are f_{inj}=21.06 MHz and f_{top}=51 MHz, respectively. For the energy gain per turn of ΔE=~400 kV the total number of turns is ~550 turns, making the total time for acceleration ~100 μs. The circumference at the central momentum is 28.5 meters for both *non-scaling* FFAG rings presented and the time of flight for protons at the central momentum is τ=0.2 μs while for fully stripped carbon ions is τ=0.378 μs at the central kinetic energy of 30.9 MeV/nucleon. Acceleration of the fully stripped carbon ions $^{+6}C^{12}$ in the super-conducting *non-scaling* FFAG shows the time of flights: at the injection energy of $\tau_{injection}$~0.262 μs, at the central kinetic energy [E_k=249 Mev/nucleon] τ_{co}~0.155 μs and at the top energy τ_{top}~0.127 μs. The RF parameters for the carbon are very similar to the selection for the proton acceleration, as the times of flights do not show significant differences.

7. Conclusions

A feasibility study for a very small *non-scaling* FFAG proton and carbon therapy accelerator is presented. Two *non-scaling* FFAG accelerators are described. One is for proton acceleration, with a kinetic energy range from 30 MeV to 250 MeV, or of the fully stripped carbon ions of the kinetic energy range from 7.83 to 68.27 MeV/n. The magnets for this proton/carbon machine have modest magnetic fields and linear gradients and could be built by using the permanent magnets reducing the cost significantly. The second *non-scaling* FFAG is designed for carbon acceleration with a kinetic energy range from 68.27 MeV/n to 501.8 GeV/n. It is the same size but it is made of super-conducting magnets.

The maximum energy of the machines could be adjusted by reducing the number of turns. The betatron tunes within the basic cell are kept between the half integer and zero, but acceleration requires ‘fast resonance crossing’. It is not clear yet if the used word “resonance” is appropriate due to the fast changes of the tunes and energies at the same time. More appropriate would be to consider this as a possibility for the emittance blow up. This is similar to the historical problems in linear accelerators or previous SLAC electron accelerator in California, USA where electrons and positrons, accelerated to the high energies, were bent and collided after passing the two arcs. This problem is still being studied.

The other future considerations are the magnetic field end effects and detail magnetic field calculations of small size magnets [L~20 and 10 cm]. In addition, the magnet length is almost of the same size as the maximum orbit offsets during acceleration [~9 cm]. A small energy spread of the beam $\delta E/E<0.1\%$ is expected providing an additional advantage with respect to the CYCLOTRONS. The high repetition rate of the spot scanning presents an additional advantage of the *non-scaling* FFAG. Recent trend in the proton and heavy ion patient treatment shows preferences towards the heavier elements than protons due their more localized energy loss and better performance less damage of the patient tissues outside the tumor region.

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