Sodium Coolant Handbook:
Thermal Hydraulic Correlations

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SODIUM COOLANT HANDBOOK: THERMAL HYDRAULIC CORRELATIONS

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The Agency's Statute was approved on 23 October 1956 by the Conference on the Statute of the IAEA held at United Nations Headquarters, New York; it entered into force on 29 July 1957. The Headquarters of the Agency are situated in Vienna. Its principal objective is "to accelerate and enlarge the contribution of atomic energy to peace, health and prosperity throughout the world''.

SODIUM COOLANT HANDBOOK: THERMAL HYDRAULIC CORRELATIONS

IAEA PROJECT ON SODIUM PROPERTIES AND SAFE OPERATION
OF EXPERIMENTAL FACILITIES IN SUPPORT OF THE DEVELOPMENT
AND DEPLOYMENT OF SODIUM COOLED FAST REACTORS (NAPRO)

INTERNATIONAL ATOMIC ENERGY AGENCY
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For further information on this publication, please contact:

Nuclear Power Technology Development Section
International Atomic Energy Agency
Vienna International Centre
PO Box 100
1400 Vienna, Austria
Email: Official.Mail@iaea.org

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FOREWORD

The IAEA supports Member States in the area of advanced fast reactor technology development by providing a central point for information exchange and collaborative research programmes. The IAEA's activities in this field are mainly carried out within the framework of the Technical Working Group on Fast Reactors (TWG-FR). The TWG-FR assists in defining and carrying out the IAEA's activities in the field of fast reactor research and technology development, promotes the exchange of information on national and multinational programmes and fosters new developments and experience. The main goals of the TWG-FR are to identify and review problems of importance and to stimulate and facilitate cooperation, development and practical application of fast reactor and subcritical hybrid system technology. To implement the activities recommended by the TWG-FR, the IAEA proposes and establishes coordinated research projects aimed at improving Member State capabilities in fast reactor design and analysis. The projects are important instruments for organizing international research work to achieve specific research objectives consistent with the IAEA's programmatic goals.

With the ongoing interest in innovative sodium cooled fast reactors and the large number of activities carried out for their development, and in order to fulfil its role in promoting efficient collaboration between organizations involved in sodium fast cooled reactor programmes at the national and international levels, in 2013 the IAEA established the CRP entitled Sodium Properties and Safe Operation of Experimental Facilities in Support of the Development and Deployment of Sodium Cooled Fast Reactors (NAPRO). The overall objective of the project was to support research programmes on sodium cooled fast reactors in IAEA Member States by providing a consistent set of data on sodium physical and chemical properties and thermal hydraulic correlations, defining design rules and best practices for sodium experimental facilities and providing guidelines for the safe handling of sodium. The work was divided into three areas: collection and assessment of consistent sodium thermophysical property data, heat transfer and pressure drop correlations; guidelines and best practices for sodium facility design and operation; and safety of sodium experimental facilities. The results of the NAPRO project are presented in two IAEA publications.

The present publication is the technical report of the NAPRO project focusing on the collection of heat transfer and pressure drop (friction factor) correlations for sodium cooled systems. The work was carried out through the collection and review of the correlations available in the open literature, the identification of data gaps and the development of recommendations for experimental programmes to support further research for closing these data gaps. The handbook collects all correlations that can be used as a common basis for the development, design, modelling, simulation and analysis of the advanced sodium cooled fast reactors and sodium experimental facilities.

Eleven organizations representing ten IAEA Member States participated in the NAPRO project and contributed to the drafting of this handbook. The IAEA expresses its appreciation to all project participants for their dedicated efforts leading to this publication, and in particular to S. Perez-Martin and E. Bubelis (Germany) for coordinating the project and compiling the draft publication. The final technical review of this manuscript was performed by V. Slobodchuk (Russian Federation). The IAEA officers responsible for this publication were V. Kriventsev, M. Khoroshev, J. Mahanes and S. Monti of the Division of Nuclear Power.

CONTENTS

1 INTRODUCTION

1.1 BACKGROUND

The International Atomic Energy Agency (IAEA) established a Coordinated Research Project (CRP) in 2013 on “Sodium properties and safe operation of experimental facilities in support of the development and deployment of sodium cooled fast reactors” (NAPRO) in order to promote an efficient collaboration between organizations involved in sodium cooled fast reactor (SFR) programmes at the national and international levels. This research programme was carried out in the time period of 2013 to 2018. Eleven organizations representing ten IAEA Member States participated in the NAPRO CRP while eight organizations contributed to the drafting of this handbook.

The overall objective of the CRP was to support the Member States’ SFR research programmes by providing a consistent set of sodium property data, to specify property uncertainties and to recommend correlations to be used as a common basis for the design, development, modelling and simulation of advanced SFRs.

This work required an extensive understanding and evaluation of the existing available data, the identification of data gaps and the development of recommendations for experimental programmes needed to close these data gaps.

The NAPRO CRP participating organizations are listed in Table 1 and the three main work packages (WP) in Table 2. Each work package was broken into many sub-WPs and Tasks, which are summarized in Section 1.3. This handbook documents the final product of WP1, which was focused on the collection, assessment and dissemination of consistent sodium datasets. In particular, the handbook includes the CRP’s review of sodium heat transfer and pressure drop correlations.

TABLE 1. IAEA NAPRO CRP PARTICIPATING ORGANIZATIONS AND COUNTRIES

	Organization Name	**Country**
1	National Atomic Energy Commission (CNEA)	Argentina
2	China Institute for Atomic Energy (CIAE)	China
3	Commissariat à l'énergie Atomique et aux Énergies Alternatives (CEA)	France
4	Helmholtz-Zentrum Dresden-Rossendorf (HZDR)	Germany
5	Karlsruhe Institute of Technology (KIT)	Germany
6	Indira Gandhi Centre for Atomic Research (IGCAR)	India
7	Japan Atomic Energy Agency (JAEA)	Japan
8	Korea Atomic Energy Research Institute (KAERI)	Republic of Korea
9	Nuclear Research and Consultancy Group (NRG)	Netherlands
10	Institute for Physics and Power Engineering (IPPE)	Russian Federation
11	Argonne National Laboratory (ANL)	United States of America

TABLE 2. IAEA NAPRO CRP WORK PACKAGES

Work Package Number	Description	Output
WP1	Collection, expert assessment and dissemination of WP1.1 Consistent sodium thermo-physical property data; WP1.2 Heat transfer and pressure drop correlations; and WP1.3 Chemical properties and compatibility	Two handbooks – the IAEA document on sodium physical and chemical properties and this publication on thermal-hydraulics correlations which could represent the common reference for all the Member States with an active sodium technology programme
WP2	Development of guidelines and best practices for sodium facility design and operation, including fill and drain, purification, out-gassing prior to filling, sodium storage, component handling, drying of sodium piping after repair, etc.	Nuclear Energy Series on guidelines and best practices for sodium facility design and operation
WP3	Development of guidelines and best practices for sodium facility safety, including prevention and mitigation of sodium leaks, prevention and detection of sodium fires, assessment of sodium impact in the environment after accidental release, hydrogen hazards in cleaning facilities, etc.	Nuclear Energy Series on guidelines and best practices for sodium facility safety

The NAPRO CRP was a collaborative enterprise. The eleven official worldwide participants brought diverse experiences and interests in the area of sodium cooled fast reactor research and development. In particular, all participants had sodium experiment capabilities or have plans to bring new experiments online. Thus, all participants were motivated to combine their collective knowledge into a comprehensive database on sodium properties for the benefit of the entire international community.

During this CRP a large literature review has been performed. The analysis portion of this handbook includes 389 publications (References [1]-[389]). The complete publication list is presented in Chapter 0.

During the four years of the NAPRO CRP, four Research Coordination Meetings (RCMs) were organized and attended by participating organizations. The objectives of these meetings were as follows:

- 1st NAPRO CRP RCM – Vienna, November 11-13, 2013: this was the kick-off meeting for the whole project in which general goals and tasks were defined and single task leaders identified.
- 2nd NAPRO CRP RCM – Vienna, May 26-28, 2014: at the second meeting elements of the methodology were refined and finalized. In addition, the property list and tasks assignment were updated and templates for the analysis distributed.

— 3rd NAPRO CRP RCM – Cadarache, October 5-9, 2015: the third meeting focused on discussing preliminary analysis results, as well as issues encountered and identified areas for improvement.
— 4th NAPRO CRP RCM – Vienna, June 12-14, 2017: the final RCM meeting focused on a review of the analysis results and on detailed discussions of the draft handbook, action items to complete the handbook, and strategies for their organization.

In between RCMs, participants communicated via email and the online collaborative workspace, Sharepoint. These efforts included collection of the reference list, analysis of the various properties, drafting of property summaries and compilation of this handbook.

Over the years, several publications have documented the structure of the CRP, its methodology as well as progress and preliminary results related to Work Package 1. In particular, the specific CRP research objectives, participating organizations and overall organization in Work Packages and specific Tasks are summarized in [1]. In addition, in [2] the implemented methodology for WP1 is described.

1.2 OBJECTIVE

The key objective of the development of two Sodium Coolant Handbooks on i) ”Physical and Chemical Properties” and ii) “Thermal-Hydraulic Correlations” was to improve the thermodynamic consistency of property equations and collect most of available correlations on friction factors and heat transfer for the sodium facilities and reactors. The exchange of data and information among international partners was demonstrated to be effective and it is anticipated that spirit of collaboration will continue beyond the conclusion of this CRP.

Despite the fact that sodium properties and related thermal-hydraulics correlations are commonly considered as ‘established’, inconsistencies and gaps have been identified by the IAEA NAPRO CRP working group. The collected data and the identified gaps and inconsistencies are particularly relevant for both computation and experimental applications. To support further development of best-estimate and high-fidelity/physics-based simulation codes an accurate, complete, consistent, precise and reliable sodium data set is essential. Reactor safety experiments are expected to require the same or better level of uncertainty quantification by regulatory bodies as for current light-water reactor technologies; therefore, gaps in sodium data on physical properties, as well as pressure drop and heat transfer correlations will need to be resolved. In addition, for condition such as natural circulation which is an important component of advanced reactor safety, thermodynamic properties and correlations uncertainties have significant impact on safety analysis.

1.3 SCOPE

This handbook on sodium coolant thermal-hydraulic correlations summarizes the results of the work performed by WP1.2 of the NAPRO CRP. The WP1.2 under the leadership of the Karlsruhe Institute of Technology (KIT) was focused on the collection of heat transfer and pressure drop (friction factor) correlations for sodium cooled systems. A big effort has been made to review the open literature and to collect all information publicly available about such thermal-hydraulic correlations including handbooks and journal papers from the fifties to recent books and proceeding contributions. By doing this extensive revision few inconsistencies were found as well as typos in publications referring to previous works. For each inconsistency found a footnote or a clarification text is included in the corresponding correlation section.

A brief revision of heat transfer and pressure drop equations, as well as the non-dimensional numbers commonly used is presented in Chapter 2.

Chapter 3 consists of six sections, describing the heat transfer correlations for various flow conditions and geometries found in sodium cooled systems.

Chapter 0 consists of three sections, describing friction factor/pressure drop correlations for both single-phase and two-phase flow conditions for sodium cooled systems.

As the list of correlations collected is very large, it is out of scope of this handbook to perform any assessments of correlations for the different conditions and geometries of sodium systems. Chapter 5, nevertheless, provides recommendations for choosing one or another correlation for an analyst working on a specific case, as well as echoes publications assessing correlations against experimental data. It also includes the identified phenomena which should be further investigated experimentally found in the course of this work.

Chapter 6 presents the conclusions of this document and finally, Chapter 0 provides the list of references [1]-[389] used for all correlations collected in this handbook.

1.4 STRUCTURE

The NAPRO CRP WP1 was divided into three work packages and corresponding tasks as detailed in Table 2.

Participation in WP1 was very collaborative with all participants providing data, references, analysis, and reviews. Overall coordination of NAPRO WP1 was led by Argonne National Laboratory (ANL; USA) along with WP1.1. WP1.2 was coordinated by Karlsruhe Institute of Technology (KIT; Germany). WP1.3 was coordinated by Indira Gandhi Centre for Atomic Research (IGCAR; India) and Institute for Physics and Power Engineering (IPPE, Russian Federation). The approach and findings documented in this handbook were written by all participants and coordinated by each work package leader.

Given the considerable amount of data and information collected during the NAPRO CRP, it was also decided to split the handbook into two volumes:

- Sodium Coolant Handbook: Physical and Chemical Properties; and
- Sodium Coolant Handbook: Thermal-Hydraulic Correlations.

1.5 USERS

The handbook represents a useful tool for the governmental, public and private sector organizations responsible for the development and/or deployment of innovative fast neutron systems, including designers, manufacturers, vendors, research institutions, academia and other organizations directly involved in technology development programmes on fast neutron systems and, more generally, on advanced nuclear energy systems.

The handbook also represents an important tool for the education and training of young engineers and scientists in the field of liquid metal coolants technology.

2 CONVECTIVE HEAT TRANSFER, PRESSURE LOSS AND NON DIMENSIONAL NUMBERS

The correlations presented in this report in most cases use non-dimensional numbers characterizing the fluid conditions, as well as other fluid variables and geometrical parameters of the cooling system. It is therefore necessary to introduce the definitions and the nomenclature used in the current work.

Following the definition of the non-dimensional numbers, the basic equations for calculating heat transfer between the wall and the fluid, as well as the pressure drop due to friction with the wall surface are presented as well.

The values of the Reynolds and Peclet numbers at which the transition between laminar and turbulent regime occurs, namely the critical values, are used in some heat transfer correlations. Therefore, the two final sections present the estimated Re and Pe critical values as found in the open literature.

2.1 NON DIMENSIONAL NUMBERS

The definition of the non-dimensional numbers used in the various heat transfer and pressure drop correlations are presented hereafter.

Reynolds number

$$Re = \frac{uD_h}{\nu} = \frac{\rho u D_h}{\mu} \tag{1}$$

where ρ is the density, u is the velocity, D_h is the equivalent hydraulic diameter, and μ and ν are the dynamic and kinematic viscosity, respectively. In general, the hydraulic diameter is estimated as proportional to the ratio of the total flow area to the wetted perimeter of the conduit:

$$D_h = \frac{4\ Flow\ Area}{Wetted\ Perimeter}$$

In case of the circular pipe D_h is equal to the internal pipe of the pipe.

Turbulent flow is characterized by high Reynolds number ($Re \geq 2300$ for pipe flow). In liquid metal fast reactor (LMFR) cores, typical Reynolds numbers are of the order of 50000, that is fully turbulent flow regime.

Prandtl number

$$Pr = \frac{\nu}{a} = \frac{c_p \mu}{\lambda} \tag{2}$$

where c_p is the heat capacity, μ the dynamic viscosity, and λ and a the thermal conductivity and diffusivity, respectively.

For liquid metals the Prandtl number is very small (generally, in the range of $0.01 - 0.001$) meaning that conductive heat transfer dominates over the momentum transfer. The low Prandtl number is due to the high thermal conductivity of metals. The Prandtl number for sodium at a typical mid-core temperature of $500°C$ is 0.0042 [3]. The molecular Prandtl numbers of NaK, mercury and lead bismuth eutectic (LBE) are similar – between 0.01 and 0.03 [4]. Figure 1 and Fig. 2 show the Prandtl number for the saturated liquid sodium and superheated sodium vapour at atmospheric pressure as a function of temperature respectively as recommended in a companion NAPRO CRP publication "Sodium Coolant Handbook: Physical and Chemical Properties" [5].

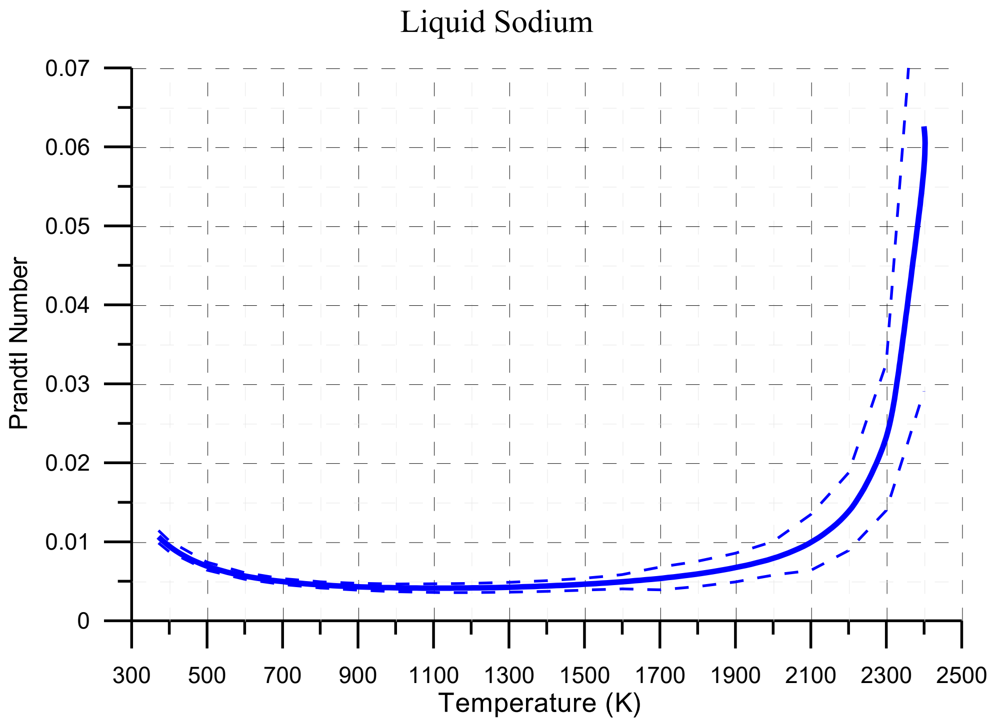


FIG. 1. Prandtl number for the saturated liquid sodium as a function of temperature and uncertainties [5].

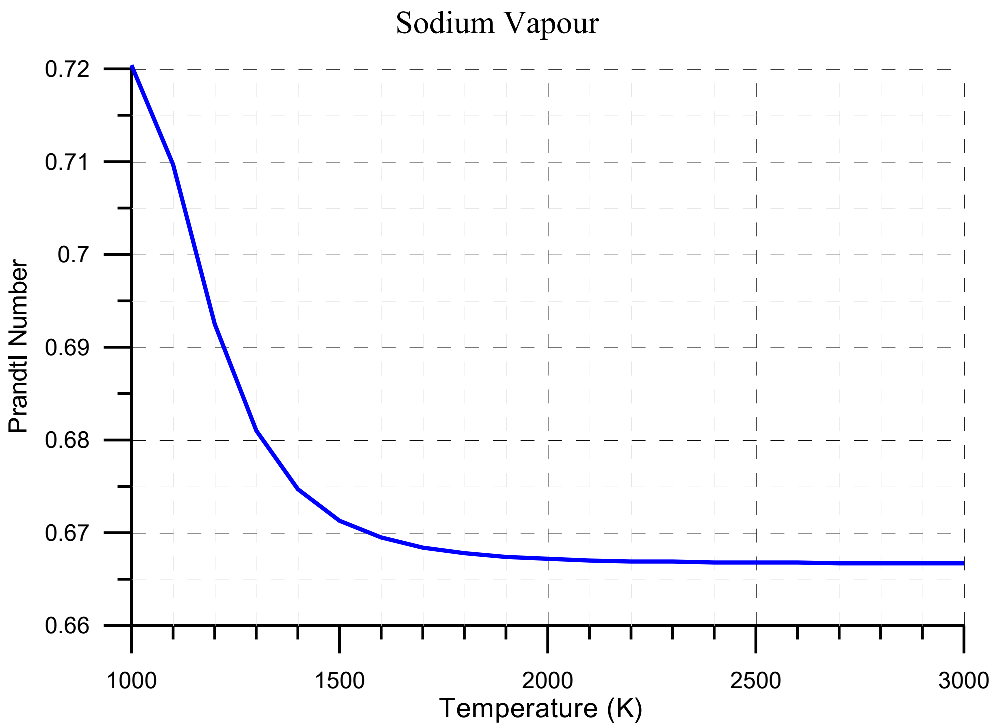


FIG. 2. Prandtl number for superheated sodium vapour at atmospheric pressure as a function of temperature [5].

For turbulent heat transfer modelling, a turbulent Prandtl number is defined as ratio of the momentum to heat transfer turbulent eddy diffusivity:

$$Pr_t = \frac{\varepsilon_M}{\varepsilon_H} = \frac{n_t}{a_t} \tag{3}$$

where ε_H is the eddy diffusivity for the heat transfer, ε_M is the eddy diffusivity for the momentum, n_t is the turbulent eddy viscosity and a_t is the turbulent thermal diffusivity.

Peclet number

$$Pe = RePr \; = \frac{uD_h}{a} = \frac{\rho c_p u D_h}{\lambda} \tag{4}$$

The turbulent Peclet number used in some correlations is defined as:

$$Pe_t = Pr \frac{\varepsilon_M}{\nu} \tag{5}$$

where ε_M is the eddy diffusivity for the momentum and n is the eddy viscosity.

Nusselt number

$$Nu = \frac{h\, l}{\lambda} \tag{6}$$

where h is the heat transfer coefficient, l is the characteristic length and λ the thermal conductivity.

Nusselt number plays an important role in the heat transfer calculations under forced convection conditions, therefore it is used in correlations assessing operation of the reactors cooled by liquid metals.

Grashof number

$$Gr = \frac{bgl^3 DT}{\nu^2} \tag{7}$$

where β is the volumetric thermal expansion coefficient, g is the gravitational acceleration, l is the characteristic length, ΔT is the temperature difference between the wall surface and bulk fluid, and ν is the kinematic viscosity.

Similarly to the Nusselt number, Grashof number plays an important role in the heat transfer calculations under natural convection conditions, therefore it is used in correlations assessing the intrinsic behaviour of the decay heat removal systems of the liquid metal cooled reactors.

Rayleigh number

$$Ra = Gr\; Pr \tag{8}$$

Following the analogy to forced convection, Rayleigh number is equivalent to the Peclet number but for natural convection conditions.

Boussinesq number

$$Bo = Gr\,Pr^2 \qquad (9)$$

Boussinesq number reflects the type of correlation found experimentally for natural circulation in liquid metal flow where the heat transfer is found to be dependent on Grashof number and Prandtl number to the power 2, contrary to ordinary fluids where the Nusselt number for natural convection is a function of the Rayleigh number $Ra = GrPr$.

Dean number

The friction factor correlation for helical coil and curved tubes is found to depend on the Dean number that is defined as

$$De = Re\sqrt{d/D} \qquad (10)$$

where D is the diameter of the coil and d is the diameter of the pipe.

TABLE 3. NON-DIMENSIONAL NUMBERS

Symbol	**Non-dimensional number**	**Definition**
Re	Reynolds number	$Re = \frac{uD_h}{\nu} = \frac{\rho u D_h}{\mu}$
Pr	Prandtl number	$Pr = \frac{\nu}{a} = \frac{c_p \mu}{\lambda}$
Pr_t	Turbulent Prandtl number	$Pr_t = \frac{\varepsilon_M}{\varepsilon_H} = \frac{\mathrm{n}_t}{\mathrm{a}_t}$
Pe	Peclet number	$Pe = RePr = \frac{uD_h}{a} = \frac{\rho c_p u D_h}{\lambda}$
Pe_t	Turbulent Peclet number	$Pe_t = Pr\frac{\varepsilon_M}{\nu}$
Nu	Nusselt number	$Nu = \frac{h\,l}{\lambda}$
Gr	Grashof number	$Gr = \frac{bgl^3DT}{\nu^2}$
Ra	Rayleigh number	$Ra = Gr\,Pr$
Bo	Boussinesq number	$Bo = Gr\,Pr^2$
De	Dean number	$De = Re\sqrt{d/D}$

TABLE 4. PARAMETERS USED IN NON-DIMENSIONAL NUMBERS

Symbol	***Parameter***	***Definition/Units***
ρ	density	kg/m^3
u	velocity	m/s
D_h	hydraulic diameter	$\frac{4\ Flow\ Area}{Wetted\ Perimeter}, m$
μ	dynamic viscosity	$Pa \times s = \frac{N \times s}{m^2} = \frac{kg}{m \times s}$
ν	kinematic viscosity	$\mu/\rho,\ \ m^2/s$
ν_t	turbulent viscosity	m^2/s
λ	thermal conductivity	$\frac{W}{m\ K}$
α	thermal diffusivity	m^2/s
α_t	turbulent thermal diffusivity	m^2/s
T_w	temperature at heated wall	K
T_f	bulk fluid temperature	K
q	heat flux	$\frac{W}{m^2}$
h	heat transfer coefficient	$\left\lvert \frac{q}{T_w - T_f} \right\rvert, \frac{W}{m^2 K}$
b	fluid expansion coefficient	$1/K$
g	gravity constant	m/s^2
l	representative length	m
ε_H	Eddy diffusivity of heat transfer	m^2/s
ε_M	Eddy diffusivity for the momentum	m^2/s
f	Darcy friction factor	$-$
P	Rod bundle pitch	m
D	Pipe diameter	m

2.2 CONVECTIVE HEAT TRANSFER COEFFICIENT

Heat transfer between a solid surface and a fluid requires a temperature difference between the wall and the coolant [6]. The rate of heat transfer Q per unit area A is given by:

$$q = \frac{Q}{A} = h(T_w - T_c) \tag{11}$$

where h is the heat transfer coefficient, T_w is the wall temperature and T_c is the bulk coolant temperature. The convective heat transfer properties of fluids are related to their Prandtl number Pr (Eq. (2)).

In order to calculate the temperature drop $(T_w - T_c)$ or to determine the heat flux per unit area q, to be expected with a given value of $(T_w - T_c)$, it is required to determine the heat transfer coefficient h. Thus, there are analytical equations and non-dimensional heat transfer coefficient (Nusselt number) based on experimental results that yield h as a function of flow conditions in most common cooling system geometries [6].

2.3 PRESSURE LOSS AND FRICTION FACTOR

As for the flow characteristics, liquid metals and non-metals fluids behave similarly, therefore conventional formulas and techniques of calculation can be used for liquid metal piping systems. The frictional pressure drop across a straight length of conduit can then be calculated as:

$$\Delta p = f \frac{L}{D} \frac{\rho u^2}{2} \tag{12}$$

where f is the Darcy-Weisbach friction factor, which is a function of the Reynolds number and geometrical parameters, normally non-dimensional ones (e.g. relative roughness $\frac{\varepsilon}{D}$, pitch-to-diameter ratio $\frac{P}{D}$); L is the length of conduit; D is the inside pipe diameter or hydraulic diameter of the duct; ρ is the fluid density and u is the bulk fluid velocity $u = {Q}/{A}$, where Q is the total volumetric flow rate, and A is the duct area.

In this document, the Darcy-Weisbach or Moody friction factor f is used. It should not be confused with the Fanning friction factor which is one-fourth of the above mentioned friction factors ($f_{Darcy} = f_{Moody} = 4f_{Fanning}$).

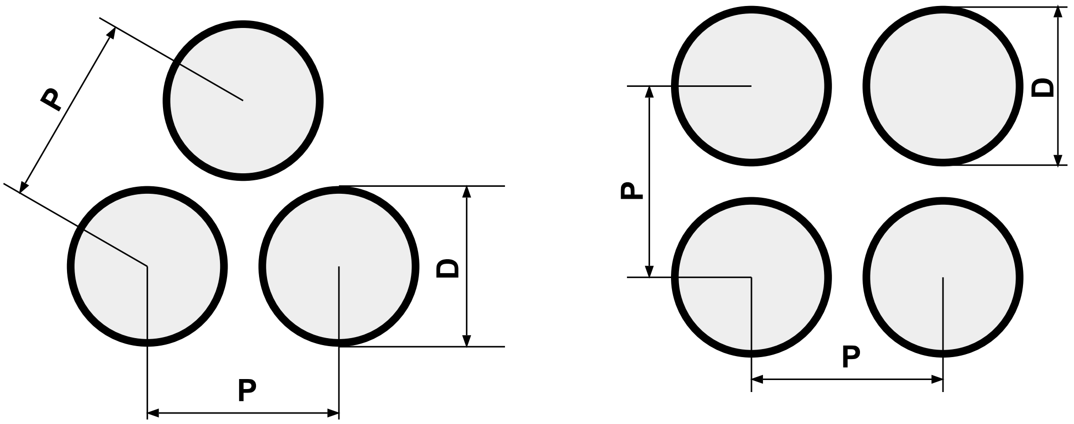
2.4 CRITICAL REYNOLDS NUMBER

The critical Reynolds number is the estimated value for the transition from laminar to turbulent flow. It depends on the flow geometry (e.g. straight or curved pipes, rod bundles, etc.). For flow in a pipe, experimental observations show that for the fully developed flow, the critical Reynolds number is about 2300. According to Cheng and Todreas [7] transition from laminar to turbulent flow occurs over a wide range of Reynolds numbers in rod bundles (Fig. 3) The authors presented simple correlations for the onset and completion of transition flow over wire-wrapped rod bundles in triangular arrays as:

$$Re_{crit} = 300 \cdot 10^{1.7\left(\frac{P}{D}-1\right)} \text{ for onset of transition} \tag{13}$$

$$Re_{crit} = 1000 \cdot 10^{0.7\left(\frac{P}{D}-1\right)} \text{ for completion of transition} \tag{14}$$

where P is the pitch between adjacent rod centerlines and D is the outer diameter of the rod. As for other geometries, the available correlations might include the critical Re values separating laminar and turbulent regimes.



Hydraulic diameter: $D_h = \frac{4\,Flow\ Area}{Wetted\ Perimeter} = \begin{cases} D\left[\frac{4}{\pi}\left(\frac{P}{D}\right)^2 - 1\right] & \text{for square array} \\ D\left[\frac{2\sqrt{3}}{\pi}\left(\frac{P}{D}\right)^2 - 1\right] & \text{for triangular array} \end{cases}$

FIG. 3. Triangular and square rod array allocations

2.5 CRITICAL PECLET NUMBER

Estimated critical Peclet numbers for different geometries and Prandtl numbers can be found in Table 5 [8].

TABLE 5. CRITICAL PECLET NUMBERS

Pr	Pe_{crit}				
	Tubes	Annuli	Rod bundle $\frac{P}{D} = 1.375$	Rod bundle $\frac{P}{D} = 1.700$	Rod bundle $\frac{P}{D} = 2.200$
0.005	117	270	460	622	770
0.01	131	300	530	720	890
0.02	144	330	582	800	1000
0.03	150	345	603	840	1056

3 HEAT TRANSFER CORRELATIONS

The first nuclear reactor producing electricity was the sodium-potassium cooled fast reactor EBR-I in 1951. Experimenters and physicists have worked together since 1930s on specific correlations for heat transfer in liquid metal. Liquid sodium has particular properties, which make it a good coolant candidate for fast reactors.

Sodium thermal behaviour is quite different from water thermal behaviour, hence the need for specific correlations to describe sodium heat transfer in thermal-hydraulic codes. Sodium allows more efficient heat removal thanks to its high heat capacity, thermal conductivity and higher boiling temperature. While the Prandtl number is quite high for water ($Pr \geq 1$), it is very low for liquid metals ($Pr << 1$) which means that thermal diffusivity dominates momentum diffusivity.

At boiling inception, large changes in heat removal properties can be expected. Sodium boiling is quite different from high-pressure water boiling. First of all, the saturation temperature is far higher than for water. Then, sodium vapour density is very small: its high liquid to vapour density ratio leads to a very specific two-phase flow behaviour.

The heat transfer coefficient between the outer surface of the pipe or plate and the bulk of the fluid can be calculated by using empirical correlations which correlate Nusselt number with both the Reynolds and the Prandtl numbers.

The collection of correlations is focused on turbulent flow regime (not laminar, except for the natural circulation) and fully developed flow (not thermal and/or velocity developing regions). This is due to the fact that sodium in SFR systems flows in turbulent regime ($Re \sim 50000$).

As liquid metals behave as ordinary fluids in laminar heat transfer, the reader is referred to other publications (e.g. Handbook of Single-Phase Convective Heat Transfer [9]) for additional laminar flow correlations.

To obtain high accuracy, different geometries have been studied (circular tubes, flat plates, concentric annuli, horizontal or vertical rod bundles, etc.) under different conditions (natural convection, forced convection, with or without impurities, boiling, etc.) that could be encountered also in sodium cooled fast reactors.

The effects of magnetic fields in sodium flow, or bends and fittings are not considered in this document. The reader is then referred to other publications, e.g. [9] Ch. 9 "*Convective Heat Transfer with Electric and Magnetic Fields*" and Ch. 10 "*Convective heat transfer in bends and fittings*".

Chapter 3 consists of six sections, describing turbulent Prandtl number correlations and heat transfer correlations for: forced convection, natural convection and two-phase sodium flow. Then one section presents the modelling of heat transfer in fuel rods and the last section is devoted to the heat transfer correlations used in the system codes for the safety analysis of sodium cooled reactor systems.

3.1 TURBULENT PRANDTL NUMBER

The ratio of the eddy diffusivity of heat transfer (ε_H) to that for momentum (ε_M) affects heat transfer in liquid metals [4] [10] [11] [12]. The turbulent Prandtl number is defined as:

$$Pr_t = \frac{\varepsilon_M}{\varepsilon_H} = \frac{1}{y} = \frac{turbulent\ eddy\ viscosity\ n_t}{turbulent\ thermal\ diffusivity\ a_t} = \frac{n_t}{\frac{l_t}{rc_p}} \quad (15)$$

y symbol utilized in some former publications refers to the diffusivity ratio although nowadays Pr_t is more frequent.

The Turbulent Prandtl number can be estimated using empirical correlations, solutions from analytical equations, or combinations of both. The simplest way is purely empirical correlations, where Pr_t is mainly fitted to various experimental data [13]. The analytic solutions give important information on the near-wall behaviour of the heat flux; however, they are not suited for numerical simulation tools.

A selection of empirical correlations are presented hereafter, but for other methods and assessment of correlations, more information can be found in ([13] Ch. 10) [14] [15] [16].

It should be noted that a particular turbulent Prandtl number expression was often derived together with a particular turbulent heat transfer correlation or set of correlations. Developing a heat transfer correlation based upon data was carried out using them together. For consistency it is then recommended to use the turbulent Prandtl number and the heat transfer correlation from the same source.

3.1.1 Dwyer (1963)

In 1963, O.E. Dwyer derived theoretically an expression for the turbulent Prandtl number [10]:

$$\frac{1}{Pr_t} = y = 1 - \frac{\left(\frac{a}{Pr}\right) - c}{\left(\frac{\varepsilon_M}{n}\right)^m} \tag{16}$$

where a, c, and m are constants, which were tentatively set at 0.2, 2 and 0.9, respectively, for liquid metal flow in pipes, annuli, parallel plates and staggered rod bundles [17]. These constants were evaluated from the experimental data available at that time and Dwyer proposed the following semi-empirical equation:

$$\frac{1}{Pr_t} = y = 1 - \frac{1.82}{Pr\left(\frac{\varepsilon_M}{n}\right)_{max}^{1.4}} \tag{17}$$

where $\left(\frac{\varepsilon_M}{n}\right)_{max}$ is the maximum value of $\frac{\varepsilon_M}{n}$ in the channel cross-section. It is a unique function of the Reynolds number Re and the channel geometry. Since y cannot fall below zero, when the previous equation yields negative value, it should be taken as zero.

FIG. 4 presents the values of $\left(\frac{\varepsilon_M}{n}\right)_{max}$ for fully developed turbulent flow in circular tubes, annuli and rod bundles with equilateral triangular spacing as a function of the Reynolds number.

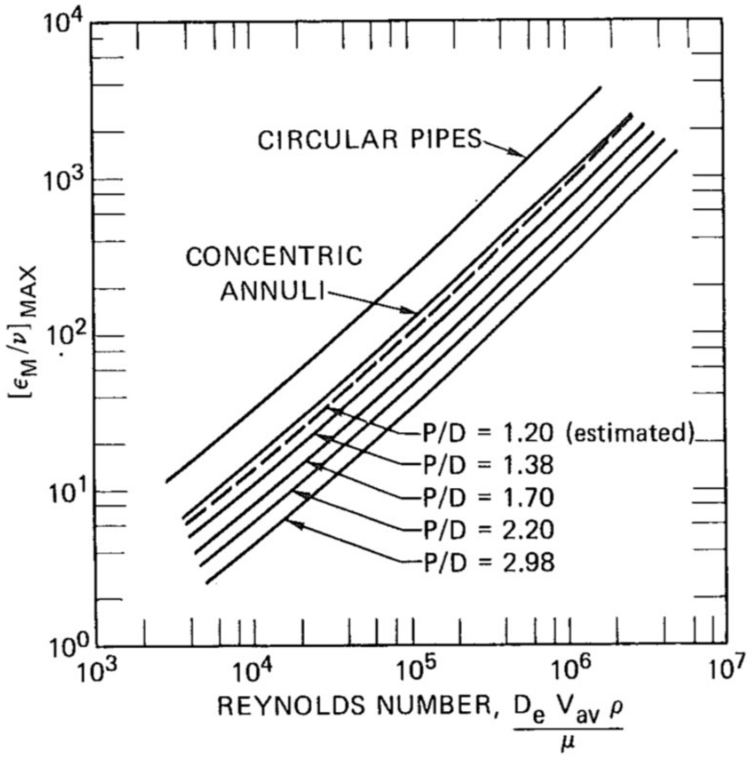


FIG. 4. Values of $\left(\varepsilon_M/\nu\right)_{max}$ as a function of Reynolds number (data from [11])

$\left(\varepsilon_M/\nu\right)_{max}$ can be also calculated for circular pipes from the relation ([9] Ch. 4):

$$\left(\frac{\varepsilon_M}{\nu}\right)_{max} = 0.037\, Re \sqrt{\frac{f}{4}} \tag{18}$$

where f is the Darcy friction factor (see Section 4.1.1 for friction factor correlations for straight circular pipes).

3.1.2 Aoki (1963)

In 1963 S. Aoki tried to modify Deissler's equation assuming that the heat transmission from the eddy travelling along two layers could be approximated by convective heat transfer from a spherical eddy. He represented the average ratio of eddy diffusivities by the following expression specific for liquid metal flows [10] [13] [18]:

$$\frac{1}{Pr_t} = 0.014\, Re^{0.45} Pr^{0.2} \left(1 - e^{-\frac{1}{0.014 Re^{0.45} Pr^{0.2}}}\right) \tag{19}$$

3.1.3 Kokorev (1963)

In 1963 L.S. Kokorev proposed the following turbulent Prandtl number correlation [19]:

$$Pr_t = \frac{1 + 0.8 c_1 (1 - f)}{1 + c_2 \frac{n}{\varepsilon_M}} \tag{20}$$

where f is the friction factor, n is the kinematic viscosity, $c_1 = 1$, $c_2 = 0.5$, and ε_M is proposed to be evaluated using Reichardt correlation [20] (Eq. (27)).

3.1.4 Subbotin et al. (1963)

In 1963 V.I Subbotin et al. proposed the following relation of non-similarity factors for turbulent transfer of heat and momentum [21]:

$$\varepsilon = \frac{(\varepsilon_H)_{max}}{(\varepsilon_M)_{max}} \tag{21}$$

Although a full similarity between turbulent transfer of heat and momentum on all cross-sections of a flow is not presented, a comparison of the maximum values of ${}^{\varepsilon_H}/_{q}$ and ${}^{\varepsilon_M}/_{n}$ obtained by the following formulas shows that the relationship between factors of turbulent transfer of heat and momentum depends on Re number:

$$\left(\frac{\varepsilon_H}{q}\right)_{max} = 7.5 \cdot 10^{-5} Re \tag{22}$$

According to Prandtl:

$$\left(\frac{\varepsilon_M}{n}\right)_{max} = 1 \cdot 10^{-2} Re^{0.875} \tag{23}$$

According to Reichardt [20]:

$$\left(\frac{\varepsilon_M}{n}\right)_{max} = 0.75 \cdot 10^{-2} Re^{0.875} \tag{24}$$

For heat transfer in liquid metals $(Pr << 1)$ a non-dimensional area parameter x is defined as $x = 0.5 - 0.9$. Therefore, weighted mean value of $\bar{\varepsilon}$ depends on Re number calculated for the given area:

$$\bar{\varepsilon} = \frac{\int_{0.5}^{0.9} \varepsilon x \, dx}{\int_{0.5}^{0.9} x \, dx} \tag{25}$$

As $\bar{\varepsilon}$ also depends on the law of the velocity distribution, using Prandtl formula yields:

$$\bar{\varepsilon} = 0.205 Re^{\frac{1}{8}}, \tag{26}$$

that is similar to Reichardt formula

$$\bar{\varepsilon} = 0.254 Re^{\frac{1}{8}} \tag{27}$$

The comparison is shown in Fig. 5.

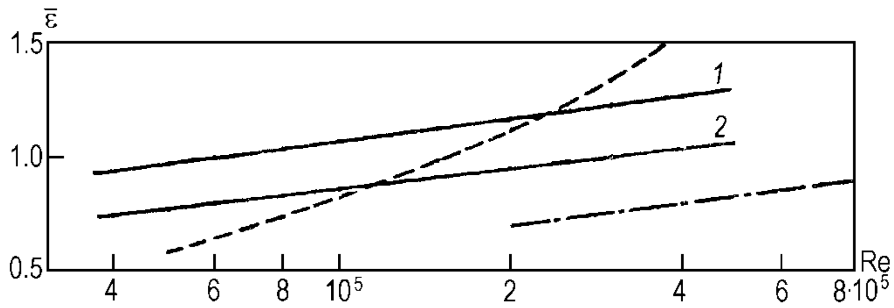


FIG. 5. An average value of $\bar{\varepsilon}$ at. radius ($x = 0.5 - 0.9$) versus Reynolds number:

***solid line**: data from* [21] *(1 and 2 for a velocity distribution by Reichardt and Prandtl formulas accordingly)*

***dashed line**: data from* [22]

***dash-dotted line**: data from* [23]

Though there are the discrepancies in the experimental data, a general trend is that ε value depends linearly on the Re number. For the high values of the Re number, the ε value becomes higher than one. It indicates that the turbulent heat transfer exceeds the turbulent momentum transfer.

3.1.5 Kunz-Yerazunis (1969)

In 1969 H.R. Kunz and S. Yerazunis derived a relation for the turbulent Prandtl number [10] [24]:

$$\frac{1}{Pr_t} = 1.5\, e^{-\frac{0.90}{\left[\frac{\varepsilon_M}{\alpha}\right]^{0.64}}} \tag{28}$$

Where $\alpha = \frac{l}{C_p\rho}$ is the thermal diffusivity.

3.1.6 Notter-Kunz-Yerazunis (1969)

R. Notter (1969) and H.R. Kunz and S. Yerazunis (1969) adopted the following equation for liquid metal flows in circular ducts, accepting deviations of $\pm 0.5\%$ from the experiments considered [13] [24] [25]:

$$Pr_t = \frac{2}{3} e^{\left[0.9\left(\frac{n_t}{n}\right)^{0.64}\right]} \tag{29}$$

where n is the kinematic viscosity and n_t is the turbulent kinematic viscosity.

3.1.7 Bobkov et al. (1970)

In 1970 V.P. Bobkov et al. obtained the following formula for the turbulent Prandtl number on the basis of the uniformity model and statistical characteristics of velocity and temperature pulsations [26] [27]:

$$Pr_t = 0.69\left(\frac{u_0}{u}\right)^{\frac{1}{2}}\left[1 - e^{-0.62\cdot 10^{-4} Re Pr^{\frac{1}{3}}}\right] \tag{30}$$

where u_0 is the mean of velocity on the channel symmetry line (maximum speed lines) and u is the mean flow velocity.

3.1.8 Quarmby-Quirk (1972, 1974)

A. Quarmby and R. Quirk (1972, 1974) concluded that, in view of the scatter of the experimental measurements, it was impossible to isolate the dependence of Pr_t on Pr and Re [13] [28] [29]. Nevertheless, they found an important radial dependence and formulated it in the following equation for their particular experiment:

$$Pr_t = \left(1 + 400^{-\frac{y}{R}}\right)^{-1} \tag{31}$$

where y is the distance to the wall and R is the pipe radius. This yields $Pr_t = 0.5$ at the wall (which is considerably too low for liquid metal flows) and $Pr_t \approx 1$ at the centreline (which is acceptable for $Re \geq 5 \cdot 10^4$).

3.1.9 Reynolds (1974)

A.J. Reynolds reviewed and assessed more than 30 studies in the open literature on turbulent Prandtl number and divided them into several groups according to their approaches, i.e. from highly analytical derivation to purely empirical methods [14]. He pointed out that empirical models show clearly higher accuracy, although the highly analytical models lead to better phenomenological understanding. Therefore, for practical application he recommended empirical models [4]. The correlation he proposed in 1974 for liquid metal flows is the following:

$$Pr_t = (1 + 100Pe^{-0.5})\left[\frac{1}{1 + 120\,Re^{-0.5}} - 0.15\right] \tag{32}$$

F. Gori et al. (1979) carried out a numerical prediction of heat transfer to low Prandtl number fluids in circular tubes where they employed a one-equation turbulence model in the near wall region and a two-equation turbulence model in the core region [4] [30]. As different expressions were selected for the turbulent Prandtl number, the calculated temperature profiles were compared with experimental data concluding that the models of Aoki (Section 3.1.2) and Reynolds (Section 3.1.9) are the best ones for $Re \leq 170000$.

3.1.10 Hubbard (1975)

In 1975 F.R. Hubbard used the graphical function proposed by O.E. Dwyer and fitted it to the expression [31] [32]:

$$\frac{1}{Pr_t} = 1 - \frac{0.942\left({}^{P}/_{D}\right)^{1.4}}{Pr\left({}^{Re}/_{1000}\right)^{1.281}} \tag{33}$$

where ${}^{P}/_{D}$ is the pitch-to-diameter ratio.

3.1.11 Jischa-Rieke (1979)

M. Jischa and H.B. Rieke (1979) concluded that the dependence of Pr_t on the radial distance from the wall and Re is of second-order importance and that the following simple expression provides a reasonable estimation for all Pr values [33] [34].

$$Pr_t = 0.85 + \frac{0.015}{Pr} \tag{34}$$

3.1.12 Yakhot et al. (1986)

Based on a renormalization group analysis, V. Yakhot et al. proposed the following equation [13] [35] [36]:

$$\left[\frac{Pr_{eff}^{-1} - 1.1793}{Pr^{-1} - 1.1793}\right]^{0.65} \left[\frac{Pr_{eff}^{-1} - 2.1793}{Pr^{-1} - 2.1793}\right]^{0.35} = \frac{1}{1 + \frac{n_t}{n}}$$

$$Pr_{eff} = \frac{1 + \frac{n_t}{n}}{\frac{\frac{n_t}{n}}{Pr_t} + \frac{1}{Pr}} \tag{35}$$

where n is the kinematic viscosity and n_t is the turbulent kinematic viscosity.

3.1.13 Kays-Crawford (1993)

W.M. Kays and M.E. Crawford (1993) developed a prediction model for Pr_t, later extended by B. Weigand et al. (1997), based on the thermal molecular conduction from an eddy moving through the mixing length [13] [37] [38]:

$$Pr_t = \left\{\frac{1}{2Pr_{t\infty}} + cPe_t\sqrt{\frac{1}{Pr_{t\infty}}} - (cPe_t)^2\left[1 - e^{-\frac{1}{cPe_t\sqrt{Pr_{t\infty}}}}\right]\right\}^{-1}$$

$$Pe_t = Pr_t\left(\frac{\nu_t}{\nu}\right) \tag{36}$$

It is valid for all molecular Prandtl numbers. It contains two empirical constants (c and Pr_t) which should be determined from the available experimental data. $Pr_{t\infty}$ is the value of Pr_t far away from the wall and $C = 0.3$ is a constant prescribing the spatial distribution of Pr_t vs. Pe_t.

The drawback of this correlation is that one should know apriori the global Reynolds number, which is not known for complex geometries.

3.1.14 Kays (1994)

An asymptotic curve for the Yakhot's expression (Eq. 35) was given by W. Kays (1994) [16]. It looks similar to the one previously suggested by Reynolds (1975) [14] but in Reynolds case Pr_t is an average throughout the whole boundary layer [13]. The recommended turbulent Prandtl number is:

$$Pr_t = 0.85 + \frac{0.7}{Pe_t} \quad (37)$$

This correlation is often used in CFD calculations.

3.1.15 Lin et al. (2000)

In 2000 B.S. Lin et al. modified the renormalization group analysis assuming a quasi-normal approximation for the statistical correlation between the velocity and temperature fields [39]. This leads to derivation of the Pr_t as a function of the Pe_t, which in turn depends on the turbulent eddy diffusivity ν_t. The functional relationship is comparable to that of Yakhot [36] presented in 3.1.12 but it also contains the spectral properties of both oscillating fields [13]:

$$\left[\frac{Pr_{eff}^{-1} - 1}{Pr^{-1} - 1}\right]^{\frac{2}{3}} \left[\frac{Pr_{eff}^{-1} + 2}{Pr^{-1} + 2}\right]^{\frac{1}{3}} = \frac{1}{1 + \frac{n_t}{n}}$$

where (38)

$$Pr_{eff} = \frac{1 + \frac{n_t}{n}}{\frac{\frac{n_t}{n}}{Pr_t} + \frac{1}{Pr}}$$

The similarity of both equations (35) and (38) is due to the fact that they have similar mathematical structures and are derived entirely from approaches other than the renormalization group analysis of turbulence.

3.1.16 Cheng-Tak (2006)

In 2006 X. Cheng and N.I. Tak [4] proposed the following correlation based on the fact that the Lyon model for heat transfer (Section 3.2.1.2) agrees well with the computational fluid dynamics (CFD) analysis for all the values of turbulent Prandtl number. This recommended correlation was derived in combination with Lyon correlation:

$$Pr_t = \begin{cases} 4.12 & Pe \leq 1000 \\ \dfrac{0.01\,Pe}{[0.018Pe^{0.8} - (7.0 - A)]^{1.25}} & 1000 < Pe \text{ £ } 6000 \end{cases}$$

where (39)

$$A = \begin{cases} 4.5 & Pe \leq 1000 \\ 5.4 - 0.0009Pe & 1000 \leq Pe \leq 2000 \\ 3.6 & Pe \geq 2000 \end{cases}$$

Authors [4] noted that this equation may not be valid in other LBE flow conditions that are not considered in their analysis (e.g., different geometries, developing flows, etc.), since the equation is derived specifically based on experimental data and calculations with the CFX code for fully developed LBE turbulent flows in tube geometries with constant heat flux.

3.1.17 Taler (2016)

In 2006 D. Taler studied the heat transfer in turbulent tube flow of liquid metals using three turbulent Prandtl models [40]. The first one was a modified Aoki's formula for the turbulent Prandtl number adjusting the parameters by the least squares method to the experimental data of N. Sheriff and D.J. O'Kane 1981 [41] obtained for liquid sodium. He proposed two similar relationships:

$$Pr_t = \left[a\, Re^{0.45} Pr^{0.2} \left\{1 - e^{-\frac{1}{aRe^{0.45}Pr^{0.2}}}\right\}\right]^{-1} \tag{40}$$

$$Pr_t = \left[b\, Re^{0.45} Pr^{0.2} \left\{1 - e^{-\frac{1}{cRe^{0.45}Pr^{0.2}}}\right\}\right]^{-1} \tag{41}$$

$$\begin{aligned} a &= 0.01592 \\ b &= 0.01171 \\ c &= 0.00712 \end{aligned} \tag{42}$$

The second model used by D. Taler is based on the Kays, Crawford (2005) [42], and Weigand (1997) [38] model:

$$Pr_t = \left\{\frac{1}{2Pr_{t\infty}} + dPe_t\sqrt{\frac{1}{Pr_{t\infty}}} - (dPe_t)^2\left[1 - e^{-\frac{1}{dPe_t\sqrt{Pr_{t\infty}}}}\right]\right\}^{-1} \tag{43}$$

where the turbulent Peclet number Pe_t is given by:

$$Pe_t = Pr\frac{\varepsilon_M}{\nu} \tag{44}$$

and $Pr_{t\infty}$ represents turbulent Prandtl number given by Jischa and Rieke [33] [43]:

$$Pr_{t\infty} = Pr_{ts} + \frac{e}{PrRe^{0.888}} \tag{45}$$

The other constants of the model are:

$$d = 0.3, Pr_{ts} = 0.85,\; e = 182.4 \tag{46}$$

Pr_{ts} represents a turbulent Prandtl number for high Reynolds numbers when the second term in Eq. 45 becomes negligible. Instead of $Pr_{ts} = 0.9$ as proposed in the original expression [33] [43], $Pr_{ts} = 0.85$ was adopted according to the recent studies [16] [38] [42].

The third model is a simple relationship:

$$Pr_t = 0.85 + \frac{f}{Pe_t} = 0.85 + \frac{f}{\frac{\varepsilon_M}{n}Pr} \tag{47}$$

where Taler assumed $f = 1.46$ instead of $f = 2.0$, as suggested by [16], as it provided better calculated Nusselt numbers compared to the experimental data.

3.1.18 Summary of correlations for the turbulent Prandtl number

The whole list of correlations collected for the turbulent Prandtl number is included in Table 6.

TABLE 6. SUMMARY OF CORRELATIONS FOR THE TURBULENT PRANDTL NUMBER

Dwyer (1963) [10][17]	$\frac{1}{Pr_t} = y = 1 - \frac{1.82}{Pr\ \left(\frac{\varepsilon_M}{n}\right)^{1.4}_{max}}$
Aoki (1963) [10] [13] [18]	$\frac{1}{Pr_t} = 0.014\ Re^{0.45} Pr^{0.2} \left(1 - \ e^{-\frac{1}{0.014 Re^{0.45} Pr^{0.2}}}\right)$
Kokorev (1963) [19]	$Pr_t = \frac{1+0.8c_1(1-f)}{1+c_2\frac{n}{\varepsilon_M}}$ with $c_1 = 1$, $c_2 = 0.5$
Subbotin et al. (1963) [21]	$\varepsilon = \frac{(\varepsilon_H)_{max}}{(\varepsilon_M)_{max}}$ $\left(\frac{\varepsilon_H}{q}\right)_{max} = 7.5 \cdot 10^{-5} Re$ $\bar{\varepsilon} = 0.205 Re^{\frac{1}{8}}$
Kunz-Yerazunis (1969) [10] [24]	$\frac{1}{Pr_t} = 1.5\ e^{-\frac{0.90}{\left[\frac{\varepsilon_M}{\alpha}\right]^{0.64}}}$
Notter-Kunz-Yerazunis (1969) [13] [24] [25]	$Pr_t = \frac{2}{3} e^{\left[0.9\left(\frac{n_t}{n}\right)^{0.64}\right]}$
Bobkov et al. (1970) [26] [27]	$Pr_t = 0.69 \left(\frac{u_0}{u}\right)^{\frac{1}{2}} \left[1 - e^{-0.62 \cdot 10^{-4} Re Pr^{\frac{1}{3}}}\right]$
Quarmby-Quirk (1972, 1974) [13] [28] [29]	$Pr_t = \left(1 + 400^{-\frac{y}{R}}\right)^{-1}$
Reynolds (1974) [4]	$Pr_t = (1 + 100 Pe^{-0.5}) \left[\frac{1}{1 + 120\ Re^{-0.5}} - 0.15\right]$
Hubbard (1975) [31] [32]	$\frac{1}{Pr_t} = 1 - \frac{0.942\left(\frac{P}{D}\right)^{1.4}}{Pr\left(\frac{Re}{1000}\right)^{1.281}}$
Jischa-Rieke (1979) [33] [34]	$Pr_t = 0.85 + \frac{0.015}{Pr}$
Yakhot et al. (1986) [13] [35] [36]	$\left[\frac{Pr_{eff}^{-1} - 1.1793}{Pr^{-1} - 1.1793}\right]^{0.65} \left[\frac{Pr_{eff}^{-1} - 2.1793}{Pr^{-1} - 2.1793}\right]^{0.35} = \frac{1}{1 + \frac{n_t}{n}}$ $Pr_{eff} = \frac{1 + {n_t}/{n}}{\frac{{n_t}/{n}}{Pr_t} + \frac{1}{Pr}}$

TABLE 6. SUMMARY OF CORRELATIONS FOR THE TURBULENT PRANDTL NUMBER

Kays-Crawford (1993) [13] [37] [38]	$Pr_t = \left\{ \frac{1}{2Pr_{t\infty}} + cPe_t \sqrt{\frac{1}{Pr_{t\infty}}} - (cPe_t)^2 \left[1 - e^{-\frac{1}{cPe_t\sqrt{Pr_{t\infty}}}} \right] \right\}^{-1}$ $Pe_t = Pr \frac{n_t}{n}$
Kays (1994) [13] [16]	$Pr_t = 0.85 + \frac{0.7}{Pe_t}$
Lin et al. (2000) [13] [39]	$\left[\frac{Pr_{eff}^{-1} - 1}{Pr^{-1} - 1} \right]^{\frac{2}{3}} \left[\frac{Pr_{eff}^{-1} + 2}{Pr^{-1} + 2} \right]^{\frac{1}{3}} = \frac{1}{1 + \frac{n_t}{n}}$ $Pr_{eff} = \frac{1 + {n_t}/{n}}{\frac{{n_t}/{n}}{Pr_t} + \frac{1}{Pr}}$
Cheng-Tak (2006) [4]	$Pr_t = \begin{cases} 4.12 & Pe \leq 1000 \\ \frac{0.01\,Pe}{[0.018Pe^{0.8} - (7.0 - A)]^{1.25}} & 1000 < Pe \text{ £ } 6000 \end{cases}$ $\mathrm{A} = \begin{cases} 4.5 & Pe \leq 1000 \\ 5.4 - 0.0009Pe & 1000 \leq Pe \leq 2000 \\ 3.6 & Pe \geq 2000 \end{cases}$

TABLE 6. SUMMARY OF CORRELATIONS FOR THE TURBULENT PRANDTL NUMBER

Taler (2016) [40]	1st correlation: $Pr_t = \left[a\, Re^{0.45} Pr^{0.2} \left\{1 - e^{-\frac{1}{aRe^{0.45}Pr^{0.2}}}\right\}\right]^{-1}$, or $Pr_t = \left[b\, Re^{0.45} Pr^{0.2} \left\{1 - e^{-\frac{1}{cRe^{0.45}Pr^{0.2}}}\right\}\right]^{-1}$ $a = 0.01592$ $b = 0.01171$ $c = 0.00712$ 2nd correlation: $Pr_t = \left\{\frac{1}{2Pr_{t\infty}} + dPe_t\sqrt{\frac{1}{Pr_{t\infty}}} - (dPe_t)^2\left[1 - e^{-\frac{1}{dPe_t\sqrt{Pr_{t\infty}}}}\right]\right\}^{-1}$ $Pr_{t\infty} = Pr_{ts} + \frac{e}{PrRe^{0.888}}$ $Pe_t = Pr\frac{\varepsilon_M}{n}$ $d = 0.3, Pr_{ts} = 0.85, e = 182.4$ 3rd correlation: $Pr_t = 0.85 + \frac{f}{Pe_t},\ f = 1.46$

3.2 FORCED CONVECTION

3.2.1 Flow in circular pipes

Most of the correlations available in the open literature for heat transfer in circular tubes have the form:

$$Nu = a + bRe^{c}Pr^{d} \tag{48}$$

where a, b, c and d are constants. A constant term of a represents the contribution of molecular conduction corresponding to laminar conditions with $Pe \rightarrow 0$, while the second term $bRe^{c}Pr^{d}$ represents the eddy conduction contribution [11].

The geometry of the heat transfer in circular pipes is shown in Fig. 6, where D –pipe diameter, T_w – wall temperature, T_f – bulk fluid temperature, q – heat flux.

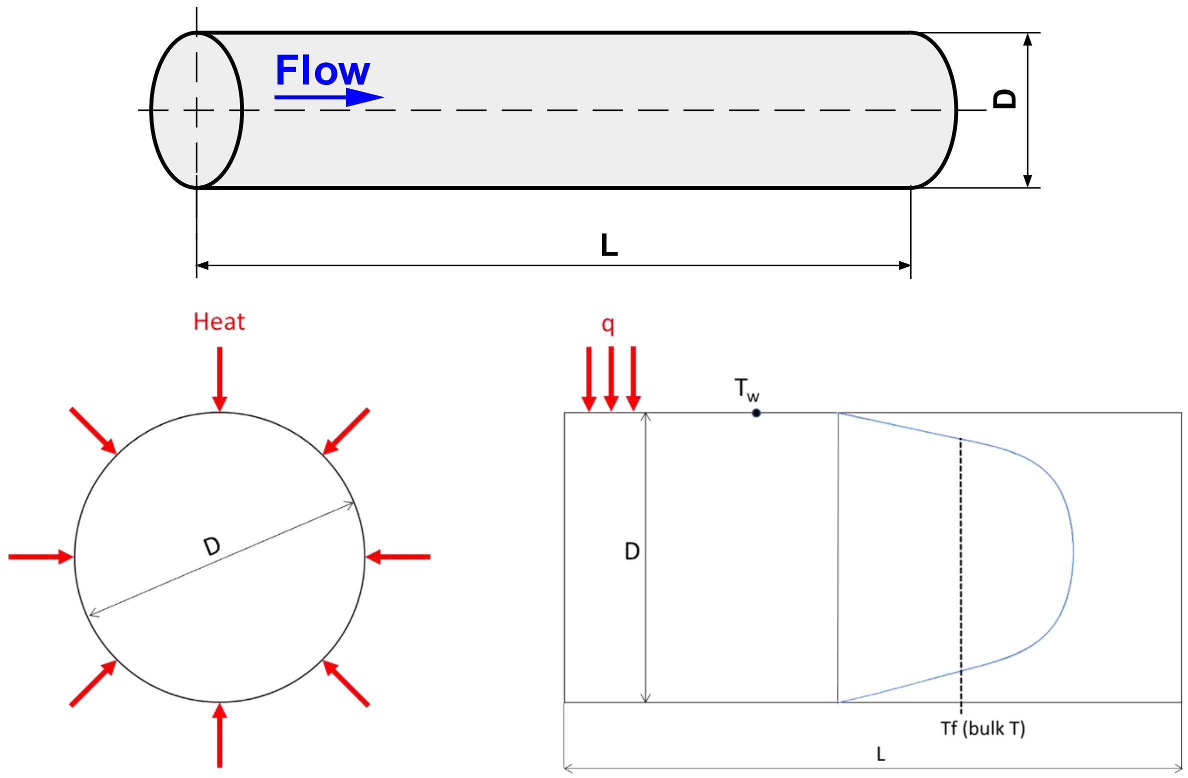


FIG. 6. Flow and heat transfer in circular pipes

3.2.1.1 Dittus-Boelter (1930)

In 1930, F.W. Dittus and L.M.K. Boelter published a correlation [44] to predict convective heat transfer in single-phase vapour conditions for pipes with length-to-diameter ratio $\frac{L}{D} \geq 60$. It takes the form [45]:

$$Nu = 0.023Re^{0.8}Pr^{n},\ n = 0.4 \text{ (heating)},\ n = 0.33 \text{ (cooling)} \tag{49}$$

It is valid for turbulent flows ($Re \geq 10^5$), thus satisfying $0.6 \leq Pr \leq 160$. The expected accuracy of this correlation is $\pm 15\%$. According to the applicability range, this correlation is not suitable for liquid sodium, however some computational tools use this correlation to estimate sodium vapour heat transfer since sodium vapour is characterized by Prandtl number values greater than 0.6 [46].

3.2.1.2 Lyon (1949)

In 1949, R.N. Lyon investigated Nusselt number for low Prandtl number fluids for circular tube geometry in forced convection situation [9] [47] [48] [49] [50]. A general equation was derived by considering the following hypothesis: steady state, system independent of angular displacement, uniform heat flux, total conductivity not a function of distance parallel to tube axis, no end effects, heat transfer only by molecular conduction, eddy conduction and forced convection, molecular conductivity at right angles to the flow of fluid unaffected by eddying of the fluid, by the velocity of the fluid or by the gradient of this velocity, and constant physical properties of the fluid for turbulent flow. He also assumed that the ratio between the eddy diffusivities of heat and momentum is constant, what he called $y = \frac{\varepsilon_H}{\varepsilon_M} = \frac{1}{Pr_t}$, which is by definition the inverse of the turbulent Prandtl number. The resulting expression is entirely rigorous for the ideal system chosen (ordinary differential equation treatment) and applies equally well to viscous or turbulent flow and for all Prandtl numbers.

As he wanted to obtain a simple relationship, he proposed the following approximation for liquid metal heat transfer in tubes considering the experimental data from Nikuradse:

$$Nu = 7 + 0.025\left(\frac{Pe}{Pr_t}\right)^{0.8} \tag{50}$$

This correlation is valid for smooth circular duct with $0 \leq Pr \leq 0.1$ and $10^4 \leq Re \leq 5 \cdot 10^6$. The predictions are within $+32.8\%$ and -6.5% of the Notter-Sleicher correlation, which is considered as the reference correlation in the Handbook of Single Phase Convective Heat Transfer from 1987 [9] (also suggested in the Heat Exchanger Design Handbook edited by Borishanskii in 1983 [51]). Lyon correlation is well-known and often served as a reference for other liquid metal single-phase heat transfer correlation developers.

3.2.1.3 Seban-Shimazaki (1951)

In 1951, R.A. Seban and T.T. Shimazaki proposed a correlation for liquid metal flow in a smooth circular tube with uniform wall temperature boundary condition based on the work done by Lyon in 1949 ([9] Ch. 4) [52]. The same correlation is also recommended in [51] by Borishanskii and reviewed in [45] by Todreas-Kazimi:

$$Nu = 5 + 0.025Pe^{0.8} \tag{51}$$

It is valid for $0 \leq Pr \leq 0.1$, $10^4 \leq Re \leq 5 \cdot 10^6$ and $10^2 \leq Pe \leq 2 \cdot 10^4$.

3.2.1.4 Deissler (1952)

In 1952 R.G. Deissler proposed the following equation for heat transfer with uniform heat flux in tubes [10] [53]:

$$Nu = 6.3 + 0.000222Pe^{1.3} \tag{52}$$

3.2.1.5 Stromquist (1953)

In 1953, W.K. Stromquist studied the effect of wetting on heat transfer characteristics of mercury over a large range of Peclet number [54]. He concluded that there is no inherent property of a non-wetted liquid metal system that causes low heat transfer coefficients as compared to a wetted system. However, he showed that for flow conditions at high Peclet number, measured heat transfer coefficients show a fluctuation behaviour in both non-wetted and wetted systems with a rough heat transfer surface. Based on a separate study using glass tubes, Stromquist found that detachment of mercury from the tube wall was observed at many locations. These zones could appear or disappear according to local conditions. Conditions, at which the detachment was observed, correspond well to that causing heat transfer fluctuation. Stable test conditions were only obtained at Peclet numbers lower than $4 \cdot 10^3$. These stable test data obtained are well correlated by the following equation [4]:

$$Nu = 3.6 + 0.018\, Pe^{0.8},\ 88 \leq Pe \leq 4 \cdot 10^3 \tag{53}$$

It is valid for uniform heat flux conditions.

3.2.1.6 Lubarsky-Kaufman (1955)

The experimental results of various investigators until 1955 of liquid metal heat transfer characteristics were examined by B. Lubarsky and S.J. Kaufman [55] [56] [57]. They also revaluated experimental data using assumptions and methods as consistent as possible and compared with theoretical results. They suggested an empirical relation for fully developed heat transfer in tubes for turbulent flow conditions as:

$$Nu = 0.625\, Pe^{0.4} \tag{54}$$

It is valid for uniform heat flux conditions in a smooth circular duct in the range $0 \leq Pr \leq 0.1$, $10^4 \leq Re \leq 10^5$ and $2 \cdot 10^2 \leq Pe \leq 9 \cdot 10^3$ ([9] Ch. 4) [55].

3.2.1.7 Hartnett-Irvine (1957)

Average Nusselt numbers for flow in channels can be estimated using J.P. Hartnett and T.F. Irvine correlation (1957) as follows [9] [58]:

$$Nu = \frac{2}{3} Nu_{slug} + 0.015\, Pe^{0.8} \tag{55}$$

where Nu_{slug} is the Nusselt number for slug flow and Pe is the Peclet number. This equation is valid when free convection effects are negligible with clean surfaces and no gas entrainment.

For a smooth circular duct, Nu_{slug} assumes the value of 5.8 in case of constant wall temperature, and 8.0 in case of constant heat input per unit length and constant wall temperature around the periphery of the duct at a given axial position. It is valid for a smooth circular duct in the range $0 \leq Pr \leq 0.1$ and $10^4 \leq Re \leq 5 \cdot 10^6$ ([9] Ch. 4).

3.2.1.8 Schleicher-Tribus (1957)

In 1957, C.A. Schleicher and M. Tribus proposed a set of Nusselt equations for heat transfer to liquid metal in a smooth circular duct ([9] Ch. 4) [59]. They are valid for $0 \leq Pr \leq 0.1$ and $10^4 \leq Re \leq 5 \cdot 10^6$:

$$Nu = 4.8 + 0.015 Re^{0.91} Pr^{1.21} \tag{56}$$

for axially and circumferentially uniform wall temperature. Its prediction is within $+19.5\%$ and -33.4% of the Notter-Sleicher correlation.

$$Nu = 6.3 + 0.016 Re^{0.91} Pr^{1.21} \tag{57}$$

valid for axially and circumferentially uniform wall heat flux. Its prediction is within $+26.3\%$ and -32.5% of the Notter-Sleicher correlation.

3.2.1.9 Lykoudis-Touloukian (1958)

In 1958 Lykoudis and Touloukian [10] [60] recommended the following correlation for uniform heat flux in round pipes:

$$Nu = 7.0 + 0.30\ Pe^{0.3} \tag{58}$$

3.2.1.10 Kutateladze et al. (1959)

In 1959 S. Kutateladze et al. proposed the following correlations for uniform heat flux [13] [61]:

$$Nu = 3.3 + 0.014\ Pe^{0.8} \tag{59}$$

$$Nu = 5.0 + 0.0021\ Pe^{0.8} \tag{60}$$

$$Nu = 5.9 + 0.015\ Pe^{0.8} \tag{61}$$

The third one was developed specifically for sodium while the other two are considered as lower limits for the Nusselt number.

3.2.1.11 Buleev (1959)

In 1959 N.I. Buleev estimated the eddy diffusivity ratio y to calculate heat transfer coefficients over the ranges $0.01 \leq Pr \leq 10$ and $5 \cdot 10^3 \leq Re \leq 10^6$. The following correlation and coefficients were obtained for the case of uniform heat flux [11] [62] [63] [64] [65]:

$$Nu = A + 4.16 \left(\frac{Re}{1000}\right)^m Pr^n \tag{62}$$

$$\begin{gathered} A = 2.5 + 1.3\ \log\left[1 + \frac{1}{Pr}\right] \\ m = 0.865 - 0.051\ \log\left[1 + \frac{1}{Pr}\right] \\ n = 0.66 \text{ for } 0.01 \leq Pr \leq 1 \\ n = 0.44 \text{ for } 1 \leq Pr \leq 10 \end{gathered} \tag{63}$$

For larger Reynolds numbers the above correlation agrees with the Lyon Eq. (50) and for small Reynolds numbers is enough smoothly extrapolated to the theoretical value of $Nu = 4.36$.

3.2.1.12 Ibragimov et al. (1960)

In 1960, M.K. Ibragimov et al. published the experimental results of heat transfer for turbulent flow of mercury and LBE (lead bismuth eutectic) in a tube [66]. The experimental results were compared with the previous correlation shown above with the conclusion that this correlation satisfactorily fits the experimental results.

$$Nu = 4.5 + 0.014Pe^{0.8} \tag{64}$$

It is used in the case of constant wall heat flux. The variation of the parameters during the tests were as follows.

For mercury tests $Pr = (22 - 27) \cdot 10^{-3}$, $Re = (40 - 400) \cdot 10^{3}$, $Pe = (1 - 10) \cdot 10^{3}$, $Nu = 7 - 27$.
For LBE tests $Pr = (20 - 27) \cdot 10^{-3}$, $Re = (8 - 290) \cdot 10^{3}$, $Pe = (0.2 - 5.8) \cdot 10^{3}$, $Nu = 6 - 19$.

3.2.1.13 Rohsenow-Cohen (1960)

In 1960, W.M. Rohsenow and L.S. Cohen proposed the following correlation [67] [68] applicable for liquid metal in the Prandtl range of $5 \cdot 10^{-3} \leq Pr \leq 5 \cdot 10^{-2}$

$$Nu = 6.7 + 0.0041(Re \cdot Pr)^{0.793} e^{41.8 \cdot Pr} \tag{65}$$

This correlation is valid for $Re \geq 10^{4}$ and applies to the case of uniform heat flux along the tube.

3.2.1.14 Azer-Chao (1961)

In 1961, N.Z. Azer and B.T. Chao investigated Nusselt number and temperature profiles for low Prandtl number fluids (liquid metals) of constant properties flowing in a smooth pipe with constant wall temperature ([9] Ch. 4) [69]. They proposed the following correlation for the estimation of the Nusselt number:

$$Nu = 5 + 0.05\, Pe^{0.77} Pr^{0.25} \tag{66}$$

It is valid for smooth circular duct with uniform wall temperature in the range $0 \leq Pr \leq 0.1$ and $10^{4} \leq Re \leq 5 \cdot 10^{6}$. This correlation fits the data with a maximum deviation of less than 11% for $Pr \leq 0.1$ and $Pe \leq 1.5 \cdot 10^{4}$ (the usual range of turbulent liquid metal heat transfer encountered in practice).

3.2.1.15 Subbotin et al. (1962)

In 1962, V.I. Subbotin et al. published the experimental results of heat transfer for the transitional and turbulent flows of liquid sodium in four tubes made of different materials: copper, St-10, nickel and 1Kh18N9T steel [70]. Two methods were used to calculate the heat transfer coefficients: processing of the measured temperature distributions in the stream along the radius of the tube (first method) and treatment of the temperature distributions in the wall along the length of the heat exchange section and in the liquid at the mixing chambers (second method) [70]. The variation of the parameters during the tests was as follows: $Pe = 40 - 1150$, $Re = 5850 - 178000$, $Pr = 0.0057 - 0.0075$. The correlation recommended for constant heat flux [11] was:

$$Nu = 5 + 0.025Pe^{0.8} \tag{67}$$

In 1966 O.E. Dwyer recommended Subbotin et al. correlation on the basis of its simplicity and apparent accuracy, for estimating heat transfer coefficients for all liquid metals where the system is clean, the wall heat flux is uniform, the velocity and temperature profiles are fully developed, and the flow is turbulent [65].

In 2001, P.L. Kirillov and P.A. Ushakov, published a journal paper about heat transfer to the liquid metals in round tubes where they recommended the same correlation for the range $Pe \leq 2 \cdot 10^4$ and $3 \cdot 10^3 \leq Re \leq 10^6$ [71].

This correlation has the same expression as the one proposed by Seban-Shimazaki in 1951 (see Section 3.2.1.3). The difference is that Seban-Shimazaki recommended it for uniform wall temperature conditions, while Subbotin et al. recommended it for uniform heat flux.

3.2.1.16 Kirillov (1962)

The most reliable data about the heat removal in liquid metals (heat transfer) can be obtained only by means of measurement of temperature fields in a liquid metal. Other methods, in which the heat release surface temperature is used, are not always exact because of the surface effects originated on a boundary line between a wall and a liquid metal. These effects are not investigated in all details. A number of the works ([72] [73] [74] [75] [76] [77] [78] [79]) about measurement of temperature in turbulent flows of liquid metals (mercury, NaK alloy, Pb-Bi alloy, sodium) is known. On the basis of these data, it is possible to carry out some generalization.

When fluid passes a flat plate, the flow velocity distribution, processed as function $u^+ = f(y^+)$ follows a universal profile that does not depend on Reynolds number, pipe diameter, viscosity, etc. The velocity distribution in a circular pipe close enough coincides with this universal profile. Between distributions of velocity and temperature on flow cross-section there is a similitude according to the hydrodynamic theory of heat exchange [80]. In [81] temperature distribution in coordinates $T^+ = f(y^+)$ is defined, that has given a family of curves with a Prandtl number as parameter. Naturally, there arises a question, how much universal is the temperature profile in coordinates $T^+ = f(y^{++})$.

y^+: non-dimensional coordinate

$$y^+ = y\frac{v^*}{\nu} \tag{68}$$

Where $v^*$is the dynamic velocity:

$$v^* = \sqrt{\frac{\tau_0}{\rho}} = \bar{u}\sqrt{\frac{f}{8}} \tag{69}$$

and ν is the kinematic viscosity, τ_0 is tangential stress on a wall, $\bar{u}$ is average linear velocity of a flow and f is the friction factor.

Experimental data of works [75] [76] [77] [78] [79] on measurement of temperature distribution in turbulent flows of sodium, mercury and lead bismuth eutectic in non-dimensional coordinates $T^+ = f(y^{++})$ provide

$$y^{++} = y\frac{v^*}{a} \tag{70}$$

The so-called «friction temperature», put into practice by H.B. Squir in 1951 [81], is chosen as a temperature scale:

$$T^* = \frac{q_0}{\rho C_p v^*} \tag{71}$$

where q_0 is heat flux on a wall, which matches temperature pulsations in the flow. Thus, experimental data on heat exchange in liquid metals can be analysed, based on analogy between transfer of momentum and heat, using the following non-dimensional variable quantities: $u^+ = \frac{u}{v^*}$, $T^+ = T/T^*$, $y^+ = y\frac{v^*}{\nu}$ and $y^{++} = y\frac{v^*}{a}$. In these coordinates data stratifications depending on a Prandtl number were not observed though its value changed within $0.06 - 0.027$. Universal profile of the temperature distribution, obtained on the basis of experimental data, is presented by following dependences which do not demand explanations:

$$T^+ = y^{++} \quad for\, 0 \le y^{++} \le 1 \tag{72}$$

$$T^+ = 1.87\, ln(y^{++} + 1) + 0.65y^{++} - 0.36 \quad for\, 1 \le y^{++} \le 11.7 \tag{73}$$

$$T^+ = 2.5\, ln\, y^{++} - 1 \quad for\, y^{++} > 11.7 \tag{74}$$

The Eq. (73) is selected so as to provide a correlation between T^+ values and $\frac{\partial T^+}{\partial y^{++}}$ derivatives on boundary lines of $y^{++} = 1$ and $y^{++} = 11.7$. Eq. (74) is a result of experimental data averaging [75] [76] [77] [78] [79].

On the basis of experimental data on temperature measurements in a turbulent flow of liquid metal the dependence $Nu = f(Pe^*)$, where Pe^* is a modified Peclet number defined as

$$Pe^* = \frac{v^* d}{a} = Pe\sqrt{\frac{f}{8}} \tag{75}$$

where v^* is the dynamic velocity or 'friction velocity' chosen as a velocity scale, d is the hydraulic diameter and a is the thermal diffusivity. The order of value v^* coincides with the value of a velocity pulsation of a flow.

Here the value of f is calculated by Filonenko formula $f = (1.82 \log Re - 1.64)^{-2}$ Eq. (324). After processing of experimental data by a least squares method the following formula for calculation of Nusselt number is obtained (at $Pe^* < 1000$) [82]:

$$Nu = 4.36 + 0.343Pe^{0.8} \tag{76}$$

The error in the formula (76) is $\pm 2\%$ and it provides a smooth transition to the laminar flow at $Nu = 4.36$. There is a good agreement between this correlation and the experimental data (sodium [77], NaK alloy [75], Pb-Bi alloy [76], mercury [72] [73] [76]).

3.2.1.17 Baker-Sesonske (1962)

In 1962, R.A. Baker and A. Sesonske proposed the following correlation for flow in pipes with uniform heat flux [10] [83]:

$$Nu = 6.05 + 0.0074Pe^{0.95} \tag{77}$$

3.2.1.18 Subbotin et al. (1963)

In 1963 data from V.I. Subbotin et al. [63] on experimental research of heat transfer using mercury in nickel and stainless-steel pipes, and a sodium-potassium alloy in a copper pipe are approximated by the following formula:

$$Nu \approx 4.3 + 0.025Pe^{0.8} \tag{78}$$

which is recommended for practical calculations over the range of $0.02 \leq Pr \leq 0.03$ and $20 \leq \mathrm{Pe} \leq 10000$ for pure heat transfer surfaces, provided the maintenance of oxides is in liquid metal at lower level than a limit of their solubility.

3.2.1.19 Dwyer (1963)

The correlation presented by O.E. Dwyer in 1963 for smooth circular duct is as follows[1] [8]([9] Ch. 4):

$$Nu = 7 + 0.025\left[Re\,Pr - \frac{1.82\,Re}{\left(\varepsilon_m/\nu\right)_{max}^{1.4}}\right]^{0.8} \tag{79}$$

It is valid for smooth circular duct with uniform heat flux and $0 \leq Pr \leq 0.1$ and $10^4 \leq Re \leq 5 \cdot 10^6$. The predictions are within $+31.4\%$ and -6.5% of the Notter-Sleicher correlation. This expression corresponds to Lyon correlation (Section 3.2.1.2), using the expression for the turbulent Prandtl number as proposed by Dwyer (Section 3.1.1).

3.2.1.20 Subbotin et al. (1963)

The calculation of Lyon integral by V.I. Subbotin et al. [21] using the Reichardt velocity profile and $\frac{\varepsilon_H}{q}$ relationships not dependent on Prandtl number, allowed to obtain the uniform formula for Nusselt number in a wide range of change of Prandtl and Peclet numbers:

$$Nu = Nu_0 + ARe^n Pr^m \tag{80}$$

where

$$Nu_0 = 7.24 - \frac{9.5}{\log Re} \tag{81}$$

$A = 0.0155, n = 0.82, m = 0.58 - 0.18 \cdot \tanh(0.81\ \log Pr)$.

It is valid for $10^4 < Re < 7 \cdot 10^5, 0.005 < Pr < 10$ and uniform heat flux conditions.

---

[1] In [9] Ch. 4 there is an error where ε_M/ν is elevated to the power of 0.14 instead of 1.4

3.2.1.21 Kokorev (1963)

In 1963 L.S. Kokorev carried out experiments measuring heat transfer in circular pipes, thus providing the following correlation [19]:

$$Nu = 5.5 + 0.025Pe^{0.8} \tag{82}$$

It is valid for $20 < Pe < 4000$.

3.2.1.22 Skupinski et al. (1965)

In 1965, E. Skupinski et al. [84] [85] reported the heat transfer behaviour of sodium-potassium alloys flowing in horizontal circular tubes with an error band of $\pm 15\%$ of the experimental data.

$$Nu = 4.82 + 0.0185Pe^{0.827} \text{ for } 58 \leq Pe \leq 1.31 \cdot 10^4 \tag{83}$$

It is valid for smooth circular duct with uniform heat flux in the range $0 \leq Pr \leq 0.1$ and $10^4 \leq Re \leq 5 \cdot 10^6$ ([9] Ch. 4).

This correlation was used in the design of Phenix reactor intermediate heat exchangers. Skupinski correlation is commonly used to model liquid sodium single-phase forced convection heat transfer in tubes and rod bundles.

3.2.1.23 Notter-Sleicher (1972)

In 1972, R.H. Notter and C.A. Sleicher proposed a set of Nusselt equations describing heat transfer to liquid metal in a pipe. These equations are solved numerically for the range $10^4 \leq Re \leq 10^6$ and $0.004 \leq Pr \leq 10^4$ ([9] Ch. 4). Heat transfer rates are predicted for both the entry and fully developed coolant flow regions in a pipe. The numerical predictions are described as follows [86] [87]:

$$Nu = 4.8 + 0.0156\, Pe^{0.85}\, Pr^{0.08} \tag{84}$$

valid for uniform wall temperature condition, and

$$Nu = 6.3 + 0.0167\, Pe^{0.85}\, Pr^{0.08} \tag{85}$$

valid for uniform heat flux wall condition.

Nusselt numbers obtained by these equations are within $\pm 5\%$ uncertainty as related to the experimental data. A third correlation proposed was:

$$Nu = 5 + 0.016\, Re^{a}\, Pr^{b} \tag{86}$$

valid for uniform heat flux or uniform temperature wall condition, where $a = 0.88 - \frac{0.24}{(4+Pr)}$ and $b = 0.33 + 0.5e^{-0.6Pr}$. Nusselt numbers obtained by this equation are within $\pm 10\%$ uncertainty as related to the experimental data.

3.2.1.24 Sleicher et al. (1973)

In 1973, C.A. Sleicher et al. investigated experimentally local heat transfer coefficients and fully developed temperature profiles in NaK eutectic mixture in a pipe at different boundary conditions, i.e. uniform heat flux and uniform wall temperature. Reynolds numbers ranged from

$2.6 \cdot 10^4 \leq Re \leq 3.02 \cdot 10^5$ and flow was fully developed. Consistency of data was affirmed by three independent heat rate measurements. Eddy diffusivity profiles were used to calculate Nusselt numbers in pipes at uniform heat flux. Results for liquid metals were correlated as follows for local and average Nusselt numbers [88]:

$$Nu(x) = Nu_\infty \left(1 + \frac{2}{x/D}\right), \; x/D \geq 4 \tag{87}$$

$$Nu_{ave} = Nu_\infty \left(1 + \frac{8}{L/D} + \frac{2}{L/D} \ln\frac{L/D}{4}\right), \; L/D \geq 4 \tag{88}$$

where x is the axial coordinate, L is the length of the pipe and D is the pipe diameter as shown in Fig. 7.

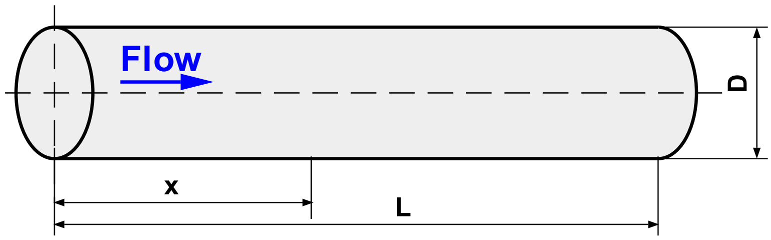


FIG. 7. Pipe geometry for evaluating local Nusselt number

For uniform wall temperature:

$$Nu_\infty = 4.8 + 0.0156\, Pe^{0.85}\, Pr^{0.08}, \; 0.004 \leq Pr \leq 0.1 \tag{89}$$

and for uniform wall heat flux:

$$Nu_\infty = 6.3 + 0.0167\, Pe^{0.85}\, Pr^{0.08}, \; 0.004 \leq Pr \leq 0.1 \tag{90}$$

According to these equations heat transfer coefficients for uniform wall temperature are smaller than for uniform heat flux conditions.

3.2.1.25 Aoki (1973)

In 1973, having analysed the work performed and the correlations proposed by Lyon 1949, Dwyer 1963, Deissler 1952, Lykoudis 1958, Baker and Sesonske 1962, Lubarsky and Kaufmann 1955, S. Aoki proposed a Nusselt correlation for heat transfer in a circular tube for the turbulent flow of liquid metal[2] [10] [89]:

$$Nu = 6.0 + 0.025\left[0.014 Re^{1.45} Pr^{1.2}\left\{1 - e^{-\frac{71.8}{Re^{0.45} Pr^{0.2}}}\right\}\right]^{0.8} \tag{91}$$

S. Aoki showed that Lyon's equation deviated considerably from experimental data. He therefore reduced the value of the molecular conduction term down to 6 and used his turbulent Prandtl number already presented in Section 3.1.2, Eq. (19). It is valid for constant wall heat flux, the same as Lyon's correlation.

[2] In Ref. [10], Aoki's correlation (Eq. 14 in the same reference, p. 573) contains an error in the exponent, as $Re^{0.45}Pr^{0.2}$ should be in the denominator of the exponent, according to Eq. (19).

3.2.1.26 Dwyer (1976)

In 1976 O.E. Dwyer being author of Chapter 2 of Volume 2 of Na-NaK Handbook edited by Foust [11] reviewed the correlations published so far and recommended the following one for the case of uniform wall temperature:

$$Nu = 4.0 + 0.025Pe^{0.8} \tag{92}$$

3.2.1.27 Chen-Chiou (1981)

C.J. Chen and J.S Chiou (1981) correlations for smooth circular duct in the fully developed flow region are as follows [9] [90].

In case of uniform wall temperature:

$$Nu = 4.5 + 0.0156\,Re^{0.85}Pr^{0.86} \tag{93}$$

In case of uniform heat flux:

$$Nu = 5.6 + 0.0165\,Re^{0.85}Pr^{0.86} \tag{94}$$

For $0 \leq Pr \leq 0.1$ and $10^4 \leq Re \leq 5 \cdot 10^6$, the predictions of Eq. (93) are within $+36.1\%$ and -1.8% of the Notter-Sleicher correlation, the predictions of Eq. (94) are within $+33.9\%$ and -7.1% of the Notter-Sleicher correlation.

As the physical properties vary from fluid to fluid at different temperature ranges, it is impossible to describe the variation of fluid flow or heat transfer characteristics due to temperature change by a single relationship and to expect that the relationship will be valid for all fluids under all conditions. Instead, one has to calculate each property under the prescribed conditions. For design convenience, C.J. Chen and J.S Chiou recommended simple formulas that approximate the variation of heat transfer coefficients. The Nusselt number was presented in the form of:

$$\frac{Nu}{Nu_0} = \left(\frac{T_b}{T_i}\right)^n \tag{95}$$

where the subscript zero refers to the values calculated under the assumption of constant physical properties, T_b represents the bulk temperature, T_i is the inlet temperature, while $T_f = \frac{1}{2}(T_b + T_w)$ is the film temperature, T_w is the wall temperature. The values of the parameter n depend on the heat transfer conditions. For liquid sodium under constant heat flux:

$n = \exp(5.9 \cdot 10^{-3}T_b - 6.91)$ $\quad 1000\,K \geq T_b \geq 600\,K$ (heating) (96)

$n = 0$ $\quad 600\,K \geq T_b \geq 370\,K$ (heating) (97)

$n = 0.25$ $\quad$ (cooling) (98)

— While for liquid sodium under constant wall temperature:

$n = 0.08 + 2.2 \cdot 10^{-4}T_b$ $\quad 1000\,K \geq T_b \geq 600\,K$ (heating) (99)

$n = 0.08$ $\quad 600\,K \geq T_b \geq 370\,K$ (heating) (100)

$n = 0.16$ $\quad$ (cooling) (101)

This correlation is valid for smooth circular duct in the range $0 \leq Pr \leq 0.1$ and $10^4 \leq Re \leq 5 \cdot 10^6$ ([9] Ch. 4).

C.J. Chen and J.S Chiou also made an extensive study on thermal entrance effects for liquid metal flows (see section 3.2.7.4).

3.2.1.28 Lee (1983)

In 1983, S.L. Lee investigated liquid metals Nusselt number for pipe geometry in turbulent forced convection situation with uniform wall flux ([9] Ch. 4) [91]. His theoretical analysis is based on the following hypothesis: circular pipe of infinite length, flow is turbulent and fully developed, fluid temperature is uniform at infinite distance, pipe wall is heated with a constant wall flux, flow is steady, Newtonian and incompressible fluids, physical properties are constant, while viscosity dissipation, free convection and tube wall thermal resistance are negligible. In this investigation, a modified form of the Azer-Chao model [92] was adopted. S.L. Lee proposed the following expression:

$$Nu = 3.01 Re^{0.0833} \tag{102}$$

This correlation is valid for $5 \leq Pe \leq 1000$, $0.001 \leq Pr \leq 0.02$, with $Re \geq 4000$. The author did a comparison with Johnson's data (mercury and lead bismuth) [93] [94] and with Notter-Sleicher's predictions being within $+24.7\%$ and -44.3% of the Notter-Sleicher correlation [86].

3.2.1.29 Borishanski (1983)

In the Heat Exchanger Design Handbook [51], V.M. Borishanski (1983) reported that the general correlations reported by R.H. Lyon (1949), as well as R.A. Seban and T.T. Shimazaki (1951) are applicable only when the impurity in the liquid metals are below the oxygen solubility limit at the operating temperature. If the impurities are above the limit, heat transfer coefficient greatly reduces because of increased resistance to heat transfer at the wall – fluid boundary. The minimum value of the Nusselt number when heating of a liquid metal contaminated with impurities may be given by the following equation reported in [95].

$$Nu = 4.3 + 0.0021 Pe \tag{103}$$

It is valid for $10^2 \leq Pe \leq 10^4$.

3.2.1.30 Reed (1987)

In 1987, C. Reed as author of Chapter 8 of the Handbook of Single-Phase Convective Heat Transfer [9] recommended the following correlation for flow in pipes in the case of uniform wall temperature [13]:

$$Nu = 3.3 + 0.02\, Pe^{0.8} \tag{104}$$

This correlation is valid for $Pe \geq 10^2$ and $\frac{L}{D_h} \geq 60$, where L is the pipe length and D_h is the pipe hydraulic diameter. This correlation corresponds to the best fit of data by Sleicher et al. 1973 [88] and Gilliland et al. 1951 [96] as it retains the simple, classical dependence on $Pe^{0.8}$.

3.2.1.31 Buleev et al. (1989)

Results of the calculations of Nusselt number for flows of various liquids in a wide range of Prandtl number, including liquid metal coolants, in a circular pipe under uniform heat flux conditions on a pipe wall in the interval of $3 \cdot 10^3 < Re < 3 \cdot 10^6$ are presented by the following interpolation formula [97]:

$$Nu = A + 3.90 \left(\frac{Re}{1000}\right)^m Pr^n \tag{105}$$

where

$$\begin{aligned} A &= 2.5 + 1.3 \log\left(1 + \frac{1}{Pr}\right) \\ m &= 0.918 - 0.05 \log\left(1 + \frac{10}{Pr}\right) \\ n &= 0.65 - 0.107 \log\left(1 + \frac{10}{Pr}\right) \end{aligned} \tag{106}$$

3.2.1.32 Siman- Tov et al. (1997)

In 1997, M. Siman-Tov et al. proposed a correlation for flow in pipes in the case of uniform heat flux [13] [98]:

$$Nu = 0.685\, Pe^{0.3726} \tag{107}$$

3.2.1.33 Tricoli (1999)

In 1999, V. Tricoli proposed a description of the heat transfer in pipes based on a surface renewal concept [13] [99]. He assumed that, for incompressible high Peclet and low Prandtl numbers flows, the ratio of local temperature gradients at the wall for uniform wall temperature to that of uniform heat flux remains constant:

$$\frac{Nu|_{T_w\, const}}{Nu|_{q'_w\, const}} = \frac{\pi^2}{12} = 0.822 \tag{108}$$

where $Nu|_{T_w\, const}$ refers to the case with uniform wall temperature and $Nu|_{q'_w\, const}$ to the case with uniform heat flux. This means that Nusselt number for uniform wall temperature conditions is about 18% smaller than in the case of uniform heat flux.

3.2.1.34 ENIN (2001)

In 2001 P.L. Kirillov and P.A. Ushakov analysed heat transfer to the liquid metals in round tubes, thus reviewing existing Nusselt correlations. They presented the formula recommended by scientist from the Krzhizhanovskii Institute ENIN for LBE flows [71]:

$$Nu = A + 0.014\, Pe^{0.8} \tag{109}$$

where $A = 3$ when there are oxide films on the wall, and $A = 4.5\text{–}5$ for clean surfaces.

3.2.1.35 TsKTI (2001)

In the same publication as the correlation just mentioned above [71], P.L. Kirillov and P.A. Ushakov also present the following expression derived on the basis of the data obtained at the Polzunov Institute TsKTI:

$$Nu = 5 + 0.0021\,Pe \tag{110}$$

where it was observed that higher values of the heat transfer coefficients were achieved for pure liquid metal conditions.

3.2.1.36 Cheng-Tak (2006)

X. Cheng and N.I. Tak [4] performed a survey of existing correlations for liquid metal heat transfer in pipe flow, and suggested a new correlation evaluated for LBE flows:

$$Nu = A + 0.018Pe^{0.80} \tag{111}$$

$$\mathrm{A} = \begin{cases} 4.5 & Pe \leq 1000 \\ 5.4 - 0.0009Pe & 1000 \leq Pe \leq 2000 \\ 3.6 & Pe \geq 2000 \end{cases}$$

3.2.1.37 Mochizuki (2010)

As the Seban-Shimazaki (1951) correlation can be used only when the Peclet number is larger than ~30, H. Mochizuki in 2010 [100] proposed a new set of correlations that are applicable also for the low Peclet number conditions. The final form of the proposed empirical correlations is as follows:

$$Nu = min(Nu_1, Nu_2), \tag{112}$$

$$Nu_1 = 5 + 0.025Pe^{0.8} \text{ (Seban-Shimazaki, 1951)}, \tag{113}$$

$$Nu_2 = 5 \cdot 10^{-3}Pe^{1.74}, \tag{114}$$

which is valid for $Pe < 55$. The above Nusselt number correlations were validated by the evaluation of the test data coming from the intermediate heat exchangers of the 50 MW steam generator facility of the experimental fast reactor Joyo and the prototype fast breeder reactor Monju.

3.2.1.38 Summary of heat transfer correlations for flow in circular pipes

Error! Reference source not found. presents the list of heat transfer correlations collected for flow in circular pipes.

TABLE 7. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN CIRCULAR PIPES

Dittus-Boelter (1930) [44] [45]	$Nu = 0.023Re^{0.8}Pr^{n}$ $n = 0.4$ (heating), $n = 0.33$ (cooling) length-to-diameter ratio $L/D \geq 60$, $Re \geq 10^5$, $0.6 \leq Pr \leq 160$
Lyon (1949) [9] [47] [48] [50]	$Nu = 7 + 0.025\left(\frac{Pe}{Pr_t}\right)^{0.8}$ $0 \leq Pr \leq 0.1$, $10^4 \leq Re \leq 5 \cdot 10^6$, uniform heat flux
Seban-Shimazaki (1951) [9] [52]	$Nu = 5 + 0.025Pe^{0.8}$ $0 \leq Pr \leq 0.1$, $10^4 \leq Re \leq 5 \cdot 10^6$, $10^2 \leq Pe \leq 2 \cdot 10^4$ uniform wall temperature
Deissler (1952) [10] [53]	$Nu = 6.3 + 0.000222Pe^{1.3}$ uniform heat flux
Stromquist (1953) [4] [54]	$Nu = 3.6 + 0.018\,Pe^{0.8}$, $88 \leq Pe \leq 4 \cdot 10^3$, uniform heat flux
Lubarsky-Kaufman (1955) [9] [55] [56] [57]	$Nu = 0.625\,Pe^{0.4}$ uniform heat flux, $0 \leq Pr \leq 0.1$, $10^4 \leq Re \leq 10^5$ $2 \cdot 10^2 \leq Pe \leq 9 \cdot 10^3$
Hartnett-Irvine (1957) [9] [58]	$Nu = \frac{2}{3}Nu_{slug} + 0.015\,Pe^{0.8}$ $Nu_{slug} = 5.8$ for constant wall temperature $Nu_{slug} = 8.0$ for constant heat flux $0 \leq Pr \leq 0.1$, $10^4 \leq Re \leq 5 \cdot 10^6$ for clean surfaces and no gas entrainment
Schleicher-Tribus (1957) [9] [59]	uniform wall temperature: $Nu = 4.8 + 0.015Re^{0.91}Pr^{1.21}$ uniform wall heat flux: $Nu = 6.3 + 0.016Re^{0.91}Pr^{1.21}$ both valid for $0 \leq Pr \leq 0.1$, $10^4 \leq Re \leq 5 \cdot 10^6$
Lykoudis-Touloukian (1958) [10] [60]	$Nu = 7.0 + 0.30\,Pe^{0.3}$ uniform heat flux

TABLE 7. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN CIRCULAR PIPES

Kutateladze et al. (1959) [13] [61]	$Nu = 5.9 + 0.015\ Pe^{0.8}$ uniform heat flux
Buleev (1959) [11] [62] [63] [64] [65]	$Nu = A + 4.16\left(\frac{Re}{1000}\right)^{m} Pr^{n}$ $A = 2.5 + 1.3\ \log\left[1 + \frac{1}{Pr}\right]$ $m = 0.865 - 0.051\ \log\left[1 + \frac{1}{Pr}\right]$ $n = 0.66$ for $0.01 \le Pr \le 1$ $n = 0.44$ for $1 \le Pr \le 10$ $0.01 \le Pr \le 10,\ 5 \cdot 10^{3} \le Re \le 10^{6}$, uniform heat flux
Ibragimov et al. (1960) [66]	$Nu = 4.5 + 0.014 Pe^{0.8}$ uniform wall heat flux
Rohsenow-Cohen (1960) [67] [68]	$Nu = 6.7 + 0.0041(Re \cdot Pr)^{0.793} e^{41.8 \cdot Pr}$ $5 \cdot 10^{-3} \le Pr \le 5 \cdot 10^{-2},\ \ Re \ge 10^{4}$, uniform heat flux
Azer-Chao (1961) [9] [69]	$Nu = 5 + 0.05\ Pe^{0.77} Pr^{0.25}$ $0 \le Pr \le 0.1,\ 10^{4} \le Re \le 5 \cdot 10^{6}$, uniform wall temperature
Subbotin et al. (1962) [70]	$Nu = 5 + 0.025 Pe^{0.8}$ $Pe \le 2 \cdot 10^{4}, 3 \cdot 10^{3} \le Re \le 10^{6}$, unifirm heat flux
Kirillov (1962) [82]	$Nu = 4.36 + 0.343 Pe^{*0.8}$ $Pe^{*} = \frac{v^{*} d}{a} = Pe\sqrt{\frac{f}{8}}$ $\mathrm{Pe}^{*} < 1000$
Baker-Sesonske (1962) [10] [83]	$Nu = 6.05 + 0.0074 Pe^{0.95}$ uniform heat flux
Subbotin et al. (1963) [63]	$Nu \approx 4.3 + 0.025 Pe^{0.8}$ $0.02 \le Pr \le 0.03, 20 \le \mathrm{Pe} \le 10000$
Dwyer (1963) [8] [9]	$\mathrm{Nu} = 7 + 0.025\left[Re\,\mathrm{Pr} - \frac{1.82\ Re}{\left(\frac{\varepsilon_m}{\nu}\right)_{max}^{1.4}}\right]^{0.8}$ uniform heat flux, $0 \le Pr \le 0.1,\ 10^{4} \le Re \le 5 \cdot 10^{6}$

TABLE 7. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN CIRCULAR PIPES

Subbotin et al. (1963) [21]	$Nu = Nu_0 + ARe^n Pr^m$ $Nu_0 = 7.24 - \frac{9.5}{\log Re}$ $A = 0.0155, n = 0.82,$ $m = 0.58 - 0.18 \cdot \tanh(0.81 \ \log Pr).$ $10^4 < Re < 7 \cdot 10^5, \ 0.005 < Pr < 10,$ uniform heat flux
Kokorev (1963) [19]	$Nu = 5.5 + 0.025 Pe^{0.8}$ $20 < Pe < 4000$
Skupinski et al. (1965) [9] [84] [85]	$Nu = 4.82 + 0.0185 Pe^{0.827}$ $58 \leq Pe \leq 1.31 \cdot 10^4, \ 0 \leq Pr \leq 0.1, \ 10^4 \leq Re \leq 5 \cdot 10^6$ uniform heat flux
Notter-Sleicher (1972) [9] [86] [87]	uniform wall temperature $Nu = 4.8 + 0.0156 \ Pe^{0.85} \ Pr^{0.08}$ uniform heat flux $Nu = 6.3 + 0.0167 \ Pe^{0.85} \ Pr^{0.08}$ uniform heat flux or uniform temperature wall: $Nu = 5 + 0.016 \ Re^a \ Pr^b$ $a = 0.88 - \frac{0.24}{(4+Pr)}, \ b = 0.33 + 0.5 e^{-0.6Pr}$ All three above are valid for $10^4 \leq Re \leq 10^6$, $0.004 \leq Pr \leq 10^4$
Sleicher et al. (1973) [88]	$Nu(x) = Nu_\infty \left(1 + \frac{2}{x/D}\right), \ x/D \geq 4$ $Nu_{ave} = Nu_\infty \left(1 + \frac{8}{L/D} + \frac{2}{L/D} \ln \frac{L/D}{4}\right), \ L/D \geq 4$ uniform wall temperature: $Nu_\infty = 4.8 + 0.0156 \ Pe^{0.85} \ Pr^{0.08}, \ 0.004 \leq Pr \leq 0.1$ uniform wall heat flux: $Nu_\infty = 6.3 + 0.0167 \ Pe^{0.85} \ Pr^{0.08}, \ 0.004 \leq Pr \leq 0.1$ $2.6 \cdot 10^4 \leq Re \leq 3.02 \cdot 10^5$
Aoki (1973) [10] [89]	$Nu = 6.0 + 0.025 \left[0.014 Re^{1.45} Pr^{1.2} \left\{1 - \frac{e^{-71.8}}{Re^{0.45} Pr^{0.2}}\right\}\right]^{0.8}$ uniform wall heat flux

TABLE 7. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN CIRCULAR PIPES

Dwyer (1976) [11]	$Nu = 4.0 + 0.025Pe^{0.8}$ uniform wall temperature
Chen-Chiou (1981) [9] [90]	uniform wall temperature $Nu = 4.5 + 0.0156\ Re^{0.85}Pr^{0.86}$ uniform heat flux $Nu = 5.6 + 0.0165\ Re^{0.85}Pr^{0.86}$ For temperature dependency: $\frac{Nu}{Nu_0} = \left(\frac{T_b}{T_i}\right)^n$ uniform heat flux: $n = \exp(5.9 \cdot 10^{-3}T_b - 6.91)\ 1000\ K \geq T_b \geq 600\ K\ heating$ $n = 0\ 600\ K \geq T_b \geq 370\ K\ heating$ $n = 0.25$ *cooling* uniform wall temperature $n = 0.08 + 2.2 \cdot 10^{-4}T_b\ 1000\ K \geq T_b \geq 600\ K\ heating$ $n = 0.08\ 600\ K \geq T_b \geq 370\ K\ heating$ $n = 0.16$ *cooling* $0 \leq Pr \leq 0.1$, $10^4 \leq Re \leq 5 \cdot 10^6$
Lee (1983) [9] [91]	$Nu = 3.01Re^{0.0833}$ $5 \leq Pe \leq 1000$, $0.001 \leq Pr \leq 0.02$, $Re \geq 4000$, uniform wall flux
Borishanski (1983) [51] [95]	$Nu = 4.3 + 0.0021Pe$ $10^2 \leq Pe \leq 10^4$ impurities in liquid metals are below the oxygen solubility limit
Reed (1987) [9] [13]	$Nu = 3.3 + 0.02\ Pe^{0.8}$ for $Pe \geq 10^2$, $\frac{L}{D_h} \geq 60$, uniform wall temperature

TABLE 7. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN CIRCULAR PIPES

Buleev et al. (1989) [97]	$Nu = A + 3.90 \left(\frac{Re}{1000}\right)^m Pr^n$ $A = 2.5 + 1.3 \log\left(1 + \frac{1}{Pr}\right)$ $m = 0.918 - 0.05 \log\left(1 + \frac{10}{Pr}\right)$ $n = 0.65 - 0.107 \log\left(1 + \frac{10}{Pr}\right)$ uniform heat flux, $3 \cdot 10^3 < Re < 3 \cdot 10^6$
Siman-Tov et al. (1997) [13] [98]	$Nu = 0.685\, Pe^{0.3726}$ uniform heat flux
Tricoli (1999) [13] [99]	$\frac{Nu]_{T_w\, const.}}{Nu]_{q'_w\, const.}} = \frac{\pi^2}{12} = 0.822$
ENIN (2001) [71]	$Nu = A + 0.014\, Pe^{0.8}$ $A = 3$ for oxide films on wall $A = 4.5$– 5 for clean wall
TsKTI (2001) [71]	$Nu = 5 + 0.0021\, Pe$, for pure liquid metals
Cheng-Tak (2006) [4]	$Nu = A + 0.018 Pe^{0.80}$ $A = \begin{cases} 4.5 & Pe \leq 1000 \\ 5.4 - 0.0009 Pe & 1000 \leq Pe \leq 2000 \\ 3.6 & Pe \geq 2000 \end{cases}$
Mochizuki (2010) [100]	$Nu = min(Nu_1, Nu_2)$ $Nu_1 = 5 + 0.025 Pe^{0.8}$ (Seban-Shimazaki, 1951) $Nu_2 = 5 \cdot 10^{-3} Pe^{1.74}$ $Pe < 55$

Several widely used heat transfer correlations for Nusselt number versus Peclet number for the pipe flows are presented in Fig. 8. When required in correlation, the Prandtl number is assumed $Pr = 4.5 \cdot 10^{-3}$ at sodium temperature $T \approx 700\ K$, and turbulent Prandtl number $Pr_t = 1.5$.

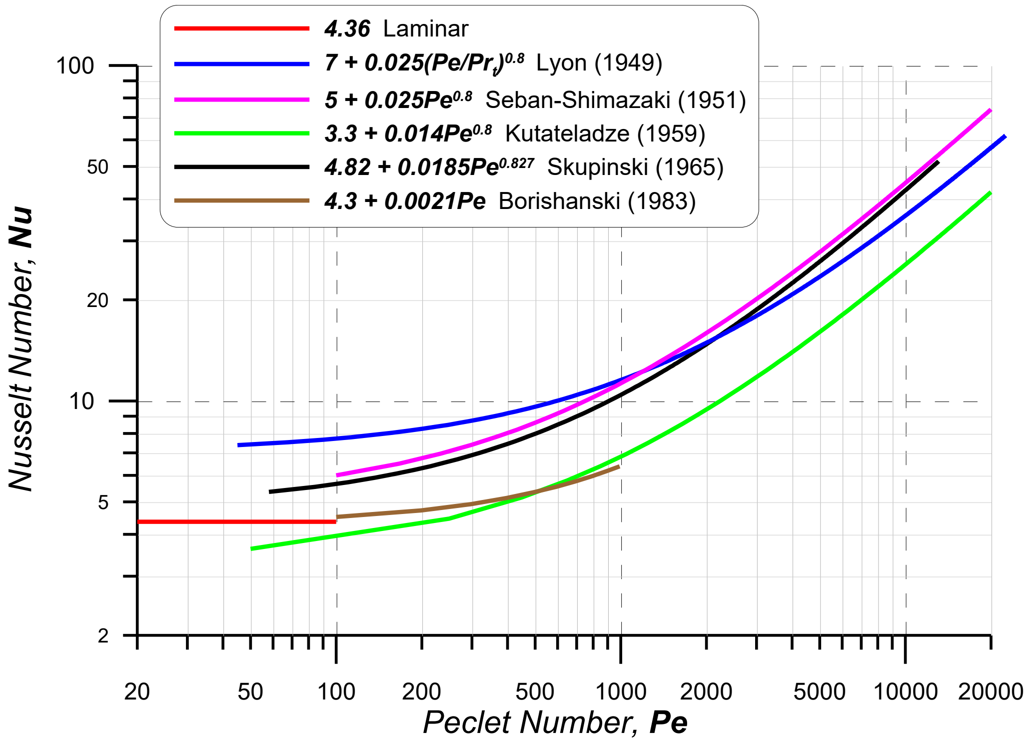


FIG. 8. Comparison of common Nusselt correlations for heat transfer in a pipe

3.2.2 Flow between parallel plates, in flat ducts and in rectangular ducts

Heat transfer in the flow between parallel plates (see Fig. 9) depends on several factors, such as single-wall (unilateral) or bilateral (both walls) heating, boundary conditions and others considered below.

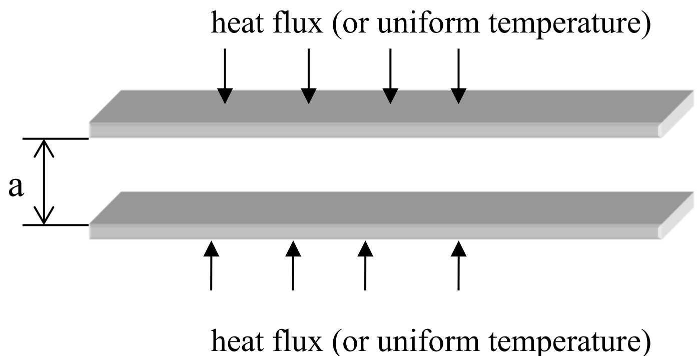


FIG. 9. Unilateral or bilateral heat transfer between parallel walls

3.2.2.1 Seban (1950)

The case of liquid metal heat transfer with constant properties flowing turbulently between parallel walls having uniform temperatures was considered on an analytical basis by R.A. Seban in 1950 [101]. Only conditions for fully developed flow from the hydrodynamic and thermal standpoint were considered. It was shown that the existence of solutions for the case of one adiabatic wall enables the specification of the temperature distribution and heat transfer coefficients for cases in which the walls have any uniform temperature value. The effect of inequality in the wall temperatures was shown to be small for Prandtl numbers greater than unity, but significant for fluids of low Prandtl number, such as liquid metals. For the case of unilateral heat transfer, constant temperature at one wall only (the other being adiabatic), the Nusselt number correlation was proposed as follows:

$$Nu = 5.8 + 0.020\, Pe^{0.8} \tag{115}$$

This equation is valid for $10^2 \leq Pe \leq 10^5$, and $0.01 \leq Pr \leq 1.0$, with an accuracy of $\pm$ 5%. For constant heat flux through both walls, a graphic correction factor for the heat transfer coefficient was also proposed by R.A. Seban [45].

3.2.2.2 Hartnett-Irvine (1957)

The first detailed analytical study of heat transfer coefficients for liquid metal flows in non-circular ducts was presented by Hartnett and Irvine in 1957 ([9] Ch. 4) [13] [58]. In their analysis, they presented a generic correlation following the functional dependence as:

$$Nu = \frac{2}{3} Nu_{slug} + \ 0.015\, Pe^{0.8} \tag{116}$$

They considered that only the direct contribution from molecular conduction is directly related to both the duct geometry and the thermal boundary conditions, while the other coefficients remain constant. This is expressed in terms of the Nusselt number corresponding to a slug flow (Nu_{slug}), in addition to a correction factor $(2/3)$ for the actual velocity profile.

They recommended values of Nu_{slug} for several common duct geometries and wall conditions (constant temperature or heat flux). They recommended the values $Nu_{slug} = \frac{\pi^2}{2} = 4.93$ and $Nu_{slug} = 7.03$ for a uniform wall temperature and uniform wall heat flux on both walls, respectively for squared ducts $(a = b)$. For rectangular ducts $(a \neq b)$, values of Nu_{slug} are plotted in Fig. 10:

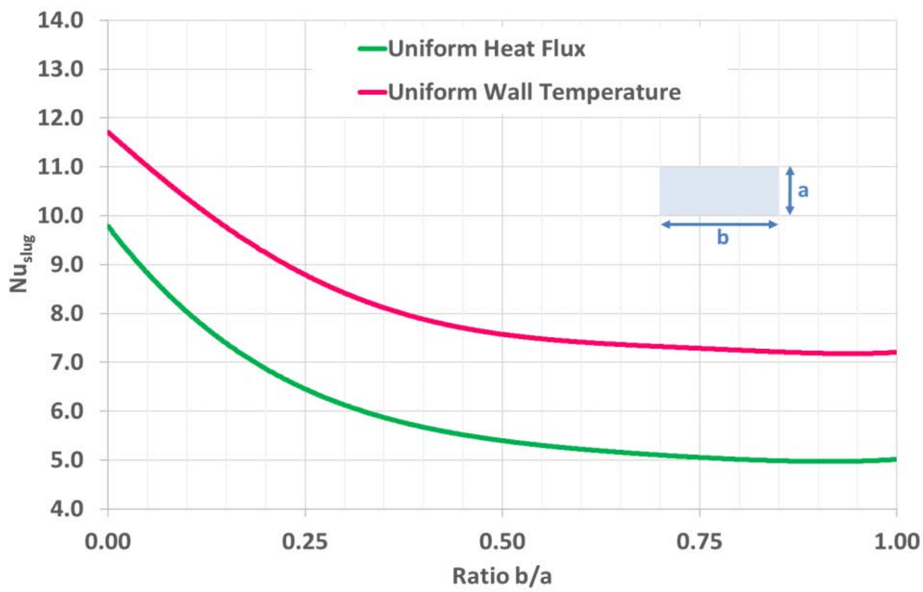


FIG. 10. Slug Nusselt number for rectangular ducts

3.2.2.3 Buleev (1959)

Some empirical correlations are developed to calculate Nu for liquid metals turbulent flows ($Pr \leq 0.03$) in a smooth flat duct. For unilateral (single-wall) heat flux, the following equation was proposed by N.I. Buleev (1959) [9] [62]:

$$\mathrm{Nu} = 5.1 + 0.02\, Pe^{0.8} \tag{117}$$

In case of the bilateral heat flux (both walls are heated) with fully-established turbulent flow when both heat fluxes are equal, Buleev proposed the empirical equation for liquid metals [11] [64]:

$$Nu = 9 + (1.41 - \ln 100\, Pr)\,(0.1 + 0.02\, Pe)^{0.8} \tag{118}$$

At $Pr = 0.01$, the correlation yields:

$$Nu = 9.14 + 0.0281 Pe^{0.8} \tag{119}$$

while for $Pr = 0.02$, it is:

$$Nu = 9.13 + 0.0266 Pe^{0.8} \tag{120}$$

These two last equations agree within less than 5% discrepancy at a Peclet number as high as 10^4.

3.2.2.4 Kays (1963)

In 1963 W.M. Kays presented a comprehensive analysis of turbulent heat transfer results for arbitrarily prescribed heat fluxes at both walls [13] [102]. Based on this analysis, the fully developed Nusselt number for the constant wall heat flux under arbitrary heat flux ratio on the two surfaces can be represented by:

$$Nu = \frac{Nu_0}{1 - \gamma\varphi} \tag{121}$$

where γ is the ratio of the prescribed heat fluxes at the two duct walls, Nu_0 is the value corresponding to $\gamma = 0$, and j is a correction factor. In that context, $\gamma = 0$ represents the case with one wall heated and the other one insulated, for $\gamma = 1$ both are heated with the same heat flux and $\gamma = -1$ one wall is heated and the other is cooled at the same rate. The parameters Nu_0 and j were obtained by means of numerical integration for different values of the Reynolds number (see Table 8).

TABLE 8. PARAMETERS OF KAYS' CORRELATION FOR SMOOTH FLAT DUCT WITH UNIFORM HEAT FLUX

Pr	$Re = 10^4$		$Re = 3 \cdot 10^4$		$Re = 10^5$		$Re = 3 \cdot 10^5$		$Re = 10^6$	
	Nu_0	j	Nu_0	j	Nu_0	j	Nu_0	j	Nu_0	j
0	5.70	0.428	5.78	0.445	5.80	0.456	5.80	0.460	5.80	0.468
0.001	5.70	0.428	5.78	0.445	5.80	0.456	5.88	0.460	6.23	0.460
0.003	5.70	0.428	5.80	0.445	5.90	0.450	6.32	0.450	8.62	0.422
0.01	5.80	0.428	5.92	0.445	6.70	0.440	9.80	0.407	21.5	0.333
0.03	6.10	0.428	6.90	0.428	11.0	0.390	23.0	0.330	61.2	0.255

This analysis was in principle developed and validated for $Pr = 0.7$ (typical for air and other gases). Asymptotic solutions were also proposed for wider ranges of Reynolds and Prandtl numbers and, in the liquid metal range. It is the only description available for predicting the heat transfer performance under generic asymmetric thermal boundary conditions.

3.2.2.5 Duchatelle-Vautrey (1964)

L. Duchatelle and L. Vautrey presented in 1964 experimental results for liquid metal heat transfer between parallel plates with unilateral uniform heat flux conditions (heat flux through one wall, adiabatic at the other wall). These results are well represented by the empirical expression [9] [65] [103]:

$$Nu = 5.85 + 0.000341\ Pe^{1.29}\ for\ Pe < 1200 \tag{122}$$

The authors also proposed the following correlation for flow in a smooth flat duct with unilateral heat transfer [9]:

$$\begin{gathered} Nu = 5.14 + 0.0127\ Pe^{0.8}\ for\ 200 < Pe < 1200 \\ Nu = 6.1\ \ for\ Pe < 200 \end{gathered} \tag{123}$$

3.2.2.6 Dwyer (1965)

For unilateral heat transfer (single heated plate), the following equation, proposed by O.E. Dwyer in 1965, is recommended[3] [9] [65] [104]:

$$Nu = 5.60 + 0.01905\ \left(\frac{Pe}{Pr_t}\right)^{0.775} \text{ for } 100 \leq \frac{Pe}{Pr_t} \leq 10^4 \tag{124}$$

where, as the author said, $\frac{1}{Pr_t}$ can be evaluated with the equation recommended by Dwyer (see Section 3.1.1).

For bilateral heat transfer (heat flux on both parallel plates) under uniform and equal heat flux conditions, the following equation proposed by O.E. Dwyer (1965), is recommended[4] [9]:

$$Nu = 9.49 + 0.0596\ \left(\frac{Pe}{Pr_t}\right)^{0.688} \tag{125}$$

In 1976 the author concluded that unilateral heat transfer from the outer wall of a narrow concentric annulus (e.g., $\frac{r_2}{r_1} \leq 2.0$) is, for all practical purposes, the same as unilateral heat transfer with parallel plates [11].

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[3] In [9] (Ch. 4) this correlation is presented with Pe in power of 0.8 instead to 0.775.

[4] In [9] (Ch. 4 pp. 4.66 Eq. 4.102) this correlation is presented with the factor of the second Peclet term as 0.00596 instead of 0.0596.

3.2.2.7 Dwyer (1966)

In 1966 O.E. Dwyer presented in [65] an equation representing the calculated Nusselt number published by Harrison and Menke in 1949 [105], where they used the velocity distribution data of Nikuradse from 1932 [106]:

$$Nu = 4.73 + 0.02768 \left(\frac{Pe}{Pr_t}\right)^{0.736} \tag{126}$$

3.2.2.8 Summary of heat transfer correlations for flow between parallel plates, in flat ducts and in rectangular ducts

Table 9 presents the list of heat transfer correlations collected for flow between parallel plates, in flat ducts and in rectangular ducts.

TABLE 9. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW BETWEEN PARALLEL PLATES, IN FLAT DUCTS AND IN RECTANGULAR DUCTS

Seban (1950) [101]	$Nu = 5.8 + 0.020\ Pe^{0.8}$ $10^2 \leq Pe \leq 10^5$, $0.01 \leq Pr \leq 1.0$, unilateral uniform heat flux
Hartnett-Irvine (1957) [9] [13] [58]	For rectangular ducts ($a \neq b$): $Nu = \frac{2}{3} Nu_{slug} + 0.015\ Pe^{0.8}$ Nu_{slug} in Fig. 10 For square ducts ($a = b$): $Nu_{slug} = 4.93$ for uniform wall temperature $Nu_{slug} = 7.03$ for uniform wall heat flux on both walls
Buleev (1959) [9] [11] [62] [64]	unilateral heat flux $\mathrm{Nu} = 5.1 + 0.02\ Pe^{0.8}$ bilateral heat flux $Nu = 9 + (1.41 - \ln 100\ Pr)\ (0.1 + 0.02\ Pe)^{0.8}$
Kays (1963) [13] [102]	$Nu = \frac{Nu_0}{1 - \gamma\varphi}$ Nu_0, j in Table 8 uniform heat flux and arbitrary heat flux ratio on the surfaces
Duchatelle-Vautrey (1964) [9] [65] [103]	$Nu = 5.85 + 0.000341\ Pe^{1.29}$ $Nu = 5.14 + 0.0127\ Pe^{0.8}$ unilateral uniform heat flux
Dwyer (1965) [9] [65] [104]	unilateral heat transfer: $Nu = 5.60 + 0.01905 \left(\frac{Pe}{Pr_t}\right)^{0.775}$ for $100 \leq \frac{Pe}{Pr_t} \leq 10^4$ bilateral uniform heat flux: $Nu = 9.49 + 0.0596 \left(\frac{Pe}{Pr_t}\right)^{0.688}$
Dwyer (1966) [65]	$Nu = 4.73 + 0.02768 \left(\frac{Pe}{Pr_t}\right)^{0.736}$

The empirical correlations for unilateral heating presented in Table 9 are also plotted in Fig. 11. For the purpose of comparison, the turbulent Prandtl number is assumed $Pr_t = 1.5$ (if required in the correlation). The value $Pr_t = 1.5$ corresponds to $Re \sim 50000$ and $Pr \sim 5 \cdot 10^{-3}$ at sodium temperature $T = 800\ K$. The visual comparison in Fig. 11 omits Harnett-Irvine (1957) [9] [13] [58] and Kays (1963) [13] [102]. The uniform heat transfer correlations are selected, as this phenomenon is best described by the correlations above.

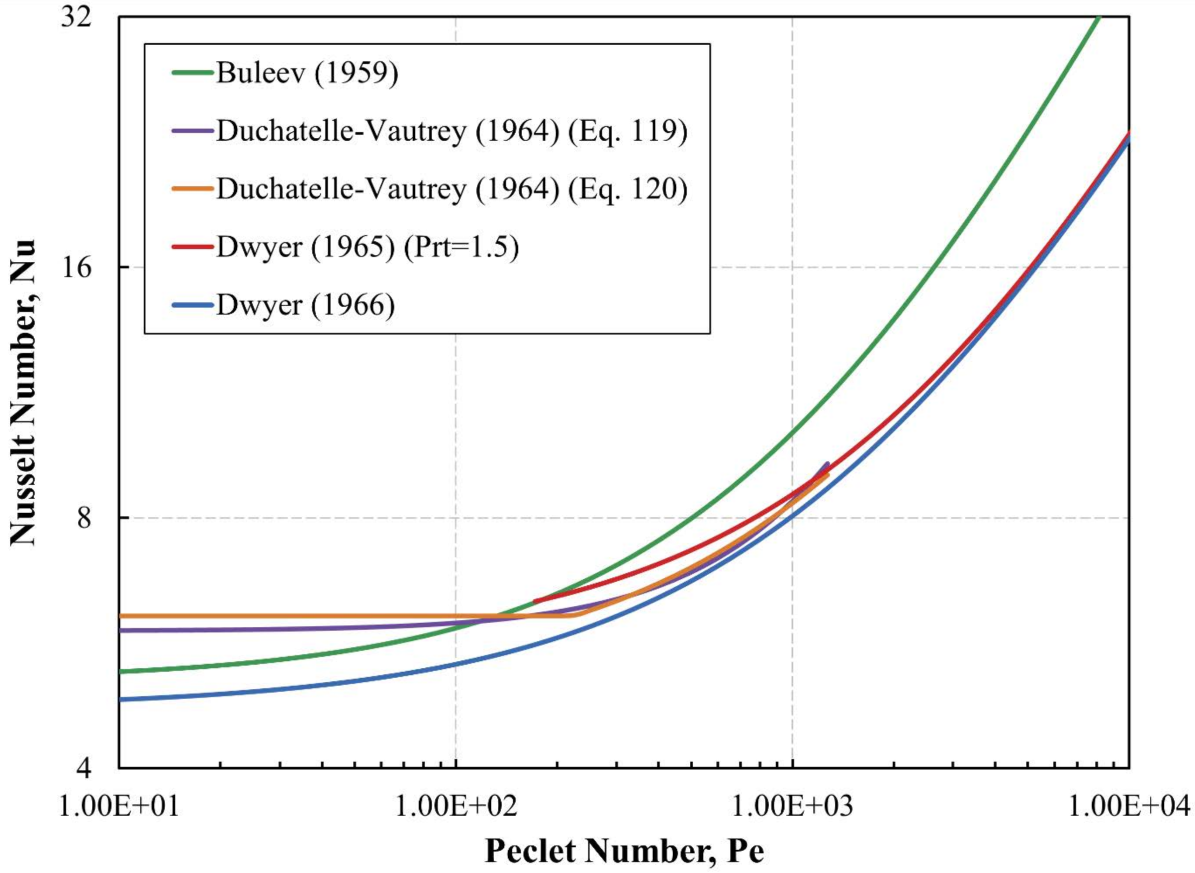

FIG. 11 Comparison of empirical heat transfer correlations for flow between parallel plates and in rectangular ducts (unilateral, one-side heating)

3.2.3 Flow in concentric annular ducts

A concentric circular annular duct is another important geometry for many sodium flow and heat transfer applications. As shown in Fig. 12 sodium flows through the gap between the inner cylinder with external diameter D_1 and the circular tube with internal diameter D_2. Heating is usually applied from the inner wall but also can be supplied from the external wall or from both.

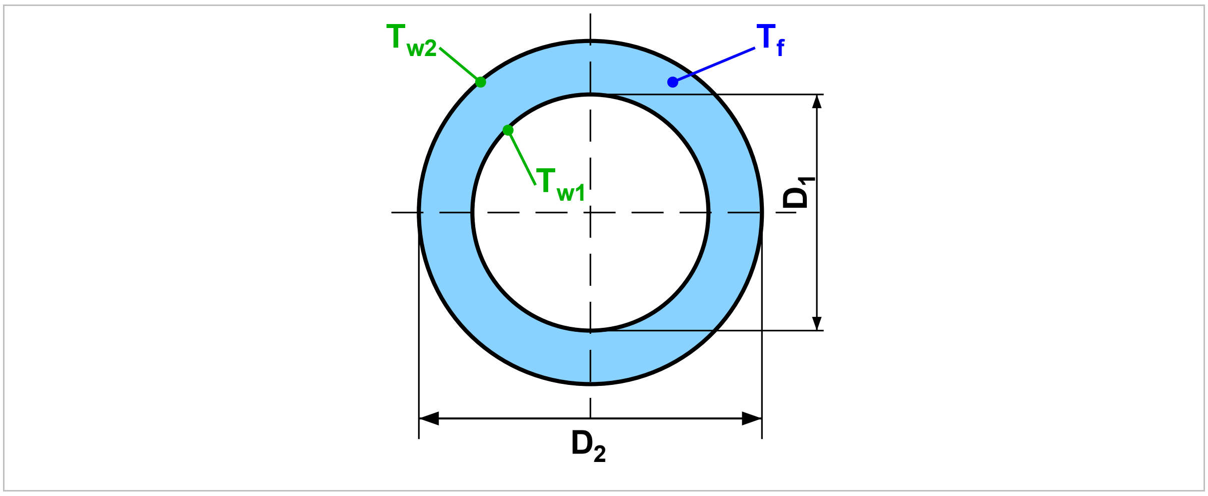

FIG. 12. Heat transfer in flow in concentric annulus

3.2.3.1 Werner et al. (1949)

In 1949 R.C. Werner et al. modified Lyon's equation for turbulent flow of liquid metals in annuli with a factor of the diameter ratio to the form [107] [108]:

$$Nu = 0.7 \left(\frac{D_2}{D_1}\right)^{0.53} [7 + 0.025 Pe^{0.8}] \tag{127}$$

where the equivalent diameter D_e of the annulus should be used to calculated Nusselt and Peclet numbers. This equation is valid for thin annuli with $\frac{D_2}{D_1} \leq 1.4$, where D_1 is the diameter of the internal annulus and D_2 the diameter of the external annulus.

3.2.3.2 Bailey (1950)

In 1950 R.V. Bailey proposed the following correlation for flow in annuli when the $\frac{D_2}{D_1} > 1.4$ [107] [109]:

$$Nu = 0.75 \left(\frac{D_2}{D_1}\right)^{0.3} [7 + 0.025 Pe^{0.8}] \tag{128}$$

For $\frac{D_2}{D_1} < 1.4$ Bailey recommended to use flat-plate correlations [110].

It should be noted that in 1993 N.E. Todreas and M. Kazimi presented for fully developed flow and uniform heat flux in the inner wall, the following Nusselt correlation for $\frac{D_2}{D_1} \geq 1.4$ [45]:

$$Nu = 5.25 + 0.0188\, Pe^{0.8} \left(\frac{D_2}{D_1}\right)^{0.3} \tag{129}$$

If $\frac{D_2}{D_1}$ is close to unity, R.A. Seban [45] [101] recommended his Eq. (115) derived for the parallel plates.

The only publication found where the correlation (129) is mentioned is the book of M.M. El-Wakil ("Nuclear Heat Transport" International Textbook Company, 1971) which presents this equation referring to Bailey [109] and Werner [108]. However, El-Wakil statement is incorrect since Bailey and Werner correlations are different as presented in the two previous sections. One should therefore be cautious about the use of Eq. (129) since the origin is doubtful.

3.2.3.3 Lyon (1952)

In 1952 R.N. Lyon presented the following correlation for uniform heat flux in the inner wall, being the outer wall insulated [110] [111]:

$$Nu = (5.25 + 0.0175\, Pe) \left(\frac{D_2}{D_1}\right)^{0.53} \tag{130}$$

Nusselt and Peclet numbers are based on the equivalent diameter $D_e = D_2 - D_1$.

3.2.3.4 Hartnett-Irvine (1957)

In 1957 J.P. Hartnett and T.F. Irvine proposed the following model ([9] Ch. 4) [13] [58]:

$$Nu = \frac{2}{3} Nu_{slug} + 0.015\, Pe^{0.8} \tag{131}$$

This correlation although recommended in the 1950s provide conservative predictions of the Nusselt numbers. Figures 13, 14 and 15 show the Slug Nusselt numbers for the cases with: i) both walls at constant temperature, ii) constant heat input per unit length with both inner and outer walls at the same temperature at a given axial position (the heat input per unit area is not the same at both wall surfaces), and iii) constant heat flux on one wall with the other wall insulated.

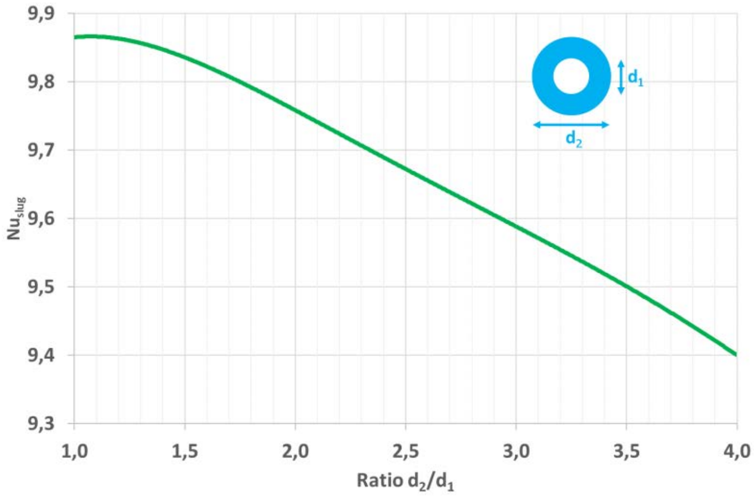

FIG. 13. Slug Nusselt number for annular duct with constant wall temperature

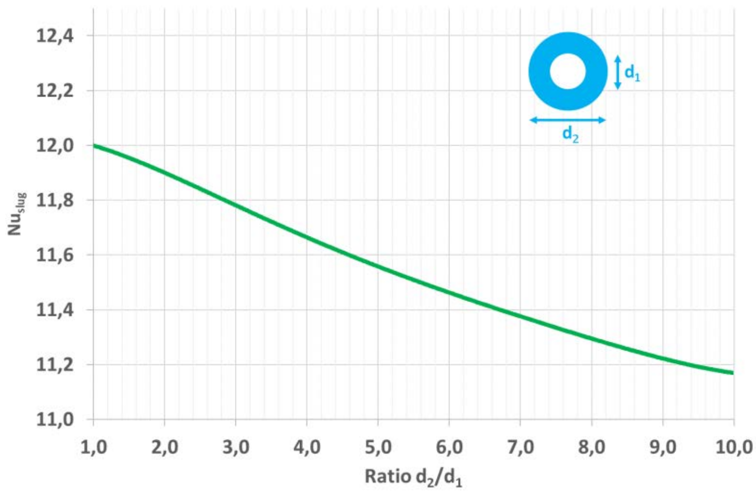

FIG. 14. Slug Nusselt number for annular duct with constant heat input on both inner and outer walls

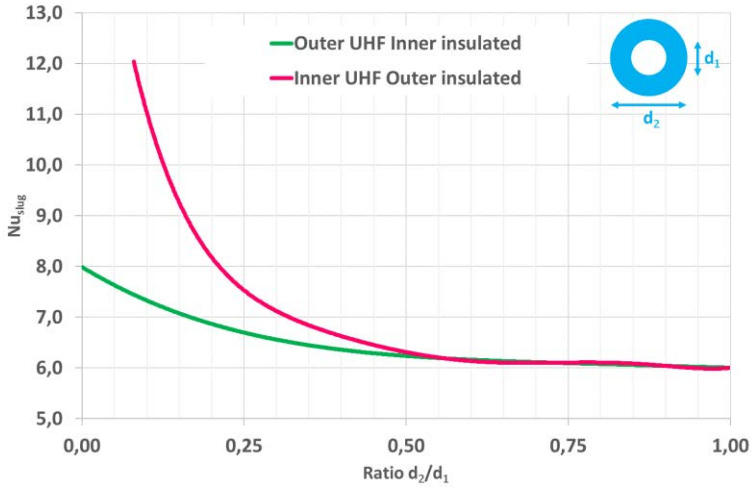

FIG. 15. Slug Nusselt numbers for annular duct with uniform heat flux (UHF) on one wall with the other wall insulated

3.2.3.5 Buleev (1959)

In 1959 N. Buleev published the following correlation for heat transfer to turbulently flowing liquid metals from the outer walls of annuli, having $\frac{r_2}{r_1} \leq 2$. It is based on uniform heat flux and fully established velocity and temperature profiles [62] [65] [104]:

$$Nu = 5.1 + 0.02Pe^{0.8} \tag{132}$$

This correlation is the same as the one presented above for one plate of a set of parallel plates Eq. (117).

3.2.3.6 Baker-Sesonske (1962)

In 1962 R.A. Baker and A. Sesonske [83] published results for counter-current flow of NaK in a stainless-steel double-pipe heat exchanger. The heat transfer coefficients on both the tube and shell sides were calculated from overall heat transfer coefficients. Furthermore, there was the problem of correcting for entrance effects. For these reasons the accuracy of the results is open to questions. The authors represented the experimental curve by the equation [65]:

$$Nu = 0.80 \left(\frac{r_2}{r_1}\right)^{0.3} [5.12 + 0.0296Pe^{0.785}] \tag{133}$$

3.2.3.7 Dwyer (1963)

In 1963 but in various publications O.E. Dwyer recommended different correlations for the heat transfer in concentric annuli. He first developed semi-empirical equations for liquid metal flow ($Pr \leq 0.03$) in concentric annuli $0 \leq \frac{r_1}{r_2} \leq 1$ with one wall subjected to uniform heat flux and the other wall insulated, where r_1 is the radius of the internal annulus and r_2 is the radius of the external annulus. For the case of outer wall heated, $Pe \leq Pe_{crit}$, the Nusselt equation is:

$$Nu = 5.52 + 0.076\frac{r_2}{r_1} \tag{134}$$

Above the critical Peclet number, the semi-empirical equation is as follows [8] [9]:

$$Nu = A_0 + B_0 \left(\frac{Pe}{Pr_t}\right)^{n_0} \tag{135}$$

where:

$$A_0 = 5.26 + 0.05\frac{r_2}{r_1}$$

$$B_0 = 0.01848 + 0.003154\frac{r_2}{r_1} - 0.0001333\left(\frac{r_2}{r_1}\right)^2 \tag{136}$$

$$n_0 = 0.78 - 0.01333\frac{r_2}{r_1} + 0.000833\left(\frac{r_2}{r_1}\right)^2$$

In the case of the inner wall heated, below the critical Peclet number the Nusselt equation is:

$$Nu = 4.92 + 0.686\frac{r_2}{r_1} \tag{137}$$

Above the critical Peclet number, the semi-empirical equation of O.E. Dwyer is as follows:

$$Nu = A_i + B_i \left(\frac{Pe}{Pr_t}\right)^{n_i} \tag{138}$$

where

$$A_i = 4.63 + 0.686\frac{r_2}{r_1}$$

$$B_i = 0.02154 - 0.000043\ \frac{r_2}{r_1} \tag{139}$$

$$n_i = 0.752 + 0.01657\frac{r_2}{r_1} - 0.000833\left(\frac{r_2}{r_1}\right)^2$$

Both Eqs. (135) and (138) are valid for Pe values above its critical values. For $Pe \leq Pe_{cr}$, the sole mode of heat transfer is molecular conduction for liquid metals. For $Pr = 0.005, 0.01, 0.02, 0.03$ the critical Pe values are $270, 300, 330, 345$, respectively (see Table 5). The critical Peclet numbers for annuli are for heat transfer at either the inner or the outer wall.

In the same year 1963 in [112] O.E. Dwyer presented another set of equations for estimating Nusselt numbers for liquid metal flowing in annuli under conditions of uniform heat flux and fully-developed flow [113]. In the case with outer wall heated and the inner wall insulated, the Nusselt equation is (also presented in [65]):

$$Nu = A_o + B_o \left(\frac{Pe}{Pr_t}\right)^{n_o} \tag{140}$$

where

$$A_o = 5.54 - 0.023\frac{r_2}{r_1} \tag{141}$$

$$B_o = 0.0189 + 0.00316\frac{r_2}{r_1} + 0.0000867\left(\frac{r_2}{r_1}\right)^2$$

$$n_o = 0.758\left(\frac{r_2}{r_1}\right)^{0.0204}$$

It is valid for $1 \leq \frac{r_2}{r_1} \leq 7$. This model is in a very good agreement with the data by Petrovichev (1961) [114] for the experiments with Hg [13].

For the case with inner wall heated and the outer wall insulated, the proposed equation is:

$$Nu = A_i + B_i\left(\frac{Pe}{Pr_t}\right)^{n_i} \tag{142}$$

where:

$$A_i = 4.82 + 0.697\frac{r_2}{r_1}$$
$$B_i = 0.0222 \tag{143}$$
$$n_i = 0.758\left(\frac{r_2}{r_1}\right)^{0.053}$$

It is valid for $1 \leq \frac{r_2}{r_1} \leq 7$. The validity of this model was demonstrated in the ranges $0.005 \leq Pr \leq 0.03$ and $3 \cdot 10^2 \leq Pe \leq 10^5$ with an accuracy being between 10 and 15% compared to the measurements by Rensen (1982) [13].

For the special case of $\frac{r_2}{r_1} = 1$ (parallel plates), the previous equations are reduced to [113]:

$$Nu = 5.52 + 0.0222\left(\frac{Pe}{Pr_t}\right)^{0.758} \tag{144}$$

In 1965 [115], 1966 [65] and later in 1976 ([11] Ch. 2) O.E. Dwyer recommended the following correlations for bilateral heat transfer to fluids flowing in annuli with uniform heat flux from each wall and equal wall temperatures at a given axial position:

$$Nu = A + B\left(\frac{Pe}{Pr_t}\right)^n \tag{145}$$

For heat transfer from inner wall the recommended equations valid for $1 \leq \frac{r_2}{r_1} \leq 7$ are [65]:

$$A = 7.82 + 1.72\frac{r_2}{r_1} - 0.051\left(\frac{r_2}{r_1}\right)^2$$
$$B = 0.0592 - 0.000342\frac{r_2}{r_1} + 0.000723\left(\frac{r_2}{r_1}\right)^2 \tag{146}$$
$$n = 0.655 + 0.0363\frac{r_2}{r_1} - 0.0037\left(\frac{r_2}{r_1}\right)^2$$

While for heat transfer from outer wall:

$$A = 7.11 + \frac{3.22}{\frac{r_2}{r_1}} - \frac{0.842}{\left(\frac{r_2}{r_1}\right)^2}$$

$$B = 0.0396 + \frac{0.0200}{\frac{r_2}{r_1}} \tag{147}$$

$$n = 0.746 - \frac{0.0864}{\frac{r_2}{r_1}} + \frac{0.0282}{\left(\frac{r_2}{r_1}\right)^2}$$

In 1966 [65] and later in 1976 ([11] Ch. 2) O.E. Dwyer recommended the following correlations for heat transfer to liquid metals flowing in concentric annuli under conditions of uniform heat fluxes and fully developed velocity and temperature profiles. For the case with equal heat fluxes from both walls, the parameters of Eq. (145) for heat transfer from inner wall are presented in Table 10.

TABLE 10. PARAMETERS OF THE INNER WALL HEAT TRANSFER CORRELATION IN CASE OF BILATERAL HEAT FLUX

$\frac{r_2}{r_1}$	A	B	n
1.00	9.49	0.0596	0.688
1.25	10.53	0.0662	0.698
1.50	11.81	0.0726	0.701
2.00	15.30	0.0855	0.704
3.00	27.0	0.1095	0.707
4.00	50.0	0.1278	0.708

While the parameters of Eq. (145) for heat transfer from outer wall are presented in Table 11.

TABLE 11. PARAMETERS OF THE OUTER WALL HEAT TRANSFER CORRELATION IN CASE OF BILATERAL HEAT FLUX

$\frac{r_2}{r_1}$	A	B	n
1.00	9.49	0.0596	0.688
1.25	8.72	0.0490	0.707
1.50	8.24	0.0420	0.723
2.00	7.60	0.0379	0.735
3.00	6.94	0.0360	0.741
4.00	6.64	0.0355	0.743

3.2.3.8 Kays-Leung (1963)

In 1963 W.M. Kays and E.Y. Leung [11] [102] published theoretical results of heat transfer in annular channels for developed turbulent flow. They presented the results for the uniform heat flux conditions from either the inner or the outer wall only in tabular form. Results were given for several Prandtl numbers, including 0.001, 0.003, 0.010, and 0.030; for Reynolds numbers of 10^4, $3 \cdot 10^4$, $1 \cdot 10^5$, $3 \cdot 10^5$, and $1 \cdot 10^6$; and for $\frac{r_2}{r_1}$ ratios of 1.00, 1.25, 2.00, 5.00, 10.00, and infinity. In the very low Peclet range, the Kays-Leung results agree well with the following equations. For the case of uniform heat flux from the inner wall only:

$$Nu = 4.98 + 0.662 \frac{r_2}{r_1} \tag{148}$$

For the case of uniform heat flux from the outer wall only:

$$Nu = 5.60 + 0.195 \left(\frac{r_2}{r_1} - 1\right)^{0.64} (log\,Re - 3.70)^{0.54} \tag{149}$$

But at higher Peclet numbers Nusselt number values are considerably lower than those predicted by the equations proposed by Dwyer. For example, at $\frac{r_2}{r_1} = 2$, $Pr = 0.02$, and heat transfer from the inner wall only, the Kays-Leung Nusselt numbers are lower by about $19, 33, and$ 38% at Peclet numbers of 600, 2000, and 6000, respectively. While the respective values for heat transfer from the outer wall only are roughly $12, 25, and$ 28% lower, respectively.

3.2.3.9 Rensen (1981)

In 1981, Q. Rensen proposed a low-Prandtl-number liquid sodium Nusselt correlation for turbulent flow in annular geometry [116]. The study was made in the thermal entrance region as well as in the thermal fully developed region. In this latter case, Rensen measured fully developed Nusselt numbers in a concentric annulus ($r_1/r_2 = 0.5409$, where r_1 and r_2 are the inside and outside radii of the two concentric walls) with the inner wall subjected to a uniform heat flux and the outer wall insulated [9]. The experiments with liquid sodium covered the ranges of $0.0047 \leq Pr \leq 0.0059$ and $6 \cdot 10^3 \leq Re \leq 6 \cdot 10^4$ (which correspond to a range of $28.2 \leq Pe \leq 354$). Under these conditions, Rensen correlated his fully developed Nusselt numbers at the inner wall within $\pm 5\%$ by the following correlation:

$$Nu = 5.75 + 0.022 Pe^{0.8} \tag{150}$$

In his study, Q. Rensen compared his correlation to that of Baker-Sesonske [83] and Borishanskiy et al. [117]. His correlation is sometimes taken as a reference for comparison with other correlations in the Handbook [9].

3.2.3.10 Summary of heat transfer correlations for flow in concentric annuli and annular ducts

Table 12 presents the list of all heat transfer correlations collected for flow in concentric annuli and annular ducts.

TABLE 12. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN CONCENTRIC ANNULI AND ANNULAR DUCTS

Werner et al. (1949) [107] [108]	$Nu = 0.7 \left(\frac{D_2}{D_1}\right)^{0.53} [7 + 0.025 Pe^{0.8}]$ $\frac{D_2}{D_1} \leq 1.4$
Bailey (1950) [107] [109]	$Nu = 0.75 \left(\frac{D_2}{D_1}\right)^{0.3} [7 + 0.025 Pe^{0.8}]$ $\frac{D_2}{D_1} > 1.4$
Lyon (1952) [110] [111]	$Nu = (5.25 + 0.0175\ Pe) \left(\frac{D_2}{D_1}\right)^{0.53}$ uniform inner wall heat flux, outer wall insulated
Hartnett-Irvine (1957) [9] [13] [58]	$Nu = \frac{2}{3} Nu_{slug} + 0.015\ Pe^{0.8}$ Nu_{slug} in Fig. 13, Fig. 14, and Fig. 15
Buleev (1959) [62] [65] [104]	$Nu = 5.1 + 0.02 Pe^{0.8}$ $\frac{r_2}{r_1} \leq 2$, uniform outer wall heat flux
Baker-Sesonske (1962) [83] [65]	$Nu = 0.80 \left(\frac{r_2}{r_1}\right)^{0.3} [5.12 + 0.0296 Pe^{0.785}]$
Dwyer (1963a) [8] [9]	uniform outer wall heat flux, inner wall insulated $Pe \leq Pe_{crit}$: $Nu = 5.52 + 0.076 \frac{r_2}{r_1}$ $Pe \geq Pe_{crit}$: $Nu = A_0 + B_0 \left(\frac{Pe}{Pr_t}\right)^{n_0}$ $A_0 = 5.26 + 0.05 \frac{r_2}{r_1}$ $B_0 = 0.01848 + 0.003154 \frac{r_2}{r_1} - 0.0001333 \left(\frac{r_2}{r_1}\right)^2$ $n_0 = 0.78 - 0.01333 \frac{r_2}{r_1} + 0.000833 \left(\frac{r_2}{r_1}\right)^2$ Pe_{crit} in Table 5

TABLE 12. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN CONCENTRIC ANNULI AND ANNULAR DUCTS

Dwyer (1963a) [8] [9]	uniform inner wall heat flux, outer wall insulated $Pe \le Pe_{crit}$: $Nu = 4.92 + 0.686\frac{r_2}{r_1}$ $Pe \ge Pe_{crit}$: $\mathrm{Nu} = A_i + B_i \left(\frac{Pe}{Pr_t}\right)^{n_i}$ $A_i = 4.63 + 0.686\frac{r_2}{r_1}$ $B_i = 0.02154 - 0.000043\frac{r_2}{r_1}$ $n_i = 0.752 + 0.01657\frac{r_2}{r_1} - 0.000833\left(\frac{r_2}{r_1}\right)^2$ Pe_{crit} in Table 5
Dwyer (1963b) [65] [112] [113]	uniform outer wall heat flux, inner wall insulated $Nu = A_0 + B_0 \left(\frac{Pe}{Pr_t}\right)^{n_0}$ $A_o = 5.54 - 0.023\frac{r_2}{r_1}$ $B_o = 0.0189 + 0.00316\frac{r_2}{r_1} + 0.0000867\left(\frac{r_2}{r_1}\right)^2$ $n_o = 0.758\left(\frac{r_2}{r_1}\right)^{0.0204}$ fully-developed flow $1 \le \frac{r_2}{r_1} \le 7, 0.005 \le Pr \le 0.03, 3 \cdot 10^2 \le Pe \le 10^5$
Dwyer (1963b) [65] [112] [113]	uniform inner wall heat flux, outer wall insulated $Nu = A_i + B_i \left(\frac{Pe}{Pr_t}\right)^{n_i}$ $A_i = 4.82 + 0.697\frac{r_2}{r_1}$ $B_i = 0.0222$ $n_i = 0.758\left(\frac{r_2}{r_1}\right)^{0.053}$ fully-developed flow, $1 \le \frac{r_2}{r_1} \le 7, 0.005 \le Pr \le 0.03,\ 3 \cdot 10^2 \le Pe \le 10^5$

TABLE 12. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN CONCENTRIC ANNULI AND ANNULAR DUCTS

Dwyer (1965) [11] [65][115] [118]	bilateral uniform heat flux from the inner wall $Nu = A + B\left(\frac{Pe}{Pr_t}\right)^n$ $A = 7.82 + 1.72\frac{r_2}{r_1} - 0.051\left(\frac{r_2}{r_1}\right)^2$ $B = 0.0592 - 0.000342\frac{r_2}{r_1} + 0.000723\left(\frac{r_2}{r_1}\right)^2$ $n = 0.655 + 0.0363\frac{r_2}{r_1} - 0.0037\left(\frac{r_2}{r_1}\right)^2$ $1 \leq \frac{r_2}{r_1} \leq 7,$
Dwyer (1965) [11] [65][115] [118]	bilateral uniform heat flux from the outer wall $Nu = A + B\left(\frac{Pe}{Pr_t}\right)^n$ $A = 7.11 + \frac{3.22}{\frac{r_2}{r_1}} - \frac{0.842}{\left(\frac{r_2}{r_1}\right)^2}$ $B = 0.0396 + \frac{0.0200}{\frac{r_2}{r_1}}$ $n = 0.746 - \frac{0.0864}{\frac{r_2}{r_1}} + \frac{0.0282}{\left(\frac{r_2}{r_1}\right)^2},$ $1 \leq \frac{r_2}{r_1} \leq 7$
Kays-Leung (1963) [11] [102]	uniform heat flux from the inner wall $Nu = 4.98 + 0.662\frac{r_2}{r_1}$ uniform heat flux from the outer wall: $Nu = 5.60 + 0.195\left(\frac{r_2}{r_1} - 1\right)^{0.64}(log\,Re - 3.70)^{0.54}$ Both are valid for $0.001 \leq Pe \leq 0.03$, $10^4 \leq Re \leq 10^6$ and $1.0 \leq \frac{r_2}{r_1} \leq 10.0$
Rensen (1981) [116]	uniform inner wall heat flux and outer wall insulated $Nu = 5.75 + 0.022Pe^{0.8}$ $0.0047 \leq Pr \leq 0.0059$, $6 \cdot 10^3 \leq Re \leq 6 \cdot 10^4$ $(28.2 \leq Pe \leq 354)$.

The empirical correlations for Nusselt number presented in Table 12 are also plotted in Fig. 16. For the purpose of comparison, a nominal value for the external-to internal ratio of the concentric radii or diameters (if required in the correlation) was selected such that ${}^{r_2}/_{r_1} = 1.5$, which is a typical value for several applications. The visual comparison in Fig. 16. omits Harnett-Irvine (1957) [9] [13] [58] and Dwyer (1965) [11] [65][115], because these correlations require additional parameters or specific conditions which may not be comparable with the other more generalized correlations. Additionally, Kays-Leung (1963) [11] [102] is omitted because the proposed correlation is not a function of Peclet Number.

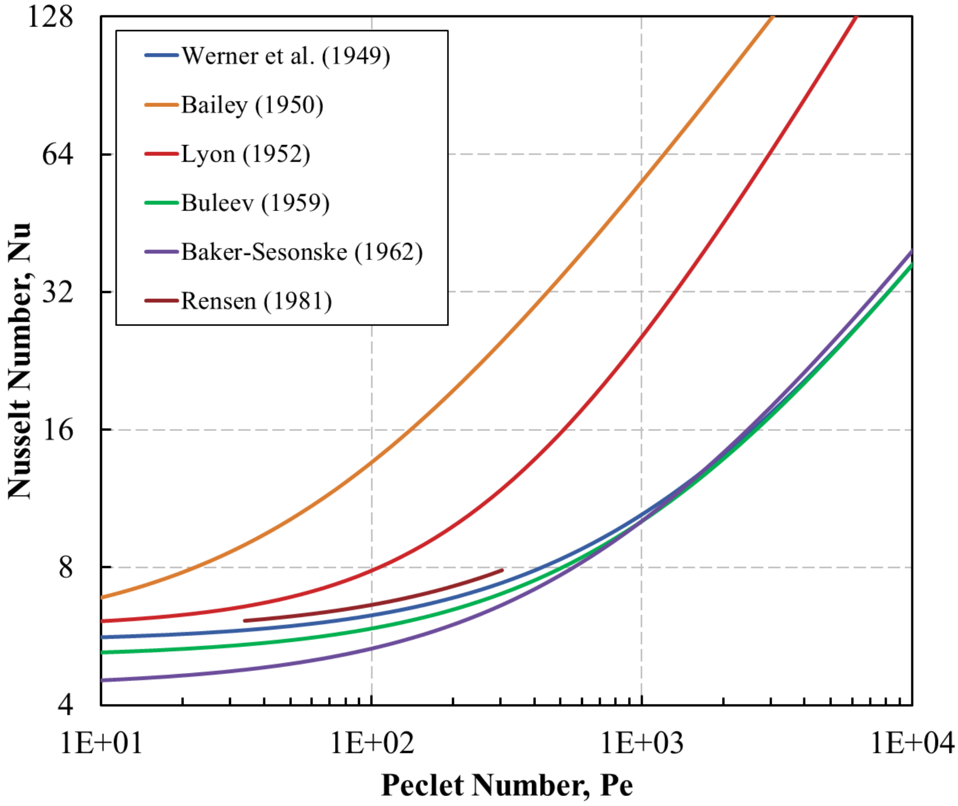

FIG. 16. Comparison of common correlations for Nusselt numbers for flow in concentric annular ducts, $r_2\ /\ r_1\ = 1.5$*, Werner (1949) at* $r_2\ /\ r_1\ = 1.4$

3.2.4 Flow in noncircular ducts

The heat transfer geometry in non-circular ducts can be seen in Fig. 17,
where *a, b – dimensions,* T_w *– wall temperature,* T_f *– film (bulk) temperature, q – heat load*

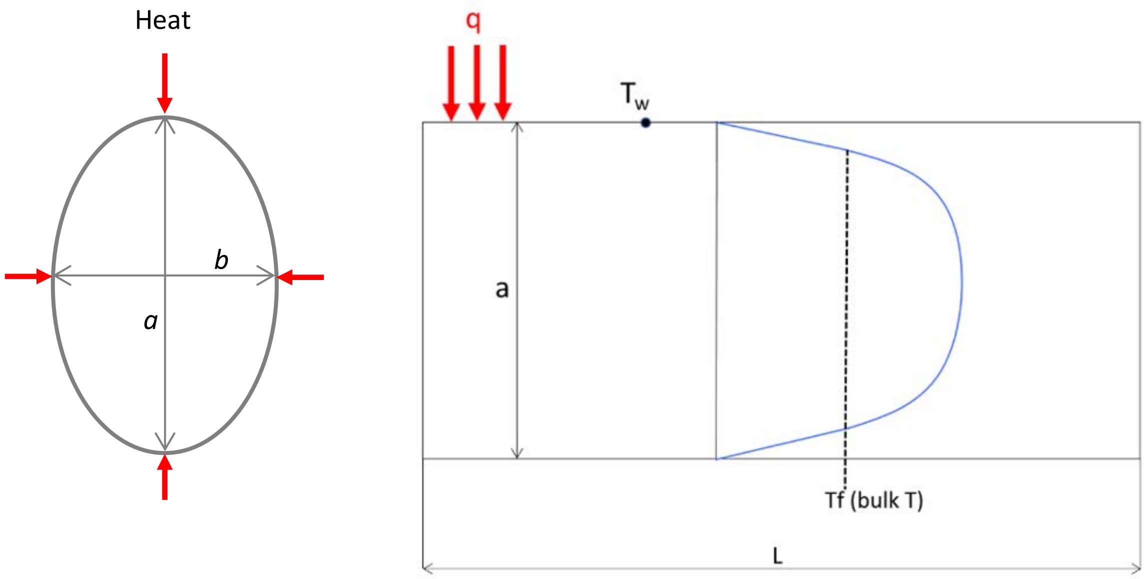

FIG. 17. Flow in non-circular ducts

3.2.4.1 Hartnett-Irvine (1957)

A simple correlation is available for estimating fully developed Nusselt numbers for turbulent flow of liquid metals in elliptical ducts with the constant wall temperature and constant axial wall heat flux boundary conditions. This correlation was derived by J.P. Hartnett and T.F. Irvine in 1957 for a uniform velocity distribution (slug flow) and a pure molecular conduction heat transfer mechanism [58]. This is a good approximation for liquid metals with $Pr \rightarrow 0$. This correlation is given by [9]:

$$Nu = \frac{2}{3} Nu_{slug} + 0.015\, Pe^{0.8} \tag{151}$$

Here Nu_{slug} is the Nusselt number corresponding to slug flow ($Pr = 0$) in ellipsoidal ducts. Figure 18 shows the slug Nusselt number for various ellipsoidal sizes.

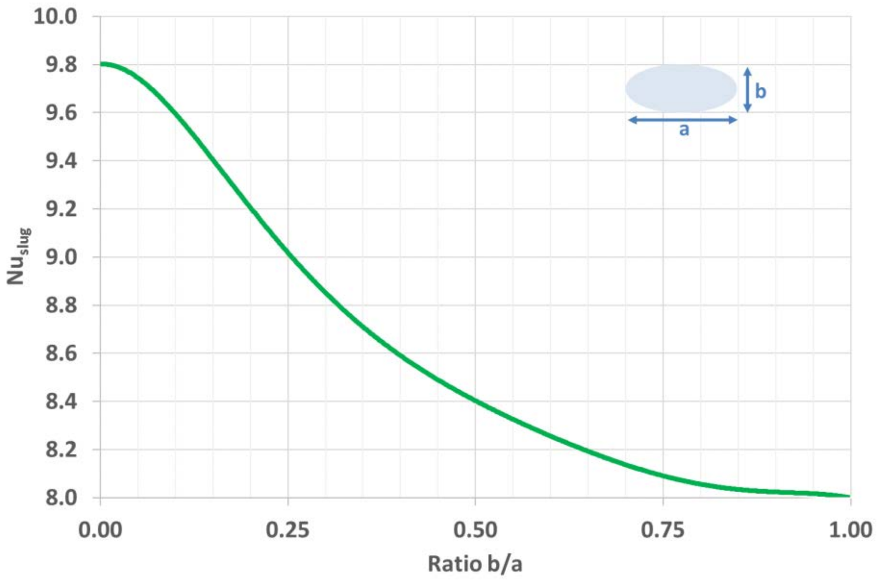

FIG. 18. Slug Nusselt numbers for elliptical ducts with uniform wall temperature conditions

3.2.4.2 Kottowski (1983)

In 1983, H.M. Kottowski investigated Nusselt number for low Prandtl number fluids for various channel shape geometry in forced convection [119]. Modifying the equation proposed by J.P. Hartnett and T.F. Irvine [58] for flow in noncircular shape channels, Kottowski proposed:

$$Nu = \frac{2}{3} Nu_{slug} + 0.025\ Pe^{0.8} \qquad (152)$$

where Nu_{slug} is the Nusselt number for slug flow. Values of Nu_{slug} have been already presented previously in the correlations recommended by Hartnett and Irvine (see Fig. 10, Fig. 13, Fig. 14, Fig. 15 and Fig. 18). This equation is valid when free convection effects are negligible, when heat transfer surfaces are clean and there is no gas entrainment. The fluid properties (e.g. Nusselt and Peclet) are evaluated at the bulk mean temperature.

3.2.4.3 Summary of heat transfer correlations for flow in noncircular ducts

Table 13 presents the list of all heat transfer correlations collected for flow in noncircular ducts.

TABLE 13. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN NONCIRCULAR DUCTS

Hartnett-Irvine (1957) [9] [58]	$Nu = \frac{2}{3} Nu_{slug} + 0.015\ Pe^{0.8}$ Nu_{slug} in Fig. 18 elliptical ducts, constant wall temperature, constant heat flux
Kottowski (1983) [119]	$Nu = \frac{2}{3} Nu_{slug} + 0.025\ Pe^{0.8}$ Nu_{slug} in Fig. 10, Fig. 13, Fig. 14, Fig. 15, Fig. 18 no free convection effects, clean surfaces, no gas entrainment

The empirical correlations for Nusselt number, Nu, presented in Table 13 are also plotted in Fig. 19. For the purpose of comparison, a nominal value for the ratio of the ellipsoidal duct's radii or diameters was selected such that $b/a = 0.5$. Note both correlations make additional conditions for validity.

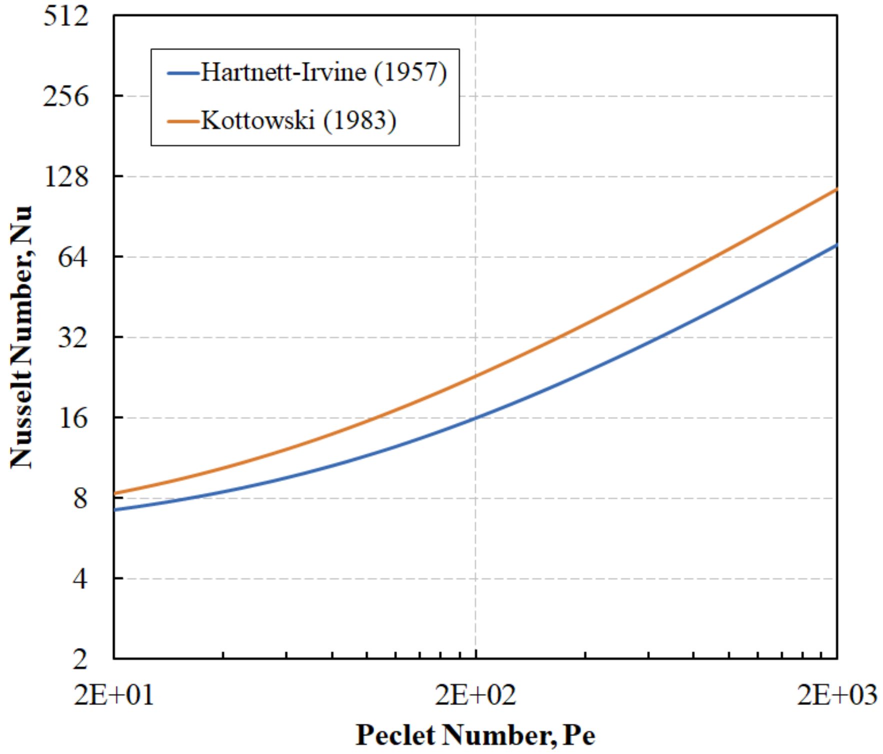

FIG. 19. Comparison of common correlations for Nusselt numbers for heat transfer in elliptical ducts, $b/a = 0.5$

3.2.5 Cross flow around circular tubes and cylinders

The geometry of cross flow around circular tubes and cylinders is shown in Fig. 20.

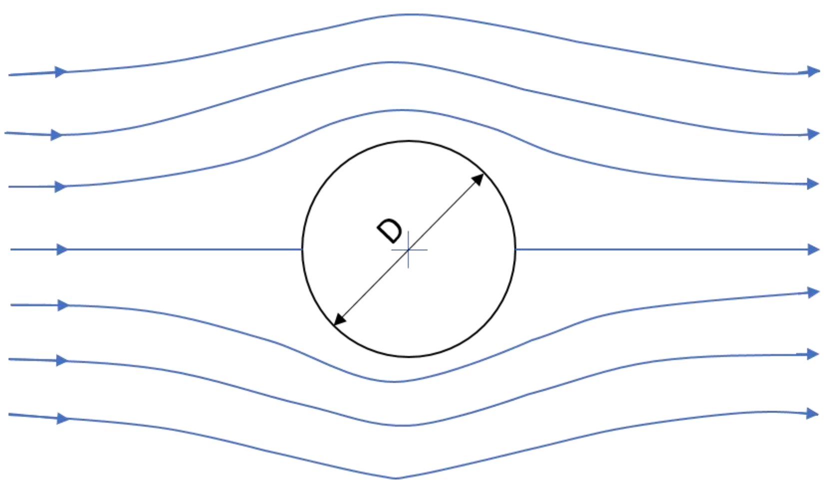

FIG. 20. Cross flow around circular tubes and cylinders

3.2.5.1 Martinelli (1948)

For turbulent cross flow around cylinders, R.C. Martinelli proposed in 1948 the following correlation [110]:

$$Nu = 0.80\, Pe^{0.5} \tag{153}$$

3.2.5.2 Andreevskii (1961)

A.A. Andreevskii (1961) proposed an averaged Nusselt number correlation for cross flow in a single cylinder, based on the heat flux, the average wall temperature $\overline{T}_w$ of the cylinder, the bulk mean temperature T_b of the fluid, and the outside diameter of the cylinder [9] [120]. The fluid properties should be evaluated at the film temperature, defined as:

$$T_f = \frac{\overline{T}_w + T_b}{2} \tag{154}$$

By using the film temperature, the effect of variable fluid properties is minimized (variables evaluated in this manner have a subscript *f*, e.g., Pe_f). The data of Andreevskii for this case are well correlated by the following equation [9]:

$$Nu_f = 0.65\, {Pe_f}^{0.5} \tag{155}$$

The empirical coefficient 0.65 is below the theoretical values (1.015 for uniform wall temperature and 1.145 for uniform wall heat flux) given by C.J. Hsu 1964 [121]. In both experimental work and commercial practice, however, the thermal boundary condition is neither of the two, but something in between. The coefficient 0.65 should represent all practical situations.

3.2.5.3 Churchill-Bernstein (1977)

S.W. Churchill and M.A. Bernstein (1977) recommended an equation for various types of fluid (e.g. different Prandtl numbers, including sodium) for flow passing in cross flow through a single cylinder, in case of a laminar boundary layer regime. The correlation postulated is as follows [122]:

$$\overline{Nu} = \frac{A\, Re^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} = A\, \Phi \tag{156}$$

The approximate, theoretical calculations of Masliyah and Epstein for $Re = 1$ and $0.7 \leq Pr \leq 4 \cdot 10^4$ indicate a value of 0.62 and the two, extrapolated, theoretical values of Jain and Goel a value of 0.64 for Eq. (156) with $A = 0.62$ provides an excellent representation for $40 \leq \mathrm{Re} \leq 10^3$ corresponding to $5 \leq \Phi \leq 80$ for $Pr = 0.7$.

In case of a creeping-flow regime ($Pe \leq 0.2$), the correlation used has the following form:

$$\overline{Nu} = \frac{1}{\left[0.8237 - \ln\left(Pe^{2/3}\right)\right]^{1/4}} \tag{157}$$

The values of $\overline{Nu}$ computed from Eq. (157) approach 0 as $Re \rightarrow 0$, as it would be expected for pure conduction from an infinitely long cylinder to surroundings of infinite extent. The finite values observed experimentally for $Re \rightarrow 0$ are presumably due to free convection, end effects and finite surroundings.

In case of intermediate regime ($Pe \geq 0.2$, $Re \leq 10^4$), a considerable gap exists between the range of applicability of Eqs. (156) and (157). This behaviour can be approximated, as suggested by Tsubouchi, Masuda and others, by adding a constant term, $\overline{Nu}_0$, to the right-hand side of Eq. (156). A proposed constant value of 0.3 for $\overline{Nu}_0$ results in:

$$\overline{Nu} = 0.3 + \frac{0.62\, Re^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \tag{158}$$

Collis and Williams asserted that an expression in the form of Eq. (158) is unsatisfactory because it cannot reproduce the discrete change in slope, which they have observed with the onset of eddy shedding ($Re = 44$ for $Pr = 0.7$).

For the complete turbulent region, the following asymptotic expression for very large Re can be derived from the data of Achenbach:

$$\overline{Nu}_\infty = 0.00091\, Re \tag{159}$$

Postulating the same dependence on Pr as in the laminar-boundary-layer regime Eq. (159) converts to:

$$\overline{Nu}_\infty = \frac{0.001168\, Re\, Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \tag{160}$$

It should be emphasized that Eqs. (159) and (160) are the apparent asymptotes for the Achenbach data, not correlations for them.

An overall correlation is obtained combining Eqs. (158) and (160) in the form suggested by Churchill and Usagi, which results in the following expression:

$$\overline{Nu} = 0.3 + \frac{0.62\, Re^{1/2}\, Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5} \tag{161}$$

and appears to provide a lower bound for $RePr \geq 0.4$ and a reasonable approximation for all Reynolds and Prandtl numbers.

Summary on the proposed equations:

- Eq. (161) is proposed as a lower bound for the computed and experimental values of heat transfer by forced convection to a cylinder in cross flow for all Re and Pr, such that $RePr \geq 0.4$.
- As a lower bound, Eq. (161) represents the behaviour for low free-stream turbulence, an isothermal surface, negligible blockage, negligible end-effects, a small temperature difference and negligible free convection. A possible exception is in the range of $10^3 \leq \mathrm{Re} \leq 10^4$ where the data of Hilpert for air and of Ishiguro et al. for sodium appear to

follow Eq. (158) rather than Eq. (161). In the range of $7 \cdot 10^4 \leq \text{Re} \leq 4 \cdot 10^5$, $\overline{Nu}$ may be significantly higher than predicted by Eq. (161) owing to a downstream shift of the point of separation of the laminar boundary layer.

- Eq. (158) can be used as an approximation for Eq. (161) for $Re \leq 4 \cdot 10^3$ and all Prandtl numbers.
- Eq. (157), which is based on the assumption of creeping flow, should provide a better representation than Eqs. (158) or (159) for $RePr \leq 0.2$ if free convection and end-effects are negligible. It agrees well with such experimental data for air but has not been tested critically for a wide range of Prandtl numbers.

Eq. (158) appears to provide reasonably good predictions even for $RePr \leq 0.2$ and can be modified to provide an even better representation for any single set of data by the proper, arbitrary choice of Nu_0.

3.2.5.4 Summary of heat transfer correlations for cross flow in circular pipes and cylinders

Table 14 summarized the list of heat transfer correlations collected for cross flow in circular pipes and cylinders.

TABLE 14. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR CROSS FLOW IN CIRCULAR PIPES AND CYLINDERS

Martinelli (1948) [110]	$Nu = 0.80\ Pe^{0.5}$
Andreevskii (1961) [9] [120]	$Nu_f = 0.65\ {Pe_f}^{0.5}$
Churchill-Bernstein (1977) [122]	For creeping-flow regime ($Pe \leq 0.2$): $\overline{Nu} = \frac{1}{\left[0.8237 - \ln\left(Pe^{2/3}\right)\right]^{1/4}}$ For intermediate regime ($Pe \geq 0.2$, $Re \leq 10^4$) $\overline{Nu} = 0.3 + \frac{0.62\ Re^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}}$ For the complete turbulent region: $\overline{Nu}_\infty = 0.00091\ Re$ $\overline{Nu}_\infty = \frac{0.001168\ Re\ Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}}$ For $RePr \geq 0.4$. $\overline{Nu} = 0.3 + \frac{0.62\ Re^{1/2}\ Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}$

The empirical correlations for Nusselt number, Nu, presented in Table 14 are also plotted in Fig. 21. For the purpose of comparison, a nominal value for the Prandtl number is assumed $Pr = 0.0045$ (if required in the correlation) as discussed in earlier sections.

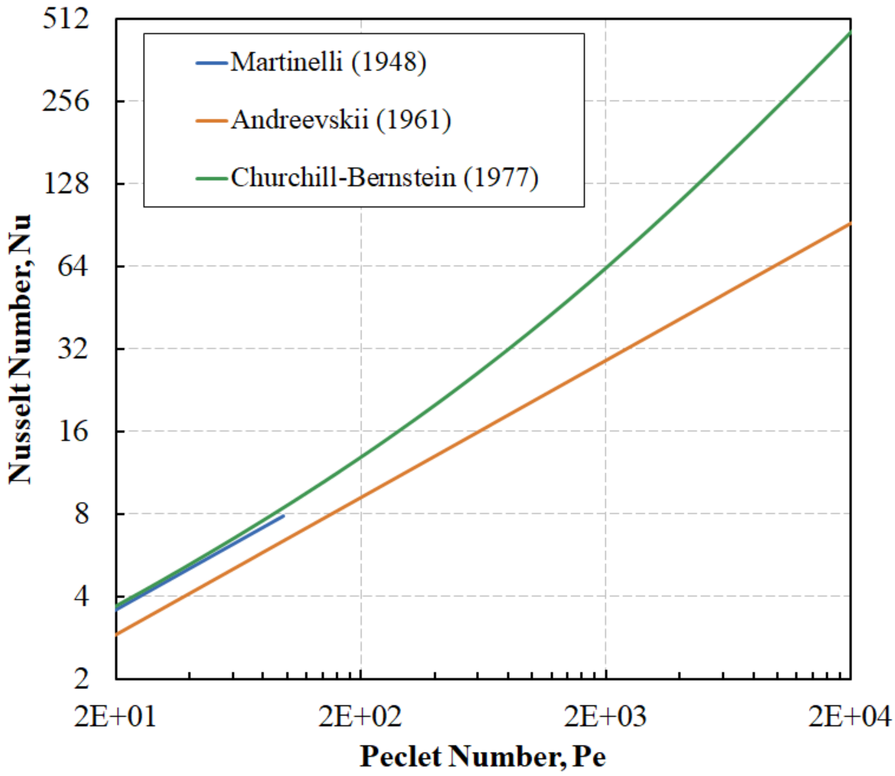

FIG. 21. Comparison of common correlations for Nusselt numbers for heat transfer around circular tubes and cylinders
Martinelli [110], Eq. (153)
Andreevskii [120], Eq. (161)
Churchill-Bernstein [122], Eq. (161)

3.2.6 Flow in the shell side of heat exchangers

The flow geometry in the shell side of heat exchangers is shown in Fig. 22 below.

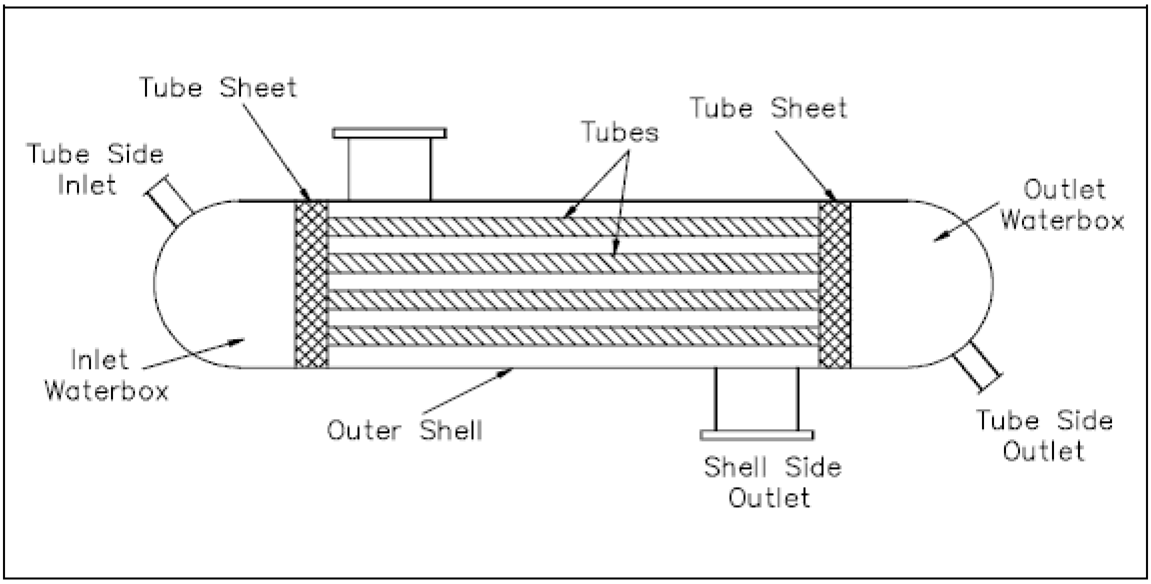

FIG. 22. Flow in the shell side of heat exchangers

3.2.6.1 Brooks-Rosenblatt (1952)

In 1952 R.D. Brooks and A.L. Rosenblatt presented the following correlation for the shell side of unbaffled heat exchangers valid for the turbulent regime [110] [123]:

$$Nu = 61.2 \left[Pe \left(\frac{A_F}{A_H} \right)^2 \right]^{0.6} \tag{162}$$

where A_F is the flow area in the shell side of a heat exchanger, parallel to tube axis and A_H is the area for heat transfer based on outside diameter tubes. In the report [124] from 1979 about the heat transfer characteristics of IHX (intermediate heat exchanger) and DHX (decay heat exchanger) for Joyo reactor, the above correlation is also presented and referred to as JSME correlation.

3.2.6.2 Schroeder-Chionohio (1959)

In 1959 R.W. Schroeder and M.A. Chionohio presented the following correlation for the shell side of heat exchangers [6] [125]:

$$Nu = 0.313 + 0.2\ \mathrm{Pe}^{0.613} \tag{163}$$

In the report [124] from 1979 about the heat transfer characteristics of IHX and DHX for Joyo reactor this correlation is presented referred to as USAEC TID-6881 correlation.

3.2.6.3 *Fast-Reactor-Technology (JOYO start-up test report, 1979)*

In the JOYO start-up test report [124] from 1979 about the heat transfer characteristics of IHX and DHX for Joyo reactor the following correlation is presented for the shell side and axial flow [6]:

$$Nu = 0.106\,(D_e\,Pe)^{0.6}, \quad (164)$$

where D_e is an equivalent diameter (in inches). This correlation is referred to as Fast-Reactor-Technology correlation in [124].

3.2.6.4 *Summary of heat transfer correlations for flow in the shell side of heat exchangers*

Table 15 presents the list of heat transfer correlations collected for flow in the shell side of heat exchangers.

TABLE 15. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN THE SHELL SIDE OF HEAT EXCHANGERS

Brooks-Rosenblatt (1952) [110] [123]	$Nu = 61.2\left[Pe\left(\frac{A_F}{A_H}\right)^2\right]^{0.6}$
Schroeder-Chionohio (1959) [6] [125]	$Nu = 0.313 + 0.2\,\mathrm{Pe}^{0.613}$
Fast-Reactor-Technology (1979) [6] [124]	$Nu = 0.106\,(De\,Pe)^{0.6}$

No summary comparison figure is included for the flow in the shell side of heat exchangers due to the inclusion of different variables across the correlations.

3.2.7 Entrance region effects

The temperature distribution and heat transfer are affected by the change of velocity profile at the channel entrance. A typical redistribution of the velocity in the entrance region is shown in Fig. 23.

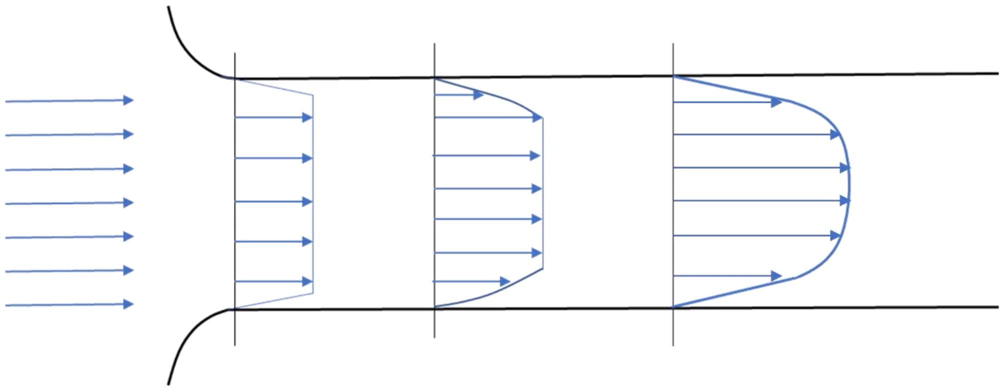

FIG. 23. Redistribution of velocity profile at the entrance region

3.2.7.1 Sidelnikov et al. (1973)

In 1973 V.N. Sidelnikov et al. recommended the following formula to determine the heat transfer coefficient and temperature behaviour at the entrance region of a pin bundle for hydraulically stable flow in turbulent and laminar regimes [126-131]:

$$\frac{F(X)}{F_{stab}} = 1 - \left[p(50p)^{-X/X_{en.}} + (1-p)(50p)^{-X/X_{en.}}\right] \quad (165)$$

which is valid for $1.02 \le P/D \le 2.0$; $0.01 \le \varepsilon_6 \le 10$; $100 \le Pe \le 2500$,

$F(X)$ represents the functions $F_1(X) = 1/Nu$ and $F_2(X) = \Delta T = \frac{t_w^{max} - t_w^{min}}{\bar{q}R}\lambda_f$ at the entrance section.

F_{stab} is the value of functions $F_1(X)$ and $F_2(X)$ at the developed heat transfer conditions.

$X_{en.Nu} = \frac{l_{Nu}}{R} \cdot Pe¢$ is the non-dimensional length of Nusselt number entrance section.

$X_{en.t} = \frac{l_t}{R} \cdot Pe¢$ is the non-dimensional length of temperature non-uniformity entrance section.

l_{Nu} and l_t are the length of Nusselt number entrance section and length of temperature non-uniformity entrance section, respectively.

e_6 is the equivalent thermal conductivity based on the sixth harmonics in Fourier (see Section 3.5)

$\bar{q}$ is the heat flux averaged around the pin perimeter.

R is the pin radius.

λ_f is the coolant conductivity.

Values $X_{en.Nu}$, $X_{en.t}$ and factors p_{Nu} and $p_t$are presented in the works of V.N. Sidelnikov et al. graphically and can be also found in [132].

For the laminar flows, the ranges of application for relationship (165) are
$1.01 \le P/D \le 2.0$; $0.01 \le e_6 \le 10$; $Pe \le 100$.

3.2.7.2 Zhukov et al. (1977)

In 1977 A.V. Zhukov et al. [133] proposed that the length of the heat transfer entrance section of a pin bundle can be evaluated by this empirical formula for a hydraulically instable flow:

$$\left(\frac{l}{D_h}\right)_{Nu} = A - \frac{B}{255 + Pe} \quad (166)$$

where

$$\begin{aligned} A &= 156.2 - 102.4\frac{P}{D} \\ B &= \left\{51 - 34.5\frac{P}{D} - 4e^{-14.27\left(\frac{P}{D}-1\right)}\right\}10^3 \end{aligned} \quad for\ 1 \le P/D \le 1.2 \quad (167)$$

$$A = 95\left(\frac{P}{D}\right)^{-5.8}$$
$$B = 25.3\left(\frac{P}{D}\right)^{-5.6} 10^3 \qquad for\ 1.2 \leq {}^{P}/_{D} \leq 1.7 \tag{168}$$

the ranges of application are:

$0.4 \leq e_6 \leq 1.0$ for $1.0 \leq {}^{P}/_{D}$ 1.20

$0.4 \leq e_6 \leq 1.6$ for $1.2 \leq {}^{P}/_{D} \leq 1.70$

For Peclet number

$15 \leq Pe \leq 800$ for ${}^{P}/_{D} = 1.0$

$30 \leq Pe \leq 2500$ for ${}^{P}/_{D} = 1.06$

$50 \leq Pe \leq\ 3000$ for $1.10\ < {}^{P}/_{D} <\ 1.70$.

In tight bundles of pins having high thermal conductivity (${}^{P}/_{D} =\ 1.10;\ e_6 \sim 10 - 15$) the length of thermal entrance section defined by Eq. (166) has to be reduced by 30– 40%.

3.2.7.3 Zhukov et al. (1977)

In 1977 A.V. Zhukov et al. proposed to evaluate the length of temperature non-uniformity entrance section of a pin bundle by the following correlation [130] [131] [133] [134]:

$$\left(\frac{l}{D_h}\right)_t = \left(\frac{l}{D_h}\right)_{Nu} (18.1 - 4.5\log Pe)\left(\frac{P}{D} - 1\right) + 1 \tag{169}$$

It is valid for $1.0 \leq {}^{P}/_{D} \leq 1.2$, $0.4 \leq \varepsilon_6 \leq 1.0$, $500 \leq Pe \leq 2000$.

3.2.7.4 Chen-Chiou (1981)

C.J. Chen and J.S. Chiou (1981) made an extensive study on thermal entrance effects for liquid metal flows. The proposed correlations for entry regions effects of a flow in a round pipe are as follows [9] [90].

Two flow regions are considered: developing thermal and developing thermal and velocity fields. In the fully developed region (FD), both velocity and temperature profiles are developed. In the developing thermal region (DT) the velocity profile is fully developed but the temperature profile is developing.

Developing thermal (DT) region

An approximate formula for the local Nusselt number Nu_x for both conditions uniform wall temperature and uniform heat flux, can be derived as follows:

$$\frac{Nu_x}{Nu} = 1 + \frac{2.4}{\frac{x}{D}} - \frac{1}{\left(\frac{x}{D}\right)^2},\ \text{for}\ \frac{x}{D} \geq 2,\ Pe \geq 500\ \text{and}\ 0.004 \leq Pr \leq 0.1 \tag{170}$$

while for the average Nusselt number Nu_m:

$$\frac{Nu_m}{Nu} = 1 + \frac{7}{\frac{L}{D}} + \frac{2.8}{\frac{L}{D}} \ln\left(\frac{L/D}{10}\right), \text{ for } \frac{L}{D} \geq 2, Pe \geq 500 \text{ and } 0.004 \leq Pr \leq 0.1 \quad (171)$$

where:

$$Nu = 5.6 + 0.0165\, Pe^{0.85} Pr^{0.01} \qquad \text{for constant heat flux} \quad (172)$$

$$Nu = 4.5 + 0.0156\, Pe^{0.85} Pr^{0.01} \qquad \text{for constant wall temperature} \quad (173)$$

The range of application is $0 \leq Pr \leq 0.1$ and $10^4 \leq Re \leq 5 \cdot 10^6$ [31]. In the above correlations the following nomenclature is used: x is the axial distance from the entrance region, D is the diameter and L is the axial length.

Eq. (172) and (173) are improvements of Sleicher's formulas Eqs. (56) and (57).

Developing thermal and velocity (DTV) region

The calculated local Nusselt number Nu_x in the DTV region may be correlated in an approximated form by:

$$\frac{Nu_x}{Nu} = 0.88 + \frac{2.4}{\frac{x}{D}} - \frac{1.25}{\left(\frac{x}{D}\right)^2} - E \text{ for } 2 \leq \frac{x}{D} \leq 35 \text{ and } 0.004 \leq Pr \leq 0.1 \quad (174)$$

and for the mean Nusselt number Nu_m:

$$\frac{Nu_m}{Nu} = 1 + \frac{5}{\frac{L}{D}} + \frac{1.86}{\frac{L}{D}} \ln\left(\frac{L/D}{10}\right) - F \text{ for } \frac{L}{D} \geq 2 \text{ and } 0.004 \leq Pr \leq 0.1 \quad (175)$$

In case of constant wall temperature, it is found that:

$$E = \frac{40 - x/D}{190}, F = 0.09 \quad (176)$$

In case of constant heat flux $E = F = 0$.

Corrections proposed in Section 3.2.1.27 by Chen and Chiou (1981) [90] for smooth circular ducts are also applicable to evaluate liquid metal properties in the entry region.

3.2.7.5 Marocco (2012)

In 2012 L. Marocco et al. [13] [135] proposed a correlation for the Nusselt number in the thermal entrance region of a vertical annulus with constant heat flux on the inner surface, based on the LBE data of Zeiniger (2009). They considered the fact that the fully developed Nusselt value Nu_∞ is well represented by the correlation proposed by Dwyer in 1963 Eqs. (138) and (139). The correlation recommended is valid for the range $400 \leq Pe \leq 6000$:

$$\frac{Nu_x}{Nu_\infty} = 1 + \frac{1.14}{\left(\frac{x}{D_h}\right)^{0.5}} \quad (177)$$

3.2.7.6 Summary of heat transfer correlations for flow in the entrance region

Table 16 presents the list of heat transfer correlations collected for flow in the entrance region.

TABLE 16. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN THE ENTRANCE REGION

Sidelnikov et al. (1973) [126] [127] [128] [129] [130] [131]	$\frac{F(X)}{F_{stab}} = 1 - \left[p(50p)^{-X/X_{en.}} + (1-p)(50p)^{-X/X_{en.}}\right]$ $F_1(X) = 1/_{Nu}$ and $F_2(X) = \Delta T = \frac{t_w^{max} - t_w^{min}}{\bar{q}R}\lambda_f$ stable flow through a pin bundle $1.02 \leq \frac{P}{D} \leq 2.0$, $0.01 \leq \varepsilon_6 \leq 10$, $100 \leq Pe \leq 2500$
Zhukov et al. (1977) [133]	$\left(\frac{l}{d_h}\right)_{Nu} = A - \frac{B}{255 + Pe}$ $A = 156.2 - 102.4\frac{P}{D}$ $B = \left\{51 - 34.5\frac{P}{D} - 4e^{-14.27\left(\frac{P}{D}-1\right)}\right\}10^3$ $for\ 1 \leq P/_D \leq 1.2$ $A = 95\left(\frac{P}{D}\right)^{-5.8}$ $B = 25.3\left(\frac{P}{D}\right)^{-5.6} 10^3$ $for\ 1.2 \leq P/_D \leq 1.7$ instable flow through a pin bound $0.4 \leq e_6 \leq 1.0$ *for* $1.0 \leq P/_D \leq 1.10$ $0.4 \leq e_6 \leq 1.6$ *for* $1.2 \leq P/_D \leq 1.70$ $15 \leq Pe \leq 800$ *for* $P/_D = 1.0$ $30 \leq Pe \leq 2500$ *for* $P/_D = 1.06$ $50 \leq Pe \leq 3000$ for $1.10 < P/_D < 1.70$
Zhukov et al. (1977) [130] [131] [133] [134]	$\left(\frac{l}{d_h}\right)_t = \left(\frac{l}{d_h}\right)_{Nu}(18.1 - 4.5\log Pe)\left(\frac{P}{D} - 1\right) + 1$ $1.0 \leq P/_D \leq 1.2$, $0.4 \leq \varepsilon_6 \leq 1.0$, $500 \leq Pe \leq 2000$

TABLE 16. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN THE ENTRANCE REGION

Chen-Chiou (1981) [9] [90]	**Developing thermal region (DT) in pipes** $\frac{Nu_x}{Nu} = 1 + \frac{2.4}{\frac{x}{D}} - \frac{1}{\left(\frac{x}{D}\right)^2},$ for $\frac{x}{D} \geq 2$, $Pe \geq 500$ and $0.004 \leq Pr \leq 0.1$ $\frac{Nu_m}{Nu} = 1 + \frac{7}{L/D} + \frac{2.8}{L/D}\ln\left(\frac{L/D}{10}\right)$, for $L/D \geq 2$ $Nu = 5.6 + 0.0165\, Pe^{0.85} Pr^{0.01}$ for constant heat flux $Nu = 4.5 + 0.0156\, Pe^{0.85} Pr^{0.01}$ for constant wall temperature **Developing thermal and velocity region (DTV) in pipes** $\frac{Nu_x}{Nu} = 0.88 + \frac{2.4}{x/D} - \frac{1.25}{(x/D)^2} - E,$ for $2 \leq x/D \leq 35$ and $0.004 \leq Pr \leq 0.1$ $\frac{Nu_m}{Nu} = 1 + \frac{5}{L/D} + \frac{1.86}{L/D}\ln\left(\frac{L/D}{10}\right) - F$ for $L/D \geq 2$
Marocco (2012) [13] [135]	$\frac{Nu_x}{Nu_\infty} = 1 + \frac{1.14}{\left(x/D_h\right)^{0.5}}$ for $400 \leq Pe \leq 6000$ with Nu_∞ from Eqs. (138) and (139) for vertical annulus with constant inner wall heat flux $400 \leq Pe \leq 6000$

No summary comparison figure is included for the entrance region due to the inclusion of different geometrical parameters across the correlations.

3.2.8 Axial flow in triangular rod array

The set of correlations presented hereafter corresponds to the configuration where the flow is parallel to the longitudinal axis of the rods arranged in the triangular pin array as shown in Fig. 24 below. The general expression of the correlations will follow the form as in the circular pipe cases Eq. (48) where the coefficients of the two terms are no more constants but functions depending on the pitch-to-diameter ratio P/D:

$$Nu = a(P/D) + b(P/D)Re^{c}Pr^{d} \quad (1)$$

Some correlations present c and d parameters depending on the P/D ratio, e.g. Eq. (191).

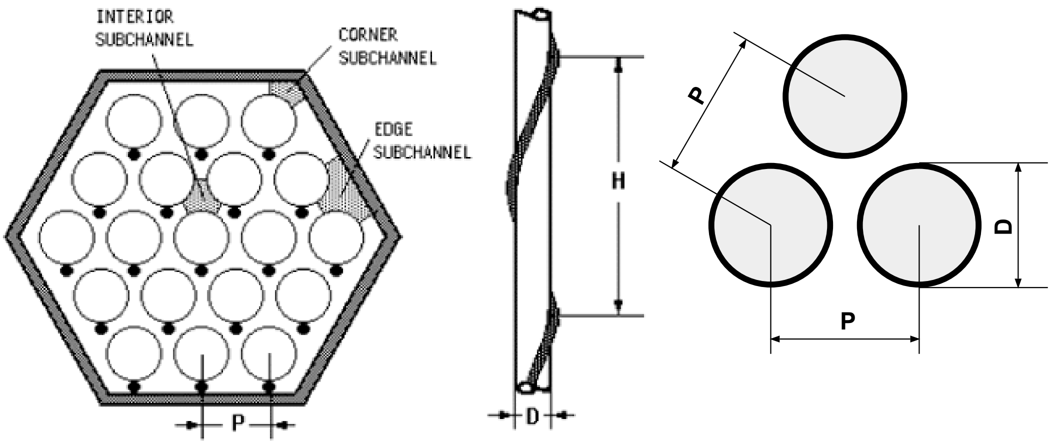

FIG. 24. Hexagonal fuel assembly and subchannels geometry

The hydraulic diameter for triangular pin array is calculated as $D_h = D\left[\frac{2\sqrt{3}}{\pi}\left(\frac{P}{D}\right)^2 - 1\right]$.

Due to the relevance of this geometry in the evaluation of SFR core designs, individual figures are added for each correlation for various P/D values.

3.2.8.1 Dwyer-Tu (1960)

A heat transfer equation for fully developed, turbulent, and parallel flow of liquid metals in staggered tube bundles in a triangular arrangement was derived by O.E. Dwyer and P.S. Tu in 1960 [136] [137]:

$$Nu = 0.93 + 10.81\frac{P}{D} - 2.01\left(\frac{P}{D}\right)^2 + 0.0252\left(\frac{P}{D}\right)^{0.273}\left(\frac{Pe}{Pr_t}\right)^{0.8} \tag{178}$$

where P is the pitch or distance between adjacent rod centerlines and D the outer diameter of the rods. It is valid for $10^2 \leq Pe \leq 10^4$, $1.375 \leq \frac{P}{D} \leq 2.2$. This equation was derived based on the experimental data obtained by the authors for flow of liquid metals outside the circular tubes arranged on an equilateral triangular pitch. A constant heat flux from the outer surfaces of the tubes was assumed, and the model of an annulus was used, i.e., the heat leaving each tube was assumed to be picked up by the flowing metal in an imaginary annulus surrounding the tube, the outer circumference of the annulus circumscribing an area equal to the total cross-sectional hexagonal area associated with each tube. The comparison of the Nusselt vs. Peclet numbers for several *P/D* ratios using $Pr = 0.0045$ is shown in Fig. 25.

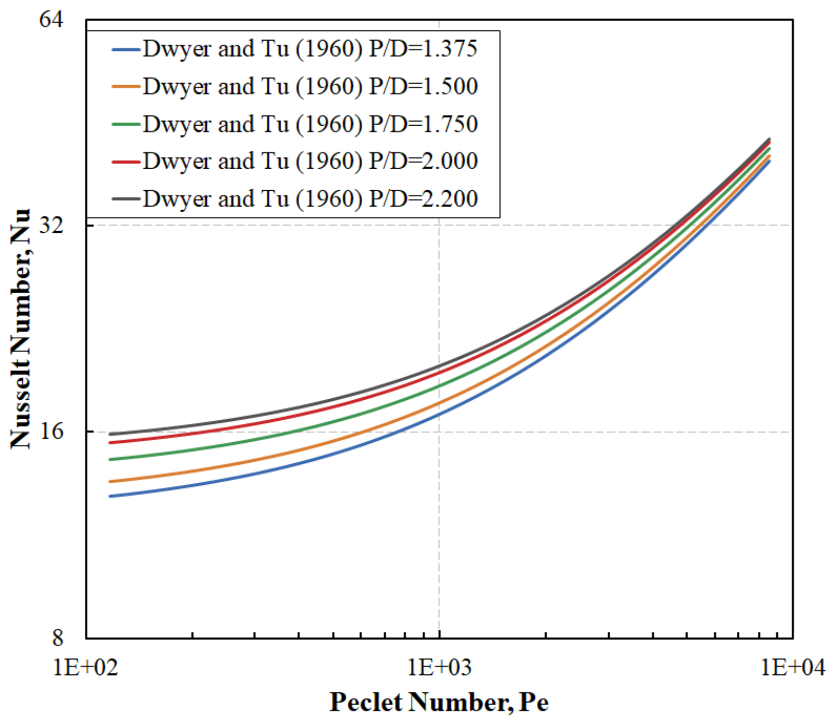

FIG. 25. Dwyer-Tu (1960) Empirical heat transfer correlations for axial flow in triangular rod array

3.2.8.2 Friedland-Bonilla (1961)

In 1961 A.J. Friedland and C.F. Bonilla made a theoretical analysis of heat transfer to liquid metals in parallel flow in a triangular tube bundle [65] [137] [138]. The equation proposed is the following:

$$Nu = 7.0 + 3.8\left(\frac{P}{D}\right)^{1.52} + 0.027\left(\frac{P}{D}\right)^{0.27}\left(\frac{Pe}{Pr_t}\right)^{0.8} \quad (179)$$

where the conditions are fully developed turbulent flow, constant heat flux at the wall, and an infinite number of tubes arranged on an equilateral triangular pitch. This equation is valid for $0 \leq Pe \leq 10^5$, $1.375 \leq \frac{P}{D} \leq 10$, $10^4 \leq Re \leq 10^6$, $0 \leq Pr \leq 0.1$. Comparison of the Nu vs. Pe numbers for several *P/D* ratios using $Pr = 0.0045$ is shown in Fig. 26 below.

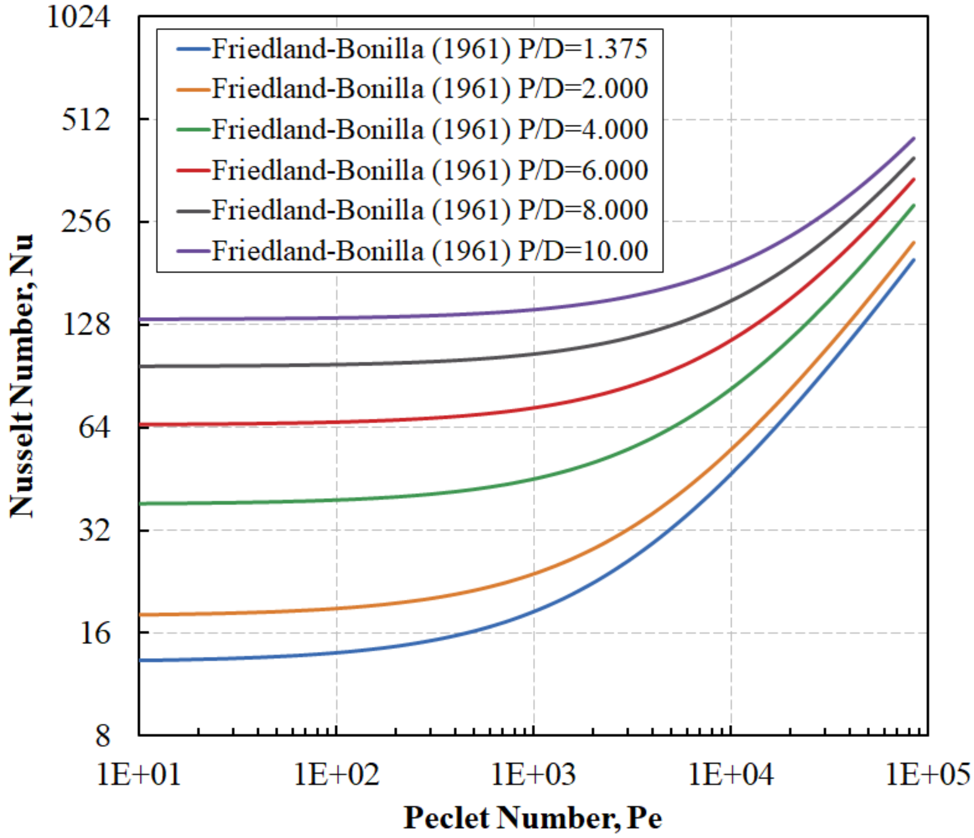

FIG. 26. Friedland-Bonilla (1961) Empirical heat transfer correlation for axial flow in triangular rod array

3.2.8.3 Borishanskii-Firsova (1964)

V.M. Borishanskii and E.V. Firsova (1964) studied the heat transfer in mercury ($Pr \sim 0.02$) flowing around rod bundles in triangular lattices with the relative spacing values of ${}^{P}/_{D} = 1.38$ and ${}^{P}/_{D} = 1.75$ (where P is the pitch and D is the outer diameter of the tube). They derived the following generalized correlation [139] (depicted in Fig. 27):

$$Nu = 6 + 0.006\,Pe \tag{180}$$

A comparison against experimental data for sodium and mercury shows a spread in the experimental points of $\pm 30\%$, where Reynolds and Peclet numbers varied over the range $2.8 \cdot 10^3 \leq Re \leq 4.3 \cdot 10^3$ and $28 \leq Pe \leq 172$. This large discrepancy may be a result of taking insufficiently the geometry of the system, such as the parameter P/D which is not present in the formula. It can be also due to a difference between the physical and chemical conditions at the liquid metal-wall interface.

This correlation can be used to model the shell side of shell-and-tube heat exchangers. However, a corrective coefficient is required to take into account the tubes length compared to the heat exchanger length.

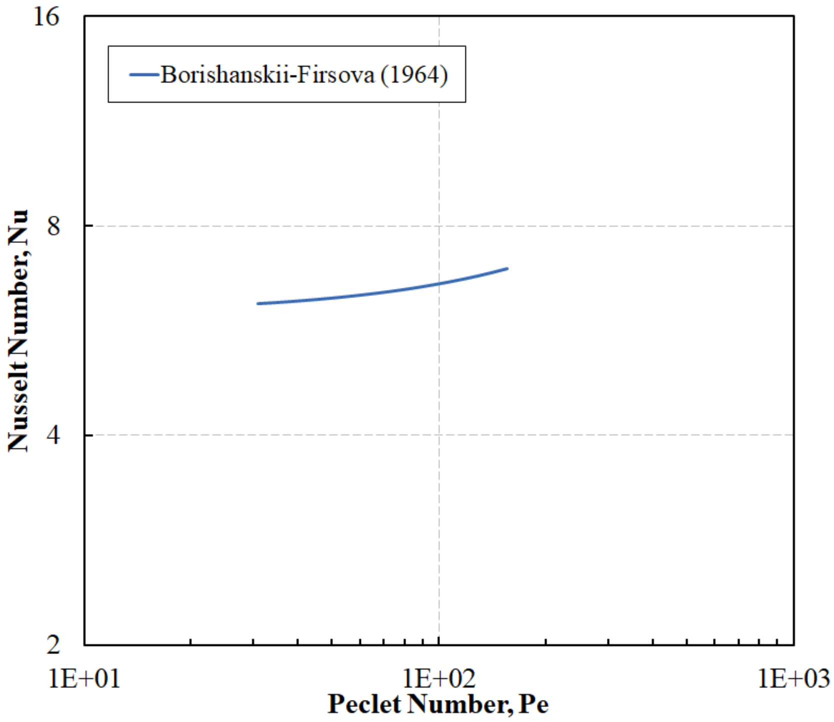

FIG. 27. Borishanskii-Firsova (1964) Empirical heat transfer correlation for axial flow in triangular rod array

3.2.8.4 Maresca-Dwyer (1964)

In the paper written by M.W. Maresca and O.E. Dwyer in 1964 [11] [137] [140], experimental results obtained for the case of in-line flow of mercury in an unbaffled bundle of circular rods were presented. The bundle consisted of 13 rods (13 mm outer diameter) arranged in an equilateral triangular pattern, the $\frac{P}{D}$ ratio being 1.750. All rods in the bundle were electrically heated to provide equal and uniform heat fluxes throughout the bundle. The total of 146 data pairs of Nu vs. Pe were presented and the following semi-empirical correlation was proposed:

$$Nu = 6.66 + 3.126\frac{P}{D} + 1.184\left(\frac{P}{D}\right)^2 + 0.0155\left(\frac{Pe}{Pr_t}\right)^{0.86} \tag{181}$$

It is valid for $70 \leq Pe \leq 10^4$, $1.3 \leq \frac{P}{D} \leq 3.0$.

In the paper of Zhukov-Subbotin-Ushakov 1969 [141] the authors referred to this correlation as the one of Dwyer and Tu from 1960 [142], however the good reference is Maresca and Dwyer from 1964 [140].

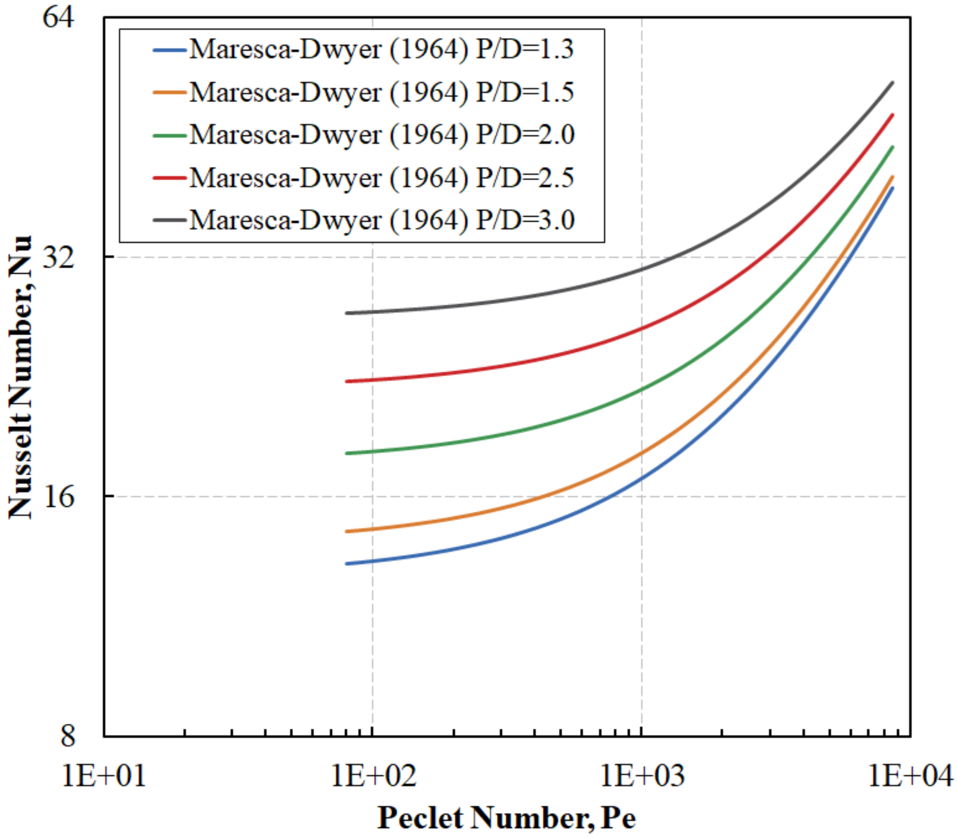

FIG. 28 Maresca-Dwyer (1964) Empirical heat transfer correlations for axial flow in triangular rod array

3.2.8.5 Subbotin et al. (1964)

In 1964 V.I. Subbotin et al. proposed a correlation for heat exchangers [65] [141] [143] [144]. The heat exchange in the experimental set-up was derived on the basis of seven tubes, where coolant inside the tubes was sodium and coolant outside the tubes was sodium-potassium in counter-current flow. The below correlation describes the shell-side Nusselt numbers between the tubes:

$$Nu = 8 \cdot \left[\frac{D_h}{L} + 0.027\left(\frac{P}{D} - 1.1\right)^{0.46}\right] \cdot Pe^{0.6} \tag{182}$$

where L is the pipe length and P is the pitch of the tubes. This correlation predicts the experimental data within the error band of $\pm 10\%$ and is valid for $200 \leq Pe \leq 1200$, $1.1 \leq \frac{P}{D} \leq 1.4$, $60 \leq \frac{L}{D_h} \leq 260$ and $6.8 \leq D \leq 7.6\ mm$.

3.2.8.6 Subbotin et al. (1965)

The following correlation was recommended by V.I. Subbotin et al. (1965) [144] for the flow of liquid metal in a triangular lattice of rods [137]:

$$Nu = 0.58\left(\frac{D_h}{D}\right)^{0.55} Pe^{0.45} \tag{183}$$

where D_h is the hydraulic diameter and D is the outer rod diameter. It is valid for $80 \leq Pe \leq 4000$, $1.1 \leq \frac{P}{D} \leq 1.5$.

This correlation is referred to as Orlov's in [145] where the author X. Cheng referred to the paper published by Subbotin et al. in the 3rd Conference for Peaceful Uses of Atomic Energy in 1965 [146].

3.2.8.7 Zhukov et al. (1969)

In a paper published in 1969 [147] A.V. Zhukov, V.I. Subbotin and P.A. Ushakov concluded that for the same flow velocity of mercury and the same tube diameters, the relative spacing practically does not affect the heat transfer coefficient. Based on this observation they recommended the expression [141]:

$$Nu = 0.58\,Pe^{0.45} \tag{184}$$

where the outer diameter of the tube is taken as the characteristic dimension being valid for the range $1.1 \leq \frac{P}{D} \leq 1.5$. The difference between experimental data for mercury and NaK and the results from the previous equation does not exceed $\pm 15\%$.

If in the previous equation the hydraulic diameter of the centre cell is taken as the characteristic linear dimension, then it will take the form of:

$$Nu = 0.58 \cdot \left(1.1\left(\frac{P}{D}\right)^2 - 1\right)^{0.55} \cdot Pe^{0.45} \tag{185}$$

It is valid for $1.1 \leq \frac{P}{D} \leq 1.5$ and $400 \leq Pe \leq 4000$.

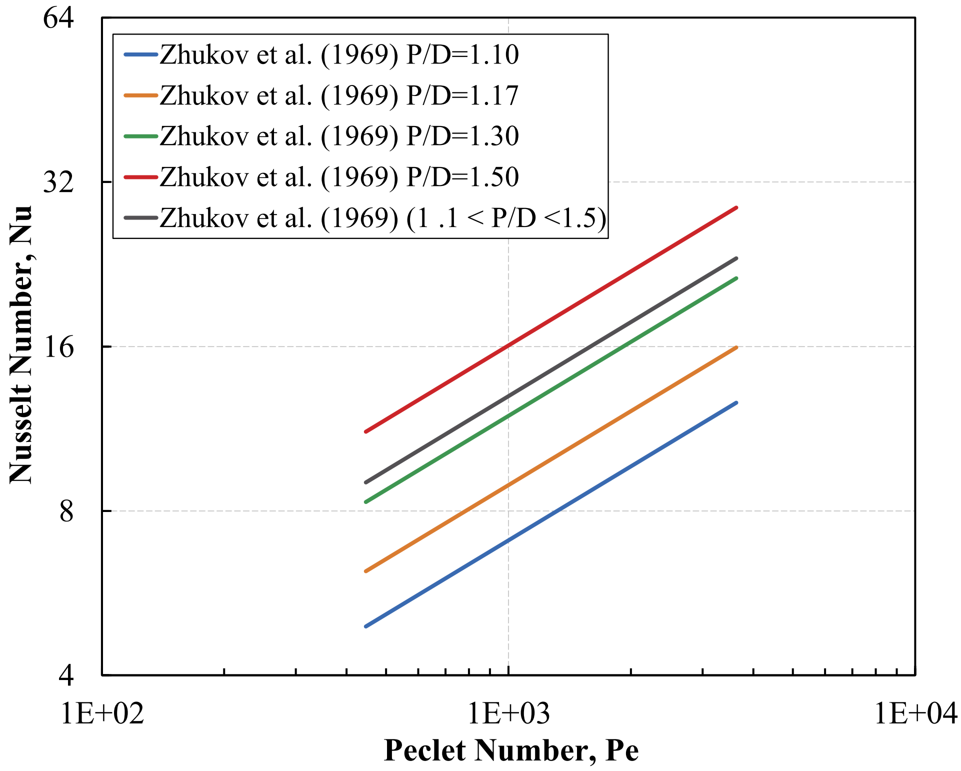

FIG. 29. Zhukov et al. (1969) Empirical heat transfer correlations for axial flow in triangular rod array

3.2.8.8 Borishanskii et al. (1969)

In 1969, V.M. Borishanskii et al. [148] published results obtained in a bundle of 7 pins with triangular arrangement with $1.1 \leq \frac{P}{D} \leq 1.5$, using coolants with Prandtl numbers of ≈ 0.007 (sodium) and ≈ 0.03 (mercury). The proposed correlation was used to verify IPPE measurements for Prandtl number 0.024 and $Pe \geq 10^3$ as well as to predict values for laminar flow ($Pe \leq 200$):

$$Nu = 24.15 \log\left[-8.12 + 12.76\frac{P}{D} - 3.65\left(\frac{P}{D}\right)^2\right] \text{ for } Pe \leq 200 \tag{186}$$

$$Nu = 24.15 \log\left[-8.12 + 12.76\frac{P}{D} - 3.65\left(\frac{P}{D}\right)^2\right] + 0.0174\left\{1 - e^{-6\left(\frac{P}{D}-1\right)}\right\}(Pe - 200)^{0.9} \text{ for } 200 \leq Pe \leq 2200 \tag{187}$$

The validity range of both equations is $1.1 \leq \frac{P}{D} \leq 1.5$

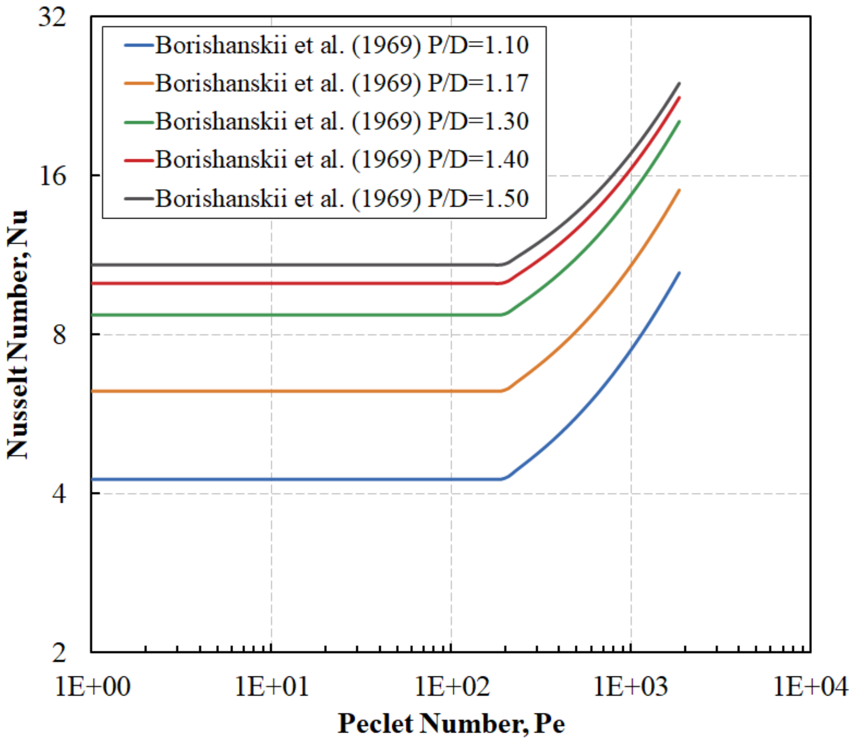

FIG. 30. Borishanskii et al. (1969) Empirical heat transfer correlations for axial flow in triangular rod array

3.2.8.9 Schad (1969)

In 1969, H.O. Schad [149] [150] proposed the following Nusselt correlation for conditions $1.1 \le \frac{P}{D} \le 1.5$ and $300 \le Pe \le 10^3$:

$$Nu = \left[-16.15 + 24.96\left(\frac{P}{D}\right) - 8.55\left(\frac{P}{D}\right)^2\right] Pe^{0.3} \tag{188}$$

Later in 1974 Carelli and Bach [151] made an extension of this correlation to lower Peclet numbers. The extended correlation is presented and plotted in Section 3.2.8.14.

3.2.8.10 West/Calamai et al./FFTF/ Westinghouse/Kazimi (1969)

In 1969 H. West [150] [152] proposed the following correlation:

$$Nu = 4.0 + 0.16\left(\frac{P}{D}\right)^5 + 0.33\left(\frac{P}{D}\right)^{3.8}\left(\frac{Pe}{100}\right)^{0.86} \tag{189}$$

which is valid for $1.1 \le \frac{P}{D} \le 1.4$ and $10 \le Pe \le 5000$.

As this correlation was considered to be conservative, it was used in the analysis of CRBRP fuel assemblies where the lower bound of the correlation uncertainty was taken as -4% (1σ level of confidence) in the range $20 \le Pe \le 1000$

According to A.E. Waltar et al. [3], Calamai et al. [153] used this correlation for Fast Flux Test Facility (FFTF) analysis in 1974. Later in 1976, M.S. Kazimi and M.D. Carelli [150] presented this correlation and called it the 'Westinghouse' correlation.

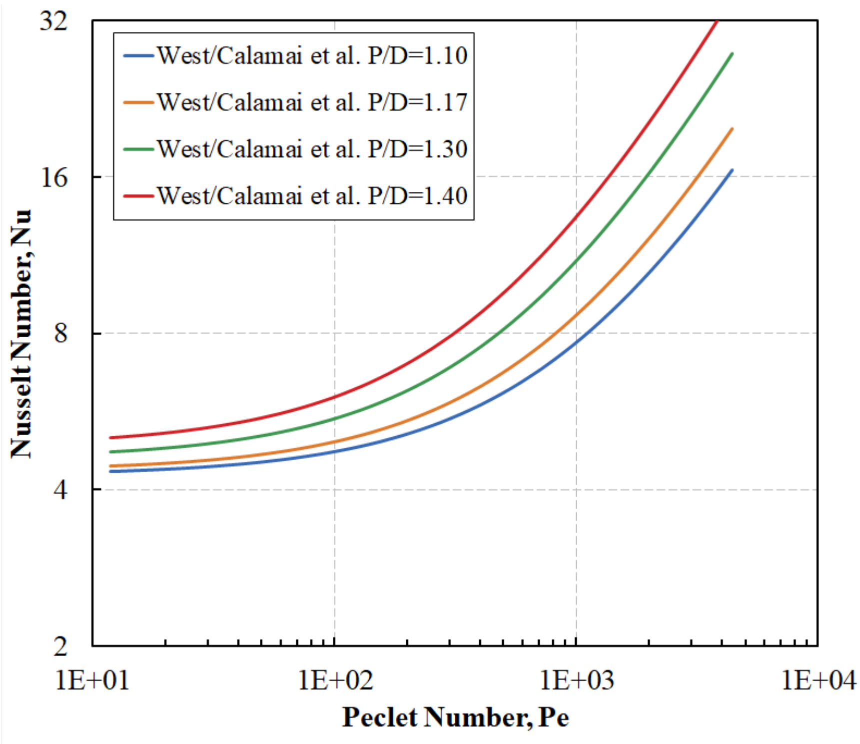

FIG. 31. West (1969) Empirical heat transfer correlation for axial flow in triangular rod array

3.2.8.11 Dwyer-Berry (1970)

The proposed correlation by O.E. Dwyer and H.C. Berry (1970) is as follows [154] [155]:

$$Nu_t = \frac{7}{8} Nu_s + 0.025 \left(\frac{Pe}{Pr_t}\right)^{0.8} \tag{190}$$

where Nu_t is the average Nusselt number for turbulent flow in channels, Nu_s is the average Nusselt number for in-line slug flow in unbaffled rod bundles and Pe is the Peclet number for in-line turbulent flow in rod bundles.

Eq. (190) is able to predict Nusselt numbers with acceptably good accuracy for turbulent flow of liquid metals in unbaffled equilateral triangular rod bundles with wide spacing. However, it was found that it predicts too high Nusselt numbers for very narrow spacing, being the narrower the spacing, the greater the error. Values of in-line slug flow Nusselt number are a function of $\frac{P}{D}$ as shown in Table 17.

TABLE 17. IN-LINE SLUG FLOW NUSSELT NUMBERS FOR VARIOUS PITCH-TO-DIAMETER RATIOS $\frac{P}{D}$

$\frac{P}{D}$	Nu_s
1.05	3.76
1.07	5.77
1.10	8.29
1.20	12.18
1.30	13.66
1.40	14.65
1.50	15.52
1.60	16.36
1.80	18.01
2.00	19.81

3.2.8.12 Gräber-Rieger (1972)

In the framework of the EURATOM-CEA association for the development of fast breeder reactors, an experimental programme was performed between 1961 and 1969 investigating NaK flow in 31 pin bundles arranged in triangular lattice. Arrangements with both heater pins and heat exchanger tubes were used. The applied $\frac{P}{D}$ ratios were 1.25, 1.6, and 1.95. In 1972, H. Gräber and M. Rieger (EURATOM, Ispra) [45] [137] [156] [157] published the final analysis of these experiments. The correlation[5] they proposed is quoted to describe also other results published at that time:

$$Nu = 0.25 + 6.2\,\frac{P}{D} + \left[0.032\,\frac{P}{D} - 0.007\right] Pe^{\left(0.8-0.024\frac{P}{D}\right)} \tag{191}$$

valid for $110 \leq Pe \leq 4000$ and $1.25 \leq \frac{P}{D} \leq 1.95$.

[5] In the IAEA report [378] from 2002 the expressions for the Gräber-Rieger correlation contain errors (0.32 and 0.07 instead of 0.032 and 0.007, respectively), as well as in Waltar book [3] where the constant 0.032 is replaced by 0.32.

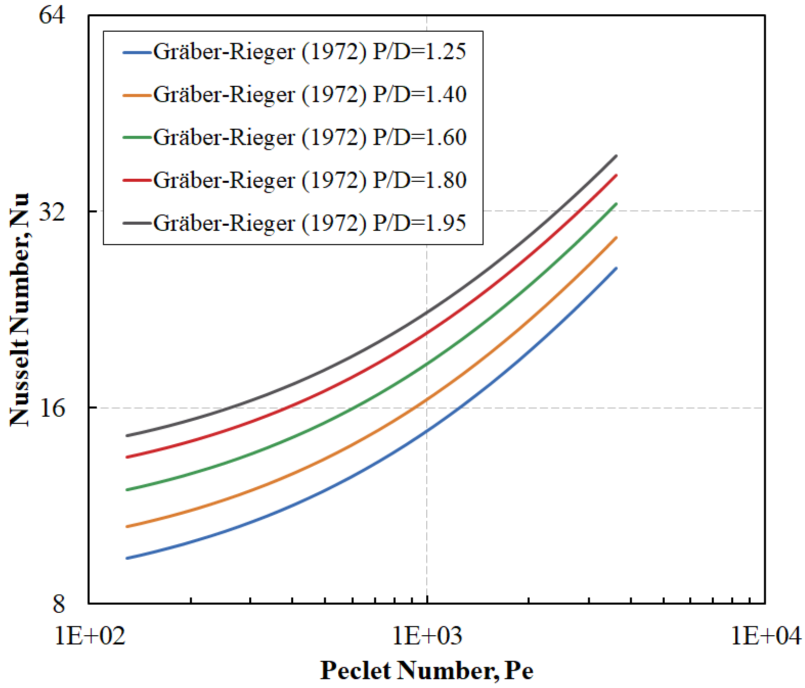

FIG. 32. Gräber-Rieger (1972) Empirical heat transfer correlation for axial flow in triangular rod array

3.2.8.13 Bobkov et al. (1973)

In 1973 V.P. Bobkov et al. published a paper recommending the following correlation [157] [158]:

$$Nu = Nu_0 + \beta Re^{0.87} Pr^m$$
$$m = 0.4 + \frac{1}{2 + 4Pr}$$
$$\beta = 0.0083\left\{1 - e^{-10.4\left(\frac{P}{D}-1\right)-0.1\sqrt{\alpha}}\right\} + 0.008\left(\frac{P}{D} - 1\right) \tag{192}$$
$$\alpha = \varepsilon_6\left[1 + \frac{4}{1 + 10Pr}\right]$$

This correlation is valid for $0 \le Pr \le 10$, $10^4 \le Re \le 10^5$, $1 \le \frac{P}{D} \le 2$, $\varepsilon_6 \ge 0.01$ (see Section 3.5), $Nu \ge 0.2$, $\beta \ge 0.001$, $\frac{R_1}{R_2} \le 0.95$. The values of Nu_0 were determined from the nomograms in [157] [159].

3.2.8.14 Schad-Kazimi-Carelli (1974)

In 1974, M.S. Kazimi and M.D. Carelli took the Schad-modified Nusselt correlation [149] for $1.1 \le \frac{P}{D} \le 1.5$ and extended it to a dog-leg shape for Peclet numbers below 150 [3] [45] [89] [150]. The complete proposed correlations are then as follows:

For $150 \le Pe \le 10^3$:

$$Nu = \left[-16.15 + 24.96\left(\frac{P}{D}\right) - 8.55\left(\frac{P}{D}\right)^2\right] Pe^{0.3} \text{ (Schad correlation)} \quad (193)$$

For $Pe \leq 150$:

$$Nu = 4.496\left[-16.15 + 24.96\left(\frac{P}{D}\right) - 8.55\left(\frac{P}{D}\right)^2\right] \text{ (Kazimi-Carelli extension)} \quad (194)$$

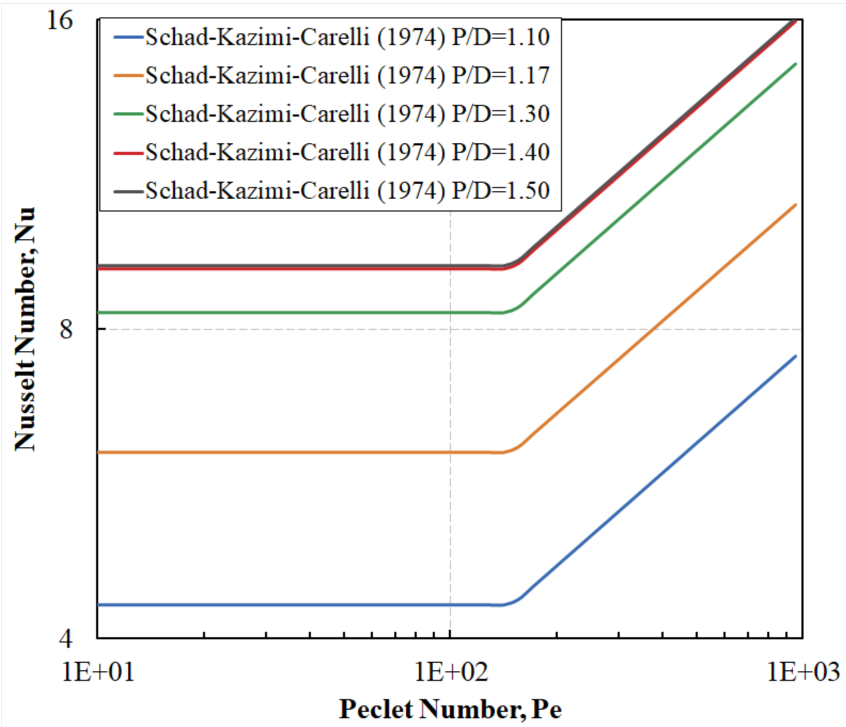

FIG. 33. Schad-Kazimi-Carelli (1974) Empirical heat transfer correlations for axial flow in triangular rod array

3.2.8.15 Subbotin et al. (1975)

The following correlation was recommended in 1975 by V.I. Subbotin, et al. [13] [49] [130] [158] [160] [161] for the flow of liquid metal in a triangular lattice of rods:

$$Nu = \mathrm{Nu}_{lam} + \frac{3.67}{90\left({}^{P}/_{D}\right)^2}\left[1 - \frac{1}{\frac{\left({}^{P}/_{D}\right)^{30} - 1}{6} + \sqrt{1.15 + 1.24\varepsilon_6}}\right] Pe^a \quad (195)$$

$$a = 0.56 + 0.19\frac{P}{D} - 0.1\left(\frac{P}{D}\right)^{-80}$$

According to the authors, the Nusselt number for laminar flow can be calculated using:

$$\mathrm{Nu}_{lam} = 7.55\frac{P}{D} - \frac{6.3}{\left({}^{P}/_{D}\right)^b}\left[1 - \frac{3.6\,{}^{P}/_{D}}{\left({}^{P}/_{D}\right)^{20}\left(1 + 2.5{\varepsilon_6}^{0.86}\right) + 3.2}\right] \quad (196)$$

$$b = 17\frac{P}{D}\left(\frac{P}{D} - 0.81\right)$$

where ε_6 (see Section 3.5) is a generalized criterion of thermal similarity of fuel elements which takes into account the effects of heat conduction in the fuel, cladding and fluid. ε_6 is calculated by the main harmonics ($k = 6$). This correlation is valid for $0.01 \leq \varepsilon_6 \leq \infty, 1 \leq Pe \leq 4000$, $1.0 \leq \frac{P}{D} \leq 2.0$ and predicts the experimental values within $\pm$ 5% [158].

For some reference values of the $P/_D$ ratio formula becomes simpler:

for $P/_D = 1$

$$Nu = \mathrm{Nu}_{lam} + 0.041\left[1 - \frac{1}{\sqrt{1.15 + 1.24\varepsilon_6}}\right]Pe^{0.65} \tag{197}$$

where

$$Nu_{lam} = 1.25\left[1 - \frac{3.6}{4.2 + 2.5{\varepsilon_6}^{0.86}}\right] \tag{198}$$

for $1.2 \leq P/_D \leq 2.0$:

$$Nu = Nu_{lam} + \frac{0.041}{\left(P/_D\right)^2}Pe^{0.56+0.19\frac{P}{D}} \tag{199}$$

where

$$Nu_{lam} = 7.55\frac{P}{D} - 20\left(\frac{P}{D}\right)^{-13} \tag{200}$$

The accuracy of this correlation is 15%.

3.2.8.16 Subbotin (1978)

In Chapter 7 of the Handbook of Single-Phase Convective Heat Transfer edited by S. Kakac, R.K. Shah and W. Aung [9], K. Rehme made an analysis of convective heat transfer over rod bundles. When presenting the correlation provided by Subbotin et al. (1975) (see Section 0) he assumed that for increasing $\frac{P}{D}$ ratios the parameter ε_6 (see Section 3.5) loses its influence on the equation. As the second term in the square brackets in Eq. (195) for $\frac{P}{D} = 1.3$ and $\varepsilon_6 = 0$ equals to 0.002 and for higher values of $\frac{P}{D}$ and ε_6 will be even smaller, so it can be neglected for the sake of simplicity. For the same reason the Peclet number power can be reduced to $0.56 + 0.19\frac{P}{D}$ [9] [49] [137] [162]. The recommended expression was [163]:

$$Nu = 7.55\frac{P}{D} - 20\left(\frac{\mathrm{P}}{D}\right)^{-13} + \frac{3.67}{90\left(\frac{\mathrm{P}}{D}\right)^2}Pe^{0.56+0.19\frac{P}{D}} \tag{201}$$

The validity range is $1 \leq Pe \leq 4000$ and $1.3 \leq \frac{P}{D} \leq 2.0$. For $\frac{P}{D} \geq 1.3$ the proposed correlation differs from Ushakov correlation Eq. (195) in less than $\pm 5\%$:

In the references [13] [164] [165] this correlation is called Ushakov (1977) correlation.

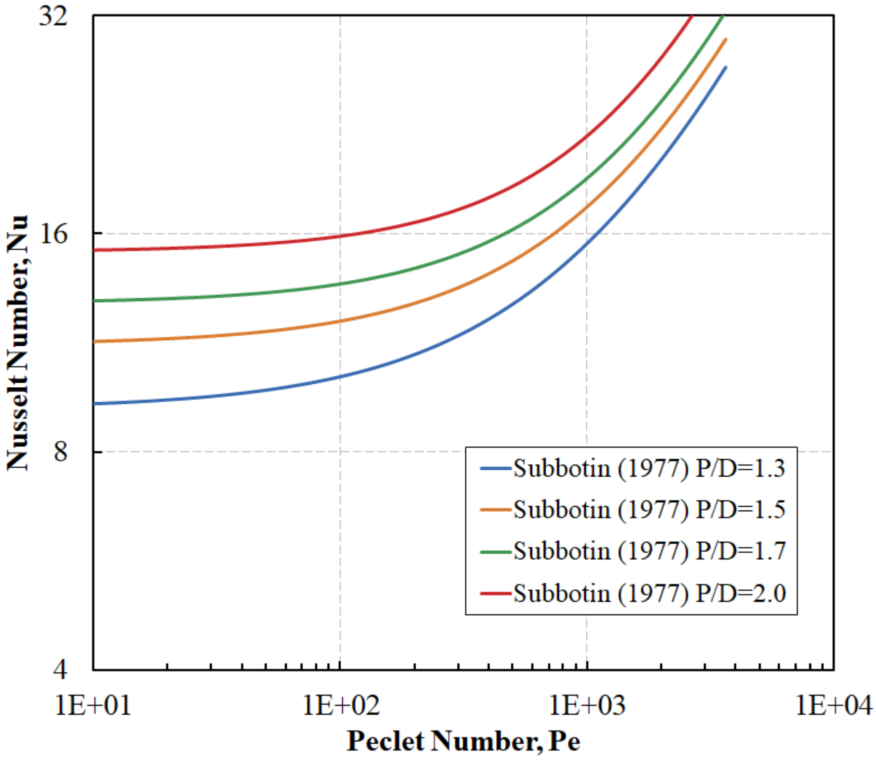

FIG. 34. Subbotin (1977) Empirical heat transfer correlations for axial flow in triangular rod array

3.2.8.17 Adamov-Orlov (2001)

In 2001 E.O. Adamov and V.V. Orlov, general editors of the report "Naturally safe lead-cooled fast reactor for large-scale nuclear power" [49] [166], presented a heat transfer correlation for lead cooled fast reactors with hexagonal fuel assemblies:

$$Nu = 7.55\frac{P}{D} - 14\left(\frac{P}{D}\right)^{-5} + \frac{3.67}{90\left(\frac{P}{D}\right)^{2}} Pe^{0.56+0.19\frac{P}{D}} \quad (202)$$

where *P* is the pitch and *D* is the pin diameter. It is very similar to previous Eq. (201) but several coefficients in the second term are different.

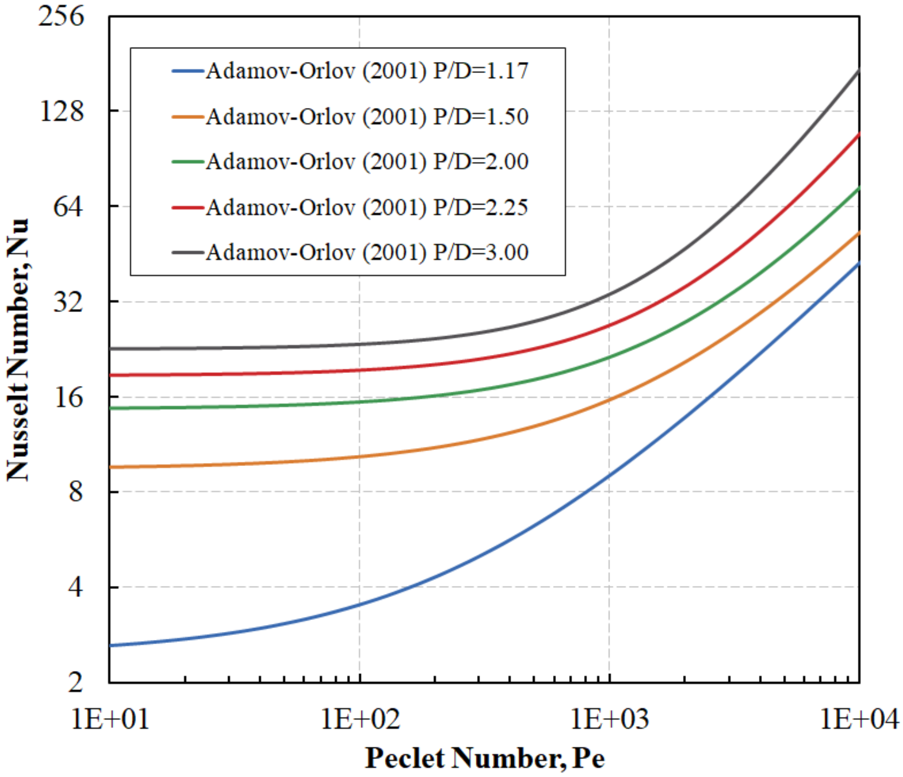

FIG. 35. Adamov-Orlov (2001) Empirical heat transfer correlations for axial flow in triangular rod array

3.2.8.18 Mikityuk (2009)

In 2009, K. Mikityuk [13] [137] reviewed the data and correlations reported for liquid metals flowing in triangular and square pin arrays or bundles of rods to estimate the quality of a number of correlations recommended for liquid metal heat transfer. A new correlation was derived as a best fit for the data analysed (with mean absolute error of -0.1% and root mean square (RMS) error of 1.9%). The correlation proposed is applicable for both triangular and square arrays:

$$Nu = 0.047\left(1 - e^{-3.8\left(\frac{P}{D}-1\right)}\right)\left(Pe^{0.77} + 250\right) \tag{203}$$

It is valid for $30 \leq Pe \leq 5000$, $1.1 \leq \frac{P}{D} \leq 1.95$.

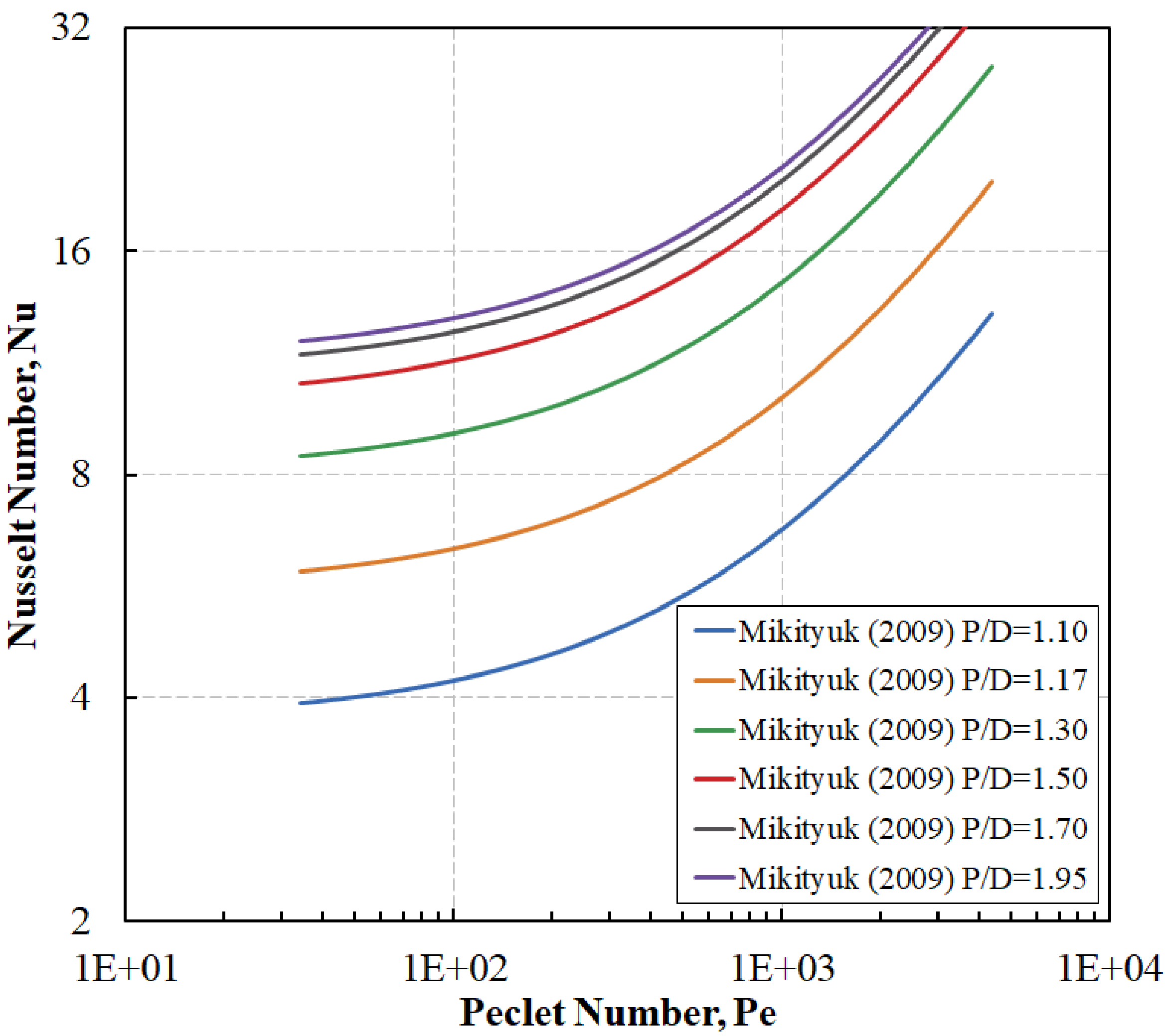

FIG. 36. Mikityuk (2009) Empirical heat transfer correlations for axial flow in triangular rod array

3.2.8.19 Choi et al. (2010)

In 2010 S.K. Choi et al. [167] performed a numerical study for the evaluation of heat transfer correlations for sodium flows in a heat exchanger of a fast breeder nuclear reactor for three different types of flows such as parallel flow, cross flow, inclined flow using five different correlations and reported that no correlation matched with numerical solutions. Then they proposed a new correlation as follows:

$$Nu = 0.16 + 4.03\frac{P}{D} + \left\{-0.005 + 0.021\frac{P}{D}\right\} Pe^{0.8-0.024\frac{P}{D}} \qquad (204)$$

valid for $40 \leq Pe \leq 400$, $1.15 \leq \frac{P}{D} \leq 1.35$.

3.2.8.20 Summary of heat transfer correlations for axial flow in triangular rod bundles

Table 18 presents the list of heat transfer correlations collected for axial flow in triangular rod bundles.

TABLE 18. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR AXIAL FLOW IN TRIANGULAR ROD BUNDLES

Dwyer and Tu (1960) [136] [137]	$Nu = 0.93 + 10.81\frac{P}{D} - 2.01\left(\frac{P}{D}\right)^2 + 0.0252\left(\frac{P}{D}\right)^{0.273}\left(\frac{Pe}{Pr_t}\right)^{0.8}$ uniform heat flux $10^2 \leq Pe \leq 10^4$, $1.375 \leq \frac{P}{D} \leq 2.2$
Friedland-Bonilla (1961) [65] [137] [138]	$Nu = 7.0 + 3.8\left(\frac{P}{D}\right)^{1.52} + 0.027\left(\frac{P}{D}\right)^{0.27}\left(\frac{Pe}{Pr_t}\right)^{0.8}$ uniform heat flux $0 \leq Pe \leq 10^5$, $1.375 \leq \frac{P}{D} \leq 10$, $10^4 \leq Re \leq 10^6$, $0 \leq Pr \leq 0.1$
Borishanskii-Firsova (1964) [139]	$Nu = 6 + 0.006\,Pe$ $2800 \leq Re \leq 4300$, $28 \leq \mathrm{Pe} \leq 172$
Maresca-Dwyer (1964) [11] [137] [140]	$Nu = 6.66 + 3.126\frac{P}{D} + 1.184\left(\frac{P}{D}\right)^2 + 0.0155\left(\frac{Pe}{Pr_t}\right)^{0.86}$ uniform heat flux $70 \leq Pe \leq 10^4$, $1.3 \leq \frac{P}{D} \leq 3.0$
Subbotin et al. (1964) [65] [141] [143] [144]	$\mathrm{Nu} = 8 \cdot \left[\frac{D_h}{L} + 0.027\left(\frac{P}{D} - 1.1\right)^{0.46}\right] \cdot Pe^{0.6}$ $200 \leq Pe \leq 1200$, $1.1 \leq \frac{P}{D} \leq 1.4$, $60 \leq \frac{L}{D_h} \leq 260$, $6.8 \leq D \leq 7.6\,mm$
Subbotin et al. (1965) [137] [144]	$Nu = 0.58\left(\frac{D_h}{D}\right)^{0.55} Pe^{0.45}$ $80 \leq Pe \leq 4000$, $1.1 \leq \frac{P}{D} \leq 1.5$
Zhukov et al. (1969) [141]	$Nu = 0.58\,Pe^{0.45}$ $Nu = 0.58 \cdot \left(1.1\left(\frac{P}{D}\right)^2 - 1\right)^{0.55} \cdot Pe^{0.45}$ $1.1 \leq \frac{P}{D} \leq 1.5$, $400 \leq Pe \leq 4000$

TABLE 18. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR AXIAL FLOW IN TRIANGULAR ROD BUNDLES

Borishanskii et al. (1969) [148]	$Nu = 24.15 \log\left[-8.12 + 12.76\frac{P}{D} - 3.65\left(\frac{P}{D}\right)^2\right]$ for $Pe \le 200$ $Nu = 24.15 \log\left[-8.12 + 12.76\frac{P}{D} - 3.65\left(\frac{P}{D}\right)^2\right] + 0.0174\left\{1 - e^{-6\left(\frac{P}{D}-1\right)}\right\}(Pe-200)^{0.9}$ for $200 \le Pe \le 2200$ $1.1 \le \frac{P}{D} \le 1.5$
Schad (1969) [149] [150]	$Nu = \left[-16.15 + 24.96\left(\frac{P}{D}\right) - 8.55\left(\frac{P}{D}\right)^2\right]Pe^{0.3}$ $1.1 \le \frac{P}{D} \le 1.5$, $300 \le Pe \le 10^3$
West/Calamai et al./FFTF/ Westinghouse/ Kazimi (1969) [150] [152]	$Nu = 4.0 + 0.16\left(\frac{P}{D}\right)^5 + 0.33\left(\frac{P}{D}\right)^{3.8}\left(\frac{Pe}{100}\right)^{0.86}$ $1.1 \le \frac{P}{D} \le 1.4$, $10 \le Pe \le 5000$
Dwyer-Berry (1970) [154] [155]	$Nu_t = \frac{7}{8}Nu_s + 0.025\left(\frac{Pe}{Pr_t}\right)^{0.8}$ turbulent flow of liquid metals, unbaffled equilateral triangular wide spacing rod bundle
Gräber-Rieger (1972) [45] [137] [156] [157]	$Nu = 0.25 + 6.2\frac{P}{D} + \left[0.032\frac{P}{D} - 0.007\right]Pe^{\left(0.8-0.024\frac{P}{D}\right)}$ $110 \le Pe \le 4000$, $1.25 \le \frac{P}{D} \le 1.95$
Bobkov et al. (1973) [157] [158]	$Nu = Nu_0 + \beta Re^{0.87}Pr^m$ $m = 0.4 + \frac{1}{2+4Pr}$ $\beta = 0.0083\left\{1 - e^{-10.4\left(\frac{P}{D}-1\right)-0.1\sqrt{\alpha}}\right\} + 0.008\left(\frac{P}{D}-1\right)$ $\alpha = \varepsilon_6\left[1 + \frac{4}{1+10Pr}\right]$ Nu_0 in Ref. [157] [159] $0 \le Pr \le 10$, $10^4 \le Re \le 10^5$, $1 \le \frac{P}{D} \le 2$, $\varepsilon_6 \ge 0.01$, $Nu \ge 0.2$, $\beta \ge 0.001$, $\frac{R_1}{R_2} \le 0.95$

TABLE 18. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR AXIAL FLOW IN TRIANGULAR ROD BUNDLES

Schad-Kazimi-Carelli (1974) [3] [45] [89] [150]	For $150 \le Pe \le 10^3$: $Nu = \left[-16.15 + 24.96\left(\frac{P}{D}\right) - 8.55\left(\frac{P}{D}\right)^2\right] Pe^{0.3}$ For $Pe \le 150$: $Nu = 4.496\left[-16.15 + 24.96\left(\frac{P}{D}\right) - 8.55\left(\frac{P}{D}\right)^2\right]$ $1.1 \le \frac{P}{D} \le 1.5$
Subbotin et al. (1975) [13] [49] [130] [158] [160] [161]	1. $Nu = Nu_{lam} + \frac{3.67}{90\left(\frac{P}{D}\right)^2}\left[1 - \frac{1}{\frac{\left(\frac{P}{D}\right)^{30} - 1}{6} + \sqrt{1.15 + 1.24\varepsilon_6}}\right] Pe^a$ $a = 0.56 + 0.19\frac{P}{D} - 0.1\left(\frac{P}{D}\right)^{-80}$ $\mathrm{Nu}_{lam} = 7.55\frac{P}{D} - \frac{6.3}{\left(\frac{P}{D}\right)^b}\left[1 - \frac{3.6\frac{P}{D}}{\left(\frac{P}{D}\right)^{20}(1 + 2.5\varepsilon_6^{0.86}) + 3.2}\right]$ $b = 17\frac{P}{D}\left(\frac{P}{D} - 0.81\right)$ $0.01 \le \varepsilon_6 \le \infty$, $1 \le Pe \le 4000$, $1.0 \le \frac{P}{D} \le 2.0$ 2. $Nu = Nu_{lam} + \frac{0.041}{(P/D)^2} Pe^{0.56 + 0.19\frac{P}{D}}$ $Nu_{lam} = 7.55\frac{P}{D} - 20\left(\frac{P}{D}\right)^{-13}$ $1 \le Pe \le 4000$, $1.2 \le P/D \le 2.0$
Subbotin (1977) [9] [49] [137] [162]	$Nu = 7.55\frac{P}{D} - 20\left(\frac{P}{D}\right)^{-13} + \frac{3.67}{90\left(\frac{P}{D}\right)^2} Pe^{0.56 + 0.19\frac{P}{D}}$ $1 \le Pe \le 4000$, $1.3 \le \frac{P}{D} \le 2.0$
Adamov-Orlov (2001) [49] [166]	$Nu = 7.55\frac{P}{D} - 14\left(\frac{P}{D}\right)^{-5} + \frac{3.67}{90\left(\frac{P}{D}\right)^2} Pe^{0.56 + 0.19\frac{P}{D}}$
Mikityuk (2009) [13] [137]	$Nu = 0.047\left(1 - e^{-3.8\left(\frac{P}{D} - 1\right)}\right)(Pe^{0.77} + 250)$ $30 \le Pe \le 5000$, $1.1 \le \frac{P}{D} \le 1.95$

TABLE 18. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR AXIAL FLOW IN TRIANGULAR ROD BUNDLES

Choi et al. (2010) [167]	$Nu = 0.16 + 4.03\frac{P}{D} + \left\{-0.005 + 0.021\frac{P}{D}\right\} Pe^{0.8-0.024\frac{P}{D}}$ $40 \leq Pe \leq 400,\ 1.15 \leq \frac{P}{D} \leq 1.35$

Several widely used heat transfer correlations for the axial flow in triangular rod arrays are show in Fig. 37. At the below plots, all Nusselt numbers are calculated for $P/D = 1.17$ that correspond to the value of pitch-to-diameter ratio in several most common designs of SFR fuel assemblies. Correlations not valid for this condition are omitted. Correlations dependent on additional variables such as rod dimensions or thermal conditions are not included.

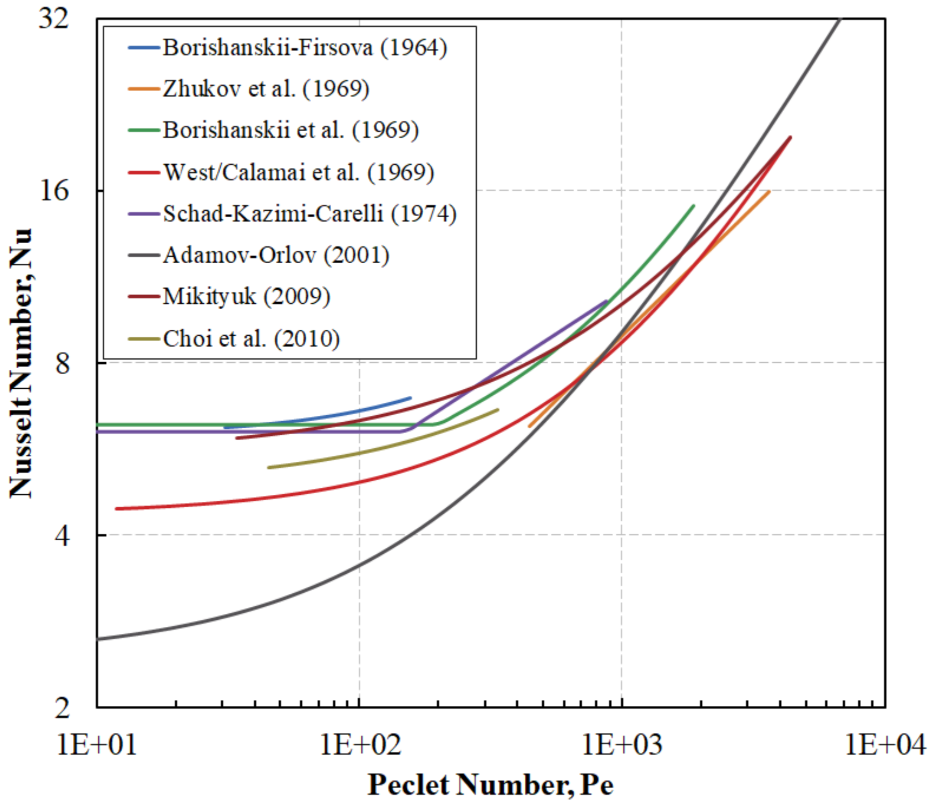

FIG. 37 Empirical heat transfer correlations for axial flow in triangular rod array with $P/D = 1.17$

3.2.9 Axial flow in square rod bundles

Relatively little heat transfer information is available on axial flow of liquid metals along square rod bundles, presumably because of lesser common use in sodium cooled reactors due to poorer heat transfer capabilities and relatively less compactness [155]. The geometry of axial flow in square rod bundles is shown in Fig. 38.

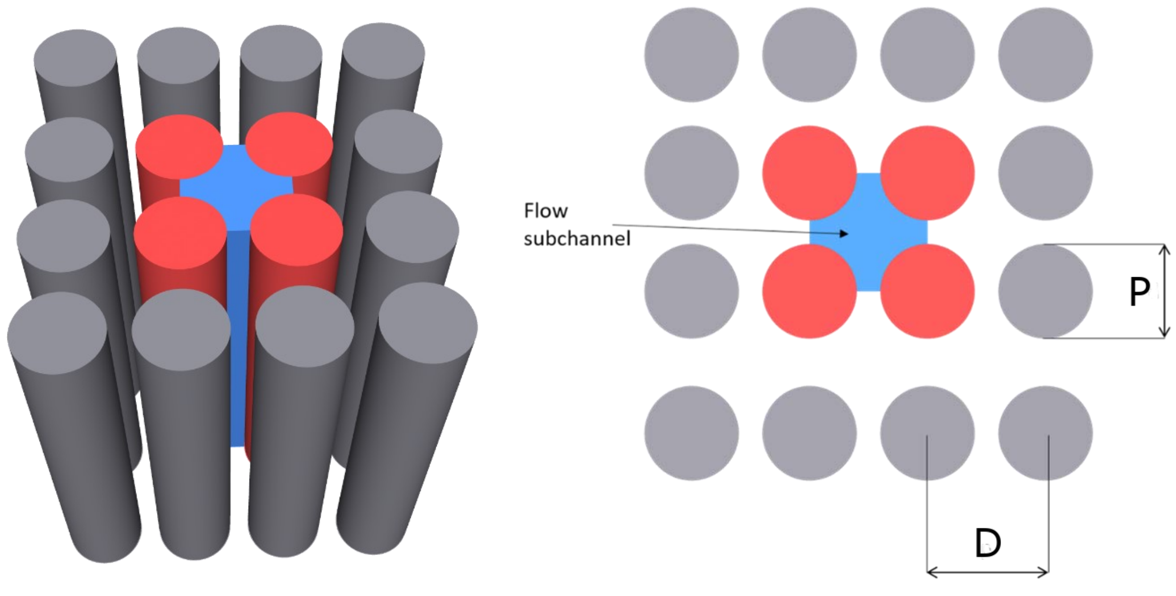

FIG. 38. Square pin array and flow subchannels geometry

The hydraulic diameter for square pin array is calculated as: $D_h = D\left[\frac{4}{\pi}\left(\frac{P}{D}\right)^2 - 1\right]$

3.2.9.1 Ushakov et al. (1960)

In 1960, P.A. Ushakov et al. published experimental data on heat transfer for flow of mercury and NaK in a square array of tightly packed rods $\left(\frac{P}{D} = 1\right)$ [168]. Their measured rod-average heat transfer coefficients published in graphical form, can be represented within $\pm 20\%$ for both coolants by the equation ([11] Ch. 2), [155]:

$$Nu = 0.48 + 0.0133\, Pe^{0.7} \tag{205}$$

This correlation was also proposed by O.E. Dwyer in [65].

3.2.9.2 Friedland-Bonilla (1961)

In 1961, A.J. Friedland and C.F. Bonilla made a theoretical analysis of heat transfer to liquid metals in parallel flow in a tube bundle [65] [138]. The equation proposed is the following:

$$Nu = 7.0 + 4.24\left(\frac{P}{D}\right)^{1.52} + 0.0275\left(\frac{P}{D}\right)^{0.27}\left(\frac{Pe}{Pr_t}\right)^{0.8} \tag{206}$$

where P is the pitch squared spacing and the conditions are fully developed turbulent flow, constant heat flux at the wall, and an infinite number of tubes arranged on an equilateral squared pitch. It is valid for $0 \le Pe \le 10^5$, $1.375 \le \frac{P}{D} \le 10$, $10^4 \le Re \le 10^6$, $0 \le Pr \le 0.1$. This equation is the modification of the correlation recommended by the authors for triangular rod bundles Eq. (179) adapted to squared rod bundles by using the relation:

$$\left(\frac{P_{triang.}}{D}\right)^2 = \frac{2}{\sqrt{3}}\left(\frac{P_{squar.}}{D}\right)^2 \tag{207}$$

where $P_{triang.}$ and $P_{squar.}$ are the pitch distances for triangular and square lattices, respectively.

3.2.9.3 Zhukov (1994)

In the frame of the BREST-300 lead cooled fast reactor project, an extensive experimental programme was conducted to study the heat transfer to liquid metal in a square lattice of round tubes [137] [169] [170][6]. The working section consisted of 25 tubes of 12 mm outer diameter. Four sets of experimental data were measured with $\frac{P}{D}$ ratios of 1.25, 1.28, 1.34 and 1.46. The working fluid was 22% Na– 78% K at a temperature of about $50°C$. The heated length of the assembly was 980 mm. A total of 36 data pairs of Nu vs. Pe were provided leading to the correlation[7] expressed as follows [49]:

$$Nu = 7.55\,\frac{P}{D} - 14\left(\frac{P}{D}\right)^{-5} + a\,Pe^{0.64+0.246\frac{P}{D}}$$

$$a = 0.007 \text{ for smooth rods (no spacers)} \qquad (208)$$

$$a = 0.009 \text{ for spacers with } \varepsilon_g = 20\%$$

$$a = 0.010 \text{ for spacers with } \varepsilon_g = 10\%$$

It is valid for $10 \leq Pe \leq 2500$ and $1.2 \leq \frac{P}{D} \leq 1.5$. The accuracy of this formula is $\pm 15\%$ [170].

3.2.9.4 Adamov and Orlov (2001)

In a report from 2001 published by E.O. Adamov and V.V. Orlov [166] the following correlation is presented as proposed and verified by the experiments performed by A.V. Zhukov et al. for square lattice which looks like a transcription of the Eq. (202):

$$Nu = 7.55\,\frac{P}{D} - 20\left(\frac{P}{D}\right)^{-5} + \frac{0.0354}{\left(\frac{P}{D}\right)^{2}} Pe^{0.56+0.204\frac{P}{D}} \qquad (209)$$

It is verified for $\frac{P}{D}$ ratios 1.28 and 1.46 in the range of $10^2 \leq Pe \leq 1.6 \cdot 10^3$. Adamov and Orlov referred to the proceeding published by Zhukov in 1994 [170]. However, in that proceeding the correlation proposed is that of Eq. (207) for smooth rods, which differs from Eq. (209).

3.2.9.5 Mikityuk (2009)

In 2009, K. Mikityuk [13] [137] proposed a correlation applicable for both square and triangular lattices (see also Section 3.2.8.18):

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[6] In Ref. [170] there is an error in the sign of the exponential in the second term, where it should be -5 instead of 5.

[7] In Ref. [160] the values of the constant a are a factor 10 smaller and should be an error. In Ref. [165] the signs of the values of the constant a are negative, as well as for the second term of the Peclet exponent.

$$Nu = 0.047\left(1 - e^{-3.8\left(\frac{P}{D}-1\right)}\right)\left(Pe^{0.77} + 250\right) \tag{210}$$

It is valid for $30 \leq Pe \leq 5000$, $1.1 \leq \frac{P}{D} \leq 1.95$.

3.2.9.6 Summary of correlations for axial al flow in square rod bundles

Table 19 presents the list of all heat transfer correlations collected for axial flow in square rod bundles.

TABLE 19. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR AXIAL FLOW IN SQUARE ROD BUNDLES

Ushakov et al. (1960) [11] [155]	$Nu = 0.48 + 0.0133\, Pe^{0.7}$ tightly packed rods $\frac{P}{D} = 1$
Friedland-Bonilla (1961) [65] [138]	$Nu = 7.0 + 4.24\left(\frac{P}{D}\right)^{1.52} + 0.0275\left(\frac{P}{D}\right)^{0.27}\left(\frac{Pe}{Pr_t}\right)^{0.8}$ $0 \leq Pe \leq 10^5$, $1.375 \leq \frac{P}{D} \leq 10$, $10^4 \leq Re \leq 10^6$, $0 \leq Pr \leq 0.1$
Zhukov et al. (1994) [137] [170]	$Nu = 7.55\,\frac{P}{D} - 14\left(\frac{P}{D}\right)^{-5} + \mathrm{a}\, Pe^{0.64+0.246\frac{P}{D}}$ $a = 0.007$ for smooth rods (no spacers) $a = 0.009$ for spacers with $\varepsilon_g = 20\%$ $a = 0.010$ for spacers with ε_g=10% $10 \leq Pe \leq 2500$, $1.2 \leq \frac{P}{D} \leq 1.5$
Adamov and Orlov (2001) [166]	$Nu = 7.55\,\frac{P}{D} - 20\left(\frac{P}{D}\right)^{-5} + \frac{0.0354}{\left(\frac{P}{D}\right)^2} Pe^{0.56+0.204\frac{P}{D}}$ $\frac{P}{D} = 1.28, 1.46$, $10^2 \leq Pe \leq 1.6 \cdot 10^3$
Mikityuk (2009) [13] [137]	$Nu = 0.047\left(1 - e^{-3.8\left(\frac{P}{D}-1\right)}\right)(Pe^{0.77} + 250)$ $30 \leq Pe \leq 5000$, $1.1 \leq \frac{P}{D} \leq 1.95$

The empirical correlations for Nusselt number, presented in Table 19 are also plotted in Fig. 39. For the purpose of comparison, a nominal value for the pitch to diameter ratio was selected such that $P/_D = 1.46$. This value meets the conditions for validity of the greatest number of correlations. This visual comparison omits Ushakov et al. (1960) [11] [155] which is only valid for tightly packed rod bundles. Note some correlations make additional conditions for validity.

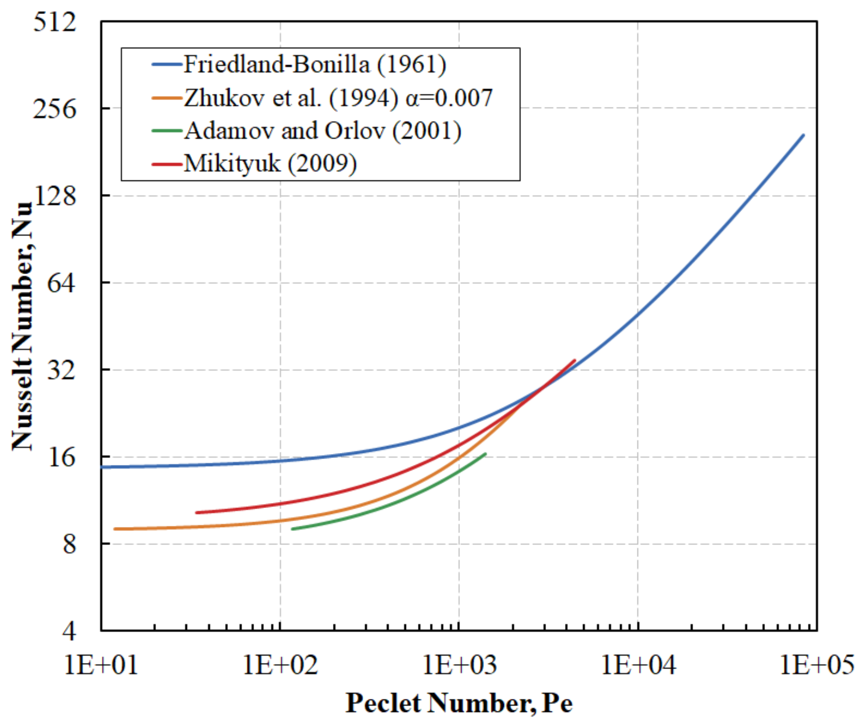

FIG. 39. Comparison of common correlations for Nusselt numbers for heat transfer in square road bundles, $P/D = 1.46$

3.2.10 Heat transfer and temperature fields in peripheral zones of hexagonal fuel assembly

The geometry of a hexagonal fuel assembly is shown in Fig. 40 below, where D is the diameter of fuel pin, P is the pitch of rod array, and H is the wire pitch.

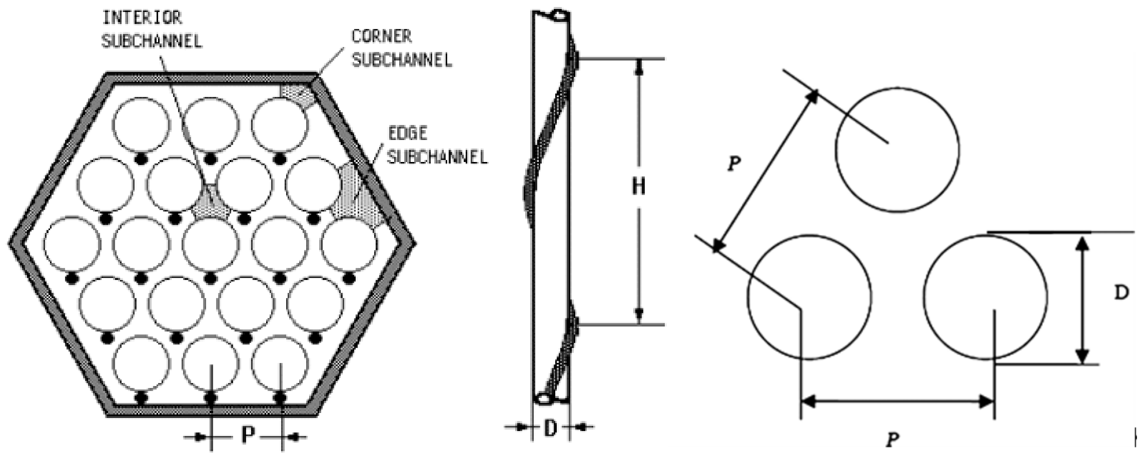

FIG. 40.Geometry of hexagonal fuel assembly

3.2.10.1 Subbotin et al. (1975)

According to V.I. Subbotin, et al. [13] [49] [130] [158] [160] [161] the maximum temperature non-uniformity around the edge pins can be calculated by the formula:

$$\Delta T = \frac{T_w^{max} - T_w^{min}}{\bar{q}R}\lambda_f = \frac{\Delta T_{lam}}{1 + \gamma(\varepsilon_6)Pe^{\beta\left(\frac{P}{D}\right)}} \tag{211}$$

where T_w^{max} and T_w^{min} are the pin wall temperatures in the internal and peripheral subchannels, respectively; $\bar{q}$ is the average heat flux; R is the pin radius; λ_f is liquid thermal conductivity; ΔT_{lam} is the temperature non-uniformity in the laminar flow obtained from pre-calculated nomograms, and

$$\gamma(\varepsilon_6) = (1 + 0.03\varepsilon_6)\,8\,10^{-3} \tag{212}$$

$$\beta\left(\frac{P}{D}\right) = 0.65 + \frac{51\log\left(\frac{P}{D}\right)}{\left(\frac{P}{D}\right)^{20}} \tag{213}$$

The deviation of the temperature distribution from a cosine law can be evaluated using the nomogram or by the formula

$$Z = \frac{T_w^{max} - \overline{T_w}}{\overline{T_w} - T_w^{min}} = 0.9 + 0.1\left[1 - e^{-10\left(\frac{P}{D}-1\right)}\right] + 0.2e^{-50\left(\frac{P}{D}-1\right)} \\ -0.49e^{-20\left(\frac{P}{D}-1\right)}\tanh\left(\log\varepsilon_6 + 0.6\left(1 - \frac{\frac{P}{D}-1}{0.1}\right)\right) \tag{214}$$

3.2.10.2 Zhukov et al. (1977)

Considerable temperature non-uniformity around the periphery of pin bundles in fast reactors cause the decrease of the heat transfer coefficient in peripheral zones as compared to the internal pins. In 1977 A.V. Zhukov et al. [130] [171] [172] recommended a special relationship to be used:

$$Nu = \frac{\bar{h}D_h}{\lambda_f} = a + bPe^n \tag{215}$$

It is valid for $1.04 \le \frac{P}{D} \le 1.3$, $0.39 \le \Psi \le 0.52$, $30 \le Pe \le 3000$,
$0 \le \frac{d_{displacer}}{D} \le 0.32$, $0.04 \le \varepsilon_1 \le 0.14$ (see Section 3.5).

Here D_h is the hydraulic diameter of the internal zone of the subassembly; $Pe = {}^{\bar{w}D_h}/_{\alpha}$ is the Peclet number calculated based on average velocity of coolant in the subassembly $\bar{w}$ and the hydraulic diameter of the internal channels; $\bar{h} = \frac{\bar{q}}{(\overline{T_w} - \overline{T_f})}$ is the mean heat transfer coefficient under stable heat transfer conditions (when there is no stable temperature difference "wall – liquid", that is often observed at the edge pins, heat transfer coefficient is evaluated at the heated outlet cross-section); $\bar{q}$, $\overline{T_w}$ are averaged heat flux and pin wall temperature, respectively; $\overline{T_f}$ is the bulk coolant temperature between adjacent channels; e_1 is equivalent thermal conductivity based on the first harmonics in Fourier series; $d_{displacer}$ is the displacer diameter; λ_f is the thermal conductivity of the coolant; α is the thermal diffusivity; $\Psi = \frac{\Delta}{P-D}$; and Δ is the gap between the wrapper and pins. The displacer refers to a cylindrical or other shape pin inserted in the peripheral subchannel next to the wrapper, which reduces the flow rate and equalizes the power-to-flow ratio between central and peripheral subchannels.

According to Zhukov's experiments, values of factors a, b, and n can be approximated by the relationships presented in Table 20. For the peripheral pins, the minimal laminar-flow value of the Nusselt number is reached at low Peclet numbers, which depend on the of the values pitch-to-diameter ratio $P/_D$.

TABLE 20. FACTORS A, B, N IN EQ. (215)

Pin location	*a*	*b*	*n*
Without displacers			
Edge	$4.69\,P/_D - 4.131$	$0.577\,P/_D - 0.566$	$3.53(P/_D)^2 - 8.71\,P/_D + 5.97$
Corner	$7.13\,P/_D - 6.972$	$0.331\,P/_D - 0.342$	$5.27(P/_D)^2 - 13.12\,P/_D + 8.83$
With displacers			
Edge	$4.81\,P/_D - 3.348$	$1.381\,P/_D - 1.376$	$1.26(P/_D)^2 - 3.35\,P/_D + 2.74$
Corner	$3.59\,P/_D - 3.189$	$1.324\,P/_D - 1.363$	$14.88 - 3.35\,P/_D + 25.43(P/_D)^2 - 6.57(P/_D)^3$

3.2.10.3 Zhukov et al. (1977)

The maximum temperature non-uniformity across the edge pin can be evaluated as recommended by A.V. Zhukov et al. [171] [172]:

$$\Delta T = \frac{T_{inner} - T_{peripheral}}{\bar{q}R}\lambda_f = A + B\Psi - \mathrm{C}e^{-D\Psi}, \tag{216}$$

where T_{inner} and $T_{peripheral}$ are the pin wall temperatures in the internal and peripheral subchannels, respectively; $\bar{q}$ is the average heat flux; $R = D/2$ is the pin radius; λ_f is the liquid thermal conductivity; and the parameter Ψ is the ratio of the gap between wrapper and pins Δ to the difference between bundle pitch *P* and pin diameter *D*:

$$\Psi = \frac{\Delta}{P - D} = \frac{\Delta}{D\left(\frac{P}{D} - 1\right)} \tag{217}$$

The value of ΔT is positive, if the maximum pin temperature is observed near the internal channels (periphery is subcooled), and negative, if the maximum pin temperature is observed near the wrapper wall (periphery is superheated).

Correlation (216) is valid for $200 \leq Pe \leq 700$ and for developed flow only $\left(\frac{l}{D_h} \geq 200\right)$, where *l* is a heated length and D_h is the hydraulic diameter of the pin array). Coefficients A, B, C, D are presented in the Table 21.

TABLE 21. COEFFICIENTS A, B, C, D IN EQ. (216)

Pe number	A	B	C	D	β
Bundle of smooth pins without displacers ($1.06 \leq P/D \leq 1.15$, $0 \leq \Psi \leq 1.05$)					
700	0.10	0.40	0.63	15	
400	0.21	0.41	0.89	12.5	
200	0.47	0.82	1.3	10	
Bundle of wire wrapped pins without displacers ($1.06 \leq P/D \leq 1.15$, $0.3 \leq \Psi \leq 1.05$)					
700	0.02	0.48	0	0	
400	0.27	0.31	0	0	
200	0.52	0.62	0	0	
Bundle of smooth pins with displacers ($1.06 \leq P/D \leq 1.25$, $0.25 \leq \Psi \leq 1.05$)					
700	0.216	0.083	1.03	4.15	
400	0.33	0.12	1.17	5.05	
200	0.40	0.5	3.66	5.07	
Bundle of wire wrapped pins with displacers ($1.06 \leq P/D \leq 1.25$, $0.25 \leq \Psi \leq 1.05$)					
700	0.0525	0.16	1.33	7.25	
400	0.132	0.17	2.25	6.72	
200	0.45	0.19	4.08	5.9	

As a rule, the temperature fields of edge pins are not stable. The maximum and minimum temperatures over pin perimeter are practically opposite each other. This complicates the heat exchange between them. The phenomenon of non-stabilized temperature fields is mostly observed in bundles with small pitch-to-diameter ratio. The introduction of displacers in these bundles basically does not change the character of the temperature field along the bundle length; if the displacer diameter is not very large as to cause the coolant overheating in peripheral channels. The increasing of the gap between pins and the wrapper Δ (for the given pin pitch) also increases the non-stabilized character of the temperatures fields.

3.2.10.4 Sorokin et al. (1984)

In 1984 A.P. Sorokin et al. [173] [174] proposed that the maximum temperature non-uniformity around the edge pin can be approximately estimated from the superposition of the coolant temperature non-uniformity and the local temperature non-uniformity for "*infinite*" triangle rod arrays and the temperature non-uniformity for square rod arrays at the peripheral pins as follows:

$$T_{inner} - T_{peripheral} = T_{f1} - T_{f11} + (T_w^{max} - T_w)_{\infty}^{trian} + \left(\overline{T_w} - T_w^{min}\right)_{\infty}^{square} \tag{218}$$

Substituting the expressions for the local non-uniformity in the "*infinite*" triangular and square rod arrays gives:

$$T_{inner} - T_{peripheral} = \Delta T_0 \left[\left(1 - \frac{1}{g_{eff}}\right)\frac{1-e^{-T_M}}{T_M} + \frac{PeD}{8L}\left(\frac{z}{z+1}(\Delta T_\infty^{max})^{trian} + \frac{1}{z+1}(\Delta T_\infty^{max})^{square}\right)\right] \tag{219}$$

where ΔT_0 is the temperature non-uniformity in the triangular internal channels and

$$g_{eff} = \frac{G_{edge}}{G_0} \tag{220}$$

$$T_M = \frac{\mu^h z}{g_{eff}} \tag{221}$$

The following values

$$\Delta T_\infty^{max} = \frac{T_w^{max} - T_w^{min}}{\bar{q}\,{}^D/_2}\lambda_f \tag{222}$$

$$Z = \frac{T_w^{max} - \overline{T_w}}{\overline{t_w} - t_w^{min}} \tag{223}$$

are calculated by correlations for the "*infinite*" triangular rod array and the nomograms for square bundle at laminar flow with the correction for the turbulent flow; G_{edge}, and G_0 are coolant mass flow rates in edge and internal channels, respectively; μ^h is the inter-channel heat exchange; and z is the distance from the beginning of the heating section.

Statistical analysis shows that experimental data for temperature non-uniformity around the edge pins are in agreement with the correlation (219).

3.2.11 Cross flow across rod bundles

The set of correlations presented hereafter corresponds to the configuration where the flow is perpendicular to the rod longitudinal axis (see Fig. 41, contrary to the previous cases where the flow is parallel to the rod bundle axis.

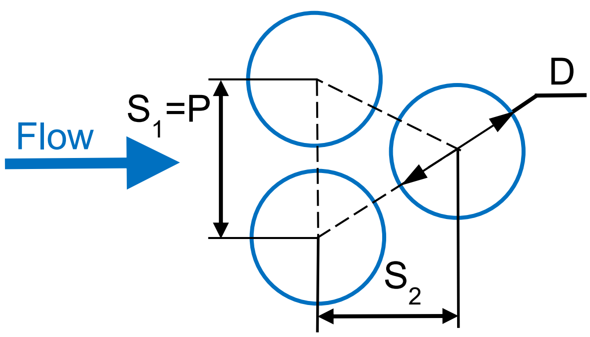

FIG. 41. Crossflow in triangular rod array

3.2.11.1 Rickard et al. (1958)

In 1958 C.L. Rickard et al. [175] participated in the liquid metal heat transfer programme at Brookhaven National Laboratory, where both local and tube-average heat transfer coefficients were obtained for the flow of mercury perpendicular to a staggered tube bank [175] [176]. The tube bank consisted of sixty ½-inch tubes, six inches wide and ten inches deep, arranged an equilateral-triangular array. The following Nusselt number correlation is proposed for the range $2 \cdot 10^4 \leq Re \leq 2 \cdot 10^5$ [68]:

$$Nu = 4.03 + 0.228 Pe^{0.67} \tag{224}$$

Despite of the fact that the correlation (224) was developed for mercury, it is generally used for sodium systems as well.

3.2.11.2 Cess-Grosh (1958)

For the boundary condition of uniform wall temperature, R. Cess and R. Grosh (1958), assuming inviscid potential flow, derived the theoretical equation for cross-flow through rod bundles ([11] Ch. 2) [177]:

$$Nu = 0.718 \left(\frac{f_1}{D}\right)^{\frac{1}{2}} \left(\frac{P - D}{P}\right)^{\frac{1}{2}} Pe_{v_{max}} \tag{225}$$

where f_1 is the unit hydrodynamic potential at the rear stagnation point of a rod, v_{max} is the average cross-flow velocity of coolant between the elements, based on minimum flow area, $Pe_{v_{max}} = \frac{D v_{max} \rho c_p}{k}$, ρ is the fluid density, c_p is the heat capacity at constant pressure, D is the equivalent diameter of the rod-bundle and k is the thermal conductivity.

3.2.11.3 Hsu (1964)

In 1964 C.J. Hsu derived the heat transfer equation based on the same assumptions as those of previous Cess-Grosh (1958) correlation ([11] Ch. 2), [65] [121]:

$$Nu = a \left(\frac{f_1}{D}\right)^{\frac{1}{2}} \left(\frac{P - D}{P}\right)^{\frac{1}{2}} \sqrt{Pe_{v_{max}}} \tag{226}$$

$a = 0.810$ for uniform heat flux from the rod surface

$a = 0.958$ for a simple cosine surface temperature distribution around the rod.

3.2.11.4 Dwyer (1966)

For a cross flow through bare fuel rod bundles or tube banks the heat transfer coefficient can be calculated from O.E. Dwyer (1966) correlation [65] [155]:

$$\mathrm{Nu} = 5.36 + 0.1974\, Pe_{v_{max}}^{0.682} \tag{227}$$

This correlation is valid for triangularly spaced elements where $S_1/D = P/D$ and S_2/D are pitch-to-diameter ratios of 1.38 and 1.19 respectively that corresponds to the triangular array with the same pitch P, where $S_1 = P$ is the rod pitch in the stream-wise direction and S_2 the rod spacing in the spam-wise direction. For any other values of P/D, corrections can be made according to the theoretical study of Hsu, but the influence is quite small and can be approximated by multiplying the P/D ratio with the Nusselt number.

3.2.11.5 Kalish-Dwyer (1967)

In 1967 S. Kalish and O.E. Dwyer [178] obtained results on NaK for rod bundles with a $\frac{P}{D}$ ratio of 1.75. On the basis of those results and adopting theoretical equations, which allow for the effects of type and degree of rod spacing, the authors developed the following equation for a rod in the interior of a bank in which all the rods were heated and for a simple cosine wall-temperature distribution around the rods ([11] Ch. 2):

$$Nu = \left(\frac{f_1}{D}\right)^{\frac{1}{2}} \left(\frac{P-D}{P}\right)^{\frac{1}{2}} \left[6.19 + 0.2665 Pe_{v_{max}}^{0.653}\right] \tag{228}$$

In case of rods with thin stainless-steel sheaths, the thermal boundary condition approaches that of a uniform wall heat flux. For that case Kalish and Dwyer recommended the equation:

$$Nu = \left(\frac{f_1}{D}\right)^{\frac{1}{2}} \left(\frac{P-D}{P}\right)^{\frac{1}{2}} \left[5.24 + 0.225 Pe_{v_{max}}^{0.653}\right] \tag{229}$$

3.2.11.6 Kottowski (1983)

For flow across a staggered tube bank, H.M. Kottowski (1983) investigated Nusselt number for low Prandtl number fluids for tube bundle geometry in forced convection conditions [119]. Results for mercury and sodium were correlated within $\pm 12\%$ over the Peclet number range of $50 \le Pe \le 4000$ by the following empirical equation given by Kottowski:

$$Nu = Pe^{0.5} \tag{230}$$

which is recommended for use in such conditions. Nusselt number Nu is based on a mean heat transfer coefficient obtained by dividing the mean heat flux from the tube by the circumferential average of the temperature difference between tube and bulk fluid at points equally spaced around the periphery of the tube. For each tube, the bulk temperature of the flowing stream is evaluated at the location in the tube bank corresponding to the axis of the tube in question. The cross flow velocity in Peclet number Pe is based on the minimum flow area [9].

3.2.11.7 Summary of heat transfer correlations for cross flow across rod bundles

Table 22 presents the list of heat transfer correlations collected for cross flow across rod bundles.

TABLE 22. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR CROSS FLOW ACROSS ROD BUNDLES

Rickard et al. (1958) [68] [175]	$Nu = 4.03 + 0.228 Pe^{0.67}$ $2 \cdot 10^4 \le Re \le 2 \cdot 10^5$
Cess-Grosh (1958) [11] [177]	$Nu = 0.718 \left(\frac{f_1}{D}\right)^{\frac{1}{2}} \left(\frac{P-D}{P}\right)^{\frac{1}{2}} Pe_{v_{max}}$ uniform wall temperature
Hsu (1964) [11] [65] [121]	$Nu = a \left(\frac{f_1}{D}\right)^{\frac{1}{2}} \left(\frac{P-D}{P}\right)^{\frac{1}{2}} \sqrt{Pe_{v_{max}}}$ $a = 0.810$ for uniform heat flux from rod surface $a = 0.958$ for cosine surface temperature around the rod
Dwyer (1966) [65] [155]	$Nu = f_{corr}\left(5.36 + 0.1974\, Pe_{v_{max}}^{0.682}\right)$ $f_{corr} = 1\ for\ S_1/D = 1.38$ and $S_2/D = 1.19$ $f_{corr} = P/D\ for\ other\ values\ of\ S_1/D\ and\ S_2/D$
Kalish-Dwyer (1967) [11] [178]	interior of a bank with cosine wall temperature: $Nu = \left(\frac{f_1}{D}\right)^{\frac{1}{2}} \left(\frac{P-D}{P}\right)^{\frac{1}{2}} \left[6.19 + 0.2665 Pe_{v_{max}}^{0.653}\right]$ uniform wall heat flux: $Nu = \left(\frac{f_1}{D}\right)^{\frac{1}{2}} \left(\frac{P-D}{P}\right)^{\frac{1}{2}} \left[5.24 + 0.225 Pe_{v_{max}}^{0.653}\right]$
Kottowski (1983) [119]	$Nu = Pe^{0.5}$ $50 \le Pe \le 4000$

No summary comparison figure is included for the cross flow across bundles due to the inclusion of different variables across the correlations.

3.3 NATURAL CONVECTION

In ordinary fluids the Nusselt number for natural convection is a function of the Rayleigh number $Ra = GrPr$, however in liquid metal flows Nusselt number becomes a function of $GrPr^2$ ([9] Ch. 8) ([11] Ch. 2). Therefore, the general natural convection heat transfer correlation is expressed in the following form:

$$Nu = aPr^b Gr^c \tag{231}$$

where a, b, and c are constants.

The criterion proposed by H.O. Buhr valid for both vertical and horizontal flows can be used to estimate whether free convection will influence the Nusselt number in pipe flow ([9] Ch. 8) [179]. The parameter Z is defined as:

$$Z = \frac{Ra'}{Re}\frac{D_h}{L} \tag{232}$$

Then Buhr criterion states:

- If $Z \leq 20 \cdot 10^{-4}$: the effect of free convection is insignificant
- If $Z \geq 20 \cdot 10^{-4}$: free convection affects forced convection

The prime in $Ra' = Gr'Pr$ is used to distinguish the Grashof number using the axial temperature difference from the usual one using the radial temperature difference:

$$Gr' = \frac{bgD_h^3 DT}{n^2} \tag{233}$$

where

$$\mathrm{D}T = \frac{dT}{dx} D_h$$

There are three distinct modes of free convection: creeping, laminar, and turbulent. These modes are associated to significantly different expressions for heat transfer and occur successively for increasing Rayleigh numbers (see Table 23) [155].

TABLE 23. RAYLEIGH NUMBERS FOR THE THREE FREE CONVECTION MODES

Creeping regime	$1700 \leq Ra < 3500$
Laminar regime	$3500 \leq Ra < 10^5$
Turbulent regime	$Ra \geq 10^5$

3.3.1 Flow on heated vertical plates

The geometry of heat transfer during flow on heated vertical plates is shown in Fig. 42. Also, the temperature profile is seen as well. T_w *– wall temperature,* T *– fluid temperature.*

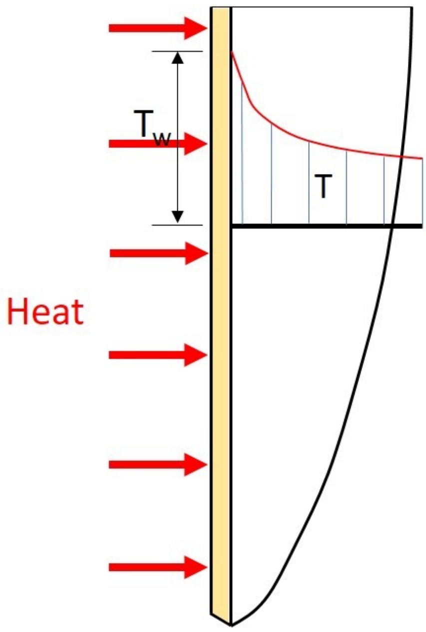

FIG. 42. Flow on heated vertical plates

3.3.1.1 Saunders (1939)

O.A. Saunders (1939) developed a correlation for a natural convection on a heated vertical plate in laminar flow [155] [180]:

$$Nu_x = 0.3{Ra_x}^{0.25} \tag{234}$$

where x is the plate vertical length. It is valid for the laminar regime, where Rayleigh number is in the range of $Ra_x \leq 1 \cdot 10^6$.

3.3.1.2 Eckert (1950)

In 1950 E.R.G. Eckert [11] [181] wrote the momentum and energy equations in an integral form for laminar boundary-layer flow and uniform wall temperature. His final result was:

$$Nu_x = 0.508 \left[\frac{Pr^2 Gr_x}{0.952 + Pr}\right]^{\frac{1}{4}} \tag{235}$$

which agrees quite closely with Ostrach's curve [182]. In [183] another correlation proposed by Eckert in 1939 is presented, where the first parameter is 0.68 instead of 0.508. It is being thought that over time Eckert refined the correlation and Eq. (235) is therefore more accurate.

3.3.1.3 Eckert-Jackson (1951)

When Gr_x becomes sufficiently large ($\sim 10^{10}$), the boundary-layer flow becomes completely turbulent. As this was a difficult case to solve, E.R.G. Eckert and T.W. Jackson in 1951 [184] employing certain simplifying assumptions, solved the momentum and energy equations by integral methods and obtained the equation [11]:

$$Nu_x = 0.0295 \left[\frac{Pr^{\frac{7}{6}} Gr_x}{1 + 0.494 Pr^{\frac{2}{3}}}\right]^{\frac{2}{5}} \tag{236}$$

for the condition of uniform wall temperature. The subscript x represents the vertical distance (measured from below) over which the boundary layer is fully turbulent. For the average Nusselt number, the coefficient 0.0295 becomes 0.0246. As this equation is based on calculations where Prandtl number was assumed to be close to unity, its applicability to liquid metals is doubtful, as indicated by the results of Bayley, Milne, and Stoddart [185].

3.3.1.4 Ostrach (1952)

In 1952 S. Ostrach was the first to solve the basic differential equations of motion and energy describing the temperature field simultaneously (along with the continuity equation), for the laminar-boundary-layer natural-convection heat transfer from vertical plates for the boundary condition of uniform wall temperature and for Prandtl numbers in the liquid metal range [11] [182]. S. Ostrach (1953) showed that the average Nusselt number for a plate of length L is equal to [9] [182]:

$$Nu_m = \frac{4}{3} \cdot Nu_L \tag{237}$$

in the case of uniform wall temperature and laminar flow. The average heat transfer coefficient is based on the average heat flux along the plate length L. In the above formula the subscript m indicates the average, and the subscript L indicates the length L of the plate. As for the local Nusselt number, the correlation reported in 3.3.1.7 is recommended.

3.3.1.5 Siegel (1954)

For the case of turbulent boundary layer flow and uniform wall heat flux, R. Siegel in 1954 [186] recommended the equation [11]:

$$Nu_x = 0.0295 \left[\frac{Pr^{\frac{7}{6}} Gr_x}{1 + 0.444 Pr^{\frac{2}{3}}} \right]^{\frac{2}{5}} \tag{238}$$

3.3.1.6 Sparrow-Gregg (1956)

In 1956 E.M. Sparrow and J.L. Gregg solved numerically the energy and velocity equations for natural convection for Prandtl numbers down to 0.1 and proposed the following local heat transfer correlation for the case of uniform wall heat flux and laminar boundary-layer flow [9] [11] [187] [188]:

$$Nu_x = \frac{({Gr_x}^*)^{1/5}}{5^{1/5} \theta(0)} \tag{239}$$

where

$${Gr_x}^* = \frac{g \beta q x^4}{\lambda n^2} \tag{240}$$

${Gr_x}^*$ is a modified Grashof number and $\theta(0)$ represents a non-dimensional temperature difference evaluated at the wall $h = 0$ $(t = t_w)$. In Eq. (240) g is the gravitational acceleration, β the volumetric thermal expansion coefficient, q the imposed heat flux, λ is the thermal conductivity, ν is the kinematic viscosity and x is the distance from the leading edge of the plate. Estimated values of $\theta(0)$ are presented in Table 24.

TABLE 24. VALUES OF $\theta(0)$ FOR EQ. (239)

Pr	$\theta(0)$
0.01	6.304
0.03	4.198
0.10	2.751

Since the modified Grashof number can be expressed as:

$$Gr_x^* = Gr_x \frac{h_x x}{\lambda} = Gr_x Nu_x \tag{241}$$

the previous correlation can be also written as:

$$Nu_x = \frac{Gr_x^{1/4}}{5^{1/4}[\theta(0)]^{5/4}} \tag{242}$$

Moreover, by fitting the estimated values of $\theta(0)$ to the following relationship:

$$\theta(0) = A \cdot Pr^{-\frac{2}{5}} \tag{243}$$

The previous heat transfer correlation gives:

$$Nu_x = \left(\frac{Gr_x^* \cdot Pr^2}{5}\right)^{\frac{1}{5}} \cdot \frac{1}{A} = \left(\frac{Gr_x \cdot Pr^2}{5}\right)^{\frac{1}{4}} \cdot \frac{1}{A^{\frac{5}{4}}} \tag{244}$$

that follows the classical dependence on the Boussinesq number Eq. (9). The value A is close to one within few percent.

E.M. Sparrow and J.L. Gregg also showed that the average Nusselt number (laminar flow) for a plate of length L is related to the local Nusselt number at $x = L$ by [9] [187]:

$$Nu_m = \frac{6}{5} \cdot Nu_L \tag{245}$$

in the case of uniform heat flux. Here the average heat transfer coefficient is based on the average temperature difference along the plate length L. As for the local Nusselt number, the correlation reported by Sparrow et al. (Section 3.3.1.7) and Chang et al. (Section 3.3.1.8) can be used.

Sparrow and Gregg [187] defined an average heat transfer coefficient and proposed the final expression valid for liquid metals [11]:

$$Nu_x = 0.707 Pr^{0.46} (Gr_x^*)^{\frac{1}{4}} \tag{246}$$

3.3.1.7 Sparrow et al. (1959)

Combining the work of E.M. Sparrow and J.L. Gregg [189], LeFevre [190] and S. Ostrach [182], they obtained solutions of the following form for local Nusselt number for uniform wall temperature, vertical plates, natural-convection, laminar flow [9] [189]:

$$Nu_x = f(Pr) \cdot (Gr_x \cdot Pr^2)^{\frac{1}{4}} \tag{247}$$

where the subscript x indicates the coordinate along the plate and the values of $f(Pr)$ are given in Table 25.

TABLE 25. VALUES OF $f(Pr)$ FOR EQ. (247)

Pr	$f\,(Pr)$
0	0.6004
0.003	0.5827
0.008	0.5729
0.01	0.5715
0.02	0.5582
0.03	0.5497
0.1	0.5160

3.3.1.8 Chang et al. (1964)

K.S. Chang et al. [188] employing a first order perturbation analysis method, succeeded in extending the Sparrow-Gregg results down to $Pr = 0.01$. Their values of $\theta(0)$ for several Prandtl numbers are given in Table 26 [11].

TABLE 26. VALUES OF $\theta(0)$ FOR EQ. (247)

Pr	$\theta(0)$
0.01	6.304
0.03	4.198
0.10	2.751
1.00	1.357

In the range of $0.01 \leq Pr \leq 0.05$, $\theta(0)$ can be expressed by the empirical equation:

$$\theta(0) = 1.147\, Pr^{-0.37} \qquad (248)$$

This gives place to the following equation, valid for liquid metals for uniform heat flux conditions:

$$Nu_x = 0.563 Pr^{0.46} (Gr_x^*)^{\frac{1}{4}} \qquad (249)$$

It can be also expressed as [188] [191]:

$$Nu_x = 0.7320 \cdot (Gr_x \cdot Pr^2)^{\frac{1}{5}} \qquad (250)$$

3.3.1.9 Drokin-Sommercales (1965)

D. Drokin and E. Sommercales (1965) [155] [198] proposed a correlation for natural convection over an enclosed liquid metal gap between plates in turbulent flow.

$$Nu_D = 0.028 \cdot (Ra_D)^{0.355} \text{ for vertical parallel plates} \qquad (251)$$

$$Nu_D = 0.043 \cdot (Ra_D)^{0.33} \text{ for horizontal parallel plates} \qquad (252)$$

where D is the distance between plates, and the turbulent regime Rayleigh number is in the range of $4 \cdot 10^4 \leq Ra \leq 1 \cdot 10^8$.

3.3.1.10 Churchill-Chu (1975)

In 1975 S.W. Churchill and H.H.S. Chu [192] developed simple expressions for the space-mean $\overline{Nu}$ (or Sh) for all Rayleigh and Prandtl (or Schmidt) numbers in terms of the model of Churchill and Usagi. Their development utilizes experimental values for Rayleigh number Ra approaching zero and infinity, and the analytical solutions obtained from laminar boundary-layer theory. The expression is applicable both to uniform heating and to uniform wall temperature conditions, as well as for mass transfer and simultaneous heat and mass transfer. For single-phase liquid sodium natural convection adjacent to a vertical plane (turbulent flow regime), the correlation recommended in [193] is:

$$\overline{Nu} = \left[0.825 + \frac{0.387 Ra^{\frac{1}{6}}}{\left(1 + (0.492/Pr)^{\frac{9}{16}}\right)^{\frac{8}{27}}}\right]^2 \tag{253}$$

It is valid for $0.1 \leq Ra \leq 10^{12}$.

Churchill and Chu also proposed the following relation valid for $Ra \leq 10^9$ [193]:

$$\overline{Nu} = 0.68 + \frac{0.670 Ra^{\frac{1}{4}}}{\left(1 + (0.492/Pr)^{\frac{9}{16}}\right)^{\frac{4}{9}}} \tag{254}$$

This equation may be modified for the case of constant heat flux if the average Nusselt number is based on the wall heat flux and the temperature difference at the centre of the plate $\left(x = \frac{L}{2}\right)$. Then the equation becomes [193]:

$$\overline{Nu}^{\frac{1}{4}}(\overline{Nu} - 0.68) = \frac{0.67 (Gr^* Pr)^{\frac{1}{4}}}{\left(1 + (0.492/Pr)^{\frac{9}{16}}\right)^{\frac{4}{9}}} \tag{255}$$

where $\overline{Nu} = \frac{qL}{(k\overline{\Delta T})}$ and $\overline{\Delta T} = T_w$ at $\frac{L}{2} - T_\infty$.

3.3.1.11 Sheriff-Davies (1979)

For natural convection, laminar flow on vertical plate for uniform heat flux, N. Sheriff and N.W. Davies (1979) [191] proposed the following relation that fits most of the available data within $\pm 7\%$ and is therefore recommended [9] [191]:

$$Nu_x = 0.732 \cdot (Gr_x^* \cdot Pr^2)^{\frac{1}{5}} \tag{256}$$

the subscript x indicates the coordinate along the plate. The properties should be evaluated at the film temperature, the average of surface temperature and bulk mean fluid temperature. Gr_x^* is the modified Grashof number defined in Eq. (240).

3.3.1.12 Summary of heat transfer correlations for flow on vertical plates

Table 27 presents the list of heat transfer correlations collected for flow in vertical plates.

TABLE 27. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN VERTICAL PLATES

Saunders (1939) [155] [180]	$Nu_x = 0.3{Ra_x}^{0.25}$ $Ra_x \leq 1 \cdot 10^6$, laminar flow
Eckert (1950) [11] [181]	$Nu_x = 0.508\left[\frac{Pr^2 Gr_x}{0.952 + Pr}\right]^{\frac{1}{4}}$ laminar flow, uniform wall temperature
Eckert-Jackson (1951) [11] [184]	$Nu_x = 0.0295\left[\frac{Pr^{\frac{7}{6}} Gr_x}{1 + 0.494 Pr^{\frac{2}{3}}}\right]^{\frac{2}{5}}$ $Pr \sim 1$, uniform wall temperature, turbulent flow
Ostrach (1953) [9] [182]	$Nu_m = \frac{4}{3} \cdot Nu_L$ laminar flow, uniform wall temperature
Siegel (1954) [11] [186]	$Nu_x = 0.0295\left[\frac{Pr^{\frac{7}{6}} Gr_x}{1 + 0.444 Pr^{\frac{2}{3}}}\right]^{\frac{2}{5}}$ turbulent flow, uniform heat flux
Sparrow-Gregg (1956) [9] [11] [187] [188]	1. $Nu_x = \frac{({Gr_x}^*)^{1/5}}{5^{1/5}\theta(0)} = \frac{Gr_x^{\frac{1}{4}}}{5^{1/4}[\theta(0)]^{5/4}}$ ${Gr_x}^* = \frac{g\beta q x^4}{\lambda n^2}$, $\theta(0)$ in Table 24, $Gr_x^* = Gr_x Nu_x$ 2. $Nu_x = \left(\frac{Gr_x^* \cdot Pr^2}{5}\right)^{\frac{1}{5}} \cdot \frac{1}{A} = \left(\frac{Gr_x \cdot Pr^2}{5}\right)^{\frac{1}{4}} \cdot \frac{1}{A^{\frac{5}{4}}}$ with $A \sim 1$ 3. $Nu_m = \frac{6}{5} \cdot Nu_L$ 4. $Nu_x = 0.707 Pr^{0.46} (Gr_x^*)^{\frac{1}{4}}$ All are valid for uniform heat flux and laminar flow

TABLE 27. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW IN VERTICAL PLATES

Sparrow et al. (1959) [9] [189]	$Nu_x = f(Pr) \cdot (Gr_x \cdot Pr^2)^{\frac{1}{4}}$ $f(Pr)$ in Table 25 uniform wall temperature, laminar flow
Chang et al. (1964) [188] [191]	$Nu_x = f(Pr) \cdot (Gr_x \cdot Pr^2)^{\frac{1}{4}}$ with $f(Pr)$ in Table 26 $Nu_x = 0.563 Pr^{0.46} (Gr_x^*)^{\frac{1}{4}}$ $Nu_x = 0.7320 \cdot (Gr_x \cdot Pr^2)^{\frac{1}{5}}$ All are valid for $0.01 \leq Pr \leq 0.05$ and uniform heat flux
Dropkin-Sommercales (1965) [155] [198]	For vertical parallel plates: $Nu_D = 0.028 \cdot (Ra_D)^{0.355}$ For horizontal parallel plates: $Nu_D = 0.043 \cdot (Ra_D)^{0.33}$ Both valid for $4 \cdot 10^4 \leq Ra \leq 1 \cdot 10^8$
Churchill-Chu (1975) [192] [193]	1. for $0.1 \leq Ra \leq 10^{12}$: $\overline{Nu} = \left[0.825 + \frac{0.387 Ra^{\frac{1}{6}}}{\left(1 + (0.492/Pr)^{\frac{9}{16}}\right)^{\frac{8}{27}}}\right]^2$ for $Ra \leq 10^9$: $\overline{Nu} = 0.68 + \frac{0.670 Ra^{\frac{1}{4}}}{\left(1 + (0.492/Pr)^{\frac{9}{16}}\right)^{\frac{4}{9}}}$ Both are valid for laminar flow and either uniform heat flux or uniform wall temperature *2.* $\overline{Nu}^{\frac{1}{4}}(\overline{Nu} - 0.68) = \frac{0.67 (Gr^* Pr)^{\frac{1}{4}}}{\left(1 + (0.492/Pr)^{\frac{9}{16}}\right)^{\frac{4}{9}}}$ laminar flow, uniform heat flux
Sheriff-Davies (1979) [9] [191]	$Nu_x = 0.732 \cdot (Gr_x^* \cdot Pr^2)^{\frac{1}{5}}$ laminar flow, uniform heat flux

3.3.2 Flow over horizontal plates and around cylinders

The flow geometry around cylinders is shown in Fig. 43, where *D – diameter,* T_w *– wall temperature,* T_s *– stream (flow) temperature*

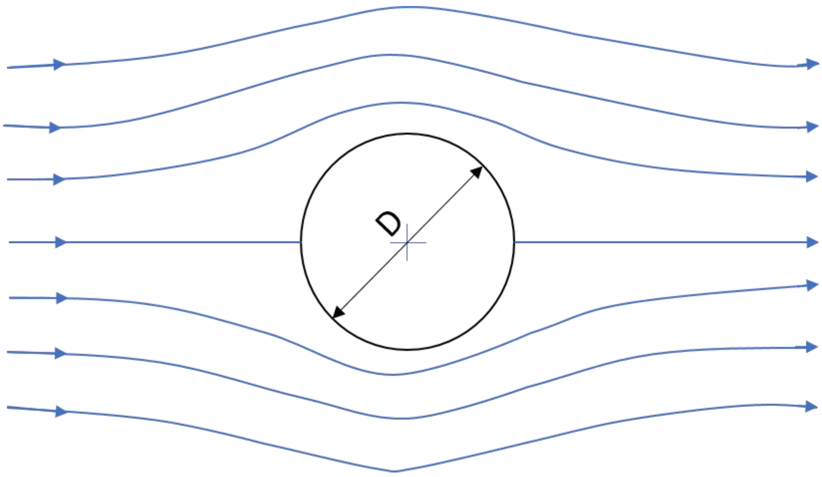

FIG. 43. Flow around cylinder

3.3.2.1 Eckert (1950)

In 1950 E.R.G Eckert proposed the following correlation for natural convection around a horizontal cylinder [110] [181]:

$$Nu_x = 0.53\left[\frac{Pr^2 Gr_x}{0.952 + Pr}\right]^{\frac{1}{4}} \tag{257}$$

where the characteristic length x in Nusselt number $Nu_x = {hx}/{\lambda}$ is the same as in Grashof Number $Gr_x = {bgx^3q}/{\nu^2}$, where x is the cylinder diameter.

3.3.2.2 Hyman et al. (1953)

S.C. Hyman et al. [194] studied natural convection on six different liquid metals and alloys under conditions of laminar-boundary-layer flow. The thermal boundary condition approached that of uniform heat flux [11]. They proposed the following correlation for natural convection around a horizontal cylinder in the laminar flow [155] [194]:

$$Nu = 0.53 \cdot (Ra \cdot Pr)^{0.25} = 0.53 \cdot (Gr \cdot Pr^2)^{0.25} \tag{258}$$

valid for laminar regime and Rayleigh number $Ra \leq 10^5$. Nu and Gr are based on the diameter of the horizontal cylinder and the fluid properties are evaluated at the average of the surface and bulk temperatures.

3.3.2.3 McAdams (1954)

For single-phase sodium natural convection over a horizontal plate in turbulent flow, McAdams (1954) [183] recommended the following correlation:

$$Nu = 0.27 Ra^{0.25} \tag{259}$$

This correlation is valid for $10^5 \leq Ra \leq 10^{10}$ for the lower surface of a heated plane or the upper surface of a cooled plane.

3.3.2.4 Levy (1955)

In 1955 S. Levy [195] using the integral method of calculation, developed a correlation [11]:

$$Nu_x = 0.372\left[\frac{Pr^2 Gr_x}{0.762 + Pr}\right]^{\frac{1}{5}} \tag{260}$$

for an infinitely long, heated, horizontal plate facing upwards; for laminar-boundary-layer flow; for uniform wall temperature; and for low Prandtl numbers. The subscript x at Nusselt number Nu_x means that h is for a point at a distance x from either edge of the plate. For liquid metals, this equation is closely approximated by:

$$Nu_x = 0.39[Pr^2 Gr_x]^{\frac{1}{5}} \tag{261}$$

For turbulent-boundary-layer flow, Levy obtained the relation:

$$Nu_x = 0.0727\left[\frac{Pr^{0.75} Gr_x}{1 + 0.441 Pr^{\frac{2}{3}}}\right]^{\frac{4}{11}} \tag{262}$$

under otherwise similar boundary conditions.

For liquid metals this can be closely approximated by:

$$Nu_x = 0.071[Pr^{0.75} Gr_x]^{0.364} \tag{263}$$

3.3.2.5 Globe-Dropkin (1959)

S. Globe and D. Dropkin (1959) [57] [196] made heat transfer measurements for horizontal spaces filled with mercury, water and silicone oils for a Prandtl number range from $0.02 \leq Pr \leq 8750$. Measurements were made in the Rayleigh range of $3 \cdot 10^5 \leq Ra \leq 7 \cdot 10^9$. The following relation correlates the test results with the reasonable accuracy:

$$Nu = 0.069 \cdot Gr^{\frac{1}{3}} \cdot Pr^{0.407}, \tag{264}$$

where properties are to be evaluated at the average of the two surface temperatures.

Globe and Dropkin [196] measured heat transfer coefficients between the centres of chrome-plated copper plates that formed the top (cooled) and bottom (heated) of a cylindrical chamber. The results obtained under turbulent-boundary-layer-flow conditions covered the Ra range of $4 \cdot 10^5 \leq Ra \leq 4 \cdot 10^7$ and were well represented by the equation [11]:

$$Nu_d = 0.052 Ra_d^{\frac{1}{3}} \tag{265}$$

where the characteristic dimension in both Nu_d and Ra_d is the distance between the plates while the value of Δt in Rayleigh number Ra is the temperature difference between two plate surfaces.

Globe and Dropkin also made measurements with water and silicone oils. All experimental results were generalized in the correlation for the turbulent flow:

$$Nu_d = 0.069 Ra_d^{\frac{1}{3}} Pr^{0.074} = 0.069 Gr_d^{\frac{1}{3}} Pr^{0.407} \tag{266}$$

that can presumably be used to predict Nu_d for liquid metals with Prandtl numbers different from 0.023, the average value for Globe and Dropkin's mercury.

3.3.2.6 McDonald-Connolly (1960)

The experimental results of J.S. McDonald and T.J. Connolly (1960) [155] [197] for the average Nusselt number for turbulent flow and uniform heat flux are correlated for heated upward facing plates cooled downward, turbulent flow and uniform wall temperature by the following equation in the range of $6 \cdot 10^8 \leq Gr_D \leq 5 \cdot 10^9$:

$$Nu = 0.262 (Gr_D Pr^2)^{0.35} \tag{267}$$

The Grashof number is based on the diameter D of the horizontal disk used in the experiment[8]. $Nu_D = \frac{hD}{\lambda}$, where D is the diameter of the horizontal plate. Turbulent regime Rayleigh number: $5 \cdot 10^6 \leq Ra \leq 4 \cdot 10^7$.

The authors also measured [155] [197] heat transfer rates to a cooled eight-inch diameter stainless-steel circular plate facing downward near the surface of a large volume of sodium. Neither uniform-wall-heat-flux nor uniform-wall-temperature conditions existed. These authors developed a correlation for the Nusselt number of liquid sodium under turbulent natural convection over a cold horizontal plate [11]:

$$Nu_D = 0.0785 (Gr_D Pr)^{0.32} = 0.0785 \cdot (Ra)^{0.32} \tag{268}$$

3.3.2.7 Dropkin-Somerscales (1965)

In 1965 D. Dropkin and E. Somerscales [198] made measurements with plates that could be rotated through a 90° angle from horizontal to the vertical position. For mercury, data were taken only for the plates in the horizontal position. The results were presented graphically, and Dwyer, the author of the corresponding Chapter in [11], estimated that mercury results could be represented by the correlation for turbulent flows:

$$Nu = 0.043 Ra^{\frac{1}{3}} \tag{269}$$

3.3.2.8 Kudryavtsev (1967)

In 1967 A.P. Kudryavtsev et al. [199] studied heat transfer to sodium by natural convection from a heated plate placed at the bottom of a vessel. Their data points showed a spread of $\pm 15\%$ from a line represented by the equation for turbulent regime [11]:

$$Nu = 0.38 (Gr_D Pr^2)^{\frac{1}{3}} \tag{270}$$

[8] Experiments were done on horizontal plates with circular geometry, i.e. on disks.

Kudryavtsev et al. observed that the transition from laminar to turbulent-boundary-layer flow occurred at a Gr_D value of $\sim 10^8$. They did not report their laminar-flow results but mentioned that they agreed satisfactorily with the equation:

$$Nu = 0.67\left[\frac{Pr^2 Gr_D}{1+Pr}\right]^{\frac{1}{4}} \tag{271}$$

Indeed, the two previous equations agree within 5% at $Gr_D = 10^8$, assuming $Pr = 0.004$, which is a representative value for their experiments.

3.3.2.9 Clifton-Chapman (1969)

J.V. Clifton and A.J. Chapman (1969) [9] [200] obtained the following average Nusselt number correlation for low Prandtl number fluids for heated downward facing plates, laminar flow and uniform wall temperature:

$$Nu_m = 0.5212 \cdot (Gr_a \cdot Pr^2)^{\frac{1}{5}} \tag{272}$$

3.3.2.10 Pera-Gebhart (1973)

The following theoretical expression for the local Nusselt number under isothermal conditions for heated upward facing plates cooled downward and laminar flow was extrapolated from the work of L. Pera and B. Gebhart (1973) [201] and N. Sheriff and N.W. Davies [9]:

$$Nu_x = 0.48 \cdot (Gr_x \cdot Pr^2)^{\frac{1}{5}} \tag{273}$$

Experimental data of Kudryavtsev et al. [199] in the range $10^6 \leq Gr_x \leq 10^8$ are $0\ to\ 25\%$ above the prediction of this correlation. It might be questionable because of possible edge effects due to the small size of the apparatus. Based on these considerations, the use of this proposed correlation is recommended with caution.

3.3.2.11 Fuji et al. (1973)

Experimental results of N. Sheriff and N.W. Davies [191] were observed to be roughly 15% higher than the approximate integral prediction made by T. Fuji et al. (1973) [9] [202] for the average Nusselt number at $Gr^* \sim 10^{10}$, for horizontal heated downward facing plates, laminar flow and uniform heat flux:

$$Nu_m = 0.522 \cdot (Gr_a^* \cdot Pr^2)^{\frac{1}{6}} \tag{274}$$

Gr_a^* is the modified Grashof number based on half width a of an infinite strip.

3.3.2.12 Churchill-Chu (1975)

In 1975 S.W. Churchill and H.H.S. Chu developed a simple empirical expression for the mean value of Nu over the cylinder for all Rayleigh and Prandtl numbers in terms of the model of Churchill and Usagi [192]. This expression is applicable for uniform heating as well as for uniform wall temperature and for mass transfer and simultaneous heat and mass transfer. These expressions improve the previous existing graphical and empirical correlations in both accuracy and convenience.

$$Nu = \left[0.60 + \frac{0.387Ra^{\frac{1}{6}}}{\left(1 + (0.559/Pr)^{\frac{9}{16}}\right)^{\frac{8}{27}}}\right]^2 \tag{275}$$

It is valid for $10^{-5} \leq GrPr \leq 10^{21}$ [193]. This equation is based on experimental values for $Ra \to 0$ and ∞ and on previous equations for the interrelationship between Ra and Pr. It fails however to take into account the discrete transitions from the laminar to the turbulent regime. It is probably a good approximation for uniform heating if the temperature difference at 90° is used in the definition of Ra and Nu.

A simpler equation but restricted to the laminar range $10^{-6} \leq GrPr \leq 10^{9}$ is [193]:

$$Nu = 0.36 + \frac{0.518(Gr^{*}Pr)^{\frac{1}{4}}}{\left(1 + (0.559/Pr)^{\frac{9}{16}}\right)^{\frac{4}{9}}} \tag{276}$$

3.3.2.13 Summary of heat transfer correlations for flow over horizontal plates and around cylinders

Table 28 presents the list of heat transfer correlations collected for flow over horizontal plates and around cylinders.

TABLE 28. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW OVER HORIZONTAL PLATES AND AROUND CYLINDERS

Eckert (1950) [110] [181]	$Nu_x = 0.53\left[\frac{Pr^2 Gr_x}{0.952 + Pr}\right]^{\frac{1}{4}}$ horizontal cylinder
Hyman et al. (1953) [155] [194]	$Nu = 0.53 \cdot (Ra \cdot Pr)^{0.25} = 0.53 \cdot (Gr \cdot Pr^2)^{0.25}$ $Ra \leq 10^5$, horizontal cylinder, laminar flow, uniform heat flux
McAdams (1954) [183]	$Nu = 0.27Ra^{0.25}$ $10^5 \leq Ra \leq 10^{10}$, turbulent flow, lower surface of a heated plane or the upper surface of a cooled plane

TABLE 28. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW OVER HORIZONTAL PLATES AND AROUND CYLINDERS

Levy (1955) [11] [195]	for laminar flow: $Nu_x = 0.372\left[\frac{Pr^2 Gr_x}{0.762 + Pr}\right]^{\frac{1}{5}}$ $Nu_x = 0.39[Pr^2 Gr_x]^{\frac{1}{5}}$ for liquid metals for turbulent flow: $Nu_x = 0.0727\left[\frac{Pr^{0.75} Gr_x}{1 + 0.441 Pr^{\frac{2}{3}}}\right]^{\frac{4}{11}}$ $Nu_x = 0.071[Pr^{0.75} Gr_x]^{0.364}$ for liquid metals All are valid for heated horizontal plate facing upwards, uniform wall temperature and low Prandtl numbers
Globe-Dropkin (1959) [11] [57] [196]	for $0.02 \leq Pr \leq 8750$, $3 \cdot 10^5 \leq Ra \leq 7 \cdot 10^9$ and for horizontal spaces: $Nu = 0.069 \cdot Gr^{\frac{1}{3}} \cdot Pr^{0.407}$ for turbulent flow, $4 \cdot 10^5 \leq Ra \leq 4 \cdot 10^7$ in the region between top (cooled) and bottom (heated) of a cylindrical chamber: $Nu_d = 0.052 Ra_d^{\frac{1}{3}}$ for turbulent flow: $Nu_d = 0.069 Ra_d^{\frac{1}{3}} Pr^{0.074} = 0.069 Gr_d^{\frac{1}{3}} Pr^{0.407}$ plate
McDonald-Connolly (1960) [11] [155] [197]	for heated upward facing plates cooled downward, uniform heat flux and uniform wall temperature, $6 \cdot 10^8 \leq Gr_D \leq 5 \cdot 10^9, 5 \cdot 10^6 \leq Ra \leq 4 \cdot 10^7$: $Nu = 0.262(Gr_D Pr^2)^{0.35}$ cold horizontal plate, turbulent flow: $Nu_D = 0.0785(Gr_D Pr)^{0.32} = 0.0785 \cdot (\mathrm{Ra})^{0.32}$
Dropkin-Somerscales (1965) [11] [198]	$Nu = 0.043 Ra^{\frac{1}{3}}$ horizontal plate, turbulent flow

TABLE 28. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR FLOW OVER HORIZONTAL PLATES AND AROUND CYLINDERS

Kudryavtsev (1967) [11] [199]	turbulent flow: $Nu = 0.38(Gr_D Pr^2)^{\frac{1}{3}}$ laminar flow: $Nu = 0.67\left[\frac{Pr^2 Gr_D}{1+Pr}\right]^{\frac{1}{4}}$ plate
Clifton-Chapman (1969) [9] [200]	$Nu_m = 0.5212 \cdot (Gr_a \cdot Pr^2)^{\frac{1}{5}}$ plate, laminar flow, uniform wall temperature
Pera-Gebhart (1973) [201]	$Nu_x = 0.48 \cdot (Gr_x \cdot Pr^2)^{\frac{1}{5}}$ plate, isothermal conditions, laminar flow
Fuji et al. (1973) [9] [202]	$Nu_m = 0.522 \cdot (Gr_a^* \cdot Pr^2)^{\frac{1}{6}}$ plates, laminar flow, uniform heat flux
Churchill-Chu (1975) [192] [193]	for $10^{-5} \leq GrPr \leq 10^{21}$: $Nu = \left[0.60 + \frac{0.387 Ra^{\frac{1}{6}}}{\left(1+(0.559/Pr)^{\frac{9}{16}}\right)^{\frac{8}{27}}}\right]^2$ for $10^{-6} \leq GrPr \leq 10^{9}$, laminar flow: $Nu = 0.36 + \frac{0.518(Gr^* Pr)^{\frac{1}{4}}}{\left(1+(0.559/Pr)^{\frac{9}{16}}\right)^{\frac{4}{9}}}$ cylinder, uniform heat flux and uniform wall temperature, all Ra and all Pr

3.3.3 Flow over inclined plate

The typical flow geometry over inclined plates is depicted in Fig. 44.

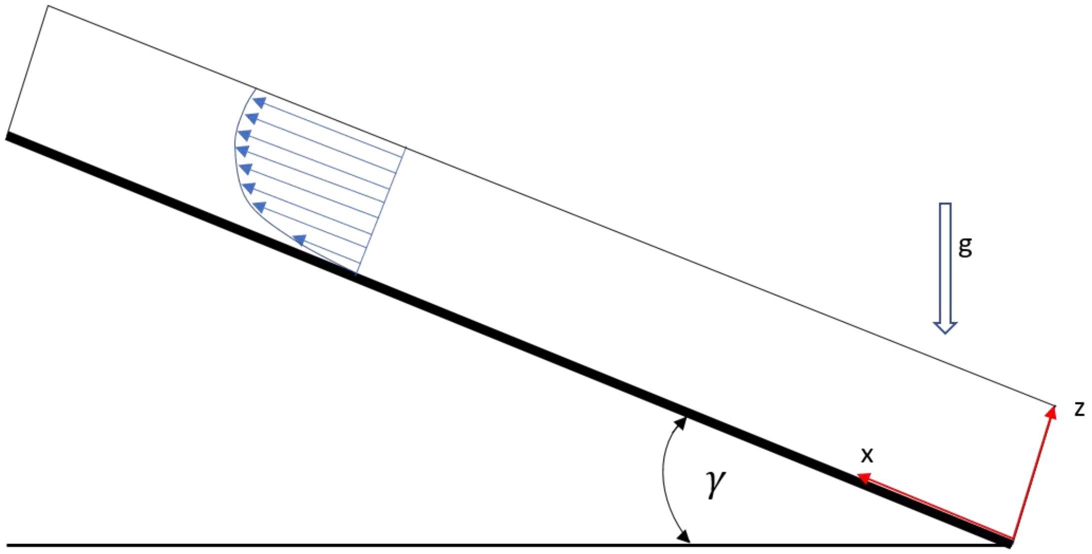

FIG. 44. Flow over inclined plate

3.3.3.1 Sheriff-Davies (1979)

A first-order estimate of the local Nusselt number for uniform heat flux and heated downward-facing surfaces inclined at an angle γ from the vertical can be determined by replacing gravity constant g with $g\cos\gamma$ in the Grashof number according to the study of N. Sheriff and N.W. Davies (1979) Eq. (256). Experimental data for uniform heat flux and laminar flow cited by N. Sheriff and N.W. Davies [191] for γ=75° were roughly 10% lower than the following relation:

$$Nu_x(\gamma) = 0.732 \cdot (Gr_x^* \cdot cos\,\gamma \cdot Pr^2)^{\frac{1}{5}} \tag{277}$$

The data were in the range of $10^5 \le Gr_x^* \le 10^{11}$. Better agreement is expected for low angles because at large angles the thermal boundary layer may actually be below the leading edge of the plate. Gr_x^* is the modified Grashof number based on the distance from the leading edge of the plate x.

3.3.4 Heat transfer in special cases

The geometry of flow inside the walls of vertical vessels is shown in Fig. 45 below.

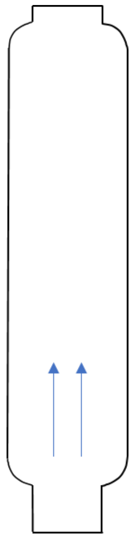

FIG. 45. Flow inside wall of vertical vessel

3.3.4.1 Bayley et al. (1961)

A correlation for the inside wall of vertical vessels under natural convection in laminar and turbulent flow was proposed by F.J. Bayley et al. (1961) [155] [185]. The correlation is of the form:

$$Nu_x = 0.16 \cdot \left[Ra_x \cdot \frac{r}{x}\right]^{0.3} \tag{278}$$

where x is the total height of cylindrical wall and r is the radius of the vessel.

3.3.4.2 Colwell-Welty (1973)

R.G. Colwell and J.R. Welty (1973) [155] [203] developed a correlation for creeping flow $(1 \cdot 10^{-3} \leq Ra \leq 25)$ in natural convection within an open-ended channel:

$$Nu_D = 0.68 \cdot (Ra)^{0.165}, \tag{279}$$

where the characteristic length D in the Nusselt number Nu_D is the distance between plates.

3.4 TWO PHASE SODIUM FLOW

The patterns of two-phase flow are described in Fig. 46 below. The pattern with the lowest void fraction is the bubbly flow. It continues with slug flow, where larger bubbles appear. In the churn flow rising numbers of larger gaseous bubbles start to propagate. In the annular flow the centre is filled with the gaseous phase and the liquid phase is on the edge which makes the annular shape. The last phase is the mist flow, where only the gaseous phase is.

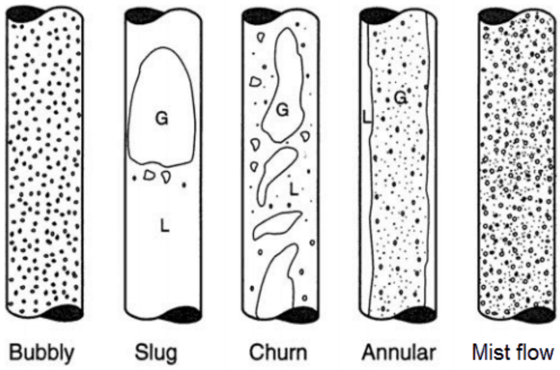

FIG. 46. Two-phase flow patterns for increasing void fractions

3.4.1 Kucherov-Rikenglaz-Silver-Simpson (1960)

A simple expression for the interfacial evaporation/condensation rate through an interface of area A was proposed by R.Y. Kucherov and L.E. Rikenglaz in 1960 [204] and also by Silver and Simpson in 1961 as referred in [205]:

$$\Gamma = A\left(\frac{2\sigma}{2-\sigma}\right)\left(\frac{M}{2\pi R}\right)^{\frac{1}{2}}\left[\frac{P_v}{\sqrt{T_v}} - \frac{P_l}{\sqrt{T_l}}\right] \tag{280}$$

with M the molecular mass of sodium, R the universal gas constant, and (P_i, T_i) the pressure and temperature of the phases ($i = l \text{ or } v$) at the interface. In principle, this correlation is valid for low condensation rates, with a condensation/evaporation coefficient $0 \leq \sigma \leq 1$ accounting for interactions between molecules approaching the interface with those leaving the interface. In practice, it has been used in sodium boiling cases with values of $\lambda = \frac{2\sigma}{2-\sigma}$ ranging between 20 and 50 [206].

The above correlation is a simple approach of the kinetic theory. As it is based on the basic mechanism of evaporation and condensation at a plane liquid-vapour interface, it is applicable to any fluid, like water or sodium. Only the correction factor A has to be different and based on resulting predictions.

The correlation can have different forms according to the condensation rate. The one written here is valid only for low condensation rates. These models ignore the non-equilibrium interactions between the cold molecules leaving the interface and the hot molecules approaching it. It is assumed that the molecular fluxes of the evaporating or condensing streams can be derived from kinetic theory for each component alone and the results superimposed to obtain the net flux, which is not correct. But there are considerable difficulties to solve a more complex problem.

This approach uses the perfect gas law and is only valid for fluids without non-condensable gas. The presence of even a small quantity of non-condensable gas in the condensing vapour has a profound influence on the resistance to heat transfer in the region of liquid-vapour interface.

3.4.2 Aladev et al. (1968)

In 1968 I.T. Aladev et al. recommended the following boiling heat transfer coefficient for potassium in pipes [207] [208]:

$$h = 0.57\, q^{0.7} p^{0.15} \tag{281}$$

It is valid for $0.1 \leq q \left(in\ \frac{MW}{m^2}\right) \leq 1.0$ h is the heat transfer coefficient in $\frac{W}{m^2K}$, q is the heat flux in $\frac{W}{m^2}$ and p is the pressure in kPa.

3.4.3 Subbotin et al. (1969)

In 1969 V.I. Subbotin et al. studied heat transfer in the boiling of metals providing important data regarding heat production and the critical thermal fluxes in the boiling of certain substances. This provided an opportunity of securing generalized relationships for calculating heat transfer in the boiling of metals [209]. In the developed boiling of liquids, including metals, the heat transfer coefficient h may be described by a relation of this type:

$$h = Aq^n p^m \tag{282}$$

where A is a coefficient, q is the specific thermal flux and p is the pressure. For liquid metals like Na, K, Cs and other alkali metals, it can take the expression:

$$h = 8\left[\frac{lr\gamma}{\sigma T^2}\right]^{\frac{1}{3}} q^{\frac{2}{3}} \left[\frac{P}{P_{crit}}\right]^{0.45} \qquad \text{for } \frac{P}{P_{crit}} \leq 10^{-3} \tag{283}$$

$$h = \left[\frac{lr\gamma}{\sigma T^2}\right]^{\frac{1}{3}} q^{\frac{2}{3}} \left[\frac{P}{P_{crit}}\right]^{0.15} \qquad \text{for } 10^{-3} \leq \frac{P}{P_{crit}} \leq 2 \cdot 10^{-2} \tag{284}$$

It is valid for the reduced pressure range $\sim 4 \cdot 10^{-5}$ to $2 \cdot 10^{-2}$, both under conditions of free convection and forced motion, when the vapour fraction in the flow is not greater than $\sim 15\ to\ 20\%$. The possibility of using the previous formulas for calculating the heat transfer coefficients in the boiling of sodium, potassium, and caesium at higher pressures, and also in the boiling of other alkali metals, requires further experimental verification according to the authors.

3.4.4 Kovalev-Zhukov (1973)

In 1973, S.A. Kovalev and V.M. Zhukov described an experimental study of heat transfer during sodium boiling on the surface of a horizontal tube under the conditions of low pressure and natural convection. In order to find out the dependence of the heat transfer coefficient on the pressure, the experimental data were treated in the coordinates of $h/_{q^{0.7}}$ and p. Within the range of 7-35 mm of Hg, all the data obtained were satisfactorily described by a straight line having a slope of 0.25. The values of the heat transfer coefficient and pressure are well described within the studied range of loads by the relation [210]:

$$h = 0.8\, p^{0.25}\, q^{0.7} \tag{285}$$

where p is in mm of Hg and h in $\frac{kcal}{m^2 h°C}$.

3.4.5 Gorlov et al. (1973)

In 1973 I.G. Gorlov et al. presented a study with heat transfer data obtained on potassium boiling in a straight tube under pressures up to $17 \cdot 10^5 \frac{N}{m^2} (T_s \leq 1460\ K)$. In boiling experiments subcooled potassium was supplied to the test section. Boiling started with great superheating relative to the saturation temperature, due to more difficult conditions of vapour formation compared to those for conventional liquids. Experimental studies of heat transfer to boiling potassium in molybdenum and stainless-steel tubes showed that $h = h(p^n)$. The latter points to the fact that the above investigations probably dealt with nucleate boiling in the liquid film wetting the tube wall. For this vaporisation mechanism of potassium, the following relationship was obtained [211]:

$$h = 0.57\ p^{0.15}\ q^{0.7} \tag{286}$$

where h units are $\frac{W}{m^2K}$. The investigation covered the following ranges of the parameters:

$p = (2 - 17) \cdot 10^5 \frac{N}{m^2}$ (T_s up to 1460 K),

$q = (0.7 - 1.8) \cdot 10^6 \frac{W}{m^2}$,

$x_a = 0.02 - 0.75$

$G = 200 - 660 \frac{kg}{m^2 s}$

3.4.6 Dwyer (1976)

O.E. Dwyer (1976) [212] compared experimental data with theoretical correlations for predicting liquid metal nucleate-boiling heat transfer rates. According to the authors the blind use of any of the so-called theoretical correlations is indeed risky. However, under certain conditions, some of them may be used with confidence if the proper value of specific coefficients (α) is known, if they are not used at low pressures (i.e. below 1/1000 of the critical pressure) and if surface coefficient for the particular heating surface (m_1) is known. A judicious choice of a theoretical equation can be made on the basis of comparison. Overall, the Kutateladze correlation as slightly modified by Minchenko appears to be the most generally dependable on h for predicting the influence of several parameters when alkali metals boil on commercially smooth stainless-steel surfaces and at a reduced pressure greater than 0.001.

$$Nu_b \equiv \frac{q}{(T_w - T_{sat})\lambda_l}\left[\frac{\sigma}{g(\rho_L - \rho_V)}\right]^{\frac{1}{2}} = \alpha\ (Re_b)^{m_1}(Pr_L)^{m_2}(K_p)^{m_3} \tag{287}$$

$$Re_b \equiv \frac{q}{c_v \rho_V \upsilon_L}\left[\frac{\sigma}{g(\rho_L - \rho_V)}\right]^{\frac{1}{2}} \tag{288}$$

$$K_p \equiv \frac{q}{[g\sigma(\rho_L - \rho_V)]^{\frac{1}{2}}} \tag{289}$$

$\alpha = 7 \cdot 10^{-4}, m_1 = 0.7, m_2 = 0.35, m_3 = 0.7$ for Kutateladze correlation

$\alpha = 7 \cdot 10^{-4},\ m_1 = 0.7,\ m_2 = 0.7,\ m_3 = 0.7$ for Minchenko correlation where q is the heat flux, T_w the wall temperature, T_{sat} the saturation temperature, λ_l the thermal conductivity of the liquid, σ the surface tension of the liquid, g the gravitational acceleration, ρ_L and ρ_v the liquid and vapour densities, respectively, c_v the latent heat of vaporisation and ν_L the liquid kinematic viscosity. The value of α, parameter depending on the liquid metal/heating surface system, is to be estimated from experimental data. Dwyer also noted that

> "unless the particular situation to be confronted is familiar and certain correlation with a given set of constants is known for adequately representing it, it is better to try to find an empirical correlation based on data from experimental conditions that match or closely simulate this data. Further, one should be cautious when applying such a correlation to a situation that represents a substantial extrapolation of one or more of the important variables beyond the limits on which the correlation is based. The results of accurate experimental data under a variety of conditions over a wide range of pressure and where all the important independent variables have been carefully controlled are needed before generalized heat transfer correlation for nucleate boiling of liquid metals can be improved further" [212]

3.4.7 Zeigarnik (1980)

In 1980 Y.A. Zeigarnik et al. performed experiments to investigate heat transfer of sodium boiling for forced flow in a tube [208] [213]. The heat flux on the wall was up to $1.1\,\frac{MW}{m^2}$, mass velocity from 150 to 400 $\frac{kg}{m^2s}$, sodium vapour quality up to 0.45 and system pressure from 100 to 200 kPa. A total temperature difference between the wall and the saturation temperatures in the range from 1.5 to 3°C was observed. The experimental data is transformed to a correlation of the temperature difference between the wall and the saturation temperatures as a function of the heat flux:

$$\Delta T = 2.12 \cdot 10^{-6} q + 0.726 \tag{290}$$

It is valid for $0.2 \leq q(in\ \frac{MW}{m^2}) \leq 1.2$ and $p = 100\ kPa$

3.4.8 Carbajo-Rose (1984)

The correlation proposed by J.J. Carbajo and S.D. Rose in 1984 [214] is of interest for dryout studies. The authors noted that

> "under certain postulated accident conditions for a Liquid Metal Fast Breeder Reactor (LMFBR), such as the failure of the shutdown heat removal system (SHRS), sodium boiling and clad dryout might occur in the fuel assemblies. It is important to predict the time from boiling inception to dryout, since sustained clad dryout will result in core damage" [214]

In general, this work is based on 21 boiling tests which resulted in dryout, a 19-pin full-length simulated LMFBR fuel assembly and from a 61-pin full-length simulated LMFBR fuel assembly. The proposed correlation "was obtained from experimental facilities with powers from *4.1* to *15.3* $\frac{kW}{pin}$, inlet velocities between *0.22* and *1.35* $\frac{m}{s}$, corresponding to inlet flows between $4.6 \cdot 10^{-6}$ and $26.3 \cdot 10^{-6}\ \frac{m^3}{s}$ per pin, test section inlet temperatures from $386°C$ to $450°C$, boiling temperatures from $913°C$ to $982°C$, bundle housing perimeters of *0.11* and *0.20 m* and flow areas of $3.7 \cdot 10^{-4}$ and $12.9 \cdot 10^{-4}\ m^2$" [214]

The authors also stated that "*the correlation was evaluated with other non-dryout boiling and non-boiling runs and good agreement was obtained. The correlation could predict if boiling occurred (by using factor K_1), and for how long it could be maintained*".

The experimental tests were performed as follows:

"for each specified bundle power, an initial steady-state high sodium flow was established, for which sodium boiling did not occur in the bundle. The temperature at the outlet of the test section was $700°C$. Then, using a programmable pump control system, the flow was reduced to a lower value and boiling occurred. The flow at the beginning of the transient is called initial flow Q. In order to correlate the data, two separate factors were chosen:

$$K_1 = \frac{P}{Q\rho_{in}\Delta h_{sub}} \tag{291}$$

and

$$K_2 = 10^3 \frac{L}{Nv} = \frac{10^3 LA}{NQ} \tag{292}$$

where P is total input power (kW), Q is initial flow (m^3/s), v is inlet velocity (m/s), ρ_{in} is the density of the sodium at the inlet (kg/m^3), Δh_{sub} is the specific inlet enthalpy of subcooling or the difference between liquid saturation and inlet enthalpies of sodium (kJ/kg), L is the perimeter of the bundle housing (m), A is the flow area (m^2) and N is the number of fuel pins in the bundle.

The factor K_1, is dimensionless, and the factor K_2 has the dimension of a time (seconds). Factor K_1 is the ratio of the total input power to the power needed to bring all the sodium inlet flow to saturated liquid at the downstream end of the heated section. A value of one for K_1 will produce liquid saturation conditions for the sodium at the end of the heated section. Values larger than one will produce boiling. Values below one will produce only local boiling or no boiling. Boiling-to-dryout times are expected to increase with decreasing value of K_1. By the trial and error method, the following correlating parameter I_d was obtained" " [214]

$$I_d = \frac{\sqrt{K_2}}{K_1} \tag{293}$$

The dryout time, t_d is defined as the time from boiling inception to dryout. Two distinct regions were identified for the forced convection and for the natural convection tests. However, most of the tests were within 25% of the estimated fitting curve.

The authors obtained a two-part correlation as follow:

> “for forced convection, using the least-squares method for the logarithm of the dryout time, the following expression was obtained:
>
> $$t_d = 10^{0.76\, I_d - 0.32} \tag{294}$$
>
> valid for: $1.6 \le I_d \le 2.5$ corresponding to a range $8\, s \le t_d \le 39\, s$. The least-squares correlation coefficient was 0.76 and the maximum deviation was 31%.
>
> For natural convection, the following curve was obtained:
>
> $$t_d = 10^{0.32\,(I_d)^3 - 0.98\,(I_d)^2 + 2.7} \tag{295}$$
>
> valid for $2.5 \le I_d \le 3.15$ corresponding to a range of $39\, s \le t_d \le 1000\, s$. The least-squares correlation coefficient was 0.85 and the maximum deviation was 29%.
>
> In order to use this correlation, factor K_1 should be calculated first. If this factor is less than one, no boiling occurs. If $K_1 \ge 1$, then factor K_2 and the parameter I_d, should be calculated.
>
> If $I_d \le 1.6$, $t_d \le 8\, s$, dryout occurs very rapidly.
>
> If $1.6 \le I_d \le 2.5$, is typical for forced convection runs.
>
> If $2.5 \le I_d \le 3.15$, is typical for natural convection runs.
>
> If $I_d \ge 3.15$, then $t_d \ge 1000$ s, a very long boiling time, which can be considered as no dryout” [214].

3.4.9 Sorokin (2002)

In 2002, G.A. Sorokin et al. proposed the following heat transfer relation for pool boiling and pin bundles for potassium and NaK [208] [215]:

$$\begin{gathered} h = A q^m p^n \\ A = 4.5 - 7.5 \\ m = 0.7 \\ n = 0.1 - 0.15 \end{gathered} \tag{296}$$

This equation is valid for $0.01 \frac{MW}{m^2} \le q \le 1.0 \frac{MW}{m^2}$ and $40\, kPa \le p \le 120\, kPa$.

3.4.10 Dunn (2012)

The sodium voiding model in SAS4A/SASSYS-1 (F.E. Dunn model) [216] is a multiple bubble slug ejection. As stated by the author,

> “The main purposes of this model are to predict the rate and extent of voiding for the voiding reactivity calculations and to predict the heat removal from the cladding surface, after the onset of voiding, for the fuel and cladding temperature calculations.

Voiding is assumed to result in the formation of bubbles that fill the whole cross section of the coolant channel, except for a film of liquid sodium that is left on the cladding and on the structure. Up to nine bubbles, separated by liquid slugs, are allowed in the channel at any time. The film is treated as a static film of a thickness that changes due to vaporisation or condensation…

…The extent of voiding is determined mainly by liquid slug motion. The bubble pressures at the bubble-slug interfaces drive the liquid slugs. Therefore, the voiding calculation couples vapour or gas pressure calculations for the bubbles with momentum equations for the liquid slugs. If a bubble is small, it is assumed that the vapour or gas pressure within the bubble is constant throughout the bubble, and the bubble pressure is computed using a uniform vapour pressure model. For larger bubbles, the vapour pressure is calculated from a pressure gradient model" [216].

In terms of the cladding-vapour resistance, the heat transfer coefficient between cladding and sodium vapour bubble

"takes the form of:

$$h = \frac{\lambda}{w_{fe}} \tag{297}$$

where λ is the thermal conductivity of liquid sodium and w_{fe} is the thickness of liquid sodium film on the cladding. Such relation is valid if the cladding is more than 100 K hotter than the vapour. Then the liquid film is assumed to be at the same temperature as the vapour, which amounts to it, thus neglecting the thermal resistance of the vapour itself. If the cladding is more than 100 K colder than the vapour, then the liquid film is assumed to be at the same temperature as the cladding and so the resistance of the film is neglected. The heat transfer coefficient from the vapour to the film is then a condensation coefficient for which a reasonable value is" [216]:

$$h_{cond} = 6 \cdot 10^4 \, \frac{W}{m^2K} \tag{298}$$

In the intermediate temperature range, "*the heat transfer coefficient is calculated as an interpolation of the condensation coefficient and the conductive film resistance according to the formula"*:

$$h = h_{cond} + \frac{\frac{\lambda}{w_{fe}} - h_{cond}}{1 + exp\left(\frac{T_{vapor} - T_{cladding}}{2}\right)} \tag{299}$$

3.4.11 Qiu et al. (2015)

In 2015 Z.C. Qiu et al. published a paper on the experimental research on the thermal-hydraulic characteristics of sodium boiling in an annulus. The authors concluded that the experimental data indicate that the heat transfer is achieved by evaporation at the vapour–liquid interface for a well-purified liquid metal under forced motion conditions. Based on the data obtained in this study, a new correlation for boiling heat transfer coefficient (in W/m^2K), was proposed as [208] being:

$$h = 5\, q^{0.7} p^{0.15} \tag{300}$$

The deviation between the boiling heat transfer coefficient data obtained from the experiment and the calculated values is within 25%.

3.4.12 Summary of heat transfer correlations for two-phase sodium flow

Table 29 presents the list of heat transfer correlations collected for two-phase sodium flow.

TABLE 29. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR TWO-PHASE SODIUM FLOW

Kucherov-Rikenglaz-Silver-Simpson (1960) [204] [205]	interfacial evaporation/condensation rate, low condensation rates $\Gamma = A\left(\frac{2\sigma}{2-\sigma}\right)\left(\frac{M}{2\pi R}\right)^{\frac{1}{2}}\left[\frac{P_v}{\sqrt{T_v}} - \frac{P_l}{\sqrt{T_l}}\right]$ $0 \le \sigma \le 1, 20 \le \lambda \le 50$
Aladev et al. (1968) [207] [208]	$h = 0.57\, q^{0.7} p^{0.15}$ $0.1 \ \le q \le 1.0\ (MW/m^2)$
Subbotin et al. (1969) [209]	for $\frac{P}{P_{crit}} \le 10^{-3}$: $h = 8\left[\frac{lr\gamma}{\sigma T^2}\right]^{\frac{1}{3}} q^{\frac{2}{3}}\left[\frac{P}{P_{crit}}\right]^{0.45}$ for $10^{-3} \le \frac{P}{P_{crit}} \le 2 \cdot 10^{-2}$: $h = \left[\frac{lr\gamma}{\sigma T^2}\right]^{\frac{1}{3}} q^{\frac{2}{3}}\left[\frac{P}{P_{crit}}\right]^{0.15}$ Valid under free convection and forced motion, when the vapour fraction $x < 15\ to\ 20\%$.
Kovalev-Zhukov (1973) [210]	$h = 0.8\, p^{0.25}\, q^{0.7}$ p in mm of Hg and q in $kcal/m^2h\ °C$
Gorlov et al. (1973) [211]	$h = 0.57\, p^{0.15}\, q^{0.7}\left(\frac{W}{m^2K}\right)$ $p = (2-17)\cdot 10^5\,\frac{N}{m^2}, T_s \le 1460\ K, q = (0.7-1.8)\cdot 10^6\,\frac{W}{m^2},$ $x_a = 0.02\ -\ 0.75, W_p = 200 - 660\,\frac{kg}{m^2 s}$

TABLE 29. SUMMARY OF HEAT TRANSFER CORRELATIONS FOR TWO-PHASE SODIUM FLOW

Dwyer (1976) [212]	$Nu_b \equiv \frac{q}{(T_w - T_{sat})\lambda_l}\left[\frac{\sigma}{g(\rho_L - \rho_V)}\right]^{\frac{1}{2}} = \alpha\,(Re_b)^{m_1}(Pr_L)^{m_2}\left(K_p\right)^{m_3}$ $Re_b \equiv \frac{q}{c_v \rho_V \upsilon_L}\left[\frac{\sigma}{g(\rho_L - \rho_V)}\right]^{\frac{1}{2}}$ $K_p \equiv \frac{q}{[g\sigma(\rho_L - \rho_V)]^{\frac{1}{2}}}$ for Kutateladze correlation: $\alpha = 7 \cdot 10^{-4}, m_1 = 0.7,\ m_2 = 0.35, m_3 = 0.7$ for Minchenko correlation: $\alpha = 7 \cdot 10^{-4},\ m_1 = 0.7,\ m_2 = 0.7,\ m_3 = 0.7$
Zeigarnik (1980) [208] [213]	$\Delta T = 2.12 \cdot 10^{-6} q + 0.726$ $0.2 \leq q \leq 1.2\ \left(\frac{MW}{m^2}\right), p = 100\ kPa$
Carbajo-Rose (1984) [214]	$K_1 = \frac{P}{Q\rho_{in}\Delta h_{sub}}$ $K_2 = 10^3\,\frac{L}{Nv} = \frac{10^3 LA}{NQ}$ $I_d = \frac{\sqrt{K_2}}{K_1}$ for forced convection: $t_d = 10^{0.76\, I_d - 0.32}$, $1.6 \leq I_d \leq 2.5$ for natural convection: $t_d = 10^{0.32\,(I_d)^3 - 0.98\,(I_d)^2 + 2.7}$, $2.5 \leq I_d \leq 3.15$
Sorokin (2002) [208] [215]	$h = Aq^m p^n$ $A = 4.5 - 7.5, m = 0.7, n = 0.1 - 0.15,$ $0.01\ \leq q \leq 1.0\ (MW/m^2)$, $40\ kPa \leq p \leq 120\ kPa$
Dunn (2012) [216]	$h_{cond} = 6 \cdot 10^4\ \frac{W}{m^2 K}$ $h = h_{cond} + \frac{\frac{\lambda}{w_{fe}} - h_{cond}}{1 + exp\left(\frac{T_{vapor} - T_{cladding}}{2}\right)}$
Qiu et al. (2015) [208]	$h = 5\ q^{0.7} p^{0.15}$

3.5 MODELLING OF HEAT TRANSFER IN FUEL PINS

The purpose of this section is to support some of the correlations presented from Russian investigation and to present the assessment needed to evaluate experimental tests studying the heat transfer in fuel pin assemblies [160][217][218]. It describes approximate equations to evaluate the temperatures in the fuel pin surfaces as well as in the coolant in a non-dimensional way. The theory and methodology of thermal modelling of fuel pins were proposed and developed by P. Ushakov in 1967 [217] and have been successfully applied to several Russian sodium cooled fast reactor designs. The approach allows to design an experimental model of the fuel pin that does not exactly reflects the pin geometry, heat source and conductivities of fuel, coolant and cladding but, nevertheless, represent nearly the same relative temperature distributions.

Assuming the fuel pin being composed of 'n' layers with contact thermal resistance between fuel and cladding or between cladding layers, (Fig. 47-a), the equation for non-dimensional temperatures in the fuel pin and coolant can be written as follows:

$$T_i = \frac{t_i \lambda_i}{\bar{q} R_{n+1}} = f_1(\xi, z, \varphi, x, \xi_1, \xi_2, \dots, \xi_n, \Lambda_0, \Lambda_1, \dots, \Lambda_n, \sigma_1, \sigma_2, \dots, \sigma_n, Re, Pe) \quad (301)$$

$$T_f = \frac{t_f \lambda_f}{\bar{q} R_{n+1}} = f_2(\xi, z, \varphi, x, \xi_1, \xi_2, \dots, \xi_n, \Lambda_0, \Lambda_1, \dots, \Lambda_n, \sigma_1, \sigma_2, \dots, \sigma_n, Re, Pe) \quad (302)$$

where T_i and T_f are non-dimensional internal cladding layer and coolant temperatures, respectively;

t_i and t_f are temperatures of internal pin layer i and coolant;

$\bar{q}$ is heat flux averaged over perimeter;

λ_f is coolant conductivity; and

R_{n+1} is the external pin radius.

It is assumed in Eqs. (301) and (302) that both non-dimensional temperatures can be defined as functions of the following quantities:

$\xi = {r}/{R_{n+1}}$, z, φ are non-dimensional coordinates in radial, axial and azimuthal directions, respectively;

$x = {P}/{2R_{n+1}}$ is the pitch-to-diameter ratio (P is the pitch of pin array);

n is the number of cladding layers in the pin;

$\xi_i = {R_i}/{R_{n+1}}$ $(i = 1, \dots, n)$ is the non-dimensional thickness of the cladding layer i;

$\Lambda_i = {\lambda_i}/{\lambda_f}$ $(i = 1, \dots, n)$ is the non-dimensional thermal conductivity of fuel (Λ_0) and claddings (${\Lambda_1}/{\Lambda_n}$);

$\sigma_i = {\phi_i \lambda_{i-1}}/{R_{n-1}}$, $(i = 1, \ldots, n)$ is a non-dimensional contact resistance between fuel and cladding (σ_1) and between claddings $(\sigma_2, \sigma_3, \ldots, \sigma_n)$;

ϕ_i is the thermal resistance;

$\bar{q} = {q_v R_1^2}/{2R_{n+1}}$ is the mean heat flux at the pin surface $(\xi = 1)$;

$Re = {\bar{w} d_h}/{v}$ is the Reynolds number;

$Pe = {\bar{w} d_h}/{\alpha}$ is the Peclet number;

$\bar{w}$ is the mean coolant velocity;

d_h is the hydraulic diameter of the channel;

v is the kinematic viscosity; and

α is the thermal diffusivity.

These parameters are also indicated in Fig. 47-a.

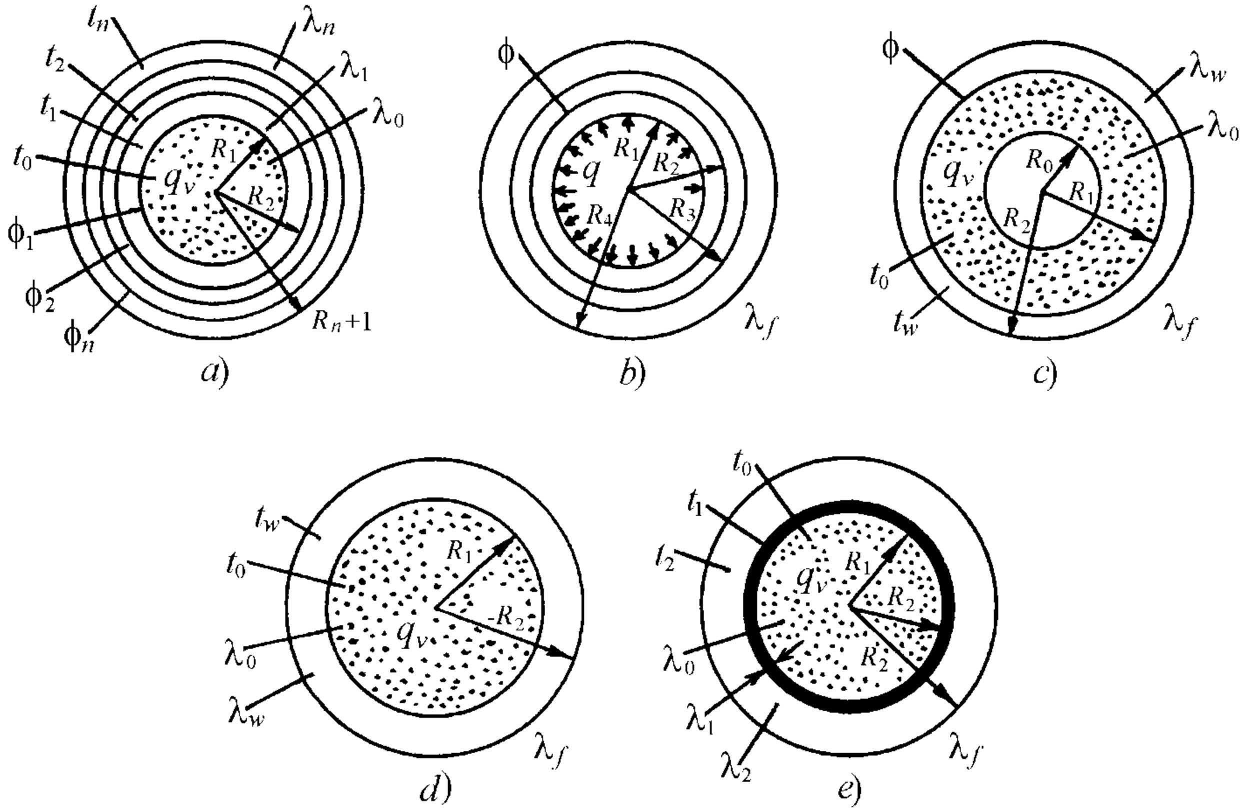

FIG. 47 Geometry and nomenclature for the fuel pin model

Equations (301) and (302) are derived from the analysis of the heat conduction equations in a heated pin and the heat transfer to the coolant assuming the following: i) heat transfer is steady-state and fully-developed; ii) coolant and pin properties do not depend on temperature; iii) the volumetric heat source in the pin is uniform and isotropic; iv) the pin is symmetrical in its properties and geometry; v) the second derivative of temperature with respect to the vertical axis z is negligible; and vi) the turbulence characteristics of the coolant flow are defined only by the geometry and properties of the channel and by the coolant velocity.

In experimental simulation, parameters $x, {}^{\Lambda_1}/_{\Lambda_n}, Re, Pe$ in a model are easy to get from the real fuel pin. Difficulties arise when the experimental pin is heated with volumetric power source (modelling Λ_0) and features contact thermal resistance (σ_i). If the pins are considered as free of contact thermal resistance, it is difficult to provide an ideal thermal contact between the cladding layers in the experimental model. Therefore, it is difficult to reproduce the relative thickness of the cladding layers in experiments.

The essence of the thermal modelling of fuel pins [160] [217] [218] consists of the integration of the parameters responsible for azimuthal thermal conductivity of the pin, namely Λ_0, ${}^{\Lambda_1}/_{\Lambda_n}$, ${}^{\xi_1}/_{\xi_n}$, ${}^{\sigma_1}/_{\sigma_n}$, into a single parameter of equivalent thermal conductivity ε_k and in the equivalence of parameters ε_{k0} for the real fuel pin and for the experimental model. These parameters are calculated on the basis of the main temperature harmonics in Fourier series expansion. Such parameter integration was confirmed by the analytical solution of the fuel pin heat conduction equation. N.I. Buleev was the first who solved this problem applying it to a cylindrical pin containing fuel and one cladding. The general solution of the problem on cylindrical pin embedded in 'n' cladding layers and having the contact interlayers was performed by P.A. Ushakov [217] where he derived the relationship for ε_{k0} .

Using of ε_{k0} allows to derive the following criterion equations:

$$T_f \approx \Psi_1(\xi, z, \varphi, \varepsilon_{k0}, Re, Pe) \tag{303}$$

$$Nu \approx \Psi_2(z, \varepsilon_{k0}, Re, Pe) \tag{304}$$

The technical concepts of thermal modelling were developed taking into account these equations.

The major practical issue is evaluating the accuracy of approximate thermal modelling of a fuel pin. Examples of well simulated fuel pins are found in the reactors (BN-350, BOR-60, BN-600, BN-800) with low fuel thermal conductivity ($l_0 \sim 1.8 - 2.9\ W/mK$). At the inner surface of the pin the uniform heat flux condition should be met. Dependence of parameter ε_k on thermal resistance or on a number of harmonics for such a pin and their simulators are similar in kind. Values of ε_k are slightly different (not more than by 5%) from each other.

To calculate ε_k for the fast reactor pin, the following relations can be used:

$$\varepsilon_k = \frac{\lambda_w}{\lambda_f} \frac{1 + x_1 + \left(\sigma + \frac{x_1 + x_0}{x_1 - x_0}\right)(1 - x_1) - m\left[1 + x_1 + \left(\sigma + \frac{x_1 + x_0}{x_1 - x_0}\right)(1 - x_1)\right]}{1 - x_1 + \left(\sigma + \frac{x_1 + x_0}{x_1 - x_0}\right)(1 + x_1) - m\left[1 - x_1 + \left(\sigma + \frac{x_1 + x_0}{x_1 - x_0}\right)(1 + x_1)\right]} \tag{305}$$

where $x_0 = \xi_0^{2k}$, $\xi_0 = R_0/R_2$, $x_1 = \xi_1^{2k}$, $\xi_1 = R_1/R_2$, $m = (\lambda_w - \lambda_0)/(\lambda_w + \lambda_0)$, $\sigma = k\lambda_w \Phi/R_1$

The parameters related to the above description are specified in Fig. 47-c. The number of the main harmonics in Fourier series is accepted to be equal $k = k_0 = 6$ for the regular part of the bundle and $k = k_0 = 1$ for the edge pins.

The value of the contact thermal resistance originated between the stainless-steel and dioxide of uranium may be defined in accordance with [219], and those between the helix and insulation were defined in experiments [220] ($R_t = 1.3 \cdot 10^{-2}\ m^2K/W$).

An internal structure of the pin has no effect on the value of ε_k for the most part, as shown in [220]. Therefore, ε_k can be predicted with the single pipe formula (see nomenclature in Fig. 47-b):

$$\varepsilon_k = \frac{\lambda_w}{\lambda_f} \frac{1 - \left({R_3}/{R_4} \right)^{2k}}{1 + \left({R_3}/{R_4} \right)^{2k}} \tag{306}$$

3.6 HEAT TRANSFER CORRELATIONS USED IN SYSTEM CODES

Table 30 summarizes the information as to what heat transfer correlations are commonly being used nowadays across the system codes used by experts performing safety analysis of the sodium cooled fast reactors for pipes or pin bundle geometries. Most codes do allow the user to specify or input other heat transfer correlation but for most applications typically default options are selected.

TABLE 30. SUMMARY OF HEAT TRANSFER CORRELATIONS USED IN SYSTEM CODES

CODE	PIPE	ROD BUNDLE
SAM	Seban-Shimazaki (1951)	Schad-Kazimi-Carelli (1974)
RELAP 5 & 7	Seban-Shimazaki (1951)	Schad-Kazimi-Carelli (1974)
TRACE	Seban-Shimazaki (1951)	Ushakov et al. (1977)
ANTEO+	Cheng-Tak (2004) and Sleicher-Rouse (1975)	Mikityuk (2009)
ATHLET	Lyon (1949), Skupinski (1965), and Notter-Sleicher (1973)	Ushakov et al. (1977), Mikityuk (2009), Gräber-Rieger (1972)
ATHENA	Seban-Shimazaki (1951)	Kazimi-Carelli (1974)
CATHARE	Dittus-Boelter (1930), Lyon (1949), Skupinski (1965), Borishanski (1983)	Dittus-Boelter (1930), Lyon (1949), Skupinski (1965)
HYDRA (IBRAE)	--	Ushakov (1977)
MARS-LMR	Aoki (1973)	Modified Schad (1976) Gräber-Rieger (1972)
SAS4A/SAS-SFR	Seban-Shimazaki (1951)	--

4 FRICTION FACTORS AND PRESSURE DROP CORRELATIONS

While thermal properties are very different between sodium and water, hydraulic properties are quite similar. For instance, sodium viscosity at nominal temperature ($\sim 400°C$) is of the same order of magnitude as water at $100°C$. Liquid density at operational temperature is quite similar as well. Hence, it is possible to realize experimental studies with water as a simulant fluid to characterize sodium pressure drop in reactor components.

As fast reactor deployment brought the development of specific technologies, used correlations have to take into account specific geometries, for example in-core (triangular-lattice rod bundles, wire spacers, plenum, etc.) or in heat exchangers (tube-side and shell-side).

For performance and safety studies, it is required to be able to characterize sodium pressure drop for single-phase flow, as well as for two-phase flow. Regarding the latter, for regimes in which evaporation/condensation kinetics do not play a large role, air/water can be used as a simulant for sodium liquid/gas, as their density ratios are quite similar. When such an approximation is valid, the use of air/water allows for accurately measuring of the void fraction, which in turn allows one to distinguish the interfacial friction and wall friction contributions to the total pressure drop in two-phase flows.

The straight tube, straight vertical shell configuration is generally preferred in IHX designs because of its greater simplicity of design and construction [221]. However, helical tube bundles such as in Superphenix steam generator or in KNK reactor were also used [45]. Therefore, two sections presenting single-phase and two-phase friction factors in helical/curved pipes have been considered in this report.

Chapter 0 consists of three sections, describing friction factor/pressure drop correlations for single-phase and two-phase sodium flow and the third one presenting the friction factor correlations used in the system codes.

Following the definition of the friction factor proposed by Darcy-Weisbach and Moody, all friction factors from different sources are transformed to the Darcy-Weisbach's form. For instance, in Kakac et al. handbook [9] (Ch. 18, Ch. 7, Ch. 4 and Ch. 5) the friction factors are given as Fanning friction factors that are four times smaller than the Darcy-Weisbach friction factor Eq. (12).

4.1 SINGLE PHASE FRICTION FACTOR AND PRESSURE DROP CORRELATIONS

This section collects friction factors and pressure drop coefficient for single-phase flows.

4.1.1 Flow in straight pipes

For the estimation of the friction factor, it is frequently distinguished between smooth tubes and tubes with a rough surface. For smooth tubes, the theory of dimensional analysis indicates that the friction factor depends on the Reynolds number only as in laminar flow case. For non-smooth tube surface, the friction factor depends both on the Reynolds number and a parameter ε/D (ε/r) that represents the relative non-dimensional roughness, where ε is the height of the wall surface roughness and D (r) is the diameter (radius) of the pipe.

Some of the correlations below presented are implicit in the friction factor, which makes them more complicate to be used than those which are explicit. For those correlation including logarithm terms, two expressions might be included, one using the natural logarithm and one using the decimal logarithm. Both expressions are related using the expression $\log x = {\ln x}/{\ln 10}$ or similarly $\ln x = {\log x}/{\log e}$.

Figure 48 depicts the velocity profile u of flow in straight pipes. Here *D – pipe diameter,* *u_{bulk} – bulk velocity, u_{max} – maximum velocity*

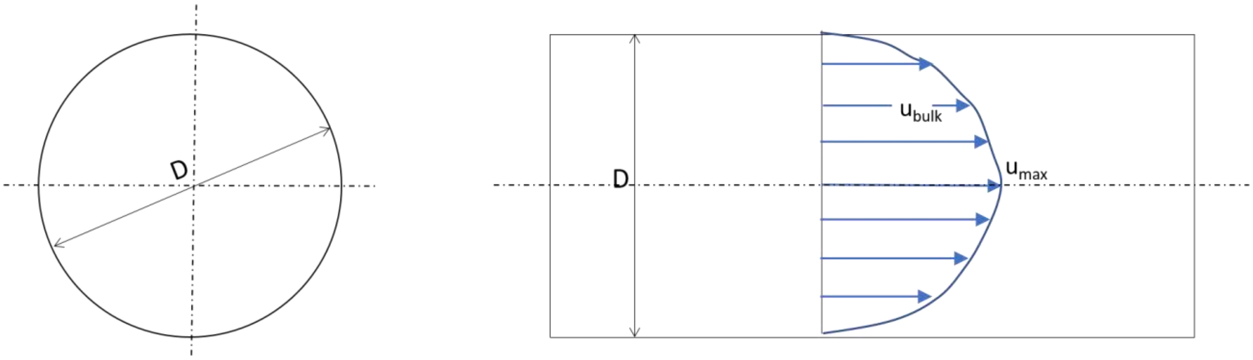

FIG. 48. Flow in straight pipes

4.1.1.1 Hagen–Poiseuille (1839, 1846)

Hagen-Poiseuille's law established that the volumetric flow rate in a pipe is directly proportional to the pressure drop and to the fourth power of the pipe radius, but inversely proportional to the viscosity of the fluid and to the length of the pipe. This relation was published first by Hagen in 1839, and then by Poiseuille in 1846 as a result of independent experiments [107] [222] [223]. For laminar flow in smooth pipes, the value of f can be derived from the Hagen-Poiseuille equation:

$$f = \frac{64}{Re} \tag{307}$$

4.1.1.2 Darcy-Weisbach (1858)

Darcy friction factor is a dimensionless quantity used in the Darcy–Weisbach equation, for the description of friction losses in pipe flow, as well as in an open channel flow [224]. It is also known as the Darcy–Weisbach friction factor, resistance coefficient or simply friction factor and is four times larger than the Fanning friction factor. Darcy friction factor for laminar flow is as follows [225]:

$$f = \frac{64}{Re}\, for\ Re \leq 2000 \tag{308}$$

This expression has been verified experimentally and is valid for engineering calculations of both smooth and rough circular pipes for $Re \leq 2000$.

Darcy friction factor for smooth pipes and turbulent flow is as follows [193]:

$$f = \frac{1}{[1.82 \cdot \log Re - 1.64]^2} \tag{309}$$

It is valid for $Re \geq 4000$.

4.1.1.3 Blasius (1912)

In 1912, P.R.H Blasius [226] [227] [228] provided a similar correlation for the friction factor of a turbulent flow ($Re \geq 3 \ \cdot 10^3$) within a circular pipe:

$$f = \frac{0.316}{Re^{0.25}} \tag{310}$$

The Blasius correlation is a simple model for liquid wall friction description. It gives reasonable results when applied to turbulent flow in smooth circular tubes. It is only a function of Reynolds number and it is a generic correlation which works for every fluid. It is often used in system codes. Then, one should be careful when using specific correlations for laminar flow and for turbulent flow in a rough tube. Blasius equation is explicit in f, therefore it is widely used for calculating turbulent flow in smooth pipes. It is valid for $4000 \leq Re \leq 1 \cdot 10^5$ ([9] Ch. 4) [225].

4.1.1.4 Von Karman (1930)

In 1930 T. von Karman [229] recommended the following correlation for rough circular duct in the range of $Re\ e/D \geq 70$ ([9] Ch. 4), where the roughness size becomes much higher that the boundary layer thickness and thus the friction factor does not depend on the Reynolds number:

$$\frac{1}{\sqrt{f}} = 1.68 - 0.8815 \ln\left(\frac{\varepsilon}{r}\right) \tag{311}$$

where ε is the surface roughness and r is the radius of the circular duct. Von Karman also presented a theoretical equation for smooth circular ducts with the constants adjusted to best fit Nikuradse's experimental data, which is valid for very high values of Reynolds numbers: $4 \cdot 10^3 \leq Re \leq 3 \cdot 10^6$. It is also referred to as the Prandtl correlation ([9] Ch. 18) [225]:

$$\frac{1}{\sqrt{\frac{f}{4}}} = 1.737 \ln\left(Re\sqrt{\frac{f}{4}}\right) - 0.4 = 4 \log\left(Re\sqrt{\frac{f}{4}}\right) - 0.4 \tag{312}$$

This expression can be approximated as:

$$f = 4 \cdot (3.64 \log Re - 3.28)^{-2} \approx 4 \cdot 0.046 \cdot Re^{-0.2} \tag{313}$$

4.1.1.5 Drew et al. (1932)

In 1932 T.B. Drew et al. [230] proposed the following correlation for smooth circular duct ([9] Ch. 4 and Ch. 18):

$$f = 0.0056 \ + 0.5 \cdot Re^{-0.32} \tag{314}$$

It is valid for the Re range of: $4 \cdot 10^3 \leq Re \leq 5 \cdot 10^6$ ([9] Ch. 18)

4.1.1.6 Nikuradse (1933)

In 1933 J. Nikuradse recommended a friction factor correlation for fully developed turbulent flow in smooth pipes [231]:

$$\frac{1}{\sqrt{f}} = 2\log\left(Re\sqrt{f}\right) - 0.8 = 0.87\ln\left(Re\sqrt{f}\right) - 0.8 \tag{315}$$

This correlation is the base for the turbulent smooth portion of the Moody diagram [232], however there are significant differences between them in the transition region from laminar to complete turbulent flow [107]. As Nikuradse equation is implicit for f, it needs an iteration procedure to be solved [233] [234]. It is also called the Karman-Nikuradse equation as in [45].

There is one similar correlation called the PKN (Prandtl-Karma-Nikuradse) valid for large Reynolds numbers $4 \cdot 10^3 \leq Re \leq 10^7$ ([9] Ch. 4) [106] [235] [236]:

$$\frac{1}{\sqrt{\frac{f}{4}}} = 1.7372\ln\left(Re\sqrt{\frac{f}{4}}\right) - 0.3946 \tag{316}$$

Explicit expressions in f were developed based on the original Nikuradse correlation, such as:

$$f = 0.0032 + 0.2232Re^{-0.237} \tag{317}$$

which is valid for smooth circular duct for the Re range: $10^5 \leq Re \leq 10^7$ ([9] Ch. 4).

For rough circular ducts J. Nikuradse also proposed a correlation ([9] Ch. 4) [225] [231]:

$$\frac{1}{\sqrt{f}} = 1.74 - 0.8686\ln\left(\frac{\varepsilon}{r}\right) \tag{318}$$

where ε/r is the relative roughness.

4.1.1.7 Colebrook-White (1939)

Colebrook developed an empirical formula for the transition zone between laminar flow and complete turbulence in smooth and rough pipes. This equation is known as the Colebrook-White formula and was used by Moody in developing his diagram. The friction coefficient for the complete turbulent zone depends only on the relative roughness ${}^{\varepsilon}/_{D}$, regardless of pipe diameter or Reynolds number [107]. The Moody diagram is a graphic representation of the C.F. Colebrook correlation (1939) [45] [237] for friction factor, which covers both rough and smooth walls and is given by:

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{{}^{\varepsilon}/_{D}}{3.70} + \frac{2.51}{Re\cdot\sqrt{f}}\right] = -0.8686\ln\left[\frac{{}^{\varepsilon}/_{D}}{3.70} + \frac{2.51}{Re\cdot\sqrt{f}}\right] \tag{319}$$

valid for $4 \cdot 10^3 \leq Re \leq 1 \cdot 10^8$, and $0 \leq \frac{\varepsilon}{D} \leq 0.05$, where $\frac{\varepsilon}{D}$ is the relative roughness.

When ${}^{\varepsilon}/_{D}$=0, the Colebrook equation is identical to the Nikuradse equation. The Nikuradse equation is the base for the turbulent smooth portion of the Moody diagram [232] while the Colebrook equation is the base for the turbulent rough portion of the Moody diagram [234]. As this correlation is implicit in f, it requires an iteration procedure to be solved [233].

In [225] Olson and Wright and in ([9] Ch. 4) Bhatti and Shah presented the Colebrook and White equation valid for rough circular duct with all turbulent flow regimes as follows:

$$\frac{1}{\sqrt{f}} = 1.74 - 0.8686\, ln\left[\frac{\varepsilon}{r} + \frac{18.7}{Re \cdot \sqrt{f}}\right] \tag{320}$$

where ε/r is the relative roughness.

In ([9] Ch. 4) Bhatti and Shah presented Colebrook correlation valid for smooth circular duct in the Re range: $4 \cdot 10^3 \leq Re \leq 1 \cdot 10^7$ as:

$$\frac{1}{\sqrt{f/_4}} = 1.5635 \ln\left(\frac{Re}{7}\right) \tag{321}$$

4.1.1.8 McAdams (1942)

A common approximate equation for the friction factor in a smooth tube is the McAdams relation ([9] Ch. 4) [45] [238]:

$$f = \frac{0.184}{Re^{0.2}} \tag{322}$$

It is valid for $3 \cdot 10^4 \leq Re \leq 1 \cdot 10^6$.

4.1.1.9 Moody (1944)

As mentioned above, the Nikuradse equation is the base for the turbulent smooth portion of the Moody diagram [232], while the Colebrook equation is the base for the turbulent rough portion of the Moody diagram [234] (see Fig. 49).

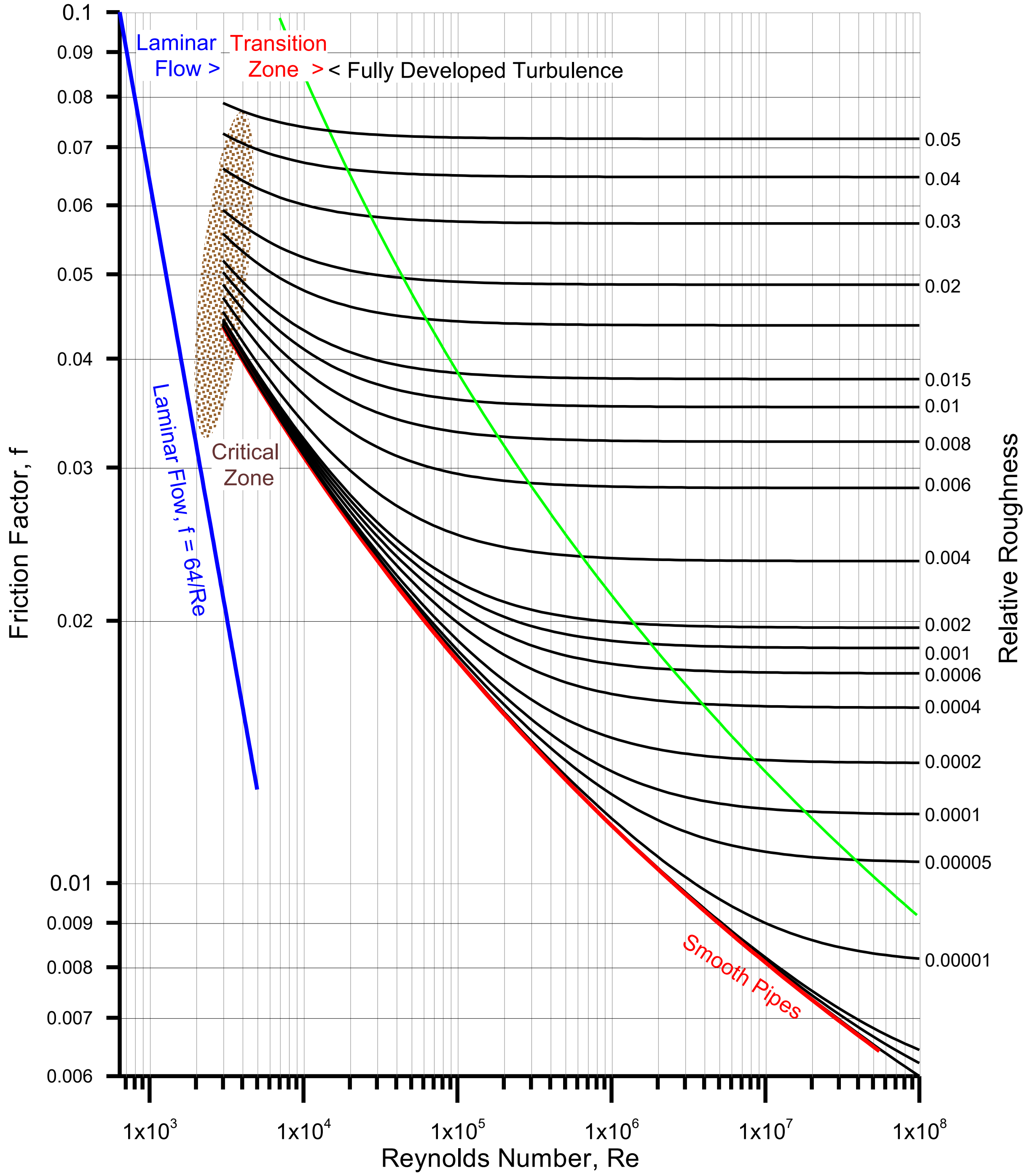

FIG. 49. Moody's diagram for friction factor f (data from [232])

4.1.1.10 Moody (1947)

In 1947 L.F. Moody proposed the following equation for flow in rough circular pipes valid for $4 \cdot 10^3 \le Re \le 5 \cdot 10^8$ and $0 \le \frac{\varepsilon}{D} \le 0.01$ [232] [239][9]:

$$f = 0.0055 \left[1 + \left(20000\frac{\varepsilon}{D} + \frac{10^6}{Re}\right)^{\frac{1}{3}}\right] \tag{323}$$

where ε/D is the relative roughness.

4.1.1.11 Filonenko (1954)

In 1954 Filonenko proposed the following equation for turbulent flow in smooth tubes ([9] Ch. 4, Ch. 18) [85] [240]:

$$f = (0.79\ \ln Re - 1.64)^{-2} = (1.82\ \log Re - 1.64)^{-2} \tag{324}$$

It is valid for the range: $4 \cdot 10^3 \le Re \le 1 \cdot 10^7$.

4.1.1.12 Techo et al. (1965)

In 1965 R. Techo et al. proposed the following correlation for turbulent flow in smooth circular ducts ([9] Ch. 4, Ch. 18) [241]:

$$\frac{1}{f} = \left(0.8686 \ln \frac{Re}{1.964\ \ln Re - 3.8215}\right)^2 \tag{325}$$

It is valid in the range: $4 \cdot 10^3 \le Re \le 1 \cdot 10^7$.

4.1.1.13 Wood (1966)

In 1966 D.J. Wood recommended the following correlation for a rough circular duct [242][10]:

$$f = 0.53\frac{\varepsilon}{D} + 0.094\left(\frac{\varepsilon}{D}\right)^{0.225} + 88\left(\frac{\varepsilon}{D}\right)^{0.44} Re^{-1.62\left(\frac{\varepsilon}{D}\right)^{0.134}} \tag{326}$$

where ε/D is the relative roughness. It is valid for $10^{-5} \le \frac{\varepsilon}{D} \le 0.04$ and $4 \cdot 10^3 \le Re \le 1 \cdot 10^8$ [233].

4.1.1.14 Churchill (1973)

In 1973 S.W. Churchill recommended the following correlation [243][11]:

[9] In Ref. ([9] Ch. 4) the correlation referred to as Moody (Table 4.3) has different parameters.

[10] In Ref. ([9] Ch. 4), Wood correlation is presented in Table 4.3. Although the Fanning friction factor is used in the whole book, Wood correlation is presented as Darcy friction factor, where $f_{Darcy} = 4f_{Fanning}$.

[11] In Ref. [234] Churchill's correlation has errors.

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7D} + \left(\frac{7}{Re}\right)^{0.9}\right) \tag{327}$$

where ε/D is the relative roughness.

In [45] this correlation is expressed as:

$$f = \frac{8}{6.0516}\left(\ln\left(\frac{\varepsilon}{3.7D} + \left(\frac{7}{Re}\right)^{0.9}\right)\right)^2 \tag{328}$$

4.1.1.15 Jain (1976)

In 1976 A.K. Jain recommended the following correlation for rough circular duct ([9] Ch. 4) [244]:

$$\frac{1}{\sqrt{f}} = 1.74205 - 0.8686\ln\left[\frac{\varepsilon}{r} + \frac{42.5}{Re^{0.9}}\right] \tag{329}$$

It is valid for $4 \cdot 10^{-5} \leq \frac{\varepsilon}{D} \leq 0.05$ where ε/D is the relative roughness. Jain's correlation is also expressed in this way in [233]:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.715D} + \frac{5.72}{Re^{0.9}}\right) \tag{330}$$

4.1.1.16 Swamee-Jain (1976)

In 1976 P.K. Swamee and A.K. Jain recommended the following correlation for rough circular duct ([9] Ch. 4) [245]:

$$\frac{1}{\sqrt{f}} = 1.73845 - 0.8686\ln\left[\frac{\varepsilon}{r} + \frac{42.48}{Re^{0.9}}\right] \tag{331}$$

where ε/r is the relative roughness. In Ref. [233], this correlation is presented as:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}}\right) \tag{332}$$

It is valid for $5 \cdot 10^3 \leq Re \leq 1 \cdot 10^8$ and $10^{-6} \leq \frac{\varepsilon}{D} \leq 0.05$.

4.1.1.17 Churchill (1977)

In 1977, S.W. Churchill proposed a single correlation [227] [246] that relates pipe friction loss to Reynolds number and surface roughness for laminar, transitional and turbulent flow, thus making fluid-flow calculation simpler[12]:

[12] In Ref. ([9] Ch. 4) and Ref. [234] the values of the constant in the expression of the parameter A are not consistent with the expression in the original paper.

$$f = 8\left[\left(\frac{8}{Re}\right)^{12} + \frac{1}{(A+B)^{\frac{3}{2}}}\right]^{\frac{1}{12}}$$

$$A = \left[2.457\, ln \frac{1}{\left(\frac{7}{Re}\right)^{0.9} + \frac{0.27\,\varepsilon}{D}}\right]^{16} \qquad (333)$$

$$B = \left(\frac{37530}{Re}\right)^{16}$$

where ε/D is the relative roughness. In the original paper, Churchill used a friction factor definition different than Darcy. Thus, he indicated how to get the Darcy friction factor – by multiplying the expressions of f in his paper by 8. This correlation not only reproduces the friction factor, but also avoids interpolation and provides unique values in the transition region.

4.1.1.18 Chen (1979)

In 1979 N.H. Chen recommended the following correlation for rough circular duct ([9] Ch. 4) [247]:

$$\frac{1}{\sqrt{f}} = 1.74 - 0.8686 \ln\left(\frac{\varepsilon}{r} - \frac{16.2426}{Re} \ln\left(\frac{(\varepsilon/r)^{1.1098}}{6.0983} + \left(\frac{7.149}{Re}\right)^{0.8981}\right)\right) \qquad (334)$$

where ε/r is the relative roughness. It is valid for $4 \cdot 10^3 \le Re \le 10^8$ and $2 \cdot 10^{-8} \le \frac{\varepsilon}{r} \le 0.1$. In [233] it is also expressed as:

$$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{\varepsilon}{3.7065\, D} - \frac{5.0452}{Re} \log\left(\frac{(\varepsilon/D)^{1.1098}}{2.8257} + \frac{5.8506}{Re^{0.8981}}\right)\right) \qquad (335)$$

It is valid for $4 \cdot 10^3 \le Re \le 4 \cdot 10^8$ and $10^{-7} \le \frac{\varepsilon}{D} \le 0.05$.

4.1.1.19 Round (1980)

In 1980 G.F. Round recommended the following correlation for rough circular duct ([9] Ch. 4) [248]:

$$\frac{1}{\sqrt{f}} = 2.1073 - 0.78175 \ln\left(\frac{\varepsilon}{r} + \frac{96.2963}{Re}\right) \qquad (336)$$

where ε/r is the relative roughness. In [233], it is presented as:

$$\frac{1}{\sqrt{f}} = -1.8 \log\left(0.135 \frac{\varepsilon}{D} + \frac{6.5}{Re}\right) \qquad (337)$$

It is valid for $4 \cdot 10^3 \le Re \le 4 \cdot 10^8$ and $0 \le \frac{\varepsilon}{D} \le 0.05$.

4.1.1.20 Barr (1981)

In 1981 D.I.H Barr recommended the following correlation [233] [249]:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/_D}{3.7} + \frac{4.518\ \log\left(\frac{Re}{7}\right)}{Re\left(1 + \frac{Re^{0.52}\left(\varepsilon/_D\right)^{0.7}}{29}\right)}\right) \tag{338}$$

4.1.1.21 Zigrang-Sylvester (1982)

In 1982 Zigrang and Sylvester recommended two similar correlations for turbulent flows in rough circular duct as an approximation that covers the entire regime with good accuracy and consideration for wall roughness (ε) ([9] Ch. 4) [250] [251]. They are based on the numerical solution of the implicit Colebrook-White formula. The first correlation proposed is:

$$\frac{1}{\sqrt{f}} = 1.73845 - 0.8686\ln\left\{\frac{\varepsilon}{r} - \frac{16.1332}{Re}\ln\left(\frac{\varepsilon}{7.4r} + \frac{13}{Re}\right)\right\} \tag{339}$$

where ε/r is the relative roughness. It is valid for $4 \cdot 10^3 \leq Re \leq 10^8$ and $2 \cdot 10^{-8} \leq \frac{\varepsilon}{r} \leq 0.1$. This correlation is implemented in RELAP 5 and 7. It can also be written as in the original paper in the following way:

$$\frac{1}{\sqrt{f}} = -2\log\left\{\frac{\varepsilon}{3.7D} - \frac{5.02}{Re}\log\left[\frac{\varepsilon}{3.7D} + \frac{13}{Re}\right]\right\} \tag{340}$$

And the second one adding a second iteration in the numerical solution of the Colebrook-White formula is:

$$\frac{1}{\sqrt{f}} = 1.73845 - 0.8686\ln\left\{\frac{\varepsilon}{r} - 16.1332\ln\left(\frac{\varepsilon}{7.4r} - 2.1802\ln\left(\frac{\varepsilon}{7.4r} + \frac{13}{Re}\right)\right)\right\} \tag{341}$$

where, as in the previous correlation, where ε/r is the relative roughness. It can be also written as [233]:

$$\frac{1}{\sqrt{f}} = -2\log\left\{\frac{\varepsilon}{3.7D} - \frac{5.02}{Re}\log\left[\frac{\varepsilon}{3.7D} - \frac{5.02}{Re}\log\left(\frac{\varepsilon}{3.7D} + \frac{13}{Re}\right)\right]\right\} \tag{342}$$

It is valid for $Re \geq 3 \cdot 10^3$ with the uncertainty of $\pm 5.5\%$.

4.1.1.22 Haaland (1983)

The recommendation from S.E. Haaland (1983) [45] [252] to estimate the friction factor for rough circular duct ([9] Ch. 4) is the following:

$$\frac{1}{\sqrt{f}} = 1.73675 - 0.78175\ln\left(\left(\frac{\varepsilon}{r}\right)^{1.11} + \frac{63.6350}{Re}\right) \tag{343}$$

where ε/r is the relative roughness. It is valid for $4 \cdot 10^3 \leq Re \leq 10^8$ and $2 \cdot 10^{-8} \leq \frac{\varepsilon}{r} \leq 0.1$.It can be also found in the literature as [225] [233]:

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/_{\mathrm{D}}}{3.70}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right] = -0.782 \ln\left[\left(\frac{\varepsilon/_{\mathrm{D}}}{3.70}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right] \tag{344}$$

valid for $10^{-6} \leq \varepsilon/_D \leq 0.05$.

4.1.1.23 Serghides (1984)

In 1984 T.K. Serghides recommended various correlations [253] for rough circular duct ([9] Ch. 4)[13]:

$$\begin{aligned} \frac{1}{\sqrt{f}} &= \frac{1}{2} \cdot \mathrm{A}_5 - \frac{1}{2} \cdot \frac{(\mathrm{A}_5 - \mathrm{B}_2)^2}{\mathrm{C}_1 - 2\mathrm{B}_2 + \mathrm{A}_5} \\ \mathrm{A}_5 &= -0.8686 \ln\left(\frac{\varepsilon}{7.4r} + \frac{12}{Re}\right) \\ \mathrm{B}_2 &= -0.8686 \ln\left(\frac{\varepsilon}{7.4r} + \frac{2.51\mathrm{A}_5}{Re}\right) \\ \mathrm{C}_1 &= -0.8686 \ln\left(\frac{\varepsilon}{7.4r} + \frac{2.51\mathrm{B}_2}{Re}\right) \end{aligned} \tag{345}$$

where ε/r is the relative roughness. It is valid for $4 \cdot 10^3 \leq Re \leq 10^8$ and $2 \cdot 10^{-8} \leq \frac{\varepsilon}{r} \leq 0.1$.

Another correlation proposed is ([9] Ch. 4):

$$\begin{aligned} \frac{1}{\sqrt{f}} &= 2.3905 - \frac{1}{2} \cdot \frac{(\mathrm{A}_5 - 4.781)^2}{4.781 - 2\mathrm{A}_5 + \mathrm{B}_2} \\ \mathrm{A}_5 &= -0.8686 \ln\left(\frac{\varepsilon}{7.4r} + \frac{12}{Re}\right) \\ \mathrm{B}_2 &= -0.8686 \ln\left(\frac{\varepsilon}{7.4r} + \frac{2.51\mathrm{A}_5}{Re}\right) \end{aligned} \tag{346}$$

where ε/r is the relative roughness. It is also valid for $4 \cdot 10^3 \leq Re \leq 10^8$ and $2 \cdot 10^{-8} \leq \frac{\varepsilon}{r} \leq 0.1$.

4.1.1.24 Bhatti-Shah (1987)

Bhatti and Shah as authors of Chapter 4 of the Handbook [9] proposed two correlations. The first one is:

$$f = \frac{0.1464}{Re^{0.1818}} \tag{347}$$

which is valid for smooth circular duct in the Re range of $4 \cdot 10^4 \leq Re \leq 1 \cdot 10^7$.

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[13] In Ref. [234] this correlation is presented in Table 1 with errors.

The second correlation is:

$$f = 0.02048 + \frac{1.8288}{Re^{0.311}} \tag{348}$$

It is valid for smooth circular duct for the Re range of $4 \cdot 10^4 \leq Re \leq 1 \cdot 10^7$ ([9] Ch. 4).

4.1.1.25 Manadilli (1997)

In 1997 Manadilli recommended the following correlation [254]:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7\,D} + \frac{95}{Re^{0.983}} - \frac{96.82}{Re}\right) \tag{349}$$

It is valid for $5.2 \cdot 10^3 \leq Re \leq 1 \cdot 10^8$, $0 \leq \frac{\varepsilon}{D} \leq 0.05$ [233].

4.1.1.26 Romeo et al. (2002)

In 2002 E. Romeo et al. recommended the following correlation [255]:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7065\,D} - \frac{5.0272}{Re}A\right)$$
$$A = \log\left\{\frac{\varepsilon}{3.827\,D} - \frac{4.567}{Re}\log\left[\left(\frac{\varepsilon}{7.7918\,D}\right)^{0.9924} + \left(\frac{5.3326}{208.815 + Re}\right)^{0.9345}\right]\right\} \tag{350}$$

It is valid for $3 \cdot 10^3 \leq Re \leq 1.5 \cdot 10^8$, $0 \leq \frac{\varepsilon}{D} \leq 0.05$ [233].

4.1.1.27 Sonnad-Goudar (2006)

In 2006 J.R. Sonnad and C.T. Goudar recommended the following correlation [256]:

$$\frac{1}{\sqrt{f}} = 0.8686\ln\left[\frac{0.4587Re}{S^{S/_{S+1}}}\right]$$
$$S = 0.124\frac{\varepsilon}{D}Re + \ln(0.4587Re) \tag{351}$$

It is valid for $4 \cdot 10^3 \leq Re \leq 1 \cdot 10^8$, $10^{-6} \leq \frac{\varepsilon}{D} \leq 0.05$ [233].

4.1.1.28 Fang et al. (2011)

In 2011 X.D. Fang et al. developed single-phase friction factor correlations for turbulent flow based on computer analysis. They used a data bank of 1056 data points covering the regime of $3 \cdot 10^3 \leq Re \leq 1 \cdot 10^8$ and $0 \leq {}^{\varepsilon}/_{D} \leq 0.05$ generated with the Colebrook equation and the Nikuradse equation.

Based on regression and optimization with software, two correlations were proposed, one for smooth pipes, and the other for both smooth and rough pipes in the range of $0.0 \leq \frac{\varepsilon}{D} \leq 0.05$.

The recommended correlation for turbulent flow in smooth pipes is [233] [234]:

$$f = 0.25\left[\log\left(\frac{150.39}{Re^{0.98865}} - \frac{152.66}{Re}\right)\right]^{-2} \tag{352}$$

While the correlation for single-phase friction factor for turbulent flow in both smooth and rough pipes is:

$$f = 1.613\left[\ln\left(0.234\left(\frac{\varepsilon}{D}\right)^{1.1007} - \frac{60.525}{Re^{1.1105}} + \frac{56.291}{Re^{1.0712}}\right)\right]^{-2} \tag{353}$$

Valid for the range of $3 \cdot 10^3 \leq Re \leq 1 \cdot 10^8$ and $0 \leq {}^{\varepsilon}/_{D} \leq 0.05$.

4.1.1.29 Summary of friction factor correlations for single-phase flow in straight pipes

Table 31 presents the list of all friction factor correlations collected for single-phase flow in straight pipes.

TABLE 31. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN STRAIGHT PIPES

Hagen–Poiseuille (1839, 1846) [107] [222] [223]	$f = \frac{64}{Re}$ laminar flow, smooth pipes
Darcy-Weisbach (1858) [193] [225] [224]	for smooth and rough circular pipes and $Re \leq 2000$: $f = \frac{64}{Re}$ for smooth pipes and $Re \geq 4000$: $f = \frac{1}{[1.82 \cdot \log Re - 1.64]^2}$
Blasius (1912) [9] [226] [227] [228]	$f = \frac{0.316}{Re^{0.25}}$ $Re \geq 3 \cdot 10^3$
Von Karman (1930) [9] [225] [229]	for rough pipes, $Re \geq 70$: $\frac{1}{\sqrt{f}} = 1.68 - 0.8815 \ln\left(\frac{\varepsilon}{r}\right)$ for smooth pipes, $4 \cdot 10^3 \leq Re \leq 3 \cdot 10^6$: $\frac{1}{\sqrt{\frac{f}{4}}} = 4 \log\left(Re\sqrt{\frac{f}{4}}\right) - 0.4$ $f = \frac{4}{(3.64 \log Re - 3.28)^2} \approx \frac{4 \cdot 0.046}{Re^{0.2}}$
Drew et al.(1932) [9] [230]	$f = 0.0056 + 0.5\, Re^{-0.32}$ $4 \cdot 10^3 \leq Re \leq 5 \cdot 10^6$, smooth pipes

TABLE 31. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN STRAIGHT PIPES

Nikuradse (1933) [9] [106] [231] [235] [236]	for turbulent flow and smooth pipes: $\frac{1}{\sqrt{f}} = 2\log\left(Re\sqrt{f}\right) - 0.8$ for $10^5 \leq Re \leq 10^7$: $f = 0.0032 + 0.2232 Re^{-0.237}$ Prandtl-Karma-Nikuradse for $4 \cdot 10^3 \leq Re \leq 10^7$: $\frac{1}{\sqrt{\frac{f}{4}}} = 1.7372 \ln\left(Re\sqrt{\frac{f}{4}}\right) - 0.3946$ for rough pipes: $\frac{1}{\sqrt{f}} = 1.74 - 0.8685 \ln\left(\frac{\varepsilon}{r}\right)$ turbulent flow
Colebrook-White (1939) [9] [107] [225] [237]	for smooth, rough pipes ($0 \leq \varepsilon/D \leq 0.05$) and $4 \cdot 10^3 \leq Re \leq 1 \cdot 10^8$: $\frac{1}{\sqrt{f}} = -2\log\left[\frac{\varepsilon/D}{3.70} + \frac{2.51}{Re \cdot \sqrt{f}}\right] = -0.8686 \ln\left[\frac{\varepsilon/D}{3.70} + \frac{2.51}{Re \cdot \sqrt{f}}\right]$ for rough pipes: $\frac{1}{\sqrt{f}} = 1.74 - 0.8686\, ln\left[\frac{\varepsilon}{r} + \frac{18.7}{Re \cdot \sqrt{f}}\right]$ for smooth pipes: $\frac{1}{\sqrt{\frac{f}{4}}} = 1.5635 \ln\left(\frac{Re}{7}\right)$ Last two are valid for $4 \cdot 10^3 \leq Re \leq 1 \cdot 10^7$
McAdams (1942) [9] [45] [238]	$f = \frac{0.184}{Re^{0.2}}$ $3 \cdot 10^4 \leq Re \leq 1 \cdot 10^6$, smooth pipes
Moody (1947) [232] [239]	$f = 0.0055\left[1 + \left(20000\frac{\varepsilon}{D} + \frac{10^6}{Re}\right)^{\frac{1}{3}}\right]$ $4 \cdot 10^3 \leq \mathrm{Re} \leq 5 \cdot 10^8$, $0 \leq \frac{\varepsilon}{D} \leq 0.01$, rough pipes

TABLE 31. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN STRAIGHT PIPES

Filonenko (1954) [9] [85] [240]	$f = (0.79 \ \ln Re - 1.64)^{-2} 4 \cdot 10^3 \leq Re \leq 1 \cdot 10^7$, smooth pipes
Techo et al. (1965) [9] [241]	$\frac{1}{f} = \left(0.8686 \ln \frac{Re}{1.964 \ \ln Re - 3.8215}\right)^2$ $4 \cdot 10^3 \leq Re \leq 1 \cdot 10^7$, smooth pipes
Wood (1966) [233] [242]	$f = 0.53 \frac{\varepsilon}{D} + 0.094 \left(\frac{\varepsilon}{D}\right)^{0.225} + 88 \left(\frac{\varepsilon}{D}\right)^{0.44} Re^{-1.62\left(\frac{\varepsilon}{D}\right)^{0.134}}$ $10^{-5} \leq \frac{\varepsilon}{D} \leq 0.04$, $4 \cdot 10^3 \leq Re \leq 1 \cdot 10^8$, rough pipes
Churchill (1973) [45] [243]	$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{\varepsilon}{3.7D} + \left(\frac{7}{Re}\right)^{0.9}\right)$ $f = \frac{8}{6.0516}\left(\ln\left(\frac{\varepsilon}{3.7D} + \left(\frac{7}{Re}\right)^{0.9}\right)\right)^2$ rough pipes
Jain (1976) [9] [233] [244]	$\frac{1}{\sqrt{f}} = 1.74205 - 0.8686 \ln\left[\frac{\varepsilon}{r} + \frac{42.5}{Re^{0.9}}\right]$ $\frac{1}{\sqrt{f}} = -2 \log\left(\frac{\varepsilon}{3.715D} + \frac{5.72}{Re^{0.9}}\right)$ $4 \cdot 10^{-5} \leq \frac{\varepsilon}{D} \leq 0.05$, rough pipes
Swamee-Jain (1976) [9] [233] [245]	$\frac{1}{\sqrt{f}} = 1.73845 - 0.8686 \ln\left[\frac{\varepsilon}{r} + \frac{42.48}{Re^{0.9}}\right]$ $\frac{1}{\sqrt{f}} = -2 \log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}}\right)$ $5 \cdot 10^3 \leq Re \leq 10^8$, $10^{-6} \leq \frac{\varepsilon}{D} \leq 0.05$ rough pipes
Churchill (1977) [227] [246]	$f = 8\left[\left(\frac{8}{Re}\right)^{12} + \frac{1}{(A+B)^{\frac{3}{2}}}\right]^{\frac{1}{12}}$ $A = \left[2.457 \ ln \frac{1}{\left(\frac{7}{Re}\right)^{0.9} + \frac{0.27 \ \varepsilon}{D}}\right]^{16}$ $B = \left(\frac{37530}{Re}\right)^{16}$ laminar, transitional and turbulent flow, rough pipes

TABLE 31. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN STRAIGHT PIPES

Chen (1979) [9] [233] [247]	$\frac{1}{\sqrt{f}} = 1.74 - 0.8686 \ln\left(\frac{\varepsilon}{r} - \frac{16.2426}{Re} \ln\left(\frac{(\varepsilon/r)^{1.1098}}{6.0983} + \left(\frac{7.149}{Re}\right)^{0.8981}\right)\right)$ $2 \cdot 10^{-8} \le \frac{\varepsilon}{r} \le 0.1$ $\frac{1}{\sqrt{f}} = -2 \log\left(\frac{\varepsilon}{3.7065\, D} - \frac{5.0452}{Re} \log\left(\frac{(\varepsilon/D)^{1.1098}}{2.8257} + \frac{5.8506}{Re^{0.8981}}\right)\right)$ $10^{-7} \le \frac{\varepsilon}{D} \le 0.05$, $4 \cdot 10^{3} \le Re \le 4 \cdot 10^{8}$, rough pipes
Round (1980) [9] [233] [248]	$\frac{1}{\sqrt{f}} = 2.1073 - 0.78175 \ln\left(\frac{\varepsilon}{r} + \frac{96.2963}{Re}\right)$ $\frac{1}{\sqrt{f}} = -1.8 \log\left(0.135 \frac{\varepsilon}{D} + \frac{6.5}{Re}\right)$ $4 \cdot 10^{3} \le Re \le 4 \cdot 10^{8}$, $0 \le \frac{\varepsilon}{D} \le 0.05$, rough pipes
Barr (1981) [233] [249]	$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{\varepsilon/D}{3.7} + \frac{4.518\ \log\left(\frac{Re}{7}\right)}{Re\left(1 + \frac{Re^{0.52}(\varepsilon/D)^{0.7}}{29}\right)}\right)$ rough pipes
Zigrang-Sylvester (1982) [9] [233] [250] [251]	$\frac{1}{\sqrt{f}} = 1.73845 - 0.8686 \ln\left\{\frac{\varepsilon}{r} - \frac{16.1332}{Re} \ln\left(\frac{\varepsilon}{7.4r} + \frac{13}{Re}\right)\right\}$ $\frac{1}{\sqrt{f}} = -2 \log\left\{\frac{\varepsilon}{3.7D} - \frac{5.02}{Re} \log\left[\frac{\varepsilon}{3.7D} + \frac{13}{Re}\right]\right\}$ $4 \cdot 10^{3} \le Re \le 10^{8}$, $2 \cdot 10^{-8} \le \frac{\varepsilon}{r} \le 0.1$ $\frac{1}{\sqrt{f}} = 1.73845 - 0.8686 \ln\left\{\frac{\varepsilon}{r} - 16.1332 \ln\left(\frac{\varepsilon}{7.4r} - 2.1802 \ln\left(\frac{\varepsilon}{7.4r} + \frac{13}{Re}\right)\right)\right\}$ $\frac{1}{\sqrt{f}} = -2 \log\left\{\frac{\varepsilon}{3.7D} - \frac{5.02}{Re} \log\left[\frac{\varepsilon}{3.7D} - \frac{5.02}{Re} \log\left(\frac{\varepsilon}{3.7D} + \frac{13}{Re}\right)\right]\right\}$ $Re \ge 3 \cdot 10^{3}$, rough pipes

TABLE 31. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN STRAIGHT PIPES

Haaland (1983) [9] [45] [225] [233] [252]	for $4 \cdot 10^3 \leq Re \leq 10^8$ and $2 \cdot 10^{-8} \leq \frac{\varepsilon}{r} \leq 0.1$: $\frac{1}{\sqrt{f}} = 1.73675 - 0.78175 \ln\left(\left(\frac{\varepsilon}{r}\right)^{1.11} + \frac{63.6350}{Re}\right)$ for $10^{-6} \leq \varepsilon/_D \leq 0.05$: $\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/_D}{3.70}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right] = -0.782 \ln\left[\left(\frac{\varepsilon/_D}{3.70}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right].$
Serghides (1984) [9] [253]	$\frac{1}{\sqrt{f}} = \frac{1}{2} \cdot \mathrm{A}_5 - \frac{1}{2} \cdot \frac{(\mathrm{A}_5 - \mathrm{B}_2)^2}{\mathrm{C}_1 - 2\mathrm{B}_2 + \mathrm{A}_5}$ $\mathrm{A}_5 = -0.8686 \ln\left(\frac{\varepsilon}{7.4r} + \frac{12}{Re}\right)$ $\mathrm{B}_2 = -0.8686 \ln\left(\frac{\varepsilon}{7.4r} + \frac{2.51\mathrm{A}_5}{Re}\right)$ $\mathrm{C}_1 = -0.8686 \ln\left(\frac{\varepsilon}{7.4r} + \frac{2.51\mathrm{B}_2}{Re}\right)$ $\frac{1}{\sqrt{f}} = 2.3905 - \frac{1}{2} \cdot \frac{(\mathrm{A}_5 - 4.781)^2}{4.781 - 2\mathrm{A}_5 + \mathrm{B}_2}$ $\mathrm{A}_5 = -0.8686 \ln\left(\frac{\varepsilon}{7.4r} + \frac{12}{Re}\right)$ $\mathrm{B}_2 = -0.8686 \ln\left(\frac{\varepsilon}{7.4r} + \frac{2.51\mathrm{A}_5}{Re}\right)$ $4 \cdot 10^3 \leq Re \leq 10^8$, $2 \cdot 10^{-8} \leq \frac{\varepsilon}{r} \leq 0.1$, rough pipes
Bhatti-Shah (1987) [9]	$f = \frac{0.1464}{Re^{0.1818}}$ $f = 0.02048 + \frac{1.8288}{Re^{0.311}}$ $4 \cdot 10^4 \leq Re \leq 10^7$, smooth pipes
Manadilli (1997) [233] [254]	$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{\varepsilon}{3.7\,D} + \frac{95}{Re^{0.983}} - \frac{96.82}{Re}\right)$ $5.2 \cdot 10^3 \leq Re \leq 1 \cdot 10^8$, $0 \leq \frac{\varepsilon}{D} \leq 0.05$, rough pipes
Romeo et al. (2002) [233] [255]	$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{\varepsilon}{3.7065\,D} - \frac{5.0272}{Re} A\right)$ $A = \log\left\{\frac{\varepsilon}{3.827\,D} - \frac{4.567}{Re} \log\left[\left(\frac{\varepsilon}{7.7918\,D}\right)^{0.9924} + \left(\frac{5.3326}{208.815 + Re}\right)^{0.9345}\right]\right\}$ $3 \cdot 10^3 \leq Re \leq 1.5 \cdot 10^8$, $0 \leq \frac{\varepsilon}{D} \leq 0.05$, rough pipes

TABLE 31. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN STRAIGHT PIPES

Sonnad-Goudar (2006) [233] [256]	$\frac{1}{\sqrt{f}} = 0.8686 \ln\left[\frac{0.4587Re}{S^{S/_{S+1}}}\right]$ $S = 0.124\frac{\varepsilon}{D}Re + \ln(0.4587Re)$ $4\cdot 10^3 \le Re \le 1\cdot 10^8$, $10^{-6} \le \frac{\varepsilon}{D} \le 0.05$
Fang et al. (2011) [233] [234]	for smooth pipes: $f = 0.25\left[\log\left(\frac{150.39}{Re^{0.98865}} - \frac{152.66}{Re}\right)\right]^{-2}$ for smooth and rough pipes ($0 \le \varepsilon/_D \le 0.05$): $f = 1.613\left[\ln\left(0.234\left(\frac{\varepsilon}{D}\right)^{1.1007} - \frac{60.525}{Re^{1.1105}} + \frac{56.291}{Re^{1.0712}}\right)\right]^{-2}$ $3\cdot 10^3 \le Re \le 1\cdot 10^8$

All friction factor correlations complied in section 4.1.1 for the turbulent flows in smooth straight pipes are compared in Fig. 50. All except Moody (1947) are in very good agreement with each other and plots cannot be visually distinguished in Fig. 50. The main difference is in the validity ranges that depend on the Reynold number.

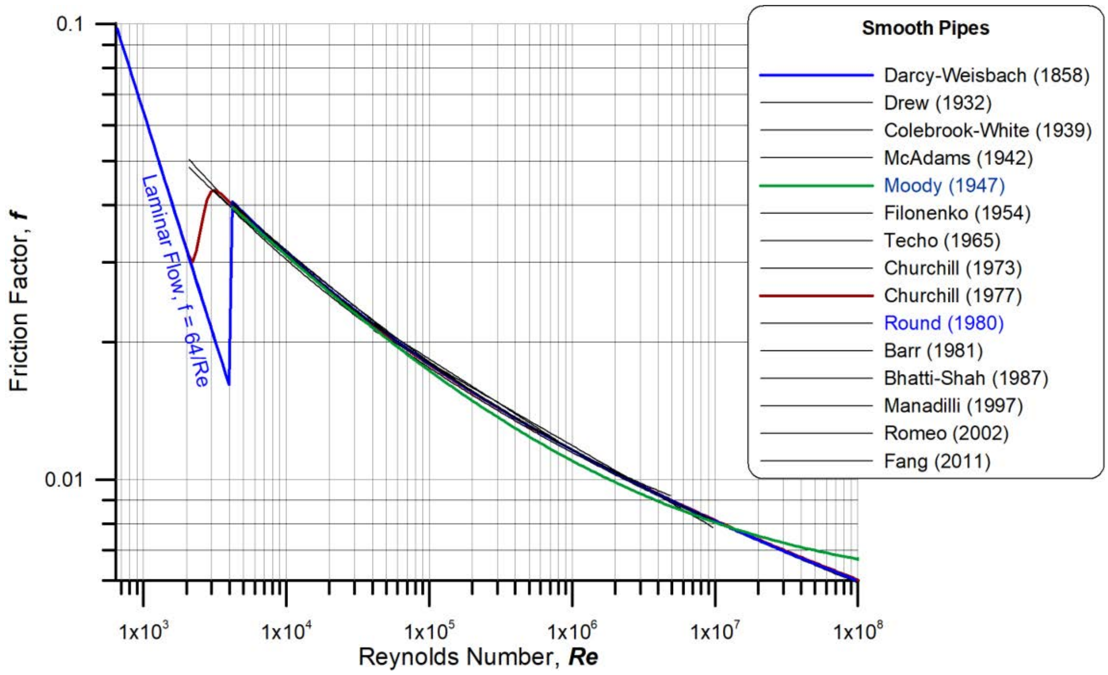

FIG. 50. Summary comparison of friction factor correlations for smooth pipes

Correlations for the rough pipes are compared in Fig. 51 for selected relative roughness $\varepsilon/D = 0.003$. All corelations are split in two groups with close results. The first group includes correlations from Moody (1947), Wood (1966) and Round (1980), while the rest are in the second group. The differences in friction fraction values within each group are negligible therefore it is nearly impossible to distinguish individual correlations in Fug. 51. Maximal deviation between two groups reaches only about 3%.

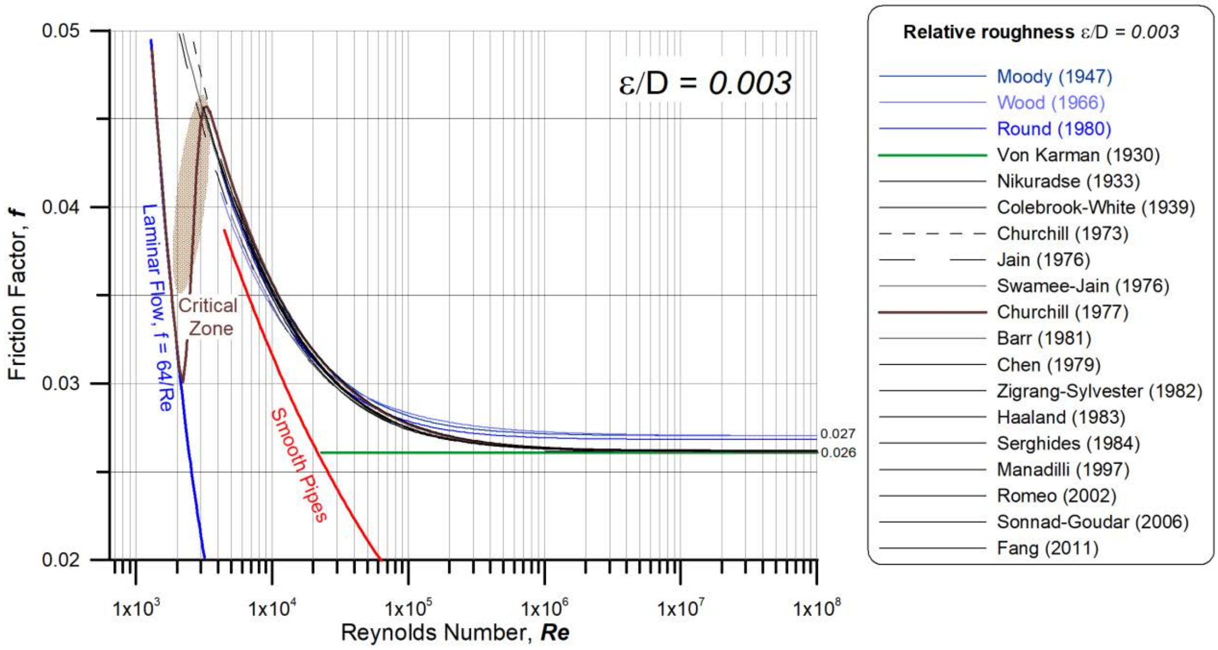

FIG. 51. Comparison of friction factor correlations for rough pipes; relative roughness ε/D = 0.003

4.1.2 Flow in curved and helical pipes

The friction factor for helical coil and curved tubes f_c is found to depend on Reynolds number and a geometrical number $\frac{d}{D}$ in the form of the dimensionless Dean number $De = Re\sqrt{\frac{d}{D}}$, where D is the diameter of the coil and d is the diameter of the pipe [257] (see Fig. 52).

As some of the correlations relate the friction factor for curved or helical coil tube f_c to the friction factor for straight tube f_s, the subscripts *c* and *s* will be used in this subsection 4.1.2 to distinguish the two tube geometries respectively. Figure 52 shows a representation of a helical pipe [258]. *d – pipe diameter, D – coil diameter, p – coil pitch, β – pipe angle.*

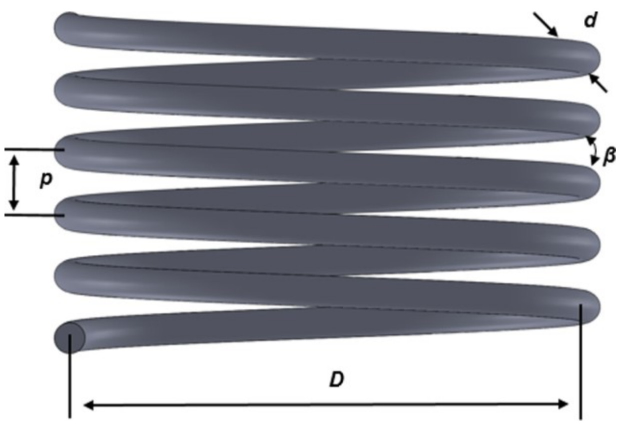

FIG. 52. Representation of helical pipe parameters (from [258])

4.1.2.1 Dean (1928)

In 1928 W.R. Dean proposed the following analytical correlation relating f_c the friction factor for curve pipes and f_s that for straight pipes [257] [259] [260]:

$$\frac{f_c}{f_s} = 1.03058\left(\frac{De^2}{288}\right)^2 + 0.01195\left(\frac{De^2}{288}\right)^4 \tag{354}$$

It is valid for laminar flow, small $\frac{d}{D}$ ratios and $De \leq 20$.

4.1.2.2 White (1929)

In 1929 C.M. White presented an empirical correlation valid for circular tubes, laminar flow and ratios $\frac{D}{d} = 15.15, 50$ and 2050 [257] [261] [262]:

$$\frac{f_s}{f_c} = 1 - \left[1 - \left(\frac{11.6}{De}\right)^{0.45}\right]^{2.22} \tag{355}$$

It is valid for $11.6 \leq De \leq 2000$. And White proposed that $f_c = f_s$ for $De \leq 11.6$.

4.1.2.3 White (1932)

Some years later, in 1932, C.M White proposed another empirical correlation based on experimental data valid for turbulent flow in helical pipes [257] [263]:

$$f_c = 0.32Re^{-\frac{1}{4}} + 0.048\sqrt{\frac{d}{D}} \tag{356}$$

It is valid in the range of $1.5 \cdot 10^4 \leq Re \leq 1 \cdot 10^5$.

4.1.2.4 Adler (1934)

In 1934 M. Adler recommended the following correlation derived from experimental results and theoretical analysis [257] [264]:

$$\frac{f_c}{f_s} = 0.1064\sqrt{De} \tag{357}$$

It is valid for laminar flow with large Dean numbers.

4.1.2.5 Prandtl (1949)

In 1949 L. Prandtl related the friction factors for laminar flow in curved and straight pipes with the following empirical correlation [235] [257]:

$$\frac{f_c}{f_s} = 0.37\left(\frac{De}{2}\right)^{0.36} \tag{358}$$

It is valid in the range of $40 \leq De \leq 2000$.

4.1.2.6 Hasson (1955)

In 1955 D. Hasson derived an empirical correlation for laminar flow in helical pipes [257] [265]:

$$\frac{f_c}{f_s} = 0.556 + 0.0969\sqrt{De} \tag{359}$$

4.1.2.7 Ito (1959)

In 1959 H. Ito proposed a set of different correlations based on the experimental data obtained in air and water experiments in curved pipes [257] [266].

For laminar flow in the range of $13.5 \leq De \leq 2000$ he recommended the following empirical correlation for the ratio between f_c and f_s:

$$\frac{f_c}{f_s} = \frac{21.5De}{(1.56 + logDe)^{5.73}} \tag{360}$$

For turbulent flow in circular curved tubes for the values $Re\left(\frac{d}{D}\right)^2 \geq 6$ Ito proposed the expression:

$$\frac{f_c}{f_s} = \left[Re \cdot \left(\frac{d}{D}\right)^2\right]^{\frac{1}{20}} \tag{361}$$

By making use of the Blasius equation, Eq. (310), Ito then derived an alternative equation for the curved friction factor:

$$f_c = \frac{0.316}{Re^{0.2}}\left(\frac{d}{D}\right)^{\frac{1}{10}} \tag{362}$$

where d is the inner diameter of the pipe and D is the helical diameter of the coil.
For turbulent flow in the range of $0.034 \leq Re\left(\frac{d}{D}\right)^2 \leq 300$ Ito recommended also the theoretical correlation ([9] Ch. 5) [266]:

$$f_c = 0.029\left(\frac{d}{D}\right)^{\frac{1}{2}} + \frac{0.304}{Re^{0.25}} \tag{363}$$

In ([9] Ch. 5) it is said that this correlation can also be used for curved rectangular ducts for $Re \geq 8000$ replacing d by D_h, where D_h is the hydraulic diameter of the rectangular duct.
Later Ito introduced a new parameter Y defined as:

$$Y^3 e^Y = Re\sqrt{\frac{D}{d}}$$

He then presented another theoretical correlation for turbulent flow:

$$f_c = 0.0324\left(\frac{d}{D}\right)^{\frac{1}{2}} + \frac{1.6}{\left(\frac{d}{D}\right)^{0.77} Y^{2.54}} \qquad \text{for } Y^2\sqrt{\frac{d}{D}} \leq 12 \qquad (364)$$

$$f_c = \frac{1.186}{Y^2} \qquad \text{for } Y^2\sqrt{\frac{d}{D}} \geq 5.3 \qquad (365)$$

4.1.2.8 Kubair-Varrier (1961)

In 1961 V. Kubair and C.B.S Varrier proposed empirical correlations for helical pipes in non-isothermal conditions and $0.037 \leq \frac{d}{D} \leq 0.097$ [257] [267]:

$$f_c = 3.0864 Re^{-0.5} e^{3.553\frac{d}{D}} \qquad \text{for } 2 \cdot 10^3 \leq Re \leq 9 \cdot 10^3 \qquad (366)$$

$$f_c = 0.014152 Re^{0.09} e^{1.887\frac{d}{D}} \qquad \text{for } 9 \cdot 10^3 \leq Re \leq 2.5 \cdot 10^4 \qquad (367)$$

4.1.2.9 Barua (1963)

In 1963 S.N. Barua derived a theoretical correlation for laminar flows in a torus [257] [268]:

$$\frac{f_c}{f_s} = 0.509 + 0.0918\sqrt{De} \qquad (368)$$

It is valid for large Dean numbers.

4.1.2.10 Seban-McLaughlin (1963)

R.A. Seban and E.F. McLaughlin presented in 1963 friction factor experimental results for turbulent flow of water in tube coils having ratios of coil to tube diameter $\frac{D}{d} = 17 \; and \; 104$ and for Reynolds numbers $12 \leq Re \leq 65000$ [262]. They compared their experimental data with the following correlation:

$$f_c = \frac{0.184}{Re^{0.2}} \cdot \left[Re \cdot \left(\frac{d}{D}\right)^2\right]^{\frac{1}{20}} \qquad (369)$$

where: d is tube inside diameter and D the coil diameter to tube centre. They observed that this formula was only about 6% lower in comparison with the experimental data, so that within this error it predicts all the turbulent friction factors measured on the two coils $\frac{D}{d} = 17 \; and \; 104$.

This equation was derived by the authors using Ito's Eq. (361) for the ratio $\frac{f_c}{f_s}$ and McAdams Eq. (322) for straight pipes.

4.1.2.11 Mori-Nakayama (1965)

In 1965 Y. Mori and W. Nakayama recommended a theoretical correlation experimentally verified with air tests for laminar flow [257] [269]:

$$\frac{f_c}{f_s} = \frac{0.108\sqrt{De}}{1 - \frac{3.253}{\sqrt{De}}} \tag{370}$$

It is valid for helical circular coils and $13.5 \leq De \leq 2000$.

4.1.2.12 Mori-Nakayama (1967)

In 1967 Y. Mori and W. Nakayama recommended a friction factor correlation for a circular curved pipe with fully developed turbulent flow [270][14]:

$$f = \left(\frac{r}{R}\right)^{0.5} \cdot \frac{0.192}{\left[Re \cdot \left(\frac{r}{R}\right)^{2.5}\right]^{\frac{1}{6}}} \cdot \left(1 + \frac{0.068}{\left[Re \cdot \left(\frac{r}{R}\right)^{2.5}\right]^{\frac{1}{6}}}\right) \tag{371}$$

As in previous equations, r is the radius of the pipe and R the radius of curvature of the coil.

4.1.2.13 Schmidt (1967)

In 1967 E.F. Schmidt proposed the following empirical correlation for curved pipes with laminar flow [271][15]:

$$\frac{f_c}{f_s} = 1 + 0.14\left(\frac{d}{D}\right)^{0.97} Re^{1-0.644\left(\frac{d}{D}\right)^{0.312}} \tag{372}$$

It is valid for $100 \leq Re \leq Re_{crit.}$. For turbulent flow, the recommended equation was:

$$\frac{f_c}{f_s} = 1 + \frac{2.88 \cdot 10^4}{Re}\left(\frac{d}{D}\right)^{0.62} \tag{373}$$

It is valid for $Re_{crit.} \leq Re \leq 2.2 \cdot 10^4$.

A similar equation was presented for turbulent flow:

$$\frac{f_c}{f_s} = 1 + 0.0823\left(1 + \frac{d}{D}\right)\left(\frac{d}{D}\right)^{0.53} Re^{0.25} \tag{374}$$

It is valid for $2 \cdot 10^4 \leq Re \leq 1.5 \cdot 10^5$.

4.1.2.14 Srinivasan et al. (1968)

In 1968 P.S. Srinivasan et al. proposed a set of empirical correlations for helical tubes with $0.0097 \leq \frac{d}{D} \leq 0.135$ [257] [272]:

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[14] In Ref. [258] this correlation is presented with errors in Table 1.

[15] In Ref. [258] this correlation is presented with errors in Table 1.

$$f_c = \frac{128}{Re} \qquad Re\sqrt{\frac{d}{D}} \leq 30 \tag{375}$$

$$f_c = 20.88\left(Re\sqrt{\frac{D}{d}}\right)^{-0.6} \qquad 30 \leq Re\sqrt{\frac{d}{D}} \leq 300 \tag{376}$$

$$f_c = 7.2\left(Re\sqrt{\frac{D}{d}}\right)^{-0.5} \qquad 300 \leq Re\sqrt{\frac{d}{D}} \leq Re_{crit.}\sqrt{\frac{d}{D}} \tag{377}$$

$$f_c = 4.336\left(Re\sqrt{\frac{D}{d}}\right)^{-0.2} \qquad Re \geq Re_{crit.} \tag{378}$$

4.1.2.15 Ito (1969)

In 1969 Ito derived the following theoretical correlation for laminar flow in curved pipes [257] [273]:

$$\frac{f_c}{f_s} = 0.1033\sqrt{De}\left[\left(1+\frac{1.729}{De}\right)^{0.5} - \frac{1.315}{De^{0.5}}\right]^{-3} \tag{379}$$

He also extended the power series to the expression:

$$\frac{f_c}{f_s} = 0.1033\sqrt{De}\left[1+\frac{3.945}{De^{0.5}}+\frac{7.782}{De}+\frac{9.097}{De^{1.5}}+\frac{5.608}{De^{2}}+\cdots\right] \tag{380}$$

Ito then presented an empirical equation deduced from experiments presenting fair accuracy for values of $De > 30$ where the numerical coefficient outside the parenthesis differed only slightly from that obtained based on the theory:

$$\frac{f_c}{f_s} = 0.1008\sqrt{De}\left[1+\frac{3.945}{De^{0.5}}+\frac{7.782}{De}+\frac{9.097}{De^{1.5}}+\frac{5.608}{De^{2}}+\cdots\right] \tag{381}$$

4.1.2.16 Srinivasan et al. (1970)

In 1970 P.S. Srinivasan et al. obtained extensive friction factor experimental data and proposed the following equation for turbulent flow in a helical smooth pipe ([9] Ch. 5) [274]:

$$f_c\left(\frac{R}{r}\right)^{0.5} = 0.336\left[Re\left(\frac{R}{r}\right)^{-2}\right]^{-0.2} \tag{382}$$

where r is the inner radius of the pipe and R is the radius of the curvature. It is valid for $Re \cdot \left(\frac{R}{r}\right)^{-2} < 700$ and $7 < \frac{R}{r} < 104$.

They also measured friction factors in five spiral coils for water and fuel-oil flow, thus recommending the experimental correlation ([9] Ch. 5):

$$f_c = \frac{0.0296\,(n_2^{0.9} - n_1^{0.9})^{1.5}}{\left[Re\left(\frac{P}{r}\right)^{0.5}\right]^{0.2}} \tag{383}$$

where P is the coil pitch (see Fig. 52), n_1 is the number of coil turns at the beginning of the spiral $n_1 = \frac{L}{2\pi PN} - \frac{N}{2}$ and n_2 is the number of coil turns at the end of the spiral $n_2 = \frac{L}{2\pi PN} + \frac{N}{2}$. N is the number of spiral coils turns $n_2 - n_1$. This correlation is valid for $40000 < Re\left(\frac{P}{r}\right)^{0.5} < 150000$ and $7.3 < \frac{P}{r} < 15.5$. In ([6] Ch. 5) it is said that this correlation can also be used for curved rectangular ducts for $Re \geq 8000$ replacing r by $0.5D_h$, where D_h is the hydraulic diameter of the rectangular duct.

Srinivasan et al. used only the Dean number in their friction factors correlations for helical coils, claiming that it alone is sufficient to account for an increase in the friction factor due to the coil curvature. They proposed the following correlation for their experimental data with several coils ($7 < R/r < 104$):

$$\frac{f_c}{f_s} = 1 \qquad \text{for } De < 30 \tag{384}$$

$$\frac{f_c}{f_s} = 0.419 De^{0.275} \qquad \text{for } 30 \leq De \leq 300 \tag{385}$$

$$\frac{f_c}{f_s} = 0.1125 De^{0.5} \qquad \text{for } De > 300 \tag{386}$$

4.1.2.17 Tarbell-Samuels (1973)

In 1973 J.M. Tarbell and M.R. Samuels solved the equations of motion and energy to study flow characteristics in helical coils by using the alternating direction implicit technique. The numerical results were compared with the experimental data of White, boundary layer analysis results of Mori and Nakayama, and numerical solution of Truesdell and Adler. A correlation of friction factor representing the data within 3% was proposed [257] [275] [276]:

$$\frac{f_c}{f_s} = 1 + \left[8.279 \cdot 10^{-4} + 7.964 \cdot 10^{-3}\frac{d}{D}\right] Re - 2.096 \cdot 10^{-7} Re^2 \tag{387}$$

It is valid for $20 \leq De \leq 500$, $3 \leq \frac{D}{d} \leq 30$.

4.1.2.18 Ramana Rao-Sadasivudu (1974)

In 1974 M.V. Ramana Rao and D. Sadasivudu proposed the following empirical correlation valid for helical pipes with $0.0159 \leq \frac{d}{D} \leq 0.0556$ [257] [277]:

$$f_c = 62\, e^{14.12\frac{d}{D}} Re^{-1} \text{ for } Re \leq 1200 \tag{388}$$

$$f_c = 6.2\, e^{14.12\frac{d}{D}} Re^{-0.64} \text{ for } 1200 \leq Re \leq Re_{crit.} \tag{389}$$

$$f_c = 0.1528\, e^{11.17\frac{d}{D}} Re^{-0.2} \text{ for } Re_{crit.} \leq Re \leq 27000 \tag{390}$$

For turbulent flow their recommendation was:

$$f_c = 0.0426\left(\frac{d^{0.94}}{D^{0.1}}\right) Re^{-0.2} \tag{391}$$

4.1.2.19 Collins-Dennis (1975)

In 1975 Collins and Dennis recommended a correlation for laminar flow and large Dean numbers [257] [278]:

$$\frac{f_c}{f_s} = 0.38 + 0.1028\sqrt{De} \tag{392}$$

4.1.2.20 Van Dyke (1978)

In 1978 M. Van Dyke presented a correlation for laminar flow in the range of $De \geq 30$ [257] [279]:

$$\frac{f_c}{f_s} = 0.47136 De^{\frac{1}{4}} \tag{393}$$

4.1.2.21 Mishra-Gupta (1979)

In 1975 P. Mishra and S.N. Gupta studied the laminar flow in helical pipes and recommended the following empirical correlation [257] [280]:

$$\frac{f_c}{f_s} = 1 + 0.033[logHe]^4$$
$$He = Re\sqrt{\frac{\frac{d}{D}}{\left[1 + \left(\frac{p}{pD}\right)^2\right]}} \tag{394}$$

It is valid for $1 \leq He \leq 3000$. For helical pipes they also recommended an empirical correlation valid for turbulent flow for $4500 \leq Re \leq 10^5$, $6.7 \leq \frac{D}{d} \leq 346$, $0 \leq \frac{P}{D} \leq 25.4$:

$$f_c = \frac{0.316}{Re^{\frac{1}{4}}} + 0.03\sqrt{\frac{d}{D}} \tag{395}$$

4.1.2.22 Dennis (1980)

In 1980 S.C.R. Dennis proposed the following correlation for laminar flow in helical pipes and large Dean numbers [257] [281]:

$$\frac{f_c}{f_s} = 0.388 + 0.1015\sqrt{De} \tag{396}$$

4.1.2.23 Manlapaz-Churchill (1980)

In 1980 R.L. Manlapaz and S.E.W. Churchill presented a correlation for helical pipes where a separated term $\frac{D}{d}$ is included in addition to a De term to account for the coil-curvature effect ([9] Ch. 5) [257] [282]:

$$\frac{f_c}{f_s} = \left[\left(1.0 - \frac{0.18}{\left[1 + \left(\frac{35}{De}\right)^2\right]^{0.5}}\right)^m + \left(1.0 + \frac{d}{3D}\right)^2 \left(\frac{De}{88.33}\right)\right]^{0.5} \tag{397}$$

$$m = 2 \text{ for } De < 20$$

$$m = 1 \text{ for } 20 < De < 40$$

$$m = 0 \text{ for } De > 40$$

Manlapaz and Churchill suggested using the helical coil number He instead of the Dean number De in the previous equation to account for changes in the friction factor due to the coil pitch. However, their own theoretical predictions, other predictions [283], and experimental data [280] demonstrate that the influence of the coil pitch on the friction factors is very small ([9] Ch. 5).

4.1.2.24 Kadambi (1983)

Kadambi's air friction factor data for $Re \geq 8000$ for two curved rectangular ducts are well predicted by a circular-tube correlation when the hydraulic diameter of the rectangular tube is used [284] [285]. However, for $Re \leq 8000$ the friction factors for a curved rectangular duct were higher than those for the curved circular tube. Higher friction factors were also observed by Butuzov et al. in 1975 [286]. Their experiments included two rectangular ducts and a square duct with water and Freon as working fluids. They correlated their extensive test results as ([9] Ch. 5) follows:

$$\frac{f_c}{f_s} = 0.435 \cdot 10^{-3}\, Re^{*0.96} \left(\frac{R}{d^*}\right)^{0.22} \tag{398}$$

where d^* represents the short channel side and is used as a characteristic dimension in Re^*. In the above equation, f_s represents the friction factor in a straight duct with the same aspect ratio as that of a curved coil. The application range for the correlation is given as $450 \leq Re\sqrt{\frac{d^*}{R}} \leq 7500$ and $25 \leq \frac{R}{d^*} \leq 164$. This equation may be used for curved rectangular ducts for $Re^* \leq 8000$.

4.1.2.25 Yanase et al. (1989)

In 1989 S. Yanase et al. recommended a theoretical correlation for laminar flow in toroidal tubes [257] [287]:

$$\frac{f_c}{f_s} = 0.557 + 0.0938\sqrt{De} \tag{399}$$

4.1.2.26 Liu-Masliyah (1993)

In 1993 S. Liu and J.M. Masliyah proposed the following numerical correlation for helical pipes and developing laminar flows [257] [288]:

$$\frac{f_c}{4}\cdot Re = \left[16 + \left(0.378De\lambda^{\frac{1}{4}} + 12.1\right)De^{\frac{1}{2}}\lambda^{\frac{1}{2}}\gamma^2\right] \times \left[1 + \left\{\frac{\left\{\left(0.0908 + 0.0233\lambda^{\frac{1}{2}}\right)De^{\frac{1}{2}} - 0.132\lambda^{\frac{1}{2}} + 0.37\lambda - 0.2\right\}}{\left(1 + {}^{49}/_{De}\right)}\right\}\right] \tag{400}$$

$$\lambda = \frac{\frac{D}{2}}{\left[\left(\frac{D}{2}\right)^2 + \left(\frac{p}{2\pi}\right)^2\right]}$$

$$\gamma = \frac{\eta}{(\lambda De)^{\frac{1}{2}}}$$

$$\eta = \frac{\frac{p}{2\pi}}{\left(\frac{D}{2}\right)^2 + \left(\frac{p}{2\pi}\right)^2}$$

where p in this last expression refers to the pitch of the coil in cm.

4.1.2.27 Xin et al. (1997)

In 1997 R.C. Xin et al. [289] studied the effects of coil geometries and the flow rates of air and water on pressure drop in both annular vertical and horizontal helical pipes with three different diameters of inner and outer tubes. On the basis of the experimental data, a correlation of the friction factor was developed [275]:

$$f_c = 0.02985 + \frac{75.89\left[0.5 - \frac{\left(\tan^{-1}\left(\frac{De - 39.88}{77.56}\right)\right)}{\pi}\right]}{\left(\frac{D}{d_{i,out} - d_{o,in}}\right)^{1.45}}, \tag{401}$$

where $d_{i,out}$ is the inner diameter of outer tube in meters and $d_{o,in}$ is the outer diameter of inner tube in meters. This correlation is valid for $35 \leq De \leq 20000$, $1.61 \leq \frac{d_{i,out}}{d_{o,in}} \leq 1.67$, $21 \leq {}^{D}/{d_{i,out} - d_{o,in}} \leq 32$.

4.1.2.28 Ju et al. (2001)

In 2001, H. Ju et al. [284] used an HTR-10 steam generator to evaluate the hydraulic performance of small bending radius helical pipe. All experimental data were regressed to obtain the friction factor correlations as follows [275]:

For $De < 11.6$, it is laminar flow:

$$f_s = \frac{64}{Re}, \frac{f_c}{f_s} = 1 \tag{402}$$

For $De > 11.6$ and $Re < Re_{crit}$ it is laminar with large vortex:

$$f_s = \frac{64}{Re}, \frac{f_c}{f_s} = 1 + 0.015Re^{0.75}\left(\frac{d}{D}\right)^{0.4} \tag{403}$$

For $De > 11.6$ and $Re > Re_{crit}$ it is turbulent flow:

$$f_s = \frac{0.316}{Re} \ (smooth\ pipe)$$
$$f_s = 0.1\left(1.46\frac{\varepsilon}{d} + \frac{100}{Re}\right)^{0.25} \ (rough\ pipe) \tag{404}$$
$$\frac{f_c}{f_s} = 1 + 0.11Re^{0.23}\left(\frac{d}{D}\right)^{0.14}$$

where ε is the roughness of the pipe.

4.1.2.29 Guo et al. (2001)

L. Guo et al. [290] studied frictional pressure drops of single-phase water flow in two helically coiled tubes at four different helix axial inclinations angles. The results indicated that the helix axial angles have insignificant effect on the single-phase frictional pressure drop. All measured data were fitted to obtain a new friction factor correlation in the following form [275]:

$$f_c = 2.552Re^{-0.15}\left(\frac{d}{D}\right)^{0.51} \tag{405}$$

4.1.2.30 Summary of friction factor correlations for single-phase flow in curved and helical pipes

Table 32 presents the list of all friction factor correlations collected for single-phase flow in curved and helical pipes.

TABLE 32. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN CURVED AND HELICAL PIPES

Dean (1928) [257] [259] [260]	$\frac{f_c}{f_s} = 1.03058\left(\frac{De^2}{288}\right)^2 + 0.01195\left(\frac{De^2}{288}\right)^4$ $De \leq 20$, laminar flow, small $\frac{d}{D}$ ratios

TABLE 32. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN CURVED AND HELICAL PIPES

White (1929) [257] [261] [262]	for $11.6 \leq De \leq 2000$: $\frac{f_s}{f_c} = 1 - \left[1 - \left(\frac{11.6}{De}\right)^{0.45}\right]^{2.22}$ for $De \leq 11.6$: $\frac{f_c}{f_s} = 1$ $\frac{D}{d} = 15.15, 50, and\ 2050$, circular tubes, laminar flow
White (1932) [257] [263]	$f_c = 0.32Re^{-\frac{1}{4}} + 0.048\sqrt{\frac{d}{D}}$ $1.5 \cdot 10^4 \leq Re \leq 1 \cdot 10^5$, helical pipes, turbulent flow
Adler (1934) [257] [264]	$\frac{f_c}{f_s} = 0.1064\sqrt{De}$ laminar flow, large Dean numbers
Prandtl (1949) [235] [257]	$\frac{f_c}{f_s} = 0.37\left(\frac{De}{2}\right)^{0.36}$ $40 \leq De \leq 2000$, laminar flow, curved pipes
Hasson (1955) [257] [265]	$\frac{f_c}{f_s} = 0.556 + 0.0969\sqrt{De}$ laminar flow, helical pipes

TABLE 32. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN CURVED AND HELICAL PIPES

Ito (1959) [9] [257] [266]	1. laminar flow, $13.5 \leq De \leq 2000$: $\frac{f_c}{f_s} = \frac{21.5De}{(1.56 + logDe)^{5.73}}$ 2. turbulent flow, $Re\left(\frac{d}{D}\right)^2 \geq 6$: $\frac{f_c}{f_s} = \left[Re \cdot \left(\frac{d}{D}\right)^2\right]^{\frac{1}{20}}$ $f_c = \frac{0.316}{Re^{0.2}}\left(\frac{d}{D}\right)^{\frac{1}{10}}$(using Blasius Eq. (310)) 3. turbulent flow, $0.034 \leq Re\left(\frac{d}{D}\right)^2 \leq 300$: $f_c = 0.029\left(\frac{d}{D}\right)^{\frac{1}{2}} + \frac{0.304}{Re^{0.25}}$ 4. turbulent flow, $Y^2\sqrt{\frac{d}{D}} \leq 12$: $f_c = 0.0324\left(\frac{d}{D}\right)^{\frac{1}{2}} + \frac{1.6}{\left(\frac{d}{D}\right)^{0.77} Y^{2.54}}$ $Y^2\sqrt{\frac{d}{D}} \geq 5.3$: $f_c = \frac{1.186}{Y^2}$ where $Y^3e^Y = Re\sqrt{\frac{D}{d}}$
Kubair-Varrier (1961) [257] [267]	for $2 \cdot 10^3 \leq Re \leq 9 \cdot 10^3$: $f_c = 3.0864Re^{-0.5}e^{3.553\frac{d}{D}}$ for $9 \cdot 10^3 \leq Re \leq 2.5 \cdot 10^4$: $f_c = 0.014152Re^{0.09}e^{1.887\frac{d}{D}}$ Both are valid for $0.037 \leq \frac{d}{D} \leq 0.097$, helical pipes
Barua (1963) [257] [268]	$\frac{f_c}{f_s} = 0.509 + 0.0918\sqrt{De}$ laminar flow, large Dean number

TABLE 32. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN CURVED AND HELICAL PIPES

Seban-McLaughlin (1963) [262]	$f_c = \frac{0.184}{Re^{0.2}} \cdot \left[Re \cdot \left(\frac{d}{D}\right)^2\right]^{\frac{1}{20}}$ $12 \leq Re \leq 65000, \frac{D}{d} = 17\ and\ 104$
Mori-Nakayama (1965) [257] [269]	$\frac{f_c}{f_s} = \frac{0.108\sqrt{De}}{1 - \frac{3.253}{\sqrt{De}}}$ $13.5 \leq De \leq 2000$, laminar flow
Mori-Nakayama (1967) [270]	$f = \left(\frac{r}{R}\right)^{0.5} \cdot \frac{0.192}{\left[Re \cdot \left(\frac{r}{R}\right)^{2.5}\right]^{\frac{1}{6}}} \cdot \left(1 + \frac{0.068}{\left[Re \cdot \left(\frac{r}{R}\right)^{2.5}\right]^{\frac{1}{6}}}\right)$ turbulent flow
Schmidt (1967) [271]	for $100 \leq Re \leq Re_{crit.}$: $\frac{f_c}{f_s} = 1 + 0.14 \left(\frac{d}{D}\right)^{0.97} Re^{1-0.644\left(\frac{d}{D}\right)^{0.312}}$ for $Re_{crit.} \leq Re \leq 2.2 \cdot 10^4$ $\frac{f_c}{f_s} = 1 + \frac{2.88 \cdot 10^4}{Re}\left(\frac{d}{D}\right)^{0.62}$ for $2 \cdot 10^4 \leq Re \leq 1.5 \cdot 10^5$: $\frac{f_c}{f_s} = 1 + 0.0823\left(1 + \frac{d}{D}\right)\left(\frac{d}{D}\right)^{0.53} Re^{0.25}$

TABLE 32. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN CURVED AND HELICAL PIPES

Srinivasan et al. (1968) [257] [272]	for $Re\sqrt{\frac{d}{D}} \leq 30$: $f_c = \frac{128}{Re}$ for $30 \leq Re\sqrt{\frac{d}{D}} \leq 300$ $f_c = 20.88\left(Re\sqrt{\frac{D}{d}}\right)^{-0.6}$ for $300 \leq Re\sqrt{\frac{d}{D}} \leq Re_{crit.}\sqrt{\frac{d}{D}}$ $f_c = 7.2\left(Re\sqrt{\frac{D}{d}}\right)^{-0.5}$ For $Re \geq Re_{crit.}$ $f_c = 4.336\left(Re\sqrt{\frac{D}{d}}\right)^{-0.2}$ All are valid for $0.0097 \leq \frac{d}{D} \leq 0.135$
Ito (1969) [257] [273]	$\frac{f_c}{f_s} = 0.1033\sqrt{De}\left[\left(1+\frac{1.729}{De}\right)^{0.5} - \frac{1.315}{De^{0.5}}\right]^{-3}$ $\frac{f_c}{f_s} = 0.1033\sqrt{De}\left[1+\frac{3.945}{De^{0.5}}+\frac{7.782}{De}+\frac{9.097}{De^{1.5}}+\frac{5.608}{De^2}+\cdots\right]$ $\frac{f_c}{f_s} = 0.1008\sqrt{De}\left[1+\frac{3.945}{De^{0.5}}+\frac{7.782}{De}+\frac{9.097}{De^{1.5}}+\frac{5.608}{De^2}+\cdots\right]$ laminar flow

TABLE 32. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN CURVED AND HELICAL PIPES

Srinivasan et al. (1970) [9] [274]	1. helical smooth pipe, turbulent flow, $Re\left(\frac{R}{r}\right)^{-2} < 700$, $7 < \frac{R}{r} < 104$: $f_c\left(\frac{R}{r}\right)^{0.5} = 0.336\left[Re\left(\frac{R}{r}\right)^{-2}\right]^{-0.2}$ 2. spiral coils, $40000 < Re\left(\frac{P}{r}\right)^{0.5} < 150000$, $7.3 < \frac{P}{r} < 15.5$: $f_c = \frac{0.0296\,(n_2^{0.9} - n_1^{0.9})^{1.5}}{\left[Re\left(\frac{P}{r}\right)^{0.5}\right]^{0.2}}$ 3. for $7 < \frac{R}{r} < 104$: $De < 30$: $\frac{f_c}{f_s} = 1$ $30 \leq De \leq 300$: $\frac{f_c}{f_s} = 0.419 De^{0.275}$ $De > 300$: $\frac{f_c}{f_s} = 0.1125 De^{0.5}$
Tarbell-Samuels (1973) [257] [275] [276]	$\frac{f_c}{f_s} = 1 + \left[8.279 \cdot 10^{-4} + 7.964 \cdot 10^{-3}\frac{d}{D}\right] Re - 2.096 \cdot 10^{-7} Re^2$ $20 \leq De \leq 500$, $3 \leq \frac{D}{d} \leq 30$
Ramana Rao-Sadasivudu (1974) [257] [277]	for $Re \leq 1200$: $f_c = 62\, e^{14.12\frac{d}{D}} Re^{-1}$ for $1200 \leq Re \leq Re_{crit.}$: $f_c = 6.2\, e^{14.12\frac{d}{D}} Re^{-0.64}$ for $Re_{crit.} \leq Re \leq 27000$: $f_c = 0.1528\, e^{11.17\frac{d}{D}} Re^{-0.2}$ turbulent flow: $f_c = 0.0426\left(\frac{d^{0.94}}{D^{0.1}}\right) Re^{-0.2}$ $0.0159 \leq \frac{d}{D} \leq 0.0556$, helical pipes

TABLE 32. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN CURVED AND HELICAL PIPES

Collins-Dennis (1975) [257] [278]	$\frac{f_c}{f_s} = 0.38 + 0.1028\sqrt{De}$ laminar flow, large Dean number
Van Dyke (1978) [257] [279]	$\frac{f_c}{f_s} = 0.47136De^{\frac{1}{4}}$ $De \geq 30$, laminar flow
Mishra-Gupta (1979) [257] [280]	for laminar flow, $1 \leq He \leq 3000$: $\frac{f_c}{f_s} = 1 + 0.033[logHe]^4$ where $He = Re\sqrt{\frac{\frac{d}{D}}{\left[1+\left(\frac{p}{pD}\right)^2\right]}}$ for $4500 \leq Re \leq 10^5$, $6.7 \leq \frac{D}{d} \leq 346$, $0 \leq \frac{P}{D} \leq 25.4$: $f_c = \frac{0.316}{Re^{\frac{1}{4}}} + 0.03\sqrt{\frac{d}{D}}$
Dennis (1980) [257] [281]	$\frac{f_c}{f_s} = 0.388 + 0.1015\sqrt{De}$ laminar flow, large Dean number
Manlapaz-Churchill (1980) [9] [257] [282]	$\frac{f_c}{f_s} = \left[\left(1.0 - \frac{0.18}{\left[1+\left(\frac{35}{De}\right)^2\right]^{0.5}}\right)^m + \left(1.0 + \frac{d}{3D}\right)^2\left(\frac{De}{88.33}\right)\right]^{0.5}$ $m = 2$ for $De < 20$ $m = 1$ for $20 < De < 40$ $m = 0$ for $De > 40$
Kadambi (1983) [284] [285]	$\frac{f_c}{f_s} = 0.435 \cdot 10^{-3}\, Re^{*0.96}\left(\frac{R}{d^*}\right)^{0.22}$ $450 \leq Re\sqrt{\frac{d^*}{R}} \leq 7500$, $25 \leq \frac{R}{d^*} \leq 164$
Yanase et al. (1989) [257] [287]	$\frac{f_c}{f_s} = 0.557 + 0.0938\sqrt{De}$ laminar flow

TABLE 32. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN CURVED AND HELICAL PIPES

Liu-Masliyah (1993) [257] [288]	$\frac{f_c}{4}Re = \left[16 + \left(0.378De\lambda^{\frac{1}{4}} + 12.1\right)De^{\frac{1}{2}}\lambda^{\frac{1}{2}}\gamma^2\right]$ $\times \left[1 + \left\{\frac{\left\{\left(0.0908 + 0.0233\lambda^{\frac{1}{2}}\right)De^{\frac{1}{2}} - 0.132\lambda^{\frac{1}{2}} + 0.37\lambda - 0.2\right\}}{\left(1 + {}^{49}/_{De}\right)}\right\}\right]$ $\lambda = \frac{\frac{D}{2}}{\left[\left(\frac{D}{2}\right)^2 + \left(\frac{p}{2\pi}\right)^2\right]}$ $\gamma = \frac{\eta}{(\lambda De)^{\frac{1}{2}}}$ $\eta = \frac{\left(\frac{p}{2\pi}\right)}{\left[\left(\frac{D}{2}\right)^2 + \left(\frac{p}{2\pi}\right)^2\right]}$ developing laminar flows
Xin et al. (1997) [275] [289]	$f_c = 0.02985 + \frac{75.89\left[0.5 - \frac{\left(\tan^{-1}\left(\frac{De - 39.88}{77.56}\right)\right)}{\pi}\right]}{\left(\frac{D}{d_{i,out} - d_{o,in}}\right)^{1.45}}$ $35 \le De \le 20000$, $1.61 \le \frac{d_{i,out}}{d_{o,in}} \le 1.67$, $21 \le {}^{D}/_{d_{i,out} - d_{o,in}} \le 32$
Ju et al.(2001) [275] [284]	for $De < 11.6$, it is laminar flow: $f_s = \frac{64}{Re}, \frac{f_c}{f_s} = 1$ for $De > 11.6$ and $Re < Re_{crit}$ it is laminar with large vortex: $f_s = \frac{64}{Re}, \frac{f_c}{f_s} = 1 + 0.015Re^{0.75}\left(\frac{d}{D}\right)^{0.4}$ for $De > 11.6$ and $Re > Re_{crit}$ it is turbulent flow: $f_s = \frac{0.316}{Re}$ $(smooth\ pipe)$ $f_s = 0.1\left(1.46\frac{\varepsilon}{d} + \frac{100}{Re}\right)^{0.25}$ $(rough\ pipe)$ $\frac{f_c}{f_s} = 1 + 0.11Re^{0.23}\left(\frac{d}{D}\right)^{0.14}$

TABLE 32. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN CURVED AND HELICAL PIPES

Guo et al. (2001) [275] [290]	$f_c = 2.552 Re^{-0.15} \left(\frac{d}{D}\right)^{0.51}$ helically coiled tubes

4.1.3 Flow in bundles with smooth pins

In Fig. 53, the geometry of the rod bundles with smooth rods (without wire wrap) are shown for hexagonal and square configurations. Here *D* – diameter of the fuel pin and *P* – pitch.

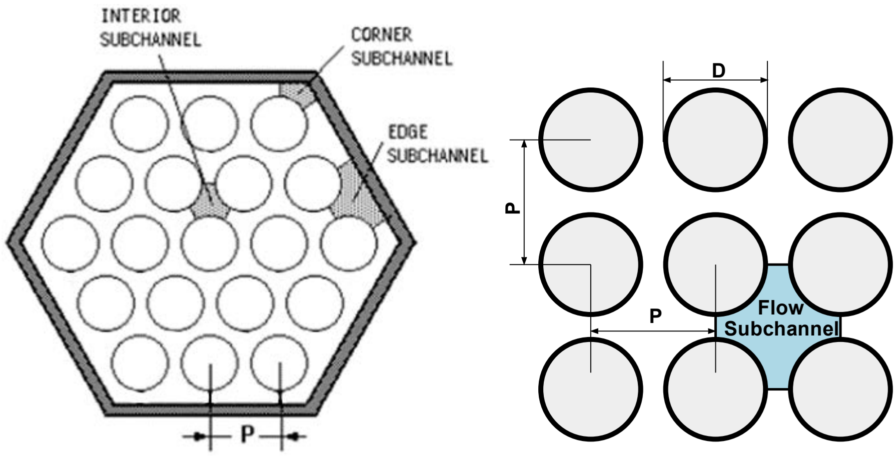


FIG. 53. Geometry of hexagonal subassembly and square rod bundle

4.1.3.1 Presser (1967)

The empirical correlations by K.H. Presser (1967) ([9] Ch. 7) [291] are recommended for an infinite triangular array and a $\frac{P}{D}$ ratio of $1 \le \frac{P}{D} \le 2$.

For $\frac{P}{D} \le 1.2$:

$$f = A_1 \cdot Re^{-0.25} \; for \; 10^4 \le Re \le 5 \cdot 10^4 \tag{406}$$

$$f = A_1 \cdot Re^{-0.2} \; for \; 5 \cdot 10^4 \le Re \le 2 \cdot 10^5 \tag{407}$$

where

$$A_1 = 0.171 + 0.012 \cdot \frac{P}{D} - 0.07 \cdot e^{-50\left(P/_D - 1\right)} \tag{408}$$

Presser's correlation agrees within 2% with the solution obtained by the laminar method. He also proposed empirical correlations for an infinite square array and $1 \le \frac{P}{D} \le 2$:

$$f = A_1 \cdot Re^{-0.25} \; for \; 10^4 \le Re \le 5 \cdot 10^4 \tag{409}$$

$$f = A_1 \cdot Re^{-0.2} \; for \; 5 \cdot 10^4 \le Re \le 2 \cdot 10^5 \tag{410}$$

where

$$A_1 = 0.181 + 0.0108 \cdot \frac{P}{D} - 0.132 \cdot e^{-20(P/_D - 1)} \tag{411}$$

4.1.3.2 Subbotin et al. (1972)

In 1972 V.I. Subbotin et al. [130] [160] [292] proposed the following friction factor for l*aminar flow* along smooth pin bundles:

$$f = \frac{64}{Re} K \tag{412}$$

where the values of the form factor *K* for smooth pins are indicated in Table 33, $Re = \frac{w \times d_h}{n}$ is the Reynolds number based on the bulk flow velocity and hydraulic diameter of the "infinite" pin array.

TABLE 33. VALUES OF FACTOR K IN LAMINAR FLOW IN SMOOTH PIN BUNDLES

Type of bundle	Relative pitch, $P/_D$								
	1.0	1.02	1.05	1.10	1.20	1.30	1.40	1.50	2.0
Triangular	0.407	0.663	0.966	1.274	1.56	1.715	1.834	1.940	2.46
Square	0.405	0.518	0.679	0.913	1.264	1.510	1.699	1.858	2.51

For approximate calculations it is possible to use the following formulas. For triangular bundles:

$$K \cong 0.41 + 1.9 \sqrt[3]{\frac{P}{D} - 1} \tag{413}$$

For square bundles:

$$K \cong 0.41 + 1.9 \sqrt{\frac{P}{D} - 1} \tag{414}$$

Eqs. (412)(413)(414) are applicable when pitch-to diameter ratio of the bundle is between $1.0 \leq P/_D \leq 2.0$.

4.1.3.3 Subbotin et al. (1971)

In 1971 V.I Subbotin et al. [130] [293] [294] [295] recommended the following friction factor for turbulent flow in triangular smooth pin bundle as follows:

$$\frac{f}{f_0} = 0.57 + 0.18\left(\frac{P}{D} - 1\right) + 0.53(1 - e^{-a}) \tag{415}$$

$$\frac{f}{f_0} = 0.59 + 0.19\left(\frac{P}{D} - 1\right) + 0.52(1 - e^{-b}) \tag{416}$$

where: $a = 0.58\left[1 - e^{-70\left(\frac{P}{D}-1\right)}\right] + 9.2\left(\frac{P}{D} - 1\right), b = 10\left(\frac{P}{D} - 1\right).$

f_0 is the friction factor for turbulent flow in smooth round tube obtained using Blasius formula Eq. (310).

For $P/_D > 1.02$, it is possible to neglect exponents in the above correlations.

4.1.3.4 Rehme (1972)

For the annular-zone solution, K. Rehme [296] recommended the following correlation for triangular arrays for $P/_D \geq 1.2$ ([9] Ch. 7):

$$\frac{f}{f_t} = 1.045 + 0.071\left(\frac{P}{D} - 1\right)\ for\ Re = 10^4 \quad (417)$$

$$\frac{f}{f_t} = 1.036 + 0.054\left(\frac{P}{D} - 1\right)\ for\ Re = 10^5 \quad (418)$$

where f_t is the friction factor of circular tubes.

A relationship between laminar and turbulent flow friction factors was also developed by Rehme [297] based on the law of the wall for the velocity profile. The equation for the turbulent friction factor can be written as follows ([9] Ch. 7):

$$\sqrt{\frac{8}{f}} = \mathrm{A}_2 \cdot \left[2.5 \ln Re \sqrt{\frac{f}{8}} + 5.5\right] - G^* \quad (419)$$

where A_2 and G^* are two geometry parameters which depend on $f\ Re$ for laminar flow:

$$\mathrm{A}_2 = 1 \text{ for } f\ Re \geq 64$$
$$\mathrm{A}_2 = 1 + 0.552 \log\left(\frac{64}{f\ Re}\right) for\ f\ Re < 64 \quad (420)$$

G^* can be determined from correlations developed by Cheng and Todreas [7] [45]

$$G^* = 2.553 + 3.872\ \log\left(\frac{fRe}{4}\right) - 1.042\ \left(\log\left(\frac{fRe}{4}\right)\right)^2 \text{ for } 24 < fRe \leq 64 \quad (421)$$

$$G^* = 6.615 - 3.376\ \log\left(\frac{fRe}{4}\right) + 2.159\ \left(\log\left(\frac{fRe}{4}\right)\right)^2 \text{ for } 64 < fRe \leq 125 \quad (422)$$

$$G^* = 1.663 + 3.151\ \log\left(\frac{fRe}{4}\right) \text{ for } 125 < fRe \leq 1000 \quad (423)$$

4.1.3.5 Zhukov et al. (1985)

In 1985 A.V. Zhukov et al. [298] [299] recommended the following correlation for transition from laminar to turbulent flow in rod bundles with smooth pins:

$$\log f = 5.2 Re^{-0.22+0.145\left(\frac{P}{D}-1\right)} - 2.35 \quad (424)$$

The same formula describes friction factor over a wide range of parameters ($10 \le Re \le 2 \cdot 10^5$, $1.0 \le P/D \le 1.5$), with accuracy $\pm$ 20%.

Considerable attention has been focused on the analysis of a rich variety of the data collected on the friction factors in infinite rod arrays with smooth pins that results in the following simple relationship:

$$f = \frac{0.210}{Re^{0.25}}\left[1 + \left(\frac{P}{D} - 1\right)^{0.32}\right] \quad (425)$$

It is valid for $1.0 \le P/D \le 1.5$, $6 \cdot 10^3 \le Re \le 2 \cdot 10^5$.

Correlation (425) is in a good agreement with the results of experimental and numerical investigations of the pressure drops in multi-pin bundles.

When arranged in square bundle, the friction factor can be found using the relationship:

$$\frac{f}{0.316Re^{-0.25}} = 0.59 + 0.19\left(\frac{P}{D} - 1\right) + 0.52\left[1 - e^{-10\left(\frac{P}{D}-1\right)}\right] \quad (426)$$

It is valid for $1.0 \le P/D \le 2.0$, $10^4 \le Re \le 5 \cdot 10^5$.

4.1.3.6 Malak et al. (1975)

In 1975 J. Malak et al. [130] [300] recommended to express the friction factor for fuel bundles of smooth pins in laminar and turbulent flow using the geometrical parameters χ_{lam} and χ_{turb} respectively as follows:

$$f = \frac{64}{Re}\chi_{lam}^2 \quad (427)$$

$$\sqrt{\frac{\chi_{turb}}{f}} = 2\log\frac{Re\sqrt{f}}{\chi_{turb}^{1.5}} - 0.8 \quad (428)$$

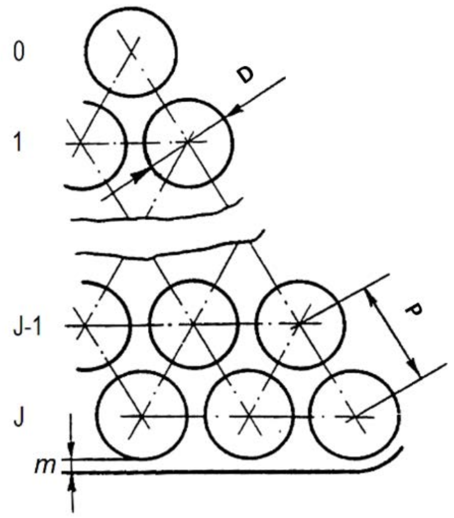
Experiments have shown that parameters χ_{lam} and χ_{turb} are related to each other as:

$$\chi_{turb} = \frac{1 + 3\chi_{lam}}{4} \quad (429)$$

$$0.25 \le \chi_{lam} \le 1.25,\ 0.45 \le \chi_{turb} \le 1.2.$$

FIG. 54 shows the values of these parameters concerning fast reactor subassembly. The wall channels are considered here and an influence of the pin number on these parameters is demonstrated.

J	n	χ_{lam}	χ_{turb}



1	7	0.774	0.830
2	9	0.982	0.987
3	37	1.059	1.044
4	61	1.098	1.073
5	91	1.120	1.090
6	127	1.135	1.101
7	169	1.145	1.109
8	217	1.152	1.114
9	271	1.158	1.119
10	331	1.163	1.122
11	397	1.166	1.125

FIG. 54. Coefficients χ_{lam} and χ_{turb} for pin bundle of BN-600 type, J: number of rows, n: number of pins, $P/D = 1.166$, $m/D = 0.1045$

4.1.3.7 Summary of friction factor correlations for single-phase flow in rod bundles with smooth pins

Friction factor correlations for single-phase flow in wire-wrapped bundle are summarized in Table 34.

TABLE 34. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN SMOOTH ROD BUNDLES

Presser (1967) [9] [291]	For $\frac{P}{D} \leq 1.2$ and hexagonal infinitive pin array: $f = A_1 \cdot Re^{-0.25}\ for\ 10^4 \leq Re \leq 5 \cdot 10^4$ $f = A_1 \cdot Re^{-0.2}\ for\ 5 \cdot 10^4 \leq Re \leq 2 \cdot 10^5$ $A_1 = 0.171 + 0.012 \cdot \frac{P}{D} - 0.07 \cdot e^{-50(P/D-1)}$ For $1 \leq \frac{P}{D} \leq 2$ and square infinitive pin array: $f = A_1 \cdot Re^{-0.25}\ for\ 10^4 \leq Re \leq 5 \cdot 10^4$ $f = A_1 \cdot Re^{-0.2}\ for\ 5 \cdot 10^4 \leq Re \leq 2 \cdot 10^5$ $A_1 = 0.181 + 0.0108 \cdot \frac{P}{D} - 0.132 \cdot e^{-20(P/D-1)}$

TABLE 34. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN SMOOTH ROD BUNDLES

Subbotin et al. (1972) [130] [160] [293]	Laminar flow in smooth rod bundles: $f = \frac{64}{Re}K$ K is given in Table 33 for both triangular and squared arrays; approximate values can be obtained as: for triangular pin arrays: $K \cong 0.41 + 1.9\sqrt[3]{\frac{P}{D} - 1}$ for square pin arrays: $K \cong 0.41 + 1.9\sqrt{\frac{P}{D} - 1}$ All are valid for $1.0 \leq P/D \leq 2.0$.
Subbotin et al. (1971) [130] [293] [294] [295]	$\frac{f}{f_0} = 0.57 + 0.18\left(\frac{P}{D} - 1\right) + 0.53(1 - e^{-a})$ $\frac{f}{f_0} = 0.59 + 0.19\left(\frac{P}{D} - 1\right) + 0.52(1 - e^{-b})$ $a = 0.58\left[1 - e^{-70\left(\frac{P}{D}-1\right)}\right] + 9.2\left(\frac{P}{D} - 1\right), b = 10\left(\frac{P}{D} - 1\right)$ f_0 is the friction factor by Blasius correlation Eq. (310) turbulent flow in triangular smooth pin bundle
Rehme (1972) [296]	$\frac{f}{f_t} = 1.045 + 0.071\left(\frac{P}{D} - 1\right)\ for\ Re = 10^4$ $\frac{f}{f_t} = 1.036 + 0.054\left(\frac{P}{D} - 1\right)\ for\ Re = 10^5$ f_t: friction factor of circular tubes for triangular arrays and $P/D \geq 1.2$
Zhukov et al. (1985) [298] [299]	Transition from laminar to turbulent flow in bundles with smooth pins: $\log f = 5.2Re^{-0.22+0.145\left(\frac{P}{D}-1\right)} - 2.35$ $10 \leq Re \leq 2 \cdot 10^5, 1.0 \leq P/D \leq 1.5$ Infinitive pin array with smooth pins: $f = \frac{0.210}{Re^{0.25}}\left[1 + \left(\frac{P}{D} - 1\right)^{0.32}\right]$ $1.0 \leq P/D \leq 1.5$, $6 \cdot 10^3 \leq Re \leq 2 \cdot 10^5$ Square rod bundles: $\frac{f}{0.316Re^{-0.25}} = 0.59 + 0.19\left(\frac{P}{D} - 1\right) + 0.52\left[1 - e^{-10\left(\frac{P}{D}-1\right)}\right]$ $1.0 \leq P/D \leq 2.0$, $10^4 \leq Re \leq 5 \cdot 10^5$

TABLE 34. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN SMOOTH ROD BUNDLES

Malak et al. (1975) [130] [300]	$f = \frac{64}{Re}\chi_{lam}^2$ $\sqrt{\frac{\chi_{turb}}{f}} = 2\log\frac{Re\sqrt{f}}{\chi_{turb}^{1.5}} - 0.8$ $\chi_{turb} = \frac{1 + 3\chi_{lam}}{4}$ $0.25 \leq \chi_{lam} \leq 1.25,\ 0.45 \leq \chi_{turb} \leq 1.2.$ bundles of smooth pins in laminar and turbulent flow χ_{lam} and χ_{turb} from Fig. 54 for hexagonal bundles

4.1.4 Flow in wire-wrapped rod bundles

The geometry of wire wrapped rod bundles in a hexagonal fuel assembly can be seen in Fig. 55 where D is the diameter of fuel pin, H *– wire pitch and* P *– pin pitch.*

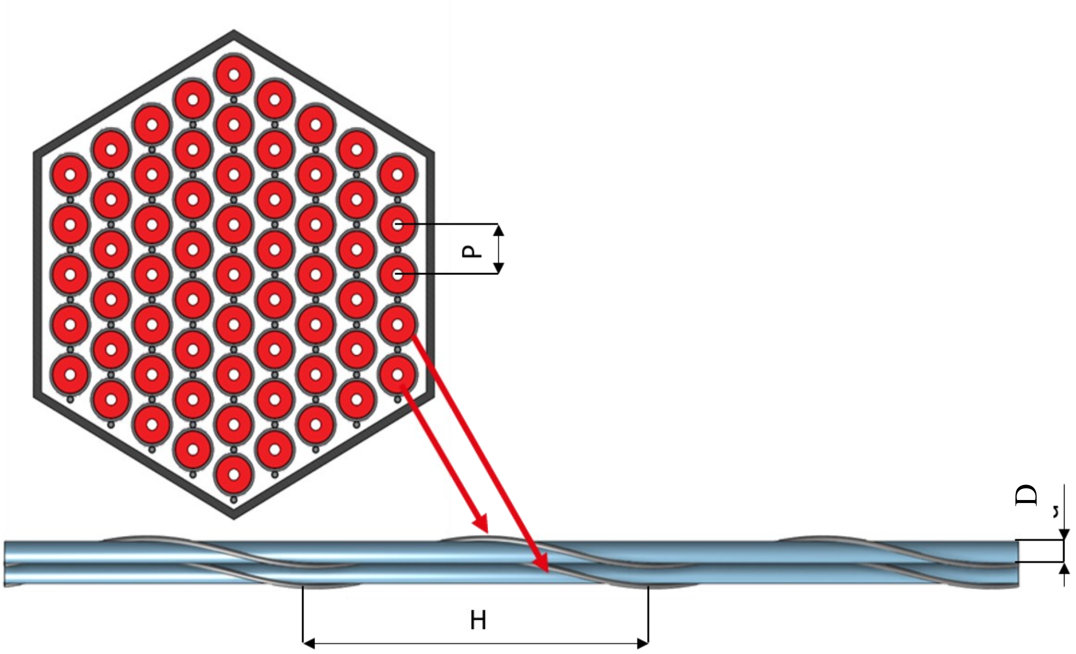


FIG. 55. Hexagonal fuel assembly and subchannels geometry

4.1.4.1 Pontier-Combe (1968)

In 1968, L. Pontier and J. Combe proposed a correlation following a programme of turbulent tests in water [301]. The tests were realized with four wire-wrapped rod bundles, each having different rod diameter and wire diameter.

$$f = \Omega_0 e^r \tag{430}$$

$$\Omega_0 = 0.12Re^{-0.16} \text{ for } \varepsilon \sim 1.6 \cdot 10^{-4} \text{ mm (as for Pontier's experiments)} \tag{431}$$

$$\Omega_0 = \left[-2log\left[\frac{\varepsilon}{3.7D_h} + \left(\frac{6.81}{Re}\right)^{0.9}\right]\right]^{-2} \text{for other roughness} \tag{432}$$

$$r = \left(1 + 4.6\left(\frac{P}{D} - 1\right)\right)\tan\alpha \text{ with } tan\,\alpha = \pi\frac{d}{H} \tag{433}$$

This correlation is valid for the following conditions: $1 \cdot 10^4 \leq Re \leq 1 \cdot 10^5$ (turbulent flow regime), $15.7 \leq \frac{H}{D} \leq \infty$, $1.1 \leq \frac{d_m}{D} \leq 1.4$, $37 \leq N_{rod} \leq 331$, $0 \leq tan\,\alpha \leq 0.2$ and $1.3 \cdot 10^{-4} \leq \varepsilon \leq 2 \cdot 10^{-4}$ where d_m is the rod diameter plus the wire diameter, P is the rod pitch, D is the rod diameter, Re is the Reynolds number using mean bundle average value, H is the wire pitch, D_h is the hydraulic diameter, N_{rod} is the rod number and ε is the roughness.

Pontier correlation was established from water turbulent flow tests, but it may be used for sodium flow as well. The test sections were horizontal, and 9 configurations were studied. It takes into account the roughness of the pins. The accuracy of this correlation in its domain of validity (turbulent region) is $\pm 10\%$. However, there is no laminar model in Pontier's correlation and moreover it is independent of the number of pins in the bundle.

4.1.4.2 Sangster (1968)

In 1968 W. Sangster proposed the friction factor correlation for rod bundles as follows [299] [302]:

$$f = 0.974\left(\frac{P}{D}\right)^{0.8}\frac{4.76}{\left({}^{H}/_{D}\right)^{0.47}}f_p \text{ for } 1.135 \leq \frac{P}{D} \leq 1.195 \tag{434}$$

$$f = 1.048\left(\frac{P}{D}\right)^{0.37}\frac{4.76}{\left({}^{H}/_{D}\right)^{0.47}}f_p \text{ for } 1.195 \leq \frac{P}{D} \leq 1.255 \tag{435}$$

$$f = 1.138\frac{4.76}{\left({}^{H}/_{D}\right)^{0.47}}f_p \text{ for } \frac{P}{D} \geq 1.255 \tag{436}$$

where f_p is a flow resistance factor for a round pipe. It is valid for $10 \leq \frac{H}{D} \leq 40$ and $4 \cdot 10^3 \leq Re \leq 10^5$.

4.1.4.3 Novendstern (1972)

E.H. Novendstern (1972) presented a model where the influence of the wire wrap is considered by means of an effective friction factor f_1 calculated as follows [3] [303] [304]:

$$f_1 = \mathrm{M}\, f_s \tag{437}$$

where f_s is the standard friction factor for smooth pipes. The friction factor will then be calculated as:

$$f = f_1 {X_1}^2\left(\frac{D_{eb}}{D_{e1}}\right) = \mathrm{M}\, f_s {X_1}^2\left(\frac{D_{eb}}{D_{e1}}\right) \tag{438}$$

The multiplier M has the expression:

$$M = \left(\frac{1.034}{\left(P/_{D}\right)^{0.124}} + \frac{29.7 \left(P/_{D}\right)^{6.94} Re_1^{0.086}}{\left(H/_{D}\right)^{2.239}} \right)^{0.885} \tag{439}$$

where Re_1 is the Reynolds number for the centre subchannel of the hot SA in the wire-wrap configuration calculated as follows [155] [305]:

$$Re_1 = \frac{\rho v_1 D_{e1}}{\mu} = X_1 Re \frac{D_{e1}}{D_{eb}} \tag{440}$$

$$Re = \frac{\rho v D_e}{\mu} \text{ and } v_1 = Xv \tag{441}$$

X_1 is the flow distribution factor calculated based on the central, side and corner subchannels:

$$X_1 = \frac{A_b}{\left(N_1 A_1 + N_2 A_2 \left(\frac{D_{e2}}{D_{e1}}\right)^{0.714} + N_3 A_3 \left(\frac{D_{e3}}{D_{e1}}\right)^{0.714} \right)} \tag{442}$$

$$A_b = N_1 A_1 + N_2 A_2 + N_3 A_3 \tag{443}$$

Flow areas A_i and equivalent diameters D_i are calculated as if the wire spacer cross-section was distributed uniformly in all subchannels:

$$D_{ei} = {}^{4A_i}/_{P_{wi}} \tag{444}$$

$$A_1 = A_1{}' - {}^{\pi D_w{}^2}/_{8} \tag{445}$$

$$A_2 = A_2{}' - {}^{\pi D_w{}^2}/_{8} \tag{446}$$

$$A_3 = A_3{}' - {}^{\pi D_w{}^2}/_{24} \tag{447}$$

$$A_b = N_1 A_1 + N_2 A_2 + N_3 A_3 \tag{448}$$

$$A_1{}' = \left({}^{\sqrt{3}}/_{4}\right) P^2 - {}^{\pi D^2}/_{8} \tag{449}$$

$$A_2{}' = P\left(W - {}^{D}/_{2}\right) - {}^{\pi D^2}/_{8} \tag{450}$$

$$A_3{}' = \left(\left(W - {}^{D}/_{2}\right)^2 \sqrt{3}\right) - {}^{\pi D^2}/_{24} \tag{451}$$

$$P_{w1} = P_{w1}{}' - {}^{\pi D_w{}^2}\Big/_{(2\ \cos\theta)} \tag{452}$$

$$P_{w2} = P_{w2}{}' - {}^{\pi D_w{}^2}\Big/_{(2\ \cos\theta)} \tag{453}$$

$$P_{w3} = P_{w3}{}' - {}^{\pi D_w{}^2}\Big/_{(6\ \cos\theta)} \tag{454}$$

$$P_{w1}' = {}^{\pi D}/_{2} \tag{455}$$

$$P_{w2}' = P + {}^{\pi D}/_{2} \tag{456}$$

$$P_{w3}' = {}^{\pi D}/_{6} - 2\left(W - {}^{D}/_{2}\right)\sqrt{3} \tag{457}$$

$$\cos\theta = {}^{H}\Big/_{\sqrt{H^2 + (\pi(D + D_w)^2)}} \tag{458}$$

The nomenclature here is as follows: P: rod pitch, D: rod diameter, f: Darcy friction factor, if no subscript means bundle average value, H: wire lead length, D_e: equivalent hydraulic diameter, D_w: wire diameter, A: area, N_i: number of subchannels in each kind of subchannel i in the bundle, W: edge pitch parameter defined as (D + gap between rod and bundle wall). Subscripts i=1, 2, 3 or b denote interior, edge, corner subchannel type, or bundle average, respectively.

This correlation is valid for the conditions [155]: $600 \le Re \le 2 \cdot 10^5$, $5 \cdot 10^{-3} \le D \le 12 \cdot 10^{-3}$, $19 \le N_{rod} \le 217$, $1.06 \le \frac{P}{D} \le 1.42$ and $8 \le \frac{H}{D} \le 90$.

4.1.4.4 Rehme (1973)

In 1973, K. Rehme proposed a correlation established on a set of experiments (most of which were conducted before 1967) combining 75 different geometries [306]:

$$f = \left[\frac{64\sqrt{F}}{Re} + \frac{0.0816F^{0.9335}}{Re^{0.133}}\right]\frac{P_b}{P_{tot}} \tag{459}$$

where P_b is the wetted perimeter of rods and wires, P_{tot} is the total wetted perimeter of the rod bundle including the wetted perimeter of the channel walls. $\sqrt{F}$ represents the ratio of the effective to average velocity:

$$F = \left(\frac{v_{eff}}{v_m}\right)^2 = \left(\frac{P}{D}\right)^{0.5} + \left[7.6\frac{d_m}{H}\left(\frac{P}{D}\right)^2\right]^{2.16} \tag{460}$$

The hydraulic diameter in the Reynolds number and the pressure drop evaluation include the cross-section and the wetted perimeter of the wires, taking into account that the cross-section of the wire perpendicular to the rod bundle axis is an ellipse. d_m is the mean diameter of the wire wraps, which is $d_m = P$ for contact between rods and wires and $d_m = D + h$ for contact among fins, with h being the height of the fins ([9] Ch. 7).

The Rehme correlation takes into account the influence of the hexagonal wrapper. This correlation is valid for the following conditions: $1 \cdot 10^3 \le Re \le 3 \cdot 10^5$ (transition or turbulent flow regime), $8 \le \frac{H}{d_m} \le 50$, $1.1 \le \frac{P}{D} \le 1.42$ and $7 \le N_{rod} \le 217$, where d_m is the rod diameter plus the wire diameter, P is the rod pitch, D the rod diameter, Re the Reynolds number, using means bundle average value, H the wire pitch, N_{rod} the rod number, P_b the rod bundle and wire friction perimeter and P_{tot} is the total (with hexagonal wrapper) friction perimeter.

Rehme correlation is a widely used correlation based on an effective velocity to take into account the swirl flow velocity around the rod. It considers the effects of $\frac{P}{D}$ and $\frac{H}{D}$ and the influence of the wrapper and the number of pins. This correlation has been built based on a large bundle of data sets. Rehme performed pressure drop experiments for 75 wire-wrapped rod bundles with different combinations of geometrical parameters. In total there were 25 combinations composed of five $\frac{P}{D}$ for each of the five $\frac{H}{D}$ ratios; further, each combination had 3 different pin number configurations, 7, 19, 37; only for $\frac{P}{D} = 1.125$ 61-pin bundles were used.

D. Tenchine in 2010 [307] concluded that, for pressure drop evaluation (the comparison of different friction factor models proposed in the literature with the pressure drop correlation provided by Superphenix subassembly tests), the best agreement with Superphenix data was obtained using Rehme and Cheng-Todreas pressure drop correlations. The advantage of the Rehme model is its relative simplicity, but the Cheng–Todreas model was validated over a larger range of bundle characteristics and flow regimes. By the way, Rehme correlation is the only correlation where the application range starts at as low as 7 pins.

Most correlations are valid for $\frac{P}{D}$ as low as 1.06, except for the Rehme correlation, which was calibrated by only Rehme's own experimental results for which a validation range for $\frac{P}{D}$ is between 1.1 and 1.42. Some evaluations [304] show that Rehme correlation can predict data of bundles with $\frac{P}{D}$ as low as 1.05, while it breaks down for $\frac{P}{D}$ approaching 1.04. Moreover, there is no laminar model in Rehme's correlation. The accuracy of this correlation compared to Rehme's own data is around $\pm 8\%$ in the turbulent region.

4.1.4.5 Subbotin et al. (1975)

For BN-600 fuel assembly, based on the results of measurements of the flow rate on models and in a reactor, the following dependence for friction factor of a pin bundle spaced by the standard (Fig. 56) spiral wire of type "single wire between pins" was derived [160]:

$$f_p = 0.117\left(\frac{\varepsilon}{d_h} + \frac{68}{Re}\right)^{0.25} \tag{461}$$

where ε is the roughness of the pin surface.

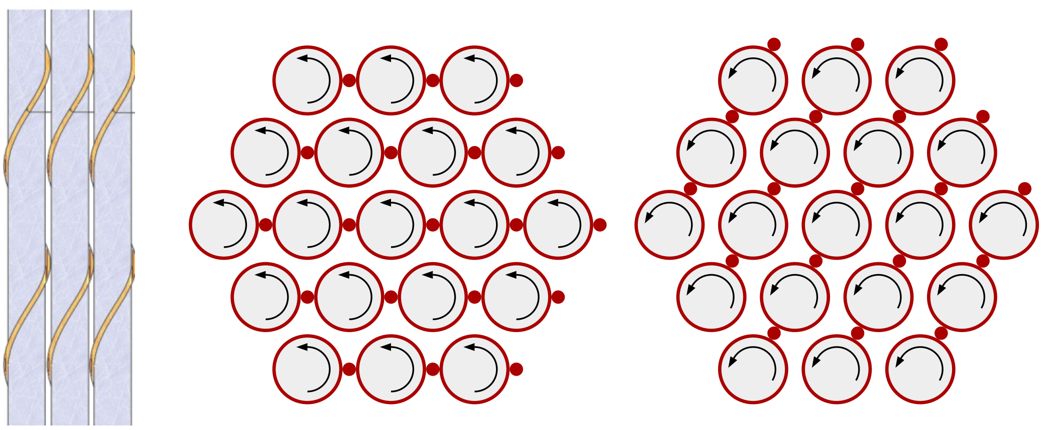


FIG. 56. Hexagonal array of pins spaced by wire wrapping in single CCW direction ("single wire between pins")

For transition from laminar to turbulent flow the following relation can be recommended:

$$f_p' = f_{p\ lam}\alpha + f_p\,(1-\alpha) \tag{462}$$

where $f_{p\ lam}$ is friction factor in the bundle of wrapped pins in laminar flow and f_p is friction factor for turbulent flow.

$$\alpha = 0.5\left\{1-\tanh\left[0.8\left({}^{Re}/_{1450}-1\right)\right]\right\} \tag{463}$$

The accuracy of this relationship is ~20%. In case the pin spacer is realized as "two wires between pins" (see Fig. 57) type of wire wrap, the experimental data for friction factor can be represented by the following formulas. For laminar flow:

$$\frac{f_p}{f} \approx 1+\frac{1.8}{\frac{H}{D}} \tag{464}$$

valid for ${}^{P}/_{D}$ from 1.13 to 1.15.

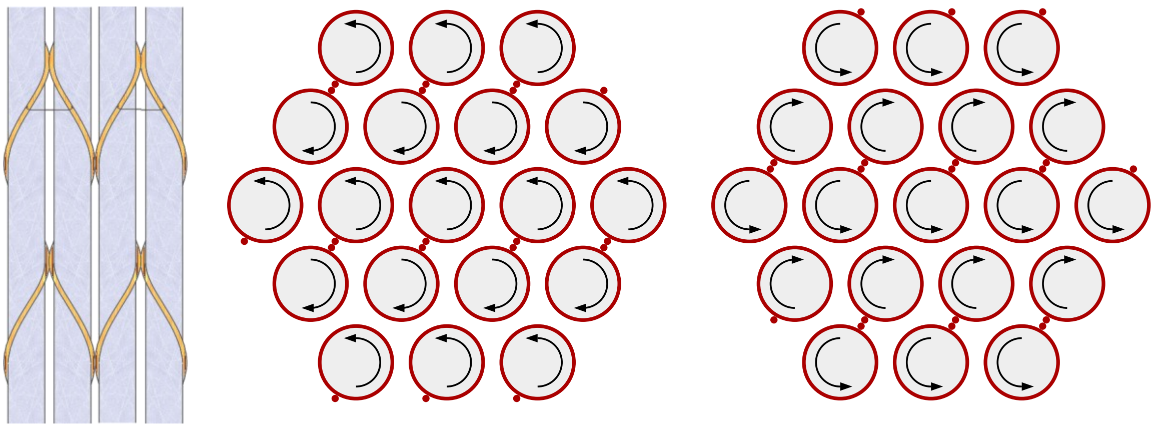


FIG. 57. Hexagonal array of pins spaced by wire wrapping in both directions (odd row in CW direction, even row in CCW; "two wires between pins")

For turbulent flow:

$$\frac{f_p}{f} \approx 1+\frac{600\left(\frac{P}{D}-1\right)}{\left(\frac{H}{D}\right)^2} \tag{465}$$

This correlation is valid for $10^4 \le Re \le 20\cdot 10^4$, $1.05 \le {}^{P}/_{D} \le 1.25$, ${}^{H}/_{D} \ge 5$ and $2 \le n \le 4$ where n is the number of entries of the fin.

4.1.4.6 Engel et al. (1979)

In 1979, F.C. Engel, et al. proposed a friction factor correlation on the basis of the 19-pin and 61-pin experiments performed in fissile-type or fertile-type fuel assembly geometries [308].

Over the domain of $50 \le Re \le 1 \cdot 10^5$ (fertile), $50 \le Re \le 400$ (fissile), $1.067 \le \frac{P}{D} \le 1.082\ (fertile), \frac{P}{D} \sim 1.2\ (fissile)$ and $19 \le N_{rod} \le 61$. This correlation takes the form of:

$$f = \frac{C_{fL}}{Re} \quad \text{for } Re \le Re_L \tag{466}$$

$$f = \frac{C_{fL}}{Re}\sqrt{1-\psi} + \frac{C_{fT}}{Re^{0.25}}\sqrt{\psi} \quad \text{for } Re_L \le Re \le Re_T \tag{467}$$

$$f = \frac{C_{fT}}{Re^{0.25}} \quad \text{for } Re \ge Re_T \tag{468}$$

$$C_{fL} = 110 \quad \text{for } 1.067 \le \frac{P}{D} \le 1.082 \tag{469}$$

$$C_{fL} = \frac{320}{\sqrt{H}}\left(\frac{P}{D}\right)^{1.5} \quad \text{for } \frac{P}{D} \sim 1.2 \tag{470}$$

$$Re_L = 400, Re_T = 5000, \psi = \frac{Re-400}{4600}, C_{fT} = 0.55$$

where P: rod pitch, D: rod diameter, Re: Reynolds number using mean bundle average value, H: wire pitch, N_{rod}: rods number. Within the given range, the accuracy of this correlation is given as $\pm 18\%$.

4.1.4.7 Markley-Engel (1976)

In 1976 R. Markley and F. Engel proposed a correlation based on the experimental data in the range of $1.067 \le \frac{P}{D} \le 1.32, \frac{H}{D} \sim 8$ and $40 \le Re \le 10^5$ [299] [309]:

$$f = \frac{110}{Re} \quad \text{for } Re \le Re_L \tag{471}$$

$$f = \frac{110}{Re}\sqrt{1-\psi} + \frac{0.48}{Re^{0.25}}\sqrt{\psi} \quad \text{for } Re_L \le Re \le Re_T \tag{472}$$

$$f = \frac{0.48}{Re^{0.25}} \quad \text{for } Re \ge Re_T \tag{473}$$

where $\psi = \frac{Re-400}{4600}$.

4.1.4.8 Engel et al. (1979)

In 1979 F.C. Engel et al. proposed the following correlation for laminar flow in rod bundles [299] [304] [308] [310]:

$$f = \frac{32}{Re\sqrt{H}}\left(\frac{P}{D}\right)^{1.5} \quad \text{for } Re \le 400 \tag{474}$$

where H is measured in meters. It is valid for $4 \le \frac{H}{D} \le 52$, $1.067 \le \frac{P}{D} \le 1.25$ and $61 \le N_{rod} \le 217$.

4.1.4.9 Roidt et al. (1980)

In 1980 R. Roidt, et al. proposed the friction factor correlation for the peripheral rod of a rod bundle [299] [311]:

$$f = \frac{2.284}{Re^{0.4183}} \tag{475}$$

It is valid for $8 \leq \frac{H}{D} \leq 52$, $1.08 \leq \frac{P}{D} \leq 1.24$ and $4.3 \cdot 10^3 \leq Re \leq 7.3 \cdot 10^4$.

4.1.4.10 Baxi-Dalle Donne (1981)

C.B. Baxi and M. Dalle Donne (1981) friction factor correlation is as follows [312]:

For laminar region, $Re \leq 400$

$$f_L = \frac{\left(\frac{T_w}{T_B}\right)\left(\frac{320}{\sqrt{H}}\right)\left({}^{P}/_{D}\right)^{1.5}}{Re} \tag{476}$$

where T_w is the wall temperature in K, T_B is the coolant bulk temperature in K and H is the wire lead length in cm.

For turbulent region, $Re \geq 5 \cdot 10^3$

$$f_T = M f_s \tag{477}$$

where f_s is the friction factor for smooth pipe and M is the multiplier proposed by Novendstern Eq. (437).

$$M = \left(\frac{1.034}{\left({}^{P}/_{D}\right)^{0.124}} + \frac{29.6\left({}^{P}/_{D}\right)^{6.94} Re^{0.086}}{\left({}^{H}/_{D}\right)^{2.239}}\right)^{0.885} \tag{478}$$

For transition region, $400 \leq Re \leq 5 \cdot 10^3$

$$f = f_L\,(1-\psi)^{0.5} + f_T\,\psi^{0.5} \tag{479}$$

Where $\psi = \frac{(Re-400)}{4600}$. Subscripts: L denotes laminar flow regime and T denotes turbulent flow regime.

4.1.4.11 Zhukov et al. (1985)

The correlation proposed by Zhukov in 1985 for the turbulent region [298] [304] [313] is as follows. For the turbulent flow, the analysis of the data on friction factor in triangular bundle of the pins spaced by the helical wire of the type "single wire between pins" (see Fig. 56) resulted in the following formula for the infinite pin bundle:

$$f = \left(\frac{0.21}{Re^{0.25}}\right)\left(1 + \frac{124\,Re^{0.06}}{\left({}^{H}/_{D}\right)^{1.65}}\right)\left(1.78 + 1.485\left({}^{P}/_{D} - 1\right)\right)\left({}^{P}/_{D} - 1\right) \tag{480}$$

It is valid for $1.0 \leq {}^{P}/_{D} \leq 1.5$, $10^4 \leq Re \leq 2 \cdot 10^5$ and $8.0 \leq {}^{H}/_{D} \leq 50$.

This formula is simple in structure, with passing on to Zhukov's Eq. (425) for smooth pins in case of their dense packing. It is in agreement with the experimental data of Rehme K. [314] and Chiu C., Todreas N. [315] with an accuracy of $\pm$ 15%.

To predict friction factor exactly, but in the lesser range for parameter ${}^{H}/_{D}$, the following formula is recommended:

$$\frac{f_p}{f} = 1 + g\left(\frac{H}{D}\right)\left(\frac{P}{D} - 1\right) Re^{0.038} \tag{481}$$

where f is defined by the Zhukov's correlation (425) for smooth pin bundles and

$$g\left(\frac{H}{D}\right) = 30.3956 - 4.5911\left(\frac{H}{D}\right) + 0.24308\left(\frac{H}{D}\right)^2 - 0.0042955\left(\frac{H}{D}\right)^3 \tag{482}$$

It is valid for $1.0 \leq {}^{P}/_{D} \leq 1.5$, $6 \cdot 10^3 \leq Re \leq 2 \cdot 10^5$ and $8.0 \leq {}^{H}/_{D} \leq 25$.

This correlation agrees with the experimental data of K. Rehme. and C. Chiu, N. Todreas in the indicated ranges of parameter change with the accuracy of $\pm 10\%$ and confirms the formulas recommended by authors Chiu, Todreas [315] and Novendstern [316].

4.1.4.12 Cheng-Todreas (1986)

In 1986, S.K. Cheng and N.E. Todreas [317] proposed correlations for the single-phase friction factor in a wire-wrapped rod bundle. In addition to a simplified correlation for the global friction factor of the bundle, a detailed correlation was also proposed, in which the friction coefficient varies within the bundle. These correlations were established over the following domain: $50 \leq Re \leq 1 \cdot 10^6$, $8 \leq \frac{H}{d_m} \leq 50$ (simplified correlation), $4 \leq \frac{H}{d_m} \leq 52$ (detailed correlation), $1.025 \leq \frac{P}{D} \leq 1.42$ (simplified), $1 \leq \frac{P}{D} \leq 1.42$ (detailed), $19 \leq N_{rod} \leq 217$, where P: rod pitch, D: rod diameter, d_m: rod diameter + wire diameter, Re: Reynolds number using mean bundle average value, H: wire pitch, N_{rod}: rod number.

Both correlations take the form:

$$f = \frac{C_{fL}}{Re} \qquad \text{for } Re \leq Re_L \tag{483}$$

$$f = \frac{C_{fL}}{Re}(1-\psi)^{\frac{1}{3}} + \frac{C_{fT}}{Re^{0.18}}\psi^{\frac{1}{3}} \qquad \text{for } Re_L \leq Re \leq Re_T \tag{484}$$

$$f = \frac{C_{fT}}{Re^{0.18}} \qquad \text{for } Re \geq Re_T \tag{485}$$

with $Re_L = 300 \cdot 10^{1.7\left(\frac{P}{D}-1\right)}$, $Re_T = 10^4 \cdot 10^{0.7\left(\frac{P}{D}-1\right)}$, and $\psi = \frac{\log\left(\frac{Re}{Re_L}\right)}{\log\left(\frac{Re_T}{Re_L}\right)}$.

For the simplified correlation:

$$C_{fL} = \left[-974.6 + 1612.0\,\frac{P}{D} - 598.5\left(\frac{P}{D}\right)^2\right]\left(\frac{H}{D}\right)^{0.06-0.085\frac{P}{D}} \tag{486}$$

$$C_{fT} = \left[0.8063 - 0.9022\,log\,\frac{H}{D} + 0.3526\left(log\,\frac{H}{D}\right)^2\right]\left(\frac{P}{D}\right)^{9.7}\left(\frac{H}{D}\right)^{1.78-2.0\frac{P}{D}} \tag{487}$$

For the detailed correlation, C_{fL} and C_{fT} take different values according to the geometric type of the liquid subchannel under consideration: triangular (between three pins), edge (between two pins and a wall), or corner (between one pin and two walls). Its complete expression is detailed in [304].

The root mean square (RMS) error of these correlations based on a database of 79 bundles is 7% (detailed) / 7.6% (simplified) for turbulent flows and 12.2% (detailed) / 13.6% (simplified) for laminar flows.

Most bundle pressure drop correlations provide a correlation for the overall bundle pressure drop. The level of description of the simplified correlation is well-suited to system-scale modelling. However, subchannel codes require correlations for the pressure drop in each of the three subchannel types encountered in SFR rod bundles (central, corner or edge subchannels): otherwise, a correlation for the flow split between subchannel types should be provided. The detailed Cheng-Todreas correlation is one of the few correlations which provide a per-type subchannel pressure drop estimate: moreover, its performance over a wide range of experimental data is considered best-in-class [304].

The per subchannel type pressure drop estimate provided by the detailed Cheng-Todreas correlation is adapted in subchannel codes but cannot be used directly in system codes. The simplified Cheng-Todreas correlation uses the same data reduction techniques used to establish the detailed Cheng-Todreas correlation to provide an estimate for the overall bundle pressure drop, in order to be more readily usable in system codes. Contrary to its detailed counterpart, the simplified Cheng-Todreas correlation does not exhibit best-in-class performance over the experimental dataset analysed in [304].

4.1.4.13 Zhukov et al. (1986)

A.V. Zhukov et al. (1986) friction factor correlation is as follows [299]:

For laminar region, $1.125 \le {}^{P}/_{D} \le 1.417$, $10^2 \le Re \le 2\cdot10^3$, $8.3 \le {}^{H}/_{D} \le 50$.

$$f = f_L = \left({}^{64}/_{Re}\right)\left(0.407 + 2\left({}^{P}/_{D} - 1\right)^{0.5}\right)\left(1 + \frac{17\left({}^{P}/_{D} - 1\right)}{{}^{H}/_{D}}\right) \tag{488}$$

For turbulent region $Re \ge 6\cdot10^3$

$$f = f_T = \left({}^{0.21}/_{Re^{0.25}}\right)\left(1 + \left({}^{P}/_{D} - 1\right)^{0.32}\right)\left(1 + M\left({}^{P}/_{D} - 1\right)Re^{0.038}\right) \tag{489}$$

where

$$M = 30.3956 - 4.5911\left({}^{H}/_{D}\right) + 0.24308\left({}^{H}/_{D}\right)^2 - 0.0042955\left({}^{H}/_{D}\right)^3 \tag{490}$$

P is rod pitch, D is rod diameter and H is wire pitch. For transition region, $2\cdot10^3 \le Re \le 6\cdot10^3$

$$f = f_{Tr} = f_L \varepsilon + f_T(1-\varepsilon) \tag{491}$$

where

$$\varepsilon = 0.5\left\{1 - \tanh\left[0.8\left({}^{Re}/_{1450} - 1\right)\right]\right\} \tag{492}$$

Subscripts: L: laminar flow regime, T: turbulent flow regime, Tr: transition flow regime.

4.1.4.14 No-Kazimi (1987)

H.C. No and M.S. Kazimi (1987) [46], when estimating total pressure drop, considered it as a sum of two terms: *Transverse wall friction force* and *Axial wall friction force*. Four multiphase regimes are distinguished:

1. Single-phase: liquid region
2. Two-phase: Pre-dryout region $0 \leq \alpha \leq 0.957$
3. Two-phase: Post-dryout region $0.957 \leq \alpha \leq 1$
4. Single-phase: vapour region

Here only the single-phase liquid regime is presented. For the multi-phase or vapour regimes the original reference gives further information [46].

The following correlations are devoted to the single-phase liquid region. The correlation for the axial wall friction force per unit volume for single-phase in the liquid region is:

For $Re_l \leq 400$

$$f_{f1} = f_L = \frac{32}{\sqrt{H}}\left(\frac{D}{P}\right)^{1.5}\frac{1}{Re_l} \tag{493}$$

For $Re_l \geq 2600$

$$f_{l1} = f_T = \frac{0.316\ \mathrm{M}}{{Re_l}^{0.25}} \tag{494}$$

where

$$M = \left(\frac{1.034}{\left(\frac{P}{D}\right)^{0.124}} + \frac{29.7\left(\frac{P}{D}\right)^{6.9}{Re_l}^{0.086}}{\left(\frac{H}{D}\right)^{2.239}}\right)^{0.885} \tag{495}$$

P is rod pitch, D is rod diameter and H is wire pitch.

For $400 \leq Re_l \leq 2600$

$$f_{l1} = f_{Tr} = f_T\sqrt{\Psi} + f_L\sqrt{1-\Psi} \tag{496}$$

where

$$\Psi = \frac{(Re_1 - 400)}{2200} \tag{497}$$

The correlation for the transverse wall friction force per unit volume for single-phase in liquid region is:

For $Re_l \leq 202.5$

$$f_{l1} = f_L = \frac{180}{Re_l} \tag{498}$$

For $Re_l \geq 202.5$

$$f_{l1} = f_T = \frac{1.92}{{Re_l}^{0.145}} \tag{499}$$

where

$$Re_l = \frac{\rho_l \left|v_{l,max}\right| D_v}{\mu_l} \tag{500}$$

and D_v is the transverse hydraulic diameter, $v_{l,max}$ is the maximum liquid velocity, and μ_l is the dynamic viscosity.

Subscripts: f: friction, L: laminar flow regime, T: turbulent flow regime, Tr: denotes transition flow regime, l: denote liquid phase.

4.1.4.15 Kirillov et al. (1990)

P.L. Kirillov et al. (1990) [304] [313] proposed a friction factor correlation as follows:

For laminar region, $Re \leq 400$

$$f = f_L = \left(\frac{64}{Re}\right)\left(0.407 + 2\left(\frac{P}{D} - 1\right)^{0.5}\right)\left(1 + \frac{17\left(\frac{P}{D} - 1\right)}{\frac{H}{D}}\right) \tag{501}$$

For turbulent region, $Re \geq 5 \cdot 10^3$

$$f = f_T = \left(\frac{0.21}{Re^{0.25}}\right)\left(1 + \left(\frac{P}{D} - 1\right)^{0.32}\right)\left(1 + 600\left(\frac{D}{H}\right)^2\left(\frac{P}{D} - 1\right)\right) \tag{502}$$

For transition region, $400 \leq Re \leq 5 \cdot 10^3$

$$f = f_{Tr} = f_L\,(1 - \psi)^{0.5} + f_T\,\psi^{0.5} \tag{503}$$

where $\psi = \frac{(Re - 400)}{4600}$, P is rod pitch, D is rod diameter and H is wire pitch.

Subscripts: L: laminar flow regime, T: turbulent flow regime, Tr: transition flow regime.

In 2006 V. Sobolev used this same correlation for the friction factor calculation in the XT-ADS project for the wire-wrapped fuel bundle [318] as follows:

$$f = \left(\frac{0.210}{Re^{0.25}}\left(1 + \left(\frac{P_t}{D} - 1\right)^{0.32}\right)\right)\left(1 + 600\left(\frac{D}{H}\right)^2\left(\frac{P_t}{D} - 1\right)\right) \tag{504}$$

where D is the rod diameter, H the wire lead length (pitch), $P_t = D + 1.0444\,D_w$ is the rod pitch for wire-wrap configuration [305], and D_w is the wire (spacer) diameter.

4.1.4.16 Summary of friction factor correlations for single-phase flow in wire-wrapped bundle

Table 35 presents the list of all friction factor correlations collected for single-phase flow in wire-wrapped bundle.

TABLE 35. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN WIRE-WRAPPED ROD BUNDLES

Pontier-Combe (1968) [301]	$f = \Omega_0 e^r$ $\Omega_0 = 0.12 Re^{-0.16}$ for $\varepsilon \sim 1.6 \cdot 10^{-4}$ mm $\Omega_0 = \left[-2log\left[\frac{\varepsilon}{3.7D_h} + \left(\frac{6.81}{Re}\right)^{0.9}\right]\right]^{-2}$ for other ε $r = \left(1 + 4.6\left(\frac{P}{D} - 1\right)\right)\tan\alpha$ $tan\,\alpha = \pi\frac{d}{H}$ $1 \cdot 10^4 \le Re \le 1 \cdot 10^5$, $15.7 \le \frac{H}{D} \le \infty$, $1.1 \le \frac{d_m}{D} \le 1.4$, $37 \le N_{rod} \le 331$, $0 \le tan\,\alpha \le 0.2$, $1.3 \cdot 10^{-4} \le \varepsilon \le 2 \cdot 10^{-4}$
Sangster (1968) [299] [302]	for $1.135 \le \frac{P}{D} \le 1.195$: $f = 0.974\left(\frac{P}{D}\right)^{0.8}\frac{4.76}{\left(\frac{H}{D}\right)^{0.47}}f_p$ for $1.195 \le \frac{P}{D} \le 1.255$: $f = 1.048\left(\frac{P}{D}\right)^{0.37}\frac{4.76}{\left(\frac{H}{D}\right)^{0.47}}f_p$ for $\frac{P}{D} \ge 1.255$: $f = 1.138\frac{4.76}{\left(\frac{H}{D}\right)^{0.47}}f_p$ $10 \le \frac{H}{D} \le 40$, $4 \cdot 10^3 \le Re \le 10^5$

TABLE 35. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN WIRE-WRAPPED ROD BUNDLES

Novendstern (1972) [3] [155] [303] [304] [305]	$f = f_1 {X_1}^2 \left(\frac{D_{eb}}{D_{e1}}\right) = \mathrm{M}\, f_s {X_1}^2 \left(\frac{D_{eb}}{D_{e1}}\right)$ $M = \left(\frac{1.034}{\left(\frac{P}{D}\right)^{0.124}} + \frac{29.7\left(\frac{P}{D}\right)^{6.94} Re_1^{0.086}}{\left(\frac{H}{D}\right)^{2.239}}\right)^{0.885}$ $Re_1 = \frac{\rho v_1 D_{e1}}{\mu} = X_1 Re \frac{D_{e1}}{D_{eb}}$ $X_1 = \frac{A_b}{\left(N_1 A_1 + N_2 A_2 \left(\frac{D_{e2}}{D_{e1}}\right)^{0.714} + N_3 A_3 \left(\frac{D_{e3}}{D_{e1}}\right)^{0.714}\right)}$ $A_b = N_1 A_1 + N_2 A_2 + N_3 A_3$ $600 \leq Re \leq 2 \cdot 10^5$, $5 \cdot 10^{-3} \leq D \leq 12 \cdot 10^{-3}$, $19 \leq N_{rod} \leq 217$, $1.06 \leq \frac{P}{D} \leq 1.42$, $8 \leq \frac{H}{D} \leq 90$
Rehme (1973) [306]	$f = \left[\frac{64\sqrt{F}}{Re} + \frac{0.0816 F^{0.9335}}{Re^{0.133}}\right] \frac{P_b}{P_{tot}}$ $F = \left(\frac{v_{eff}}{v_m}\right)^2 = \left(\frac{P}{D}\right)^{0.5} + \left[7.6 \frac{d_m}{H} \left(\frac{P}{D}\right)^2\right]^{2.16}$ $1 \cdot 10^3 \leq Re \leq 3 \cdot 10^5$, $8 \leq \frac{H}{d_m} \leq 50$, $1.1 \leq \frac{P}{D} \leq 1.42$, $7 \leq N_{rod} \leq 217$

TABLE 35. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN WIRE-WRAPPED ROD BUNDLES

Subbotin et al. (1975) [160]	1.For standard wire wrapping “single wire between pins” (see Fig. 56) $f_p = 0.117\left(\frac{\varepsilon}{d_h} + \frac{68}{Re}\right)^{0.25}$ transition from laminar to turbulent: $f'_p = f_{p\ lam}\alpha + f_p\,(1-\alpha)$ $\alpha = 0.5\left\{1 - \tanh\left[0.8\left({}^{Re}/_{1450} - 1\right)\right]\right\}$ 2. For bi-directional wire wrapping “two wires between pins” (Fig. 57): a. for laminar flow: $\frac{f_p}{f} \approx 1 + \frac{1.8}{\frac{H}{D}}$ Valid for $1.13 \le \frac{P}{D} \le 1.15$ b. for turbulent flow: $\frac{f_p}{f} \approx 1 + \frac{600\left(\frac{P}{D} - 1\right)}{\left(\frac{H}{D}\right)^2}$ $10^4 \le Re \le 20 \cdot 10^4$, $1.05 \le \frac{P}{D} \le 1.25$, $\frac{H}{D} \ge 5$, $2 \le n \le 4$
Engel et al. (1979) [308]	for $Re \le Re_L$: $f = \frac{C_{fL}}{Re}$ for $Re_L \le Re \le Re_T$: $f = \frac{C_{fL}}{Re}\sqrt{1-\psi} + \frac{C_{fT}}{Re^{0.25}}\sqrt{\psi}$ for $Re \ge Re_T$: $f = \frac{C_{fT}}{Re^{0.25}}$ where $Re_L = 400$, $Re_T = 5000$, $\psi = \frac{Re-400}{4600}$ $C_{fT} = 0.55$, $19 \le N_{rod} \le 61$ $C_{fL} = 110$ for $1.067 \le \frac{P}{D} \le 1.082$, $C_{fL} = \frac{320}{\sqrt{H}}\left(\frac{P}{D}\right)^{1.5}$ for $\frac{P}{D} \sim 1.2$.

TABLE 35. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN WIRE-WRAPPED ROD BUNDLES

Markley-Engel (1979) [299] [309]	for $Re \leq 400$: $f = \frac{110}{Re}$ for $400 \leq Re \leq 5000$: $f = \frac{110}{Re}\sqrt{1-\psi} + \frac{0.48}{Re^{0.25}}\sqrt{\psi}$ for $Re \geq 5000$ $f = \frac{0.48}{Re^{0.25}}$ $\psi = \frac{Re - 400}{4600}$ $1.067 \leq \frac{P}{D} \leq 1.32$, $\frac{H}{D} \sim 8$, $40 \leq Re \leq 10^5$
Engel et al. (1979) [299] [304] [308] [310]	$f = \frac{32}{Re\sqrt{H}}\left(\frac{P}{D}\right)^{1.5}$ $Re \leq 400$, $4 \leq \frac{H}{D} \leq 52$, $1.067 \leq \frac{P}{D} \leq 1.25$, $61 \leq N_{rod} \leq 217$
Roidt et al. (1980) [299] [311]	$f = \frac{2.284}{Re^{0.4183}}$ $8 \leq \frac{H}{D} \leq 52$, $1.08 \leq \frac{P}{D} \leq 1.24$, $4.3 \cdot 10^3 \leq Re \leq 7.3 \cdot 10^4$, for peripheral rods of a bundle
Baxi-Dalle Donne (1981) [312]	for $Re \leq 400$: $f_L = \frac{\left(\frac{T_w}{T_B}\right)\left(\frac{320}{\sqrt{H}}\right)\left(P/D\right)^{1.5}}{Re}$ for $Re \geq 5 \cdot 10^3$: $f_T = M f_s$ $M = \left(\frac{1.034}{\left(\frac{P}{D}\right)^{0.124}} + \frac{29.6\left(\frac{P}{D}\right)^{6.94} Re^{0.086}}{\left(\frac{H}{D}\right)^{2.239}}\right)^{0.885}$ f_s : friction factor for smooth pipe for $400 \leq Re \leq 5 \cdot 10^3$: $f = f_L\,(1-\psi)^{0.5} + f_T\,\psi^{0.5}$ $\psi = \frac{(Re - 400)}{4600}$

TABLE 35. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN WIRE-WRAPPED ROD BUNDLES

Zhukov et al. (1985) [298] [304] [313]	1. $f = \left(\frac{0.21}{Re^{0.25}}\right)\left(1 + \frac{124\,Re^{0.06}}{\left(H/D\right)^{1.65}}\right)\left(1.78 + 1.485\left(\frac{P}{D} - 1\right)\right)\left(\frac{P}{D} - 1\right)$ $1.0 \le \frac{P}{D} \le 1.5,\ 10^4 \le Re \le 2 \cdot 10^5,\ 8.0 \le \frac{H}{D} \le 50$ 2. $\frac{f_p}{f} = 1 + g\left(\frac{H}{D}\right)\left(\frac{P}{D} - 1\right) Re^{0.038}$ f is defined by Eq. (425) $g\left(\frac{H}{D}\right) = 30.3956 - 4.5911\left(\frac{H}{D}\right) + 0.24308\left(\frac{H}{D}\right)^2 - 0.0042955\left(\frac{H}{D}\right)^3$ $1.0 \le \frac{P}{D} \le 1.5,\ 6 \cdot 10^3 \le Re \le 2 \cdot 10^5$ and $8.0 \le \frac{H}{D} \le 25.$
Cheng-Todreas (1986) [317]	for $Re \le Re_L$: $f = \frac{C_{fL}}{Re}$ for $Re_L \le Re \le Re_T$: $f = \frac{C_{fL}}{Re}(1-\psi)^{\frac{1}{3}} + \frac{C_{fT}}{Re^{0.18}}\psi^{\frac{1}{3}}$ for $Re \ge Re_T$: $f = \frac{C_{fT}}{Re^{0.18}}$ $Re_L = 300 \cdot 10^{1.7\left(\frac{P}{D}-1\right)},\ Re_T = 10^4 \cdot 10^{0.7\left(\frac{P}{D}-1\right)}$ $\psi = \frac{\log\left(\frac{Re}{Re_L}\right)}{\log\left(\frac{Re_T}{Re_L}\right)}$ $C_{fL} = \left[-974.6 + 1612.0\,\frac{P}{D} - 598.5\left(\frac{P}{D}\right)^2\right]\left(\frac{H}{D}\right)^{0.06-0.085\frac{P}{D}}$ $C_{fT} = \left[0.8063 - 0.9022\,log\,\frac{H}{D} + 0.3526\left(log\,\frac{H}{D}\right)^2\right]\left(\frac{P}{D}\right)^{9.7}\left(\frac{H}{D}\right)^{1.78-2.0\frac{P}{D}}$ $50 \le Re \le 1 \cdot 10^6,\ 8 \le \frac{H}{d_m} \le 50,\ 1.025 \le \frac{P}{D} \le 1.42$

TABLE 35. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN WIRE-WRAPPED ROD BUNDLES

Zhukov et al. (1986) [299]	for laminar region, $1.125 \leq P/D \leq 1.417$, $10^2 \leq Re \leq 2 \cdot 10^3$ and $8.3 \leq H/D \leq 50$: $f = f_L = \left(\frac{64}{Re}\right)\left(0.407 + 2\left(\frac{P}{D} - 1\right)^{0.5}\right)\left(1 + \frac{17\left(\frac{P}{D} - 1\right)}{\frac{H}{D}}\right)$ for turbulent region $Re \geq 6 \cdot 10^3$: $f = f_T = \left(\frac{0.21}{Re^{0.25}}\right)\left(1 + \left(\frac{P}{D} - 1\right)^{0.32}\right)\left(1 + M\left(\frac{P}{D} - 1\right)Re^{0.038}\right)$ $M = 30.3956 - 4.5911\left(\frac{H}{D}\right) + 0.24308\left(\frac{H}{D}\right)^2 - 0.0042955\left(\frac{H}{D}\right)^3$ for transition region, $2 \cdot 10^3 \leq Re \leq 6 \cdot 10^3$: $f = f_{Tr} = f_L\varepsilon + f_T(1 - \varepsilon)$ $\varepsilon = 0.5\left\{1 - \tanh\left[0.8\left(\frac{Re}{1450} - 1\right)\right]\right\}$

TABLE 35. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN WIRE-WRAPPED ROD BUNDLES

No-Kazimi (1987) [46]	axial wall friction force: for $Re_l \leq 400$: $f_{f1} = f_L = \frac{32}{\sqrt{H}} \left(\frac{D}{P}\right)^{1.5} \frac{1}{Re_l}$ for $Re_l \geq 2600$: $f_{l1} = f_T = \frac{0.316\ \mathrm{M}}{{Re_l}^{0.25}}$ $M = \left(\frac{1.034}{\left(\frac{P}{D}\right)^{0.124}} + \frac{29.7\ \left(\frac{P}{D}\right)^{6.9} Re_l^{0.086}}{\left(\frac{H}{D}\right)^{2.239}}\right)^{0.885}$ for $400 \leq Re_l \leq 2600$: $f_{l1} = f_{Tr} = f_T \sqrt{\Psi} + f_L \sqrt{1 - \Psi}$ $\Psi = \frac{(Re_1 - 400)}{2200}$ transverse wall friction: for $Re_l \leq 202.5$: $f_{l1} = f_L = \frac{180}{Re_l}$ for $Re_l \geq 202.5$: $f_{l1} = f_T = \frac{1.92}{{Re_l}^{0.145}}$ $Re_l = \frac{\rho_l \left\|v_{l,max}\right\| D_v}{\mu_l}$
Kirillov et al. (1990) [304] [313]	for laminar region, $Re \leq 400$: $f = f_L = \left(\frac{64}{Re}\right)\left(0.407 + 2\ \left(\frac{P}{D} - 1\right)^{0.5}\right)\left(1 + \frac{17\ \left(\frac{P}{D} - 1\right)}{\frac{H}{D}}\right)$ for turbulent region, $Re \geq 5 \cdot 10^3$: $f = f_T = \frac{0.21}{Re^{0.25}} \left(1 + \left(\frac{P}{D} - 1\right)^{0.32}\right)\left(1 + 600 \left(\frac{D}{H}\right)^2 \left(\frac{P}{D} - 1\right)\right)$ for transition region, $400 \leq Re \leq 5 \cdot 10^3$: $f = f_{Tr} = f_L\ (1 - \psi)^{0.5} + f_T\ \psi^{0.5}$ $\psi = \frac{(Re - 400)}{4600}$

4.1.5 Flow in grid-spaced rod bundles

The pressure losses across grid spacers can be calculated using Rehme's formulation as follows [227] [296] [319]:

$$\Delta p_{grid\,spacer} = C_v\,\varepsilon^2 \frac{1}{2}\,\rho\,v^2 = K\frac{1}{2}\,\rho\,v^2 \tag{505}$$

where v is the average coolant velocity in the rod bundle, $K = C_v\varepsilon^2$, and C_v is the modified loss coefficient. Rehme assumed that the relative plugging $\varepsilon = \frac{A_v}{A_s}$ (where A_v is the projected grid cross-section and A_s is the undisturbed flow section) constitutes the main factor influencing the pressure drop.

The geometry of the spacer grid is shown in Fig. 58 below.

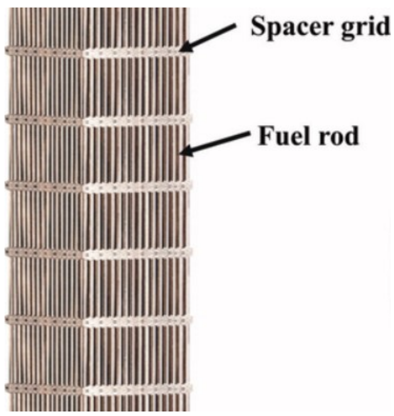


FIG. 58. Grid-spaced rod bundle

4.1.5.1 Voj et al. (1971)

P. Voj et al. in 1971 developed a correlation based on sodium flow experiments in a SNR-300 reactor like grid-spaced bundle [320]:

$$K = C_v\varepsilon^2 = \frac{1-\varepsilon}{\varepsilon}\left(2.12 + \frac{10^4\varepsilon^2}{Re}\right) \tag{506}$$

where ε: blockage factor of the grid spacer (ratio of areas) calculated as: $\varepsilon = \frac{A_{grid\,spacer}}{A_{flow}}$, $A_{grid\,spacer}$ is the cross-section area of the grid spacer in the flow path (m^2) and A_{flow} is the unobstructed coolant flow area (m^2) (parameter ε can be based on the unit cell, per pin or per subassembly).

4.1.5.2 Rehme-Cigarini-Dalle Donne (1973)

M. Cigarini and M. Dalle Donne recommended the following modified loss coefficient [306] [319] [321]:

$$C_v = 3.5 + \frac{73.14}{Re^{0.264}} + \frac{2.79\,\cdot 10^{10}}{Re^{2.79}} \tag{507}$$

having a maximum value:

$$C_v = \frac{2}{\varepsilon^2} \tag{508}$$

Finally, C_v can be written as:

$$C_v = \min\left[3.5 + \frac{73.14}{Re^{0.264}} + \frac{2.79 \cdot 10^{10}}{Re^{2.79}}, \frac{2}{\varepsilon^2}\right] \tag{509}$$

where ε: is the blockage factor of the grid spacer, which ranges from 0.15 to 0.5 for typical grid spacer designs. It is calculated as:

$$\varepsilon = \frac{A_{grid\ spacer}}{A_{flow}} \tag{510}$$

$A_{grid\ spacer}$ is the cross-section area of the grid spacer in the flow path (m^2), and A_{flow} is the unobstructed coolant flow area (m^2). The blockage factor ε is sensitive to the particular design characteristics of the sub-assemblies and the grid spacers. Typical numerical examples of ε can be found in [306] ranging from $\varepsilon \sim 0.15$ for transversally connected tube spacers to $\varepsilon \sim 0.44$ for honeycomb-type grid spacers. Under ideal conditions, ε should be determined for each particular grid spacer design. Then,

$$\Delta p_{grid\ spacers} = N_{spacers}\, \Delta p_{grid\ spacer} \tag{511}$$

where $N_{spacers}$ is the number of grid spacers in a fuel assembly.

4.1.5.3 Savatteri et al. (1986)

In 1986 C. Savatteri et al. developed a correlation based on sodium flow experiments in a 12-pin grid-spaced bundle [227] [322]. They proposed the following correlation:

$$K = C_v \varepsilon^2 = \left(9 + \frac{3.8}{(10^{-4}Re)^{0.25}} + \frac{0.82}{(10^{-4}Re)^2}\right)\varepsilon^2 \tag{512}$$

where ε: blockage factor of the grid spacer.

4.1.5.4 Cevolani (1995)

In 1995 S. Cevolani proposed the following correlation for triangular bundles and spacers with rounded leading edges [227] [323]:

$$K = C_v \varepsilon^2 = \min\left[\varepsilon^2\, exp\left(7.69 - 0.9421\, ln(Re) + 0.0379\, ln^2(Re)\right), 2\right] \tag{513}$$

where ε is the blockage factor of the grid spacer.

4.1.5.5 Epiney et al. (2010)

In order to improve the prediction of the spacer loss for sharp-edged spacers, a new correlation was proposed by A. Epiney et al. in 2010 [324]:

$$K = C_v \varepsilon^{0.2} = \left(1.104 + \frac{791.8}{Re^{0.748}} + \frac{3.348 \cdot 10^9}{Re^{5.652}}\right)\varepsilon^{0.2} \tag{514}$$

where ε is the blockage factor of the grid spacer. The Reynolds number range covered by the sharp-edge spacer experiments, taken into account when deriving this correlation, is $1 \cdot 10^3 \leq Re \leq 5 \cdot 10^4$.

4.1.5.6 Summary of friction factor correlations for single-phase flow in grid-spaced bundle

Table 36 presents the list of friction factor correlations collected for single-phase flow in grid-spaced bundle.

TABLE 36. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR SINGLE-PHASE FLOW IN GRID-SPACED BUNDLES

Voj et al. (1971) [320]	$K = C_v \varepsilon^2 = \frac{1-\varepsilon}{\varepsilon}\left(2.12 + \frac{10^4 \varepsilon^2}{Re}\right)$ $\varepsilon = \frac{A_{grid\ spacer}}{A_{flow}}$
Rehme-Cigarini-Dalle Donne (1973) [306] [319] [321]	$C_v = \min\left[3.5 + \frac{73.14}{Re^{0.264}} + \frac{2.79 \cdot 10^{10}}{Re^{2.79}}, \frac{2}{\varepsilon^2}\right]$
Savatteri et al. (1986) [227] [322]	$K = C_v \varepsilon^2 = \left(9 + \frac{3.8}{(10^{-4} Re)^{0.25}} + \frac{0.82}{(10^{-4} Re)^2}\right) \varepsilon^2$
Cevolani (1995) [227] [323]	$K = C_v \varepsilon^2 = \min\left[\varepsilon^2 \, exp\left(7.69 - 0.9421 \, ln(Re) + 0.0379 \, ln^2(Re)\right), 2\right]$
Epiney et al. (2010) [324]	$K = C_v \varepsilon^{0.2} = \left(1.104 + \frac{791.8}{Re^{0.748}} + \frac{3.348 \cdot 10^9}{Re^{5.652}}\right) \varepsilon^{0.2}$ $1 \cdot 10^3 \leq Re \leq 5 \cdot 10^4$

4.1.6 Transverse flow in a rod bundle

The representation of transverse flow in a rod bundle geometry is shown in Fig. 59.

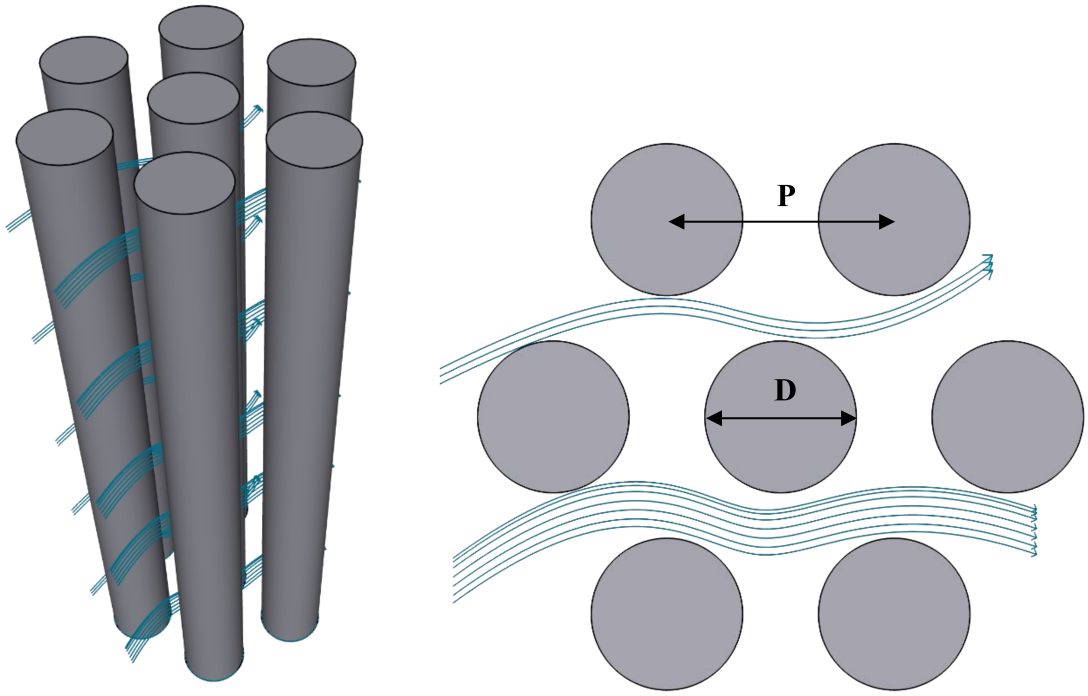


FIG. 59. Transverse flow in a rod bundle

4.1.6.1 Subbotin et al. (1975)

In 1975 V.I. Subbotin et al. proposed the following friction factor for the transverse flow in triangular bundle having an accuracy of 20% [160]:

$$f_r = \left(\frac{P}{D} - 1\right)^{-0.125} K_p \, 10^{3.14Re^{-0.22} - 0.42} \tag{515}$$

where parameter K_p accounts for an influence of the– I -entries of wire wrap with the wire pitch H, $x = {}^{P}/_{D}$ is relative pins pitch in the array.

$$K_p = \left\{ I - \frac{I}{\pi} \sqrt{\left[\left(2 - \frac{1}{x}\right)^2 - 1\right]\left[1 + \left(\frac{\pi x D}{H}\right)^2\right]} \right\}^{-2} \tag{516}$$

4.1.7 Inclined flow in a rod bundle

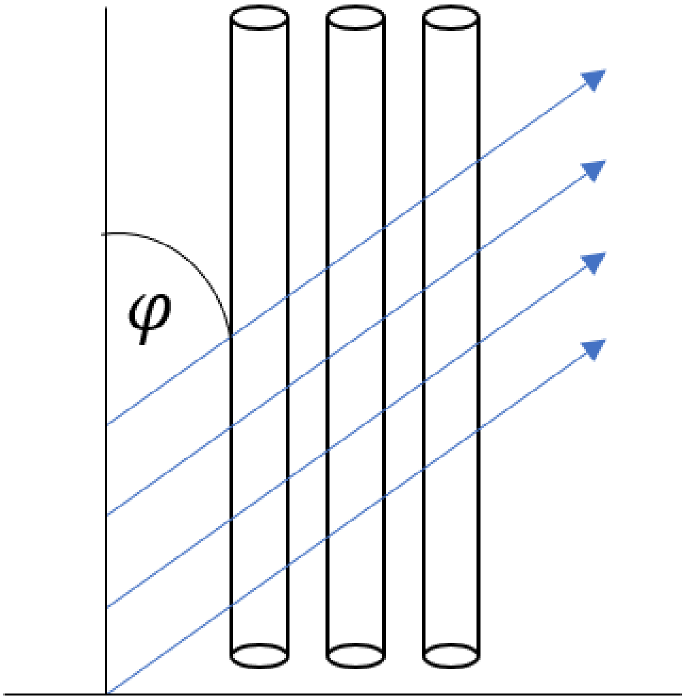


FIG. 60. Inclined flow in a rod bundle

4.1.7.1 Subbotin et al. (1975)

In 1975 V.I. Subbotin et al. proposed the following friction factor for inclined flow in a triangular bundle which depends on the slope as follows [160]:

$$\frac{f_\varphi}{f_{90º}} = \sin^2\varphi + \frac{f_z}{f_r}\cos^2\varphi \tag{517}$$

where f_z is the friction factor in longitudinal flow:

$$f_z = \frac{\Delta p 2 d_h}{L\rho\overline{w}^2} \tag{518}$$

and $f_r = f_{90º}$ is the friction factor in transverse flow

$$f_r = \frac{\Delta p 2 d_h}{L\rho\overline{w}^2} \tag{519}$$

where L is the length of the rod bundle; $\overline{w} = {}^{\overline{v}}/_{\varepsilon}$ is the liquid velocity averaged over the bundle; $\overline{v}$ is the free flow velocity on the bundle; and ε is porosity.

4.2 TWO PHASE FRICTION FACTORS AND PRESSURE DROP CORRELATIONS

It has been experimentally observed in two-phase flow that, for a given mass flow, the pressure drop can be much greater than for a corresponding single-phase flow. In general, the friction factor depends on flow patterns, which are classified as bubbly, slug, plug, churn, annular, wavy and mist flows as show in Fig. 61.

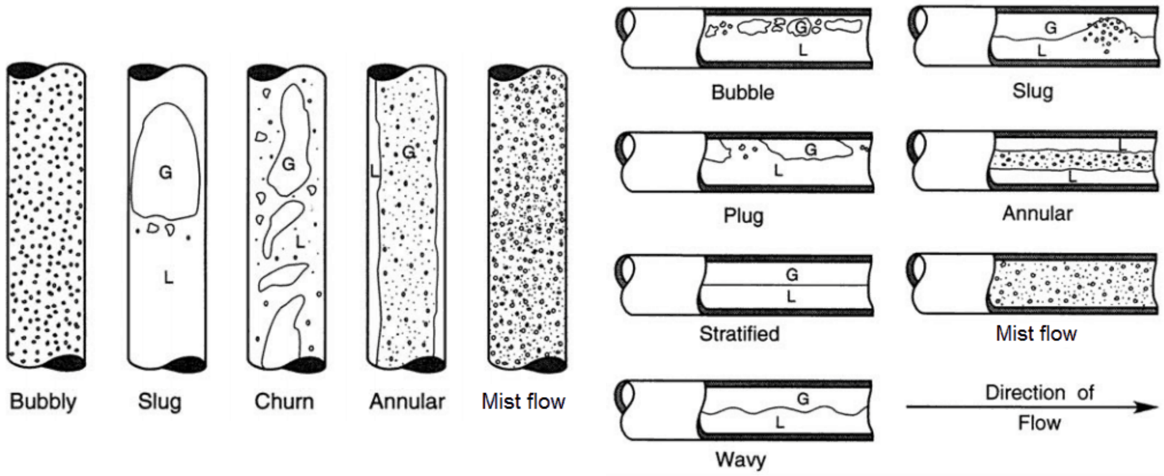


FIG. 61. Two-phase flow patterns in vertical and horizontal pipes (adapted from [325])

In order to correlate two-phase frictional losses, the classical approach is to consider the friction factor for single-phase flow f_l, at the same mass flux as in the two-phase case, and to use a multiplier ϕ_l^2 to account for the two-phase effects [227]. Then the two-phase pressure gradient is expressed as a multiple of the pressure gradient that would occur if the liquid phase flowed alone in the duct:

$$\left(\frac{\Delta p}{\Delta L}\right)_{2\phi} = \phi_l^2 \left(\frac{\Delta p}{\Delta L}\right)_l \tag{520}$$

where

$\left(\frac{\Delta p}{\Delta L}\right)_{2\phi}$ is the negative two-phase pressure gradient

$\left(\frac{\Delta p}{\Delta L}\right)_l$ is the negative pressure gradient for the liquid alone

$$\Delta P_{2\phi} = \phi_l^2 f_l \frac{L}{D_h} \frac{\rho_l V_l^2}{2} \tag{521}$$

where: l is the liquid single-phase, 2ϕ: two-phase, ϕ: two-phase friction multiplication factor, f: friction factor, L: integrated fuel element length, D_h: hydraulic diameter [326]. The two-phase friction multiplication factor ϕ_l can also be expressed as [339]:

$$\phi_l = f(X_{LM}) = \left\{ \frac{\left(\frac{\Delta p}{\Delta L}\right)_{2\phi\, fric}}{\left(\frac{\Delta p}{\Delta L}\right)_{1\phi}} \right\}^{\frac{1}{2}} \tag{522}$$

where p is the pressure, L is the axial coordinate and $fric$ stands for 'friction'.

In the following v represents the void fraction or vapour volume fraction and x represents the vapour quality or vapour mass fraction.

4.2.1 Flow in straight pipes

The methods developed so far can be divided into two groups: homogeneous and separated flow approaches.

Homogeneous models treat two-phase flow as a pseudo single-phase fluid characterized by averaged properties of the liquid and vapour phases, where the two phases are assumed to have the same velocity.

The separated flow model considers the two-phase flow to be artificially separated into two streams, each flowing in separate zones, as in the annular and stratified regimes. In this model the two phases generally have different velocities, but constant for each phase and can interact with each other. Under most conditions, the separated flow model provides a better representation of the pressure drop in a pipe flow [234] [326].

The separated flow approach can be further classified in two categories: the f_l^2, f_g^2 based method and the f_{l0}^2, f_{g0}^2 based method, where f_l^2, f_g^2, f_{l0}^2, and f_{g0}^2 are two-phase friction multipliers.

As for the f_l^2, f_g^2 based method, Lockhart and Martinelli (1949) proposed the concept of two-phase friction multipliers f_l^2, f_g^2 [327]. f_l^2 is defined as the ratio of the two-phase frictional pressure gradient to the frictional pressure gradient, which would exist if the liquid phase is assumed to flow alone. Correspondingly, f_g^2 is defined as the ratio of the two-phase frictional pressure gradient to the frictional pressure gradient, which would exist if the vapour phase is assumed to flow alone [234].

$$\mathrm{f}_l^2 = \frac{\left(\frac{\Delta p}{\Delta \mathrm{L}}\right)_{2\phi}}{\left(\frac{\Delta p}{\Delta \mathrm{L}}\right)_l} \tag{523}$$

$$\mathrm{f}_g^2 = \frac{\left(\frac{\Delta p}{\Delta \mathrm{L}}\right)_{2\phi}}{\left(\frac{\Delta p}{\Delta \mathrm{L}}\right)_g} \tag{524}$$

$$\left(\frac{\Delta p}{\Delta \mathrm{L}}\right)_l = \frac{\left[G_{2\phi}(1-x)\right]^2}{2D\rho_l} f_l \tag{525}$$

$$\left(\frac{\Delta p}{\Delta \mathrm{L}}\right)_g = \frac{\left[G_{2\phi}x\right]^2}{2D\rho_g} f_g \tag{526}$$

where f is calculated with a single-phase friction factor correlation using the single-phase properties and mass flux.

As for the f_{l0}^2, f_{g0}^2 based method, f_{l0}^2 is defined as the ratio of the two-phase frictional pressure gradient to the frictional pressure gradient, which would exist if the total mixture were assumed to be liquid. f_{g0}^2 is defined as the ratio of the two-phase frictional pressure gradient to the frictional pressure gradient, which would exist if the total mixture were assumed to be vapour [234].

$$f_{lo}^2 = \frac{\left(\frac{\Delta p}{\Delta L}\right)_{2\phi}}{\left(\frac{\Delta p}{\Delta L}\right)_{lo}} \tag{527}$$

$$f_{go}^2 = \frac{\left(\frac{\Delta p}{\Delta L}\right)_{2\phi}}{\left(\frac{\Delta p}{\Delta L}\right)_{go}} \tag{528}$$

$$\left(\frac{\Delta p}{\Delta L}\right)_{lo} = \frac{{G_{2\phi}}^2}{2D\rho_l} f_{lo} \tag{529}$$

$$\left(\frac{\Delta p}{\Delta L}\right)_{go} = \frac{{G_{2\phi}}^2}{2D\rho_g} f_{go} \tag{530}$$

where f_{lo} can be calculated with the single-phase friction factor correlations (presented in Section 4.1) using $G_{2\phi}$ and the liquid phase properties, and f_{go} can be calculated with same single-phase correlations using $G_{2\phi}$ and the vapour phase properties.

Concerning the homogeneous model, it determines two-phase frictional pressure drop using the following equation:

$$\left(\frac{\Delta p}{\Delta \mathrm{L}}\right)_{2\phi} = \frac{G_{2\phi}^2}{2D\rho_{2\phi}} f_{2\phi} \tag{531}$$

where $f_{2\phi}$ can be calculated with single-phase friction factor correlations, using $G_{2\phi}$ and the two-phase properties, while $\rho_{2\phi}$ is commonly calculated as:

$$\frac{1}{\rho_{2\phi}} = \frac{x}{\rho_g} + \frac{1-x}{\rho_l} \tag{532}$$

The main difference among homogeneous correlations concerns the estimation of the two-phase viscosity [234]. The homogeneous model generally gives reliable results for the mist flow and bubbly flow regimes, where the velocities of the two phases do not differ greatly. For other flow regimes, it tends to overestimate the void fraction v causing the underestimation of the two-phase density [326].

When assuming a homogeneous flow, both phases have the same velocity and the slip ratio $SR = \frac{u_g}{u_l}$ is one, then the void volume fraction is [326]:

$$v = \frac{x}{x + (1-x)\,{}^{\rho_g}/_{\rho_l}} \tag{533}$$

which is the fraction of the total volume that is occupied by the vapour phase [326]. As the two-phase density is the total mass (vapour and liquid) of fluid divided by the total volume of the fluid, it can be expressed as:

$$\rho_{2\phi} = v\rho_g + (1-v)\rho_l \tag{534}$$

The void fraction is also the fractional area of the pipe cross-section occupied by the vapour which relates the void fraction to the vapour mass fraction, or quality x. By denoting the average vapour velocity by u_g, the mass flux of vapour can be written as [326]:

$$\rho_g u_g = \frac{\dot{m}_g}{A_g} \tag{535}$$

where A_g is the cross-sectional area occupied by the gas phase. Since $x = {}^{\dot{m}_g}/_{\dot{m}}$, it follows that:

$$\frac{x}{\rho_g u_{gv}} = \frac{\dot{m}_g}{\dot{m}}\frac{A_g}{\dot{m}_g} = \frac{A_g}{\dot{m}} \tag{536}$$

As for the liquid phase, it can be written:

$$\frac{1-x}{\rho_l u_l} = \frac{A_l}{\dot{m}} \tag{537}$$

where A_l is the cross-sectional area occupied by the liquid. Combining both previous equations it leads to the void fraction v as follows [326]:

$$\frac{\frac{x}{\rho_g u_{gv}}}{\frac{x}{\rho_g u_g} + \frac{1-x}{\rho_l u_l}} = \frac{\frac{A_g}{\dot{m}}}{\frac{A_g}{\dot{m}} + \frac{A_l}{\dot{m}}} = \frac{A_g}{A_g + A_l} = v \tag{538}$$

Void fraction can be also written as:

$$v = \frac{x}{x + SR(1-x)\,{}^{\rho_g}/_{\rho_l}} \tag{539}$$

where $SR = \frac{u_g}{u_l}$ is the slip ratio. It would be then needed a flow model or empirical correlation to evaluate SR [326].

As for the slip model, assuming a slip between the vapour and liquid phases, the two-phase pressure loss multiplier can be obtained from [227]:

$$\phi = \frac{\rho_l}{\alpha\,\rho_g + (1-\,\alpha)\rho_l} \tag{540}$$

$$\alpha \equiv \frac{1}{1 + \left(\frac{1-x}{x}\right)\,SR\,\frac{\rho_g}{\rho_l}} \tag{541}$$

The slip model becomes the homogeneous model when the slip ratio $SR = \frac{u_g}{u_l}$ is equal to 1.

4.2.1.1 McAdams et al. (1942)

In the early 1942 W. McAdams et al. proposed a very simple homogeneous model to estimate the two-phase dynamic viscosity [234] [238]:

$$\frac{1}{\mu_{2\phi}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_l} \tag{542}$$

It was proposed based on an analogy to the expression for the two-phase flow density.

4.2.1.2 Lockhart-Martinelli (1949)

In 1949 R.W. Lockhart and R.C. Martinelli suggested that the two-phase friction factor multipliers f_l^2 and f_g^2 are functions of the non-dimensional variable X_{LM}. This conclusion was based on experimental data with air, water and other liquids, such as benzene, kerosene, as well as various oils in pipes. They studied the values of f^2 as a function of X_{LM} for four flow types: turbulent liquid–turbulent vapour (tt), turbulent liquid–viscous vapour (tv), viscous liquid–turbulent vapour (vt), viscous liquid–viscous vapour (vv), where the turbulent, viscous and transitional regimes where assumed to exist when $Re \geq 2000$, $Re \leq 1000$, and $1000 \leq Re \leq 2000$, respectively [234].

The variable X_{LM} is a function of the ratio of densities of the liquid and vapour, the ratio of viscosities of the liquid and vapour and tube diameter [327].

$$X_{LM}^2 = \frac{\left(\frac{\Delta p}{\Delta \mathrm{L}}\right)_l}{\left(\frac{\Delta p}{\Delta \mathrm{L}}\right)_g} = \frac{Re_{gp}^m}{Re_{lp}^n}\frac{C_l}{C_g}\left(\frac{W_l}{W_g}\right)^2\frac{\rho_g}{\rho_l} \tag{543}$$

where

$$Re_{gp} = \frac{4W_g}{\pi D_g \mu_g}$$

$$Re_{lp} = \frac{4W_l}{\pi D_l \mu_l}$$

C_k is the constant in Blasius equation for friction factor for the phase k,

W_k is the mass flow rate for phase k,

D_k is relative to the tube diameter and the flow conditions (it is always lower than the pipe diameter),

m and n are equal to 1 for viscous flow regime and are equal to 0.2 for turbulent flow regime.

The relationship between f_l^2 and f_g^2 and X_{LM} was given in graphical form by Lockhart and Martinelli. From the definition of the parameter X_{LM}, the following expression can be deduced:

$$X_{LM} = \left(\frac{1-x}{x}\right)^{\left(\frac{2-n}{2}\right)}\left(\frac{\rho_g}{\rho_l}\right)^{0.5}\left(\frac{\mu_l}{\mu_g}\right)^{\frac{n}{2}} \tag{544}$$

where x is the vapour mass fraction. For the turbulent-turbulent flow Lockhart and Martinelli assumed the value n=0.2, obtaining:

$$X_{LM}^{tt} = \left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\rho_g}{\rho_l}\right)^{0.5}\left(\frac{\mu_l}{\mu_g}\right)^{0.1} \tag{545}$$

Although the exponent n is closer to 0.25 for commercial pipe and tubing, the difference is insignificant compared with the inherent uncertainty associated with the Lockhart-Martinelli correlation. It should be noted that X_{LM}^{tt} is zero, when the vapour quality x equals 1.0 and the two-phase multiplier, f_l^2 becomes infinite.

In their classic paper from 1949, Lockhart and Martinelli presented a single correlation for void fraction v covering all four laminar-turbulent combinations [326]. Their graphical correlation can be represented analytically by the following equation:

$$v = \frac{\mathrm{f}_l - 1}{\mathrm{f}_l} \tag{546}$$

where f_l is calculated for the turbulent-turbulent case. Substituting the above expression for v in the expression for the two-phase density, it leads to:

$$\rho_{2\phi} = \frac{\rho_l + (\mathrm{f}_l - 1)\rho_v}{\mathrm{f}_l} \tag{547}$$

The Lockhart–Martinelli methodology is the basis of the two-phase model based on the functions f_l^2 and f_g^2. And the correlation proposed for the turbulent-turbulent flow is considered as a reference. The model which provides a correlation for the two-phase pressure drop wall friction multiplier may be considered as a *best-estimate mean value* [328].

The Lockhart-Martinelli correlation, used in tubes, provides quite good results for bubbly flow, but doesn't work as well for churn flow (typical void fraction overestimation of 20% to 30%).

4.2.1.3 Lottes-Flinn (1956)

A simple correlation for ϕ_l^2 was proposed by P.A. Lottes and W.A. Flinn in 1956 [329]. According to the authors, for $(1 - x)$ close to one, constant heat flux and constant slip ratio $SR = \frac{u_g}{u_l}$ along the streaming channel, the local liquid velocity $\frac{v}{(1-v)}$ and $\frac{1}{(1-v)}$ are all linear with the length of the channel, so the ratio of the two-phase friction to single-phase friction at the same mass flow rate for the entire boiling length can be expressed as follows:

$$\phi_l^2 = \frac{1}{3}\left[1 + \frac{1}{1 - v} + \frac{1}{(1 - v)^2}\right] \tag{548}$$

where v is vapour volume fraction.

Compared to the Lockhart-Martinelli correlation described above, this correlation was found to provide a better estimate for the two-phase multiplier within a single subchannel in the framework of 3D codes [206].

The Lottes-Flinn model provides a simple correlation for the two-phase pressure drop wall friction multiplier: it is obtained by assuming that the liquid remains in contact with the wall in two-phase regime, and thus that wall-to-liquid friction predominates. This model is expected to remain accurate unless film entrainment by the gas core occurs (typically at very high void fraction): then, the Lottes-Flinn correlation is likely to overestimate the two-phase pressure drop.

Because of its simplicity, this model should be considered as a first step towards the establishment of a more elaborate two-phase pressure drop model based on feedback from experimental validation.

4.2.1.4 Cicchitti et al. (1960)

In 1960 Cicchitti et al. proposed the following homogeneous model correlation [234] [330]:

$$\mu_{2\phi} = x\mu_g + (1-x)\mu_l \tag{549}$$

4.2.1.5 Dukler et al. (1964)

In 1964 Dukler et al. proposed the following homogeneous model correlation [234] [331]:

$$\mu_{2\phi} = \rho_{2\phi}\left[x\frac{\mu_g}{\rho_g} + (1-x)\frac{\mu_l}{\rho_l}\right] \tag{550}$$

based on the averaged value of kinematic viscosity.

4.2.1.6 Chisholm (1967)

A milestone of the f_l^2 and f_g^2 based method is the D. Chisholm (1967) method, where f_l^2 is expressed analytically as the function of X_{LM} and a constant C [332] based on the Lockhart–Martinelli graphs:

$$\mathrm{f}_l^2 = 1 + \frac{\mathrm{C}}{X_{LM}} + \frac{1}{{X_{LM}}^2} \tag{551}$$

$$\begin{array}{ll} \mathrm{C} = 20 & \text{for turbulent liquid /turbulent vapour} \\ \mathrm{C} = 12 & \text{for laminar liquid /turbulent vapour} \\ \mathrm{C} = 10 & \text{for turbulent liquid /laminar vapour} \\ \mathrm{C} = 2 & \text{for laminar liquid /laminar vapour} \end{array} \tag{552}$$

Chisholm in [332] proposed four constants for the different flow patterns following the same criterion for the Reynolds numbers in the turbulent, viscous, and transitional regimes.

In practice, if the liquid phase is turbulent, the vapour phase will usually be turbulent as well, and this is by far the most important case. Therefore, the Chisholm equation is restated for the turbulent-turbulent case as follows [326]:

$$\mathrm{f}_l^2 = 1 + \frac{20}{X_{LM}^{tt}} + \frac{1}{{X_{LM}^{tt}}^2} \tag{553}$$

After Chisholm, many correlations with this same functional form were proposed [234].

4.2.1.7 Premoli et al. (1970)

The CISE correlation was proposed by A. Premoli et al. [333] in 1970 to provide the correct asymptotic behaviour for the slip ratio, as various fluid properties and flow parameters approach their theoretical limits. The correlation is based on experimental data for upward flow in vertical channels [326]. The equation for the slip ratio is:

$$SR = 1 + K\sqrt{\frac{Y}{1+CY} + CY} \tag{554}$$

$$Y = \frac{v}{1-v} \tag{555}$$

$$v = \frac{\rho_l x}{\rho_l x + \rho_g (1-x)} \quad \text{is the homogeneous void fraction} \tag{556}$$

$$K = 1.578 Re^{-0.19} \left(\frac{\rho_l}{\rho_g}\right)^{0.22} \tag{557}$$

$$C = 0.0273 We Re^{-0.51} \left(\frac{\rho_l}{\rho_g}\right)^{-0.08} \tag{558}$$

$$Re = \frac{Gd}{\mu_l} \quad \text{is the Reynolds number} \tag{559}$$

$$We = \frac{G^2 d}{\sigma \rho_l} \quad \text{is the Weber number} \tag{560}$$

In the above equations μ_l and μ_g are the liquid and gas dynamic viscosity, respectively; G is the total mass flux and σ is the surface tension.

Both the Reynolds and Weber numbers in this correlation are computed using the total mass flux, G, and physical properties of the liquid phase. As mentioned in [326], in the article by Premoli et al. [333], the last term in square brackets was incorrectly printed with a minus sign instead of the plus sign, and this typographical error has been repeated many times in the subsequent papers. The plus sign is required based on theoretical grounds in order for the slip ratio to exhibit the correct asymptotic behaviour, and to prevent the square root of negative numbers from occurring at low Reynolds numbers or high Weber numbers. It is also needed to reproduce the graphical results presented in the original paper.

4.2.1.8 Chen-Kalish (1970)

The correlation proposed by J.C. Chen and S. Kalish in 1970 was derived on the basis of the measurements performed with potassium [227] [334]:

$$\ln\left(\frac{1}{\phi_l}\right) = -1.59 + 0.518 \ln X_{LM} - 0.0867 \left(\ln X_{LM}\right)^2 \tag{561}$$

where X_{LM} is the Lockhart-Martinelli parameter.

4.2.1.9 Chisholm (1973)

In 1973 D. Chisholm proposed the following correlation for the two-phase multiplier in the frame of the separated flow model [234] [335], based on empirical results from the literature:

$$\mathrm{f}_{l0}^2 = 1 + (Y^2 - 1)\left\{B[x(1-x)]^{\frac{2-n}{2}} + \mathrm{x}^{2-n}\right\} \tag{562}$$

where x is the vapour fraction, n is 0.25 for turbulent-turbulent flow, B is a parameter depending on the mass flux G and Y and defined as:

$$Y^2 = \frac{\left(\frac{\Delta p}{\Delta L}\right)_{g0}}{\left(\frac{\Delta p}{\Delta L}\right)_{lo}} = \frac{\rho_l}{\rho_g}\left(\frac{\mu_g}{\mu_l}\right)^n \tag{563}$$

Y is analogous to the reciprocal of the Lockhart-Martinelli parameter X_{LM}. Chisholm then adjusted the correlation to obtain a conservative estimate of pressure drop for design purposes. As a result, his final correlation for B is a discontinuous function that tends to over-predict the pressure drop in certain ranges of G and Y.

In [234] the graphical procedure of Baroczy (1966) [336] was transformed to equations for predicting pressure drop during the turbulent flow of two-phase mixtures in smooth tubes to enable their more convenient application to evaporating flow [234] in the following way:

For $0 \leq Y < 9.5$:

$$B = \frac{55}{G_{2\phi}^{0.5}}\ for\ G_{2\phi} \geq 1900\frac{kg}{m^2s} \tag{564}$$

$$B = 2400\ for\ 500 < G_{2\phi} < 1900\frac{kg}{m^2s} \tag{565}$$

$$B = 4.8\ for\ G_{2\phi} \leq 500\frac{kg}{m^2s} \tag{566}$$

For $9.5 < Y < 28$:

$$B = \frac{520}{YG_{2\phi}^{0.5}}\ for\ G_{2\phi} \leq 600\frac{kg}{m^2s} \tag{567}$$

$$B = \frac{21}{Y}\ for\ G_{2\phi} > 600\frac{kg}{m^2s} \tag{568}$$

For $Y > 28$:

$$B = \frac{15000}{Y^2G_{2\phi}^{0.5}} \tag{569}$$

In [326] it is proposed to follow Hewitt's [337] [338] recommendation and use Chisholm's unadjusted correlation for B, which is represented by similar equations for B as shown above.

4.2.1.10 Kaiser et al. (1974)

On the basis of the measurements in a sodium loop with an induction heated round test section of 9 mm inner diameter and 200 mm heated length, A. Kaiser et al. in 1974 derived the following correlation for the two-phase friction pressure drop multiplier [227] [339]:

$$\phi_l = 8.2\ X_{LM}^{-0.55} \tag{570}$$

where X_{LM} is the Lockhart-Martinelli parameter.

4.2.1.11 Friedel (1979)

In 1979 L. Friedel derived a separated flow model correlation for frictional two-phase pressure gradient for $\frac{\mu_l}{\mu_g} \leq 1000$, which is given by [234] [326] [340]:[16]

$$f_{l0}^2 = (1-x)^2 + x^2 \frac{f_{g0}}{f_{l0}} \frac{r_l}{r_g} + 3.24 x^{0.78} (1-x)^{0.224} \frac{H}{Fr_{2\phi}^{0.045} We_{2\phi}^{0.035}} \tag{571}$$

$$H = \left(\frac{\rho_l}{\rho_g}\right)^{0.91} \left(\frac{\mu_g}{\mu_l}\right)^{0.19} \left(1 - \frac{\mu_g}{\mu_l}\right)^{0.7} \tag{572}$$

$$Fr_{2\phi} = \frac{G_{2\phi}^2}{g D \rho_{2\phi}^2} \quad \text{is the Froude number} \tag{573}$$

$$We = \frac{G^2 D}{\rho_{2\phi} \sigma} \quad \text{is the Weber number} \tag{574}$$

where f_{go} and f_{lo} are the friction factors for the total mass flux flowing with the gas and the liquid, respectively. σ is the surface tension and g is the acceleration due to gravity. In addition, G is the mass flux (product of velocity and density or, alternatively is the mass flow rate divided by the channel area), D is the channel internal diameter, x is the vapour quality and $\rho_{2\phi}$ is the two-phase density:

$$\frac{1}{\rho_{2\phi}} = \frac{x}{\rho_g} + \frac{1-x}{\rho_l} \tag{575}$$

The correlation can be rewritten in a more convenient form to eliminate the friction factors:

$$f_{l0}^2 = (1-x)^2 + x^2 \left(\frac{\mu_g}{\mu_l}\right)^n \frac{r_l}{r_g} + 3.24 x^{0.78} (1-x)^{0.224} \frac{H}{Fr_{2\phi}^{0.045} We_{2\phi}^{0.035}} \tag{576}$$

with $n = 0.25$ for turbulent-turbulent flows. The correlation as given here is valid for horizontal and vertical upward flows. This correlation is based on 25000 experimental data points and considers the effects of the gravity and the surface tension [326].

4.2.1.12 Gronnerud (1979)

In 1979 Gronnerud proposed the following separated flow model correlation [234] [341]:

$$\mathrm{f}_{l0}^2 = 1 + \left(\frac{\Delta p}{\Delta L}\right)_{Fr} \left[\left(\frac{\rho_l}{\rho_g}\right) \left(\frac{\mu_g}{\mu_l}\right)^{0.25} - 1\right] \tag{577}$$

[16] In Ref. [339] the Weber number expression for this correlation is presented as $We = \frac{G^2 D}{g \rho_{2\phi} \sigma}$, which is not correct and should be $We = \frac{G^2 D}{\rho_{2\phi} \sigma}$.

$$\left(\frac{\Delta p}{\Delta L}\right)_{Fr} = f_{Fr}[x + 4(x^{1.8} - x^{10} f_{Fr}^{0.5})] \tag{578}$$

$$f_{Fr} = 1 \qquad \text{for } Fr_{l0} \geq 1 \tag{579}$$

$$f_{Fr} = Fr_{l0}^{0.3} + 0.0055 \left[ln\left(\frac{1}{Fr_{l0}}\right)\right]^2 \qquad \text{for } Fr_{l0} \leq 1 \tag{580}$$

$$Fr_{l0} = \frac{G_{2\phi}^2}{gD\rho_l^2} \tag{581}$$

4.2.1.13 Beattie-Whalley (1982)

In 1982 Beattie-Whalley proposed the following homogeneous model correlation [234] [342]:

$$\mu_{2\phi} = \beta\mu_g + (1-\beta)(1+2.5\beta)\mu_l \tag{582}$$

$$\beta = \frac{x}{x + (1-x)\frac{\rho_g}{\rho_l}} \tag{583}$$

The structure of this equation is consistent with the form that might be expected for some gravity dominated flows.

4.2.1.14 Chisholm (1983)

D. Chisholm [326] [343] presented the following simple correlation for the slip ratio:

$$SR = \left(\frac{\rho_l}{\rho_{2\phi}}\right)^{0.5} \qquad \text{for } X_{LM} > 1 \tag{584}$$

$$SR = \left(\frac{\rho_l}{\rho_g}\right)^{0.25} \qquad \text{for } X_{LM} \leq 1 \tag{585}$$

where X_{LM} is the Lockhart-Martinelli parameter and $\rho_{2\phi}$ is the homogeneous two-phase density given by:

$$\frac{1}{\rho_{2\phi}} = \frac{x}{\rho_g} + \frac{1-x}{\rho_l} \tag{586}$$

This correlation is valid for flow in both vertical and horizontal tubes over a wide range of conditions, including void fractions from zero to unity. Chisholm stated that in a test of 14 methods, it proved to be the best for density prediction.

4.2.1.15 Kottowski-Savatteri (1984)

In 1984, H.M. Kottowski and C. Savatteri derived a correlation on the basis of the round tube quasi steady-state experiments. The least-square fit correlation derived from the measurement data is given by [227] [325]:

$$\log \phi_l = 0.1046\,(\log X_{LM})^2 - 0.5098 \log X_{LM} + 0.6252 \tag{587}$$

for $0.07 \leq X_{LM} \leq 30$. X_{LM} is the Lockhart-Martinelli parameter.

4.2.1.16 Muller-Steinhagen- Heck (1986)

The correlation proposed in 1986 by H. Muller-Steinhagen and K. Heck can be reformulated in the format of Chisholm functional with the two-phase multiplier given by the following equation [234] [326] [344]:

$$\mathrm{f}_{l0}^2 = Y^2 x^3 + (1-x)^{\frac{1}{3}}[1 + 2x(Y^2 - 1)]$$
$$Y^2 = \frac{\left(\frac{\Delta p}{\Delta L}\right)_{g0}}{\left(\frac{\Delta p}{\Delta L}\right)_{lo}} \tag{588}$$

where x is the vapour mass fraction and Y is the Chisholm parameter. This is an interpolation formula between all liquid flow ($x = 0$) and all vapour flow ($x = 1$). For $x = 0$, f_{l0}^2 is one and the negative two-phase pressure gradient becomes $\left(\frac{\Delta p}{\Delta L}\right)_{l0}$. For $x = 1$, f_{l0}^2 is Y^2 and the negative two-phase pressure gradient becomes $\left(\frac{\Delta p}{\Delta L}\right)_{g0}$.

The Friedel correlation (section 4.2.1.11) is widely accepted as the most reliable general method for computing two-phase pressure losses. However, in two studies [345] [346], the performance of the simpler Muller-Steinhagen and Heck method was found to be superior to that of the Friedel method. It should be recognized that the uncertainty associated with all of these methods is much greater than for single-phase pressure drop calculations, and relatively large errors are possible in any given application [326].

4.2.1.17 Lin et al. (1991)

In 1991 S. Lin et al. proposed the following homogeneous model correlation [234] [348]:

$$\mu_{2\phi} = \frac{\mu_g \mu_l}{\mu_g} + x^{1.4}\left(\mu_l - \mu_g\right) \tag{589}$$

It is based on experimental data of vaporisation of R12 in capillary tubes.

4.2.1.18 Lahey-Moody (1993)

In 1993, based on extensive experimental data, R.T. Lahey and F.J. Moody [227] [349] reported that a homogeneous multiplier does a fairly good job of correlating the two-phase pressure drop data for a wide range of grid-type spacers, and proposed the following correlation:

$$\phi = \left(\frac{x}{\rho_g} + \frac{1-x}{\rho_l}\right) \rho_l \tag{590}$$

where x is the flow quality.

4.2.1.19 Lobo de Souza-de Mattos Pimenta (1995)

In 1995 A. Lobo de Souza and M. de Mattos Pimenta proposed the following separated flow model correlation [234] [350]:

$$\mathrm{f}_{l0}^{2} = 1 + (\Gamma^2 - 1)x^{1.75}(1 + 0.9524\,\Gamma\, X_{tt}^{0.4126}) \tag{591}$$

$$\Gamma = \left(\frac{\rho_l}{\rho_g}\right)^{0.5} \left(\frac{\mu_g}{\mu_l}\right)^{0.125} \tag{592}$$

$$X_{tt} = \frac{1}{\Gamma}\left(\frac{1-x}{x}\right)^{0.875} \tag{593}$$

It is based on experimental data for R12, R22, R134a, MP39, and R32/125 refrigerants.

4.2.1.20 Mishima-Hibiki (1996)

In 1996 K. Mishima and T. Hibiki, recommended a correlation based on the Chisholm functional relationship for viscous liquid and viscous gas flow [234] [351]:

$$C = 21(1 - e^{-0.319D}) \tag{594}$$

where D is the hydraulic diameter in meters.

4.2.1.21 Wang et al. (1997)

In 1997 C.C. Wang et al. proposed the following correlation based on the Chisholm functional form [234] [352] for different values of the mass flux G:

$$C = 4.566 \cdot 10^{-6} X^{0.128} Re_{lo}^{0.938} \left(\frac{\rho_l}{\rho_g}\right)^{-2.15} \left(\frac{\mu_l}{\mu_g}\right)^{5.1} \text{ for } G = 50\text{–}100\,\frac{kg}{m^2 s} \tag{595}$$

$$f_g^2 = 1 + 9.397X^{0.62} + 0.564X^{2.45} \text{for } G \geq 200\,\frac{kg}{m^2 s} \tag{596}$$

This correlation is based on experimental data for R22, R134a and R407C refrigerants inside a 6.5 mm smooth tube.

4.2.1.22 Tran et al. (2000)

In 2000 T. Tran et al. proposed the following separated flow model correlation [234] [353]:

$$f_{l0}^2 = 1 + (4.3Y^2 - 1)\{[x(1-x)]^{0.875} La + x^{1.75}\} \tag{597}$$

It is applicable for R134a, R113 and R12 in smooth tubes, with p from 138 to 864 kPa, G from 33 to 832 $\frac{kg}{m^2 s}$, q from 2.2 to 90.8 $\frac{kW}{m^2}$, and x from 0 to 0.95.

4.2.1.23 Zhang-Webb (2001)

In 2001 M. Zhang and R.L. Webb proposed the following separated flow model correlation [234] [354]:

$$f_{lo}^2 = (1-x)^2 + 2.87x^2 \left(\frac{p}{p_c}\right)^{-1} + 1.68x^{0.8}(1-x)^{0.25} \left(\frac{p}{p_c}\right)^{-1.64} \quad (598)$$

where p_c is the critical pressure. It is based on experimental data of R134a, R22 and R404A flowing in a multi-port extruded aluminum tube with diameter of 2.13 mm, and in two cooper tubes having diameter or 6.25 and 3.25 mm.

4.2.1.24 Chen et al. (2001)

In 2001 Y. Chen et al. proposed the following separated flow model correlation for $D \leq 10\ mm$ [234] [355]:

$$\left(\frac{\Delta p}{\Delta L}\right)_{2\phi} = \Omega \left(\frac{\Delta p}{\Delta L}\right)_{2\phi\ Friedel} \quad (599)$$

$$\Omega = 0.0333 \frac{Re_{lo}^{0.45}}{Re_g^{0.09}} [1 + 0.4e^{-B_0}]\ for\ B_0 \leq 2.5 \quad (600)$$

$$\Omega = \frac{We_{2\phi}^{0.2}}{2.5} + 0.06B_0\ for\ B_0 \geq 2.5 \quad (601)$$

$$B_0 = g(\rho_l - \rho_g) \frac{\left({}^{D}/_{2}\right)^2}{\sigma} \quad (602)$$

where $\left({}^{Dp}/_{DL}\right)_{tp,Friedel}$ is the two-phase friction pressure gradient predicted using the Friedel correlation, and the W correlation is obtained from the measured data of air–water and R410A refrigerant.

They also proposed another homogeneous model correlation for $D \leq 10\ mm$ [234] [355]:

$$\left(\frac{\Delta p}{\Delta L}\right)_{2\phi} = \Omega_{Hom.} \left(\frac{\Delta p}{\Delta L}\right)_{2\phi\ Hom.} \quad (603)$$

$$\Omega_{Hom.} = 1.2 - 0.9e^{-B_0}\ for\ B_0 \leq 2.5 \quad (604)$$

$$\Omega_{Hom.} = 1 + \frac{We^{0.2}}{e^{B_0{}^{0.3}}} - 0.9e^{-B_0}\ for\ B_0 \geq 2.5 \quad (605)$$

$$B_0 = g(\rho_l - \rho_g) \frac{\left({}^{D}/_{2}\right)^2}{\sigma} \quad (606)$$

where $\left({}^{Dp}/_{DL}\right)_{2\phi\ Hom.}$ is the two-phase friction pressure gradient predicted using a homogeneous model, and the W correlation is obtained from the measured data of air–water and R410A refrigerant.

4.2.1.25 Lee-Lee (2001)

Based on the Chisholm functional relation, in 2001 H.J. Lee and S.Y. Lee proposed the following correlation [234] [356]:

$$C = Al^{q}Y^{R}Re_{lo}^{S} \tag{607}$$

where $l = \frac{\mu_l}{\rho_l \sigma D}$ and $Y = \frac{\mu_l j}{D}$. Table 37 presents the values of Re_l, Re_g, A, q, R and S.

TABLE 37. VALUES OF PARAMETERS OF LEE-LEE CORRELATION

Re_l	Re_g	A	q	R	S
≤ 2000	≤ 2000	$6.833 \cdot 10^{-8}$	-1.317	0.719	0.557
≤ 2000	> 2000	$6.185 \cdot 10^{-2}$	0	0	0.726
> 2000	≤ 2000	3.627	0	0	0.174
> 2000	> 2000	0.408	0	0	0.451

It is based on 305 experimental data points of horizontal rectangular channels with small heights.

4.2.1.26 Cavallini et al. (2002)

In 2002 A. Cavallini et al. proposed the following separated flow model correlation [234] [357]:

$$\mathrm{f}_{l0}^{2} = (1-x)^2 + x^2 \frac{r_l f_{g0}}{r_g f_{l0}} + 1.262 x^{0.6978} \frac{H}{We_{g0}^{0.1458}}$$

$$H = \left(\frac{\rho_l}{\rho_g}\right)^{0.3278} \left(\frac{\mu_g}{\mu_l}\right)^{-1.184} \left(\frac{1-\mu_g}{\mu_l}\right)^{3.477} \tag{608}$$

$$We_{go} = \frac{G_{2\phi}^{2} D}{\rho_g \sigma}$$

It is based on experimental data of condensation of halogenated refrigerants inside smooth tubes.

4.2.1.27 Yu et al. (2002)

In 2002 W. Yu et al. recommended a correlation [234] [358]:

$$\mathrm{f}_{l}^{2} = \left[18.65 \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \left(\frac{1-x}{x}\right) \frac{Re_g^{0.1}}{Re_l^{0.5}}\right]^{-1.9} \tag{609}$$

It is based on experimental data of water in a horizontal tube of 2.98 mm inside diameter.

4.2.1.28 Wilson et al. (2003)

In 2003 M. Wilson et al. proposed the following separated flow model correlation [234] [359]:

$$f_{l0}^{2} = 12.82\,(1-x)^{1.8} X_{tt}^{-1.47}$$

$$X_{tt} = \left(\frac{1-x}{x}\right)^{0.9} \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \left(\frac{\mu_g}{\mu_l}\right)^{0.1} \tag{610}$$

It is based on experimental data of R134a and R410A in horizontal flattened tube, with G from 75 to 400 $\frac{kg}{m^2 s}$ and x from 0.1 to 0.8.

4.2.1.29 Lee-Mudawar (2005)

In 2005 J. Lee and I. Mudawar, published a correlation based on the Chisholm functional relation [234] [360]:

$$C = \begin{cases} 2.16\mathrm{Re}_{lo}^{0.047}\mathrm{We}_{lo}^{0.6} \text{ for vv} \\ 1.45\mathrm{Re}_{lo}^{0.25}\mathrm{We}_{lo}^{0.23} \text{ for vt} \end{cases} \tag{611}$$

$$Re_{lo} = \frac{G_{2\phi} D}{\mu_l} \tag{612}$$

$$We_{lo} = \frac{G_{2\phi}^{2} D}{\rho_l \sigma} \tag{613}$$

It is based on experimental data of R134a in a micro-channel of 231 µm wide × 713 µm deep groove at high heat flux of q=31.6–93.8 $\frac{W}{cm^2}$.

4.2.1.30 Hwang-Kim (2006)

In 2006, following the functional relation proposed by Chisholm, Y.W. Hwang and M.S. Kim proposed a correlation for the value of the C parameter [234] [361]:

$$C = 0.227 Re_{lo}^{0.452} X^{-0.32}\, La^{-0.82} \tag{614}$$

$$La = \frac{\sqrt{\frac{\sigma}{g(\rho_l - \rho_g)}}}{D} \tag{615}$$

They recommended this correlation based on experimental data of R134a in 0.244, 0.430 and 0.792 mm pipes.

4.2.1.31 Awad-Muzychka (2008)

In 2008 M. Awad and Y. Muzychka proposed the following homogeneous model correlations [234] [362]:

$$\mu_{2\phi} = \frac{\mu_l\left[2\mu_l + \mu_g - 2(\mu_l - \mu_g)x\right]}{2\mu_l + \mu_g + (\mu_l - \mu_g)x} \tag{616}$$

$$\mu_{2\phi} = \frac{\mu_g\left[2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-x)\right]}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-x)} \tag{617}$$

It was proposed using an analogy between thermal conductivity of porous media and dynamic viscosity of two-phase flow.

4.2.1.32 Shannak (2008)

In 2008 B.A. Shannak proposed the following homogeneous model correlation [234] [363]:

$$Re_{2\phi} = \frac{G_{2\phi}D\left[x^2 + (1-x)^2\frac{\rho_g}{\rho_l}\right]}{\mu_g x + \mu_l(1-x)\frac{\rho_g}{\rho_l}} \tag{618}$$

It is given as the ratio of the sum of inertial force of each phase and that of the sum of viscous force of each phase.

4.2.1.33 Sun-Mishima (2009)

In 2009 L. Sun and K. Mishima proposed the following correlation for viscous flow based on the Chisholm method [234] [364]:

$$C = 26\left(1 + \frac{Re_l}{1000}\right)\left(1 - e^{-\frac{0.153}{0.8+0.28La}}\right) \tag{619}$$

For turbulent flow they recommended:

$$f_l^2 = 1 + \frac{C}{X^{1.19}} + \frac{1}{X^2} \tag{620}$$

$$C = 1.79\left(\frac{Re_g}{Re_l}\right)^{0.4}\left(\frac{1-x}{x}\right)^{0.5} \tag{621}$$

$$Re_g = \frac{G_{2\phi}xD}{\mu_g} \text{ and } Re_l = \frac{G_{2\phi}(1-x)D}{\mu_l} \tag{622}$$

These correlations are based on 2092 experimental data points of R123, R134a, R22, R236ea, R245fa, R404A, R407C, R410A, R507, CO_2, water and air in 0.506–12 mm tubes.

4.2.1.34 Zhang et al. (2010)

In 2010 W. Zhang et al. proposed the following correlation for viscous liquid and viscous gas flow of the Chisholm-type [234] [365]:

$$C = 21\left(1 - e^{-\frac{0.674}{La}}\right) \quad \text{for adiabatic gas–liquid} \tag{623}$$

$$C = 21\left(1 - e^{-\frac{0.142}{La}}\right) \quad \text{for adiabatic vapour–liquid} \tag{624}$$

$$C = 21\left(1 - e^{-\frac{0.358}{La}}\right) \qquad \text{for flow boiling} \qquad (625)$$

4.2.1.35 Pamitran et al. (2010)

In 2010 A. Pamitran et al. recommended a two-phase correlation of the Chisholm-type [234] [366]:

$$C = 3 \cdot 10^{-3}\, Re_{2\phi}^{1.23} We_{2\phi}^{-0.433} \qquad (626)$$

$$Re_{2\phi} = \frac{G_{2\phi} D}{\mu_{2\phi}} \qquad (627)$$

$$We_{2\phi} = \frac{G_{2\phi}^2 D}{\rho_{2\phi} \sigma} \qquad (628)$$

It is based on experimental data of R22, R134a, R410A, R290 and R744 in horizontal tubes of 0.5, 1.5 and 3.0 mm inside diameter.

4.2.1.36 Qiu et al. (2015)

In 2015 Z.C. Qiu et al. studied the thermal-hydraulic characteristics of sodium boiling by boiling experiments on sodium, flowing through the annulus of 1000 mm length, 8 mm inner diameter and 12 mm outer diameter. The heat flux varied from 80 to 500 $\frac{kW}{m^2}$, with inlet subcooling from 63 to 285ºC, inlet flow velocity from 0.02 to 0.5 $\frac{m}{s}$ and system pressure from 3.67 to 103 kPa. They proposed the following correlation for the two-phase friction multiplier factor [208]:

$$f_l^2 = 1 + \frac{8.57}{X} + \frac{1}{X^2} \qquad (629)$$

4.2.1.37 Summary of friction factor correlations for two-phase flow in straight pipes

Table 38 presents the list of all friction factor correlations collected for two-phase flow in straight pipes.

TABLE 38. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN STRAIGHT PIPES

McAdams et al. (1942) [234] [238]	$\frac{1}{\mu_{2\phi}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_l}$ homogeneous model
Lockhart-Martinelli (1949) [234] [326] [327]	$X_{LM}^2 = \frac{\left(\frac{\Delta p}{\Delta L}\right)_l}{\left(\frac{\Delta p}{\Delta L}\right)_g} = \frac{Re_{gp}^m}{Re_{lp}^n}\frac{C_l}{C_g}\left(\frac{W_l}{W_g}\right)^2\frac{\rho_g}{\rho_l}$ $Re_{gp} = \frac{4W_g}{\pi D_g \mu_g}$ $Re_{lp} = \frac{4W_l}{\pi D_l \mu_l}$ $X_{LM} = \left(\frac{1-x}{x}\right)^{\left(\frac{2-n}{2}\right)}\left(\frac{\rho_g}{\rho_l}\right)^{0.5}\left(\frac{\mu_l}{\mu_g}\right)^{\frac{n}{2}}$ for the turbulent-turbulent flow n=0.2: $X_{LM}^{tt} = \left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\rho_g}{\rho_l}\right)^{0.5}\left(\frac{\mu_l}{\mu_g}\right)^{0.1}$ $v = \frac{\mathrm{f}_l - 1}{\mathrm{f}_l}$ $\rho_{2\phi} = \frac{\rho_l + (\mathrm{f}_l - 1)\rho_v}{\mathrm{f}_l}$ separated flow model
Lottes-Flinn (1956) [329]	$\phi_l^2 = \frac{1}{3}\left[1 + \frac{1}{1-v} + \frac{1}{(1-v)^2}\right]$ separated flow model
Cicchitti et al. (1960) [234] [330]	$\mu_{2\phi} = x\mu_g + (1-x)\mu_l$ homogeneous model
Dukler et al. (1964) [234] [331]	$\mu_{2\phi} = \rho_{2\phi}\left[x\frac{\mu_g}{\rho_g} + (1-x)\frac{\mu_l}{\rho_l}\right]$ homogeneous model

TABLE 38. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN STRAIGHT PIPES

Chisholm (1967) [326] [332]	$f_l^2 = 1 + \frac{C}{X_{LM}} + \frac{1}{{X_{LM}}^2}$ $C = 20$ for turbulent liquid /turbulent vapour $C = 12$ for laminar liquid /turbulent vapour $C = 10$ for turbulent liquid /laminar vapour $C = 2$ for laminar liquid /laminar vapour separated flow model
Premoli et al. (1970) [326] [333]	$SR = 1 + K\sqrt{\frac{Y}{1+CY} + CY}$ $Y = \frac{v}{1-v}$ $v = \frac{\rho_l x}{\rho_l x + \rho_g (1-x)}$ $K = 1.578Re^{-0.19}\left(\frac{\rho_l}{\rho_g}\right)^{0.22}$ $C = 0.0273WeRe^{-0.51}\left(\frac{\rho_l}{\rho_g}\right)^{-0.08}$ slip ratio model
Chen-Kalish (1970) [227] [334]	$\ln\left(\frac{1}{\phi_l}\right) = -1.59 + 0.518\ln X_{LM} - 0.0867\,(\ln X_{LM})^2$ separated flow model

TABLE 38. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN STRAIGHT PIPES

Chisholm (1973) [234] [335]	$f_{l0}^2 = 1 + (Y^2 - 1)\left\{B[x(1-x)]^{\frac{2-n}{2}} + x^{2-n}\right\}$ n is 0.25 for turbulent-turbulent flow $Y^2 = \frac{\left(\frac{\Delta p}{\Delta L}\right)_{g0}}{\left(\frac{\Delta p}{\Delta L}\right)_{lo}} = \frac{\rho_l}{\rho_g}\left(\frac{\mu_g}{\mu_l}\right)^n$ for $0 \leq Y < 9.5$: $B = \frac{55}{G_{2\phi}^{0.5}}\ for\ G_{2\phi} \geq 1900\frac{kg}{m^2 s}$ $B = 2400\ for\ 500 < G_{2\phi} < 1900\frac{kg}{m^2 s}$ $B = 4.8\ for\ G_{2\phi} \leq 500\frac{kg}{m^2 s}$ for $9.5 < Y < 28$: $B = \frac{520}{YG_{2\phi}^{0.5}}\ for\ G_{2\phi} \leq 600\frac{kg}{m^2 s}$ $B = \frac{21}{Y}\ for\ G_{2\phi} > 600\frac{kg}{m^2 s}$ for $Y > 28$: $B = \frac{15000}{Y^2 G_{2\phi}^{0.5}}$ separated flow model
Kaiser et al. (1974) [227] [339]	$\phi_l = 8.2\ X_{LM}^{-0.55}$ separated flow model

TABLE 38. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN STRAIGHT PIPES

Friedel (1979) [234] [326] [340]	$f_{l0}^2 = (1-x)^2 + x^2 \frac{f_{g0}}{f_{l0}} \frac{r_l}{r_g} + 3.24x^{0.78}(1-x)^{0.224} \frac{H}{Fr_{2\phi}^{0.045} We_{2\phi}^{0.035}}$ $H = \left(\frac{\rho_l}{\rho_g}\right)^{0.91} \left(\frac{\mu_g}{\mu_l}\right)^{0.19} \left(1 - \frac{\mu_g}{\mu_l}\right)^{0.7}$ $Fr_{2\phi} = \frac{G_{2\phi}^2}{gD\rho_{2\phi}^2}$ $We = \frac{G^2 D}{\rho_{2\phi}\sigma}$ alternatively: $f_{l0}^2 = (1-x)^2 + x^2 \left(\frac{\mu_g}{\mu_l}\right)^n \frac{r_l}{r_g} + 3.24x^{0.78}(1-x)^{0.224} \frac{H}{Fr_{2\phi}^{0.045} We_{2\phi}^{0.035}}$ $n = 0.25$ for turbulent-turbulent flows separated flow model with $\frac{\mu_l}{\mu_g} \leq 1000$
Gronnerud (1979) [234] [341]	$\mathrm{f}_{l0}^2 = 1 + \left(\frac{\Delta p}{\Delta L}\right)_{Fr} \left[\left(\frac{\rho_l}{\rho_g}\right) \left(\frac{\mu_g}{\mu_l}\right)^{0.25} - 1\right]$ $\left(\frac{\Delta p}{\Delta L}\right)_{Fr} = f_{Fr}[x + 4(x^{1.8} - x^{10} f_{Fr}^{0.5})]$ $f_{Fr} = 1$ for $Fr_{l0} \geq 1$ $f_{Fr} = Fr_{l0}^{0.3} + 0.0055 \left[ln\left(\frac{1}{Fr_{l0}}\right)\right]^2$ for $Fr_{l0} \leq 1$ $Fr_{l0} = \frac{G_{2\phi}^2}{gD\rho_l^2}$ separated flow model
Beattie-Whalley (1982) [234] [342]	$\mu_{2\phi} = \beta\mu_g + (1-\beta)(1+2.5\beta)\mu_l$ $\beta = \frac{x}{x + (1-x)\frac{\rho_g}{\rho_l}}$ homogeneous model
Chisholm (1983) [234] [343]	$\mathrm{SR} = \left(\frac{\rho_l}{\rho_{2\phi}}\right)^{0.5}$ for $X_{LM} > 1$ $\mathrm{SR} = \left(\frac{\rho_l}{\rho_g}\right)^{0.25}$ for $X_{LM} \leq 1$ $\frac{1}{\rho_{2\phi}} = \frac{x}{\rho_g} + \frac{1-x}{\rho_l}$ slip ratio model

TABLE 38. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN STRAIGHT PIPES

Kottowski-Savatteri (1984) [227] [325]	$\log \phi_l = 0.1046 (\log X_{LM})^2 - 0.5098 \log X_{LM} + 0.6252$ $0.07 \leq X_{LM} \leq 30$ separated flow model
Muller-Steinhagen-Heck (1986) [234] [326] [344]	Chisholm functional relationship $f_{l0}^2 = Y^2 x^3 + (1-x)^{\frac{1}{3}}[1 + 2x(Y^2 - 1)]$ separated flow model
Lin et al. (1991) [234] [348]	$\mu_{2\phi} = \frac{\mu_g \mu_l}{\mu_g} + x^{1.4}(\mu_l - \mu_g)$ homogeneous model
Lahey-Moody (1993) [227] [349]	$\phi = \left(\frac{x}{\rho_g} + \frac{1-x}{\rho_l}\right) \rho_l$ valid for a wide range of grid-type spacers separated flow model
Lobo de Souza-de Mattos Pimenta (1995) [234] [350]	$f_{l0}^2 = 1 + (\Gamma^2 - 1)x^{1.75}(1 + 0.9524\, \Gamma\, X_{tt}^{0.4126})$ $\Gamma = \left(\frac{\rho_l}{\rho_g}\right)^{0.5} \left(\frac{\mu_g}{\mu_l}\right)^{0.125}$ $X_{tt} = \frac{1}{\Gamma}\left(\frac{1-x}{x}\right)^{0.875}$ separated flow model
Mishima-Hibiki (1996) [234] [351]	Chisholm functional relationship $C = 21(1 - e^{-0.319D})$ for viscous liquid and viscous gas flow separated flow model
Wang et al. (1997) [234] [352]	for $G = 50\text{–}100 \frac{kg}{m^2 s}$ $C = 4.566 \cdot 10^{-6} X^{0.128} Re_{lo}^{0.938} \left(\frac{\rho_l}{\rho_g}\right)^{-2.15} \left(\frac{\mu_l}{\mu_g}\right)^{5.1}$ for $G \geq 200 \frac{kg}{m^2 s}$ $f_g^2 = 1 + 9.397X^{0.62} + 0.564X^{2.45}$ separated flow model

TABLE 38. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN STRAIGHT PIPES

Tran et al. (2000) [234] [353]	$f_{l0}^2 = 1 + (4.3Y^2 - 1)\{[x(1-x)]^{0.875} La + x^{1.75}\}$ for R134a, R113 and R12 in smooth tubes $138 \leq p \leq 864 kPa$, $33 \leq G \leq 832 \frac{kg}{m^2 s}$, $2.2 \leq q \leq 90.8 \frac{kW}{m^2}$, and $0 \leq x \leq 0.95$ separated flow model
Zhang-Webb (2001) [234] [354]	$f_{l0}^2 = (1-x)^2 + 2.87x^2 \left(\frac{p}{p_c}\right)^{-1} + 1.68x^{0.8}(1-x)^{0.25} \left(\frac{p}{p_c}\right)^{-1.64}$ separated flow model
Chen et al. (2001) [234] [355]	1. separated flow model: $\left(\frac{\Delta p}{\Delta L}\right)_{2\phi} = \Omega \left(\frac{\Delta p}{\Delta L}\right)_{2\phi\ Friedel}$ $\Omega = 0.0333 \frac{Re_{lo}^{0.45}}{Re_g^{0.09}} [1 + 0.4e^{-B_0}]\ for\ B_0 \leq 2.5$ $\Omega = \frac{We_{2\phi}^{0.2}}{2.5} + 0.06B_0\ for\ B_0 \geq 2.5$ $B_0 = g(\rho_l - \rho_g) \frac{\left(D/2\right)^2}{\sigma}$ 2. homogeneous model $\left(\frac{\Delta p}{\Delta L}\right)_{2\phi} = \Omega_{Hom.} \left(\frac{\Delta p}{\Delta L}\right)_{2\phi\ Hom.}$ $\Omega_{Hom.} = 1.2 - 0.9e^{-B_0}\ for\ B_0 \leq 2.5$ $\Omega_{Hom.} = 1 + \frac{We^{0.2}}{e^{{B_0}^{0.3}}} - 0.9e^{-B_0}\ for\ B_0 \geq 2.5$ Valid for $D \leq 10\ mm$
Lee-Lee (2001) [234] [356]	Chisholm functional relationship $C = Al^q Y^R Re_{lo}^S$ $l = \frac{\mu_l}{\rho_l \sigma D}$ $Y = \frac{\mu_l j}{D}$ Re_l, Re_g, A, q, R and S are presented in Table 37 separated flow model

TABLE 38. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN STRAIGHT PIPES

Cavallini et al. (2002) [234] [357]	$f_{l0}^2 = (1-x)^2 + x^2 \frac{r_l f_{g0}}{r_g f_{l0}} + 1.262 x^{0.6978} \frac{H}{We_{g0}^{0.1458}}$ $H = \left(\frac{\rho_l}{\rho_g}\right)^{0.3278} \left(\frac{\mu_g}{\mu_l}\right)^{-1.184} \left(\frac{1-\mu_g}{\mu_l}\right)^{3.477}$ $We_{go} = \frac{G_{2\phi}^2 D}{\rho_g \sigma}$ separated flow model
Yu et al. (2002) [234] [358]	$f_l^2 = \left[18.65 \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \left(\frac{1-x}{x}\right) \frac{Re_g^{0.1}}{Re_l^{0.5}}\right]^{-1.9}$ separated flow model
Wilson et al. (2003) [234] [359]	$f_{l0}^2 = 12.82\,(1-x)^{1.8} X_{tt}^{-1.47}$ $X_{tt} = \left(\frac{1-x}{x}\right)^{0.9} \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \left(\frac{\mu_g}{\mu_l}\right)^{0.1}$ horizontal flattened tube, $75 \le G \le 400 \frac{kg}{m^2 s}$ and $0.1 \le x \le 0.8$ separated flow model
Lee-Mudawar (2005) [234] [360]	Chisholm functional relation $C = \begin{cases} 2.16 \mathrm{Re}_{lo}^{0.047} \mathrm{We}_{lo}^{0.6} \text{ for vv} \\ 1.45 \mathrm{Re}_{lo}^{0.25} \mathrm{We}_{lo}^{0.23} \text{ for vt} \end{cases}$ $Re_{lo} = \frac{G_{2\phi} D}{\mu_l}$ $We_{lo} = \frac{G_{2\phi}^2 D}{\rho_l \sigma}$ micro-channel at high heat flux of $31.6 \le q\ (in\ \frac{W}{cm^2}) \le 93.8$ separated flow model
Hwang-Kim (2006) [234] [361]	Chisholm functional relation $C = 0.227 Re_{lo}^{0.452} X^{-0.32}\, La^{-0.82}$ $La = \frac{\sqrt{\frac{\sigma}{g(\rho_l - \rho_g)}}}{D}$ separated flow model

TABLE 38. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN STRAIGHT PIPES

Awad-Muzychka (2008) [234] [362]	$\mu_{2\phi} = \frac{\mu_l[2\mu_l + \mu_g - 2(\mu_l - \mu_g)\mathrm{x}]}{2\mu_l + \mu_g + (\mu_l - \mu_g)\mathrm{x}}$ $\mu_{2\phi} = \frac{\mu_g[2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-\mathrm{x})]}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-\mathrm{x})}$ homogeneous model
Shannak (2008) [234] [363]	$Re_{2\phi} = \frac{G_{2\phi}D\left[x^2 + (1-x)^2\frac{\rho_g}{\rho_l}\right]}{\mu_g x + \mu_l(1-\mathrm{x})\frac{\rho_g}{\rho_l}}$ homogeneous model
Sun-Mishima (2009) [234] [364]	Chisholm functional relation for viscous flow: $C = 26\left(1 + \frac{Re_l}{1000}\right)\left(1 - e^{-\frac{0.153}{0.8+0.28La}}\right)$ for turbulent flow: $f_l^2 = 1 + \frac{C}{X^{1.19}} + \frac{1}{X^2}$ $C = 1.79\left(\frac{Re_g}{Re_l}\right)^{0.4}\left(\frac{1-x}{x}\right)^{0.5}$ $Re_g = \frac{G_{2\phi}xD}{\mu_g}$ and $Re_l = \frac{G_{2\phi}(1-x)D}{\mu_l}$ separated flow model
Zhang et al. (2010) [234] [365]	Chisholm functional relationship for viscous liquid and viscous gas flow $C = 21\left(1 - e^{-\frac{0.674}{La}}\right)$ for adiabatic gas–liquid $C = 21\left(1 - e^{-\frac{0.142}{La}}\right)$ for adiabatic vapour–liquid $C = 21\left(1 - e^{-\frac{0.358}{La}}\right)$ for flow boiling separated flow model
Pamitran et al. (2010) [234] [366]	Chisholm functional relationship $C = 3 \cdot 10^{-3}\, Re_{2\phi}^{1.23} We_{2\phi}^{-0.433}$ $Re_{2\phi} = \frac{G_{2\phi}D}{\mu_{2\phi}}$ and $We_{2\phi} = \frac{G_{2\phi}^2 D}{\rho_{2\phi}\sigma}$ separated flow model

TABLE 38. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN STRAIGHT PIPES

Qiu et al. (2015) [208]	$f_l^2 = 1 + \frac{8.57}{X} + \frac{1}{X^2}$ $80 \leq q \leq 500 \frac{kW}{m^2}$, $63 \leq inlet\ subcooling \leq 285℃$ $0.02 \leq inlet\ flow\ velocity \leq 0.5 \frac{m}{s}$ and $3.67 \leq p \leq 103 kPa$ separated flow model

4.2.2 Flow in helical and curved pipes

As for the pressure drop for two-phase flow in helically coiled tubes, there are fewer studies compared to the case of a single-phase flow. Scientists during their experimental and computational studies [258] investigated the two-phase frictional pressure drop with steam-water flow boiling, R134a refrigerant flows and air-water flows in helically coiled tubes.

In the Fig. 62 below the representation of a helical pipe can be seen, where *d – pipe diameter, D – coil diameter, p – coil pitch, β – pipe angle.*

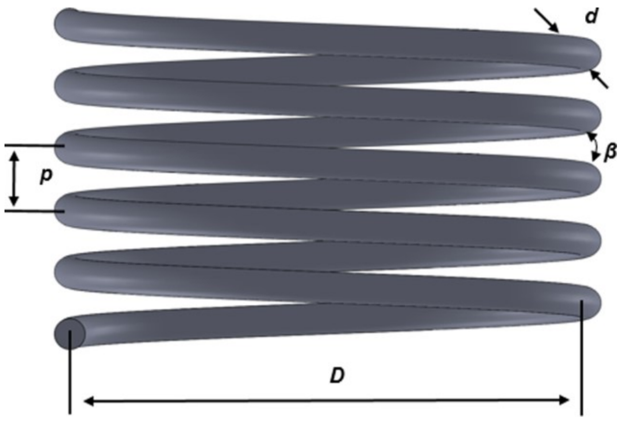


FIG. 62. Representation of helical pipe parameters (adopted from [258])

4.2.2.1 Kozeki et al. (1970)

In 1970 M. Kozeki et al. proposed a correlation derived based on the widely used two-phase flow pressure drop correlations for straight tubes ($p < 3.5\ MPa$ and $d \geq 12\ mm$). They concluded that the pressure drop is greater than that for a straight tube and it increases with the vapour quality and mass flux [258] [367]. The numerical model based on the Martinelli and Nelson prediction for two-phase flow in straight tubes is:

$$\phi_{g,tt}^2 = 0.895 + (X_{tt} + 0.076)^{0.875} + 1.21 \cdot 10^{-0.334(\log X_{tt} + 0.668)^2} \quad (630)$$

$$\phi_{l,tt} = \frac{\phi_{g,tt}}{X_{tt}^{0.875}} \quad (631)$$

The main experimental parameters for the vertical helical coil test were:

$$0.032 < \delta < 0.035$$
$$0.5 < p < 2.1\ MPa$$
$$151 < q < 348$$

$$161 < G < 486$$
$$21.7\ mm\ OD \text{ (outer pipe diameter)}$$
$$628 < D < 682\ mm$$
$$0 < x < 1$$

where δ is the curvature ratio (internal tube diameter d_i/mean coil diameter D), q is the heat flux $\left(\frac{kW}{m^2}\right)$, G is the mass flux $\left(\frac{kg}{m^2 s}\right)$, D is the helix diameter (mm) and x is the steam quality.

4.2.2.2 Ruffell (1974)

In 1974 A. E. Ruffell recommended a correlation for high system pressures ($p > 3.5\ MPa$) [258] [368] [369]. The final form of the new correlation is obtained considering the whole data bank, including the experimental data from Santini et al. (2008) [370] and Zhao et al. (2003) [371]:

$$\phi_l^2 = (1 + F)\frac{\rho_l}{\rho_{2\phi}} \quad (632)$$

$$F = \sin\frac{1.16G}{1000}\left[0.875 - 0.314y - \frac{0.74G}{1000}(0.152 - 0.07y) - x\left(\frac{0.155G}{1000} + 0.7 - 0.19y\right)\right][1 - 12(x - 0.3)(x - 0.4)(x - 0.5)(x - 0.6)] \quad (633)$$

$$y = \frac{D}{100d} \quad (634)$$

The main experimental parameters for the helical coil tests were:

$$10.7 < ID \text{ (inner pipe diameter)} < 18.6\ mm$$
$$0.0054 < \delta < 0.16$$
$$6 < p < 18\ MPa$$
$$41 < q < 731$$
$$300 < G < 1800$$
$$0 < x < 1$$

4.2.2.3 Unal et al. (1981)

In 1981 H.C. Unal et al. proposed the following correlation for high system pressures ($p > 3.5\ MPa$) [258] [372]:

$$\Delta P_{f,2\phi} = \frac{2(1 + b_1 b_2) f_l G^2}{d\rho_l} \quad (635)$$

$$b_1 = 3850 x^{0.01} Pr^{-1.515} Re_l^{-0.758} \quad (636)$$

$$b_2 = 1 + Re_l^{0.1}(3.67 - 3.04p_b)^{-0.014\delta^{-1} - 2\delta^{-1}} \quad (637)$$

$$p_b = \frac{p}{p_{crit}} \quad (638)$$

The authors used as single-phase friction factor correlation of Ito (see Eq. (363)):

$$f_l = 0.304Re^{-0.25} + 0.029\delta^{0.5} \quad (639)$$

The main experimental parameters for the vertical helical coil test were:

$$18\ mm\ I$$

$$700\ and\ 1500\ mm = D$$

$$0.0054 < \delta < 0.022$$

$$14.7 < p < 20.2\ MPa$$

$$41 < q < 731$$

$$112 < G < 1829$$

$$0.08 < x < 1$$

4.2.2.4 Chen and Zhou (1981)

In 1981 X.J. Chen and F.D. Zhou presented a correlation for high system pressures ($p > 3.5\ MPa$) [258] [373]:

$$\Delta P_{f,2\phi} = \xi \Delta P_l \quad (640)$$

$$\xi = 2.06\delta^{0.05} Re_{2\phi}^{-0.025} \left[1 + VF\left(\frac{\rho_g}{\rho_l} - 1\right)\right]^{0.8} \left[1 + x\left(\frac{\rho_l}{\rho_g} - 1\right)\right]^{0.8} \left[1 + VF\left(\frac{\mu_g}{\mu_l} - 1\right)\right]^{0.2} \quad (641)$$

where VF refers to void fraction. The main experimental parameters for the vertical helical coil test were:

$$18\ mm\ ID$$

$$235, 446\ and\ 907\ mm = D$$

$$0.02 < \delta < 0.076$$

$$4.2 < p < 22\ MPa$$

$$400 < G < 2000$$

$$0 < x < 1$$

4.2.2.5 Nariai et al. (1982)

In 1982 H. Nariai et al. recommended a correlation derived based on the widely used two-phase flow pressure drop correlations for straight tubes ($p < 3.5\ MPa$ and $d \geq 12\ mm$) [258] [374]. Pressure drop increases with mass flux and vapour quality:

$$\Delta P_{f,2\phi} = R_{MN} \Delta P_l \quad (642)$$

$$R_{MN} = (1 - x)^{1.75}\ \phi_{l,tt}^2 = \phi_l^2(P, x) \quad (643)$$

The experimental values for ϕ_l^2 were given in a table as a function of the system pressure P and the vapour quality x. As for $\phi_{g,tt}^2$, Kozeki et al. [367] prediction was found to be better:

$$\phi_{g,tt}^2 = 0.895 + (X_{tt} + 0.076)^{0.875} + 1.21 \cdot 10^{-0.334(\log X_{tt} + 0.668)^2} \tag{644}$$

$$\phi_{l,tt} = \frac{\phi_{g,tt}}{X_{tt}^{0.875}} \tag{645}$$

The main experimental parameters for the vertical helical coil test were:

$$14.3 \ and \ 20 \ mm \ ID$$

$$D = 595 \ mm$$

$$0.024 < \delta < 0.034$$

$$2 < p < 3.5 \ MPa$$

$$0.7 \cdot 10^5 < q < 1.8 \cdot 10^5$$

$$150 < G < 850$$

$$0.1 < x < 0.9$$

4.2.2.6 Guo et al. (1994)

In 1994 L.J. Guo published a correlation for large tube diameters ($d \geq 12 \ mm$) [258] [375]:

$$\phi_l^2 = 1 + (4.25 - 2.55x^{1.5})G^{0.34} \tag{646}$$

The main experimental parameters for the horizontal helical coil test were:

$$20 \ mm \ ID$$

$$240, 480 \ and \ 960 \ mm = D$$

$$0.021 < \delta < 0.083$$

$$1.5 < p < 3 \ MPa$$

$$150 < G < 1400$$

$$0 < x < 0.8$$

4.2.2.7 Bi et al. (1994)

In 1994 Q.C. Bi et al. recommended the following correlation for small tube and helix diameters ($d < 12 \ mm$) [258] [376]:

$$\phi_l^2 = 1 + \left(\frac{\rho_l}{\rho_g} - 1\right)(C + x^2) \tag{647}$$

$$C = 0.14691x^{1.3297}(1 - x)^{0.59884}\delta^{-1.2864} \tag{648}$$

They concluded that coil orientation had no significant effect on the two-phase frictional pressure drop. The two-phase frictional pressure drop was not influenced by the conditions of the thermodynamic system, i.e. adiabatic or electrically heated tubes. The main experimental parameters for both horizontal and vertical helical coil tests were:

$$10 \ and \ 12 \ mm \ ID$$

$$D = 115 \ mm$$

$$0.087 < \delta < 0.104$$
$$4 < p < 14\ MPa$$
$$0 < q < 750$$
$$400 < G < 2000$$
$$0 < x < 1$$

4.2.2.8 Awwad et al. (1995)

Awwad et al. [377] (in 1995) and Xin et al. [289] (in 1997) studied the air-water two-phase flow in horizontal and vertical helical pipes, respectively. Four different inside diameters of pipes and two different outside diameters of the cylindrical concrete forms were used for the various configurations of the helical pipe [275]. For horizontal helical pipes they concluded that the superficial velocities of air or water had a significant influence on the pressure drop multiplier, while the helix angle had insignificant effect. As for the pipe and coil diameters, they had a certain influence only at low flow rates. For vertical helical pipes at low flow rates in small aspect ratios, both the Lockhart–Martinelli multiplier and the flow rates affected the two-phase pressure drop. The void fraction was influenced by geometric parameters affecting the frictional pressure drop. Based on their experimental data for horizontal helical pipes, correlations of the frictional pressure drop multiplier for two-phase flow were proposed as follows [258] [369]:

$$\phi_l = \left(1 + \frac{x}{CF_d^n}\right)\left(1 + \frac{12}{X} + \frac{1}{X}\right)^{0.5} \tag{649}$$

$$F_d = Fr\left(\frac{d}{D}\right)^{0.1} \tag{650}$$

$$\begin{aligned} &For\ F_d \leq 0.3{:}\ C = 7.79, n = 0.576 \\ &For\ F_d > 0.3{:}\ C = 13.56, n = 1.3 \end{aligned} \tag{651}$$

4.2.2.9 Xin et al. (1997)

In 1997 R.C. Xin et al. recommended [275] [289] [369] the following correlation for vertical helical pipes:

$$\phi_l = \left(1 + \frac{X}{CF_d^n}\right)\left(1 + \frac{20}{X} + \frac{1}{X}\right)^{0.5} \tag{652}$$

$$F_d = Fr\left(\frac{d}{D}\right)^{0.5}(1 + \tan\beta)^{0.2} \tag{653}$$

$$\begin{aligned} &For\ F_d \leq 0.1{:}\ C = 65.45, n = 0.6 \\ &For\ F_d > 0.1{:}\ C = 434.8, n = 1.7 \end{aligned} \tag{654}$$

where d=19.1 mm, D=340 mm and $\beta = 0.5°$.

4.2.2.10 Ju et al. (2001)

In 2001 H. Ju et al. determined the two-phase flow pressure drop in small bending radius helical coil-pipe used in an HTR-10 steam generator [258] [275] [284]. Based on the uniform flow formula with a correction factor, a formula for frictional pressure drop was proposed:

$$\Delta P_{f,2\phi} = f\left(\frac{L}{d}\right)\left(\frac{\rho v^2}{2}\right)\left[1 + x\left(\frac{\rho_g}{\rho_l - 1}\right)\right]\Psi \quad (655)$$

$$\Psi = (1.29 + A_n x^n)\left[1 + x\left[\left(\frac{\mu_l}{\mu_g}\right)^{0.25} - 1\right]\right] \quad (656)$$

$$A_1 = 2.19\ A_2 = -3.61\ A_3 = 7.35\ A_4 = -5.93 \quad (657)$$

where: f is the friction factor, v is the flow velocity, x the average steam content, Ψ the unevenness correction factor. It is valid for $2.5 < P < 4.5\ MPa$ and $8 < \frac{D}{d} < 9.3$.

The main experimental parameters for the helical coil tests were:

$$18\ mm\ OD$$
$$D = 112\ mm$$
$$\delta = 0.161$$
$$p = 3\ MPa$$
$$2500 < Re < 23000$$
$$0 < x < 1$$

4.2.2.11 Guo et al. (2001)

In 2001 L. Guo et al. studied the pressure drops of steam–water two-phase flows in two helical coiled tubes with four different helix-axial inclinations. The results showed that the system pressure and mass quality had significant effect on the two-phase pressure drop, as well as the coil orientation. They recommended a correlation based on the work of Chen for boiling two-phase flow frictional pressure drop in helical coiled tubes [258] [275] [290] [369]:

$$\phi_l^2 = \psi_1 \psi\left[1 + x\left(\frac{\rho_l}{\rho_g} - 1\right)\right] \quad (658)$$

$$\psi = 1 + \frac{x(1-x)\left(\frac{1000}{G} - 1\right)\frac{\rho_l}{\rho_g}}{1 + x\left(\frac{\rho_l}{\rho_g} - 1\right)} \qquad \text{for } G \leq 1000 \quad (659)$$

$$\psi = 1 + \frac{x(1-x)\left(\frac{1000}{G} - 1\right)\frac{\rho_l}{\rho_g}}{1 + (1-x)\left(\frac{\rho_l}{\rho_g} - 1\right)} \qquad \text{for } G > 1000 \quad (660)$$

$$\psi_1 = 142.2\left(\frac{P}{P_{crit}}\right)^{0.62}\delta^{1.04} \quad (661)$$

$$\delta = \frac{d}{D}$$

The main experimental parameters for both horizontal/vertical and inclined helical coil tests were:

$$10, 11\ mm\ ID$$

$$D = 132\ and\ 256\ mm$$

$$0.043 < \delta < 0.076$$

$$3 < p < 3.5\ MPa$$

$$0 < q < 540$$

$$150 < G < 1760$$

$$0.01 < x < 1.2$$

4.2.2.12 Zhao et al. (2003)

In 2003 L. Zhao et al. recommended [258] [369] [371] a correlation for small tube and helix diameters ($d < 12\ mm$):

$$\phi_{lo}^2 = 1 + \left(\frac{\rho_l}{\rho_g} - 1\right)[0.303x^{1.63}(1-x)^{0.885} Re_{lo}^{0.282} + x^2] \tag{662}$$

Frictional pressure drop is a function of the mass flux, vapour quality and the system pressure. Heat flux has no effect on the pressure drop. The main experimental parameters for the horizontal helical coil tests were:

$$9mm\ ID$$

$$D = 292\ mm$$

$$\delta = 0.031$$

$$0.5 < p < 3.5\ MPa$$

$$0 < q < 900$$

$$236 < G < 943$$

$$10000 < Re < 80000$$

$$0.1 < x < 0.2$$

4.2.2.13 Cioncolini et al. (2008)

A. Cioncolini et al. presented in 2008 a correlation for small tube and helix diameters ($d < 12\ mm$) [258][379], where they proposed a correction factor:

$$\phi_{lo}^2 = \left[1 + \frac{C}{X_{tt}} + \frac{1}{X_{tt}^2}\right]\left[1 + 0.0044\left(\frac{q}{G}\right)^{0.7}\right] \tag{663}$$

They observed a minimal effect of the coil curvature on the frictional pressure drop. It corresponds to the Lockhart and Martinelli correlation for straight tubes corrected for heating effects. The main experimental parameters for vertical helical coil tests with saturated flow boiling were:

$$4.03\ and\ 4.98\ mm\ ID$$
$$130 < D < 376$$
$$0.011 < \delta < 0.038$$
$$120 < p < 660\ kPa$$
$$50 < q < 440$$
$$290 < G < 690$$
$$10000 < Re < 60000$$
$$2 < Fr < 14$$
$$0 < x < 0.9$$

4.2.2.14 Santini et al. (2008)

In 2008 L. Santini et al. recommended a correlation for high system pressures ($p > 3.5\ MPa$) [258] [369] [370]:

$$\Delta P_{f,2\phi} = K(x)\frac{G^{1.91}v_m}{d^{1.2}}\Delta z \tag{664}$$

$$K(x) = -0.0373x^3 + 0.0387x^2 - 0.00479x + 0.0108 \tag{665}$$

The authors concluded that the frictional pressure drop increases with the vapour quality and mass flux, while it decreases with the system pressure. The main experimental parameters for vertical helical coil tests were:

$$12.53\ mm\ ID$$
$$D = 1000\ mm$$
$$\delta = 0.019$$
$$1.1 < p < 6.3\ MPa$$
$$50 < q < 200$$
$$192 < G < 824$$
$$0 < x < 1$$

4.2.2.15 Colombo et al. (2015)

In 2015 M. Colombo et al. proposed a correlation for high system pressures ($p > 3.5\ MPa$), derived from the Lockhart and Martinelli equation for straight tubes [258] [369]:

$$\phi_l^2 = 0.0986\phi_{LM}^2 De_l^{0.19}\left(\frac{\rho_m}{\rho_l}\right)^{-0.40} \tag{666}$$

The main experimental parameters for vertical helical coil tests were:

$$9\ and\ 12.53\ mm = ID$$
$$292\ and\ 1000\ mm = D$$
$$0.013\ and\ 0.031 = \delta$$
$$5 < p < 6.5\ MPa$$

$$0 < q < 900$$
$$200 < G < 800$$
$$0 < x < 1$$

4.2.2.16 Summary of friction factor correlations for two-phase flow in helical and curve pipes

Table 39 presents the list of all friction factor correlations collected for two-phase flow in helical and curve pipes.

TABLE 39. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN HELICAL AND CURVE PIPES

Kozeki et al. (1970) [258] [367]	$\phi_{g,tt}^2 = 0.895 + (X_{tt} + 0.076)^{0.875} + 1.21 \cdot 10^{-0.334(\log X_{tt}+0.668)^2}$ $\phi_{l,tt} = \frac{\phi_{g,tt}}{X_{tt}^{0.875}}$ $0.032 < \delta < 0.035$, $0.5 < P < 2.1 MPa$, $151 < q < 348$, $161 < G < 486$, $21.7\ mm\ OD$, $628 < D < 682\ mm$, $0 < x < 1$ separated flow model
Ruffell (1974) [258] [368] [369]	$\phi_l^2 = (1 + F)\frac{\rho_l}{\rho_{2\phi}}$ $F = \sin\frac{1.16G}{1000}\left[0.875 - 0.314y - \frac{0.74G}{1000}(0.152 - 0.07y) - x\left(\frac{0.155G}{1000} + 0.7 - 0.19y\right)\right][1 - 12(x - 0.3)(x - 0.4)(x - 0.5)(x - 0.6)]$ $y = \frac{D}{100d}$ $10.7 < ID < 18.6\ mm$, $0.0054 < \delta < 0.16$, $6 < P < 18\ MPa$, $41 < q < 731$, $300 < G < 1800$, $0 < x < 1$ separated flow model
Unal et al. (1981) [258] [372]	$\Delta P_{f,2\phi} = \frac{2(1 + b_1 b_2) f_l G^2}{d\rho_l}$ $b_1 = 3850x^{0.01} Pr^{-1.515} Re_l^{-0.758}$ $b_2 = 1 + Re_l^{0.1}(3.67 - 3.04P_b)^{-0.014\delta^{-1} - 2\delta^{-1}}$ $P_b = \frac{P}{P_{crit}}$ $f_l = 0.304Re^{-0.25} + 0.029\delta^{0.5}$ $18\ mm\ ID$, $700\ \&\ 1500\ mm = D$, $0.0054 < \delta < 0.022$, $14.7 < P < 20.2\ MPa$, $41 < q < 731$, $112 < G < 1829$, $0.08 < x < 1$

TABLE 39. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN HELICAL AND CURVE PIPES

Chen and Zhou (1981) [258] [373]	$\Delta P_{f,2\phi} = \xi \Delta P_l$ $\xi = 2.06\delta^{0.05} Re_{2\phi}^{-0.025} \left[1 + VF\left(\frac{\rho_g}{\rho_l} - 1\right)\right]^{0.8} \left[1 + x\left(\frac{\rho_l}{\rho_g} - 1\right)\right]^{0.8} \cdot \left[1 + VF\left(\frac{\mu_g}{\mu_l} - 1\right)\right]^{0.2}$ $18\ mm\ ID$, $235, 446, 907\ mm = D$, $0.02 < \delta < 0.076$, $4.2 < P < 22\ MPa$, $400 < G < 2000$, $0 < x < 1$
Nariai et al. (1982) [258] [367] [374]	$\Delta P_{f,2\phi} = R_{MN} \Delta P_l$ $R_{MN} = (1 - x)^{1.75}\ \phi_{l,tt}^2 = \phi_l^2(P, x)$ $\phi_{g,tt}^2 = 0.895 + (X_{tt} + 0.076)^{0.875} + 1.21 \cdot 10^{-0.334(\log X_{tt} + 0.668)^2}$ $\phi_{l,tt} = \frac{\phi_{g,tt}}{X_{tt}^{0.875}}$ $14.3, 20\ mm\ ID$, $D = 595mm$, $0.024 < \delta < 0.034$, $2 < P < 3.5\ MPa$, $0.7 \cdot 10^5 < q < 1.8 \cdot 10^5$, $150 < G < 850$, $0.1 < x < 0.9$
Guo et al. (1994) [258] [375]	$\phi_l^2 = 1 + (4.25 - 2.55x^{1.5})G^{0.34}$ $20\ mm\ ID$, $240, 480, 960\ mm = D$, $0.021 < \delta < 0.083$, $1.5 < P < 3\ MPa$, $150 < G < 1400$, $0 < x < 0.8$
Bi et al. (1994) [258] [376]	Chisholm functional relationship $\phi_l^2 = 1 + \left(\frac{\rho_l}{\rho_g} - 1\right)(C + x^2)$ $C = 0.14691x^{1.3297}(1 - x)^{0.59884}\delta^{-1.2864}$ $10, 12\ mm\ ID$, $D = 115mm$, $0.087 < \delta < 0.104$, $4 < P < 14MPa$, $0 < q < 750$, $400 < G < 2000$, $0 < x < 1$
Awwad et al. (1995) [258] [369] [377]	$\phi_l = \left(1 + \frac{x}{CF_d^n}\right)\left(1 + \frac{12}{X} + \frac{1}{X}\right)^{0.5}$ $F_d = Fr\left(\frac{d}{D}\right)^{0.1}$ $F_d \leq 0.3$: $C = 7.79, n = 0.576$ $F_d > 0.3$: $C = 13.56, n = 1.3$ horizontal helical pipes

TABLE 39. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN HELICAL AND CURVE PIPES

Xin et al. (1997) [275] [289] [369]	$\phi_l = \left(1 + \frac{X}{CF_d^n}\right)\left(1 + \frac{20}{X} + \frac{1}{X}\right)^{0.5}$ $F_d = Fr\left(\frac{d}{D}\right)^{0.5}(1 + \tan\beta)^{0.2}$ $F_d \leq 0.1: C = 65.45, n = 0.6$ $F_d > 0.1: C = 434.8, n = 1.7$ d=19.1 mm, D=340 mm and $\beta = 0.5°$ vertical helical pipes
Ju et al. (2001) [258] [275] [284]	$\Delta P_{f,2\phi} = f\left(\frac{L}{d}\right)\left(\frac{\rho v^2}{2}\right)\left[1 + x\left(\frac{\rho_g}{\rho_l - 1}\right)\right]\Psi$ $\Psi = (1.29 + A_n x^n)\left[1 + x\left[\left(\frac{\mu_l}{\mu_g}\right)^{0.25} - 1\right]\right]$ $A_1 = 2.19\ A_2 = -3.61\ A_3 = 7.35\ A_4 = -5.93$ $2.5 < P < 4.5\ MPa$, $8 < \frac{D}{d} < 9.3$, $18\ mm\ OD$, $D = 112\ mm$, $\delta = 0.161$, $P = 3\ MPa$, $2500 < Re < 23000$, $0 < x < 1$
Guo et al. (2001) [258] [275] [290] [369]	$\phi_l^2 = \psi_1 \psi\left[1 + x\left(\frac{\rho_l}{\rho_g} - 1\right)\right]$ For $G \leq 1000\ \psi = 1 + \frac{x(1-x)\left(\frac{1000}{G} - 1\right)\frac{\rho_l}{\rho_g}}{1 + x\left(\frac{\rho_l}{\rho_g} - 1\right)}$ For $G > 1000\ \psi = 1 + \frac{x(1-x)\left(\frac{1000}{G} - 1\right)\frac{\rho_l}{\rho_g}}{1 + (1-x)\left(\frac{\rho_l}{\rho_g} - 1\right)}$ $\psi_1 = 142.2\left(\frac{P}{P_{crit}}\right)^{0.62}\delta^{1.04}$ $\delta = \frac{d}{D}$ $10, 11\ mm\ ID$, $D = 132, 256\ mm$, $0.043 < \delta < 0.076$, $3 < P < 3.5\ MPa$, $0 < q < 540$, $150 < G < 1760$, $0.01 < x < 1.2$
Zhao et al. (2003) [258] [369] [371]	$\phi_{l0}^2 = 1 + \left(\frac{\rho_l}{\rho_g} - 1\right)[0.303x^{1.63}(1 - x)^{0.885}Re_{lo}^{0.282} + x^2]$ $9mm\ ID$, $D = 292\ mm$, $\delta = 0.031$, $0.5 < P < 3.5\ MPa$, $0 < q < 900$, $236 < G < 943$, $10000 < Re < 80000$, $0.1 < x < 0.2$

TABLE 39. SUMMARY OF FRICTION FACTOR CORRELATIONS FOR TWO-PHASE FLOW IN HELICAL AND CURVE PIPES

Cioncolini et al. (2008) [258] [379]	Chisholm functional relationship $\phi_{l0}^2 = \left[1 + \frac{C}{X_{tt}} + \frac{1}{X_{tt}^2}\right]\left[1 + 0.0044\left(\frac{q}{G}\right)^{0.7}\right]$ $4.03, 4.98\ mm\ ID,\ 130 < D < 376,\ 0.011 < \delta < 0.038,\ 120 < P < 660\ kPa,\ 50 < q < 440,\ 290 < G < 690,\ 10000 < Re < 60000,\ 2 < Fr < 14,\ 0 < x < 0.9$
Santini et al. (2008) [258] [369] [370]	$\Delta P_{f,2\phi} = K(x)\frac{G^{1.91}v_m}{d^{1.2}}\Delta z$ $K(x) = -0.0373x^3 + 0.0387x^2 - 0.00479x + 0.0108$ $12.53\ mm\ ID,\ D = 1000\ mm,\ \delta = 0.019,\ 1.1 < P < 6.3\ MPa,\ 50 < q < 200,\ 192 < G < 824,\ 0 < x < 1$
Colombo et al. (2015) [258] [369]	$\phi_l^2 = 0.0986\phi_{LM}^2 De_l^{0.19}\left(\frac{\rho_m}{\rho_l}\right)^{-0.40}$ $9, 12.53\ mm = ID,\ 292, 1000\ mm = D,\ 0.013, 0.031 = \delta,$ $5 < P < 6.5\ MPa,\ 0 < q < 900,\ 200 < G < 800,\ 0 < x < 1$

4.2.3 Flow in rod bundles

The two-phase pressure loss due to local flow disturbances, such as grid spacers is treated as frictional pressure losses [227]. Thus, as in Eq. (520), the corresponding single-phase pressure drop is corrected using an appropriate two-phase multiplier to yield the local two-phase pressure drop:

$$\Delta P_{2\phi_{local}} = K_{1\phi}\ \frac{\rho_l\, v_l^2}{2}\ \Phi \qquad (667)$$

where Φ is the two-phase local loss multiplier.

Figure 63 shows a typical representation of the hexagonal fuel assembly.

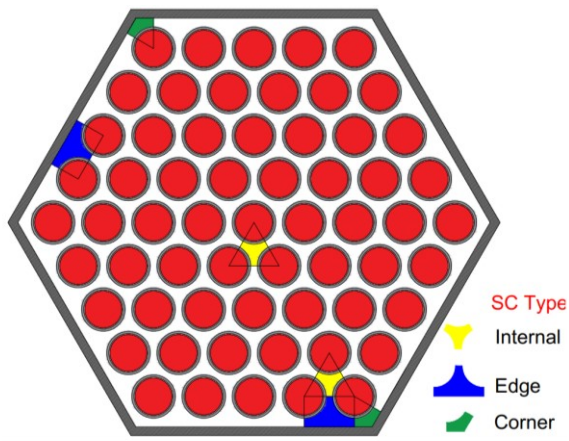


FIG. 63. Hexagonal fuel assembly

4.2.3.1 Kaiser et al. (1988)

A. Kaiser et al. (1988) correlation [380] was obtained from quasi steady-state sodium boiling experiments carried out in an electrically heated 7-pin test section. The two-phase pressure multiplier ϕ, related to the liquid phase, is calculated using the equation:

$$\ln \phi = 1.48 - 1.05\, ln \sqrt{X_{LM}} + 0.09 \left(\ln \sqrt{X_{LM}}\right)^2 \tag{668}$$

where X_{LM} is the Lockhart-Martinelli parameter. This correlation is valid for rod bundles.

4.2.4 Interfacial friction correlations

4.2.4.1 Wallis (1969)

A correlation for the interfacial friction factor of thin annular flows in pipes was proposed by G.B. Wallis in 1969 [381]. For a ratio of a film thickness to diameter $\frac{\delta}{D}$ lower than 0.04 (corresponding to $x \geq 0.8464$), experiments by Martinelli, Dukler, Sze Foo Chien and Charvonia cluster around the relationship:

$$\left(C_f\right)_i = 0.005 \left(1 + 300\frac{\delta}{D}\right) = 0.005\left(1 + 150(1 - \sqrt{x})\right) \tag{669}$$

where x is the vapour volume fraction.

Given the high liquid-to-vapour density ratio of sodium $\left({}^{\rho_l}/_{\rho_v} \approx 2000\ at\ 1\ bar\right)$, the annular flow regimes covered by this correlation are encountered in most sodium boiling cases. However, one should note that it does not take into account the droplet entrainment and deposition phenomena that may occur at even higher x values.

Wallis correlation is used to calculate the interfacial friction between liquid and vapour. This correlation is based on the assumption that annular flow is the main flow regime when the two phases co-exist. One should note that this correlation is valid only for flow in tubes. The use of Wallis correlation in a rod bundle overestimates the interfacial friction.

Wallis correlation permits avoiding discontinuity problems when moving from co-current to counter-current flows. Hence, this correlation is more preferred than Lockhart and Martinelli interfacial friction correlation, which is available only in co-current flow situations. In SAS4A code the two-phase friction factor multiplier used is based on a correlation by Wallis.

4.3 FRICTION FACTOR CORRELATIONS USED IN THE SYSTEM CODES

Below in Table 40 the information is summarized as to what friction factor correlations are being used nowadays across the system codes used by experts performing safety analysis of the sodium cooled fast reactors.

TABLE 40. FRICTION FACTOR CORRELATIONS USED IN THE SYSTEM CODES

CODE	**PIPE**			**ROD BUNDLE**
	Laminar	**Transition**	**Turbulent**	
SAM	Darcy	Reciprocal Interp.	Blasius-McAdams	--
RELAP 5 & 7	Darcy	Reciprocal Interp.	Zigrang-Sylvester	Cheng-Todreas (1986) and Rehme (1973)
TRACE	Churchill (1977)			--
ANTEO+	--	--	Blasius (1913)	Cheng-Todreas (1986) and Rehme (1973)
ATHLET	Darcy	Max of lam. and turb.	Colebrook	--
ATHENA	Darcy	Reciprocal Interp.	Zigrang-Sylvester	--
CATHARE	Darcy	Max. of lam. and turb.	Blasius (1913)	Pontier-Combe (1968) and Rehme (1973)
HYDRA (IBRAE)	--	--	--	Zhukov (1986)
MARS-LMR	Darcy	MARS Interp.	Zigrang-Sylvester	Cheng-Todreas (1986)
SAS4A/SAS-SFR	Darcy	--	Blasius Moody (1947)	--

5 RECOMMENDATIONS FOR CHOOSING A CORRELATION AND RESEARCH GAPS IDENTIFIED

In the absence of experimental data, simple correlations derived from physical considerations should be preferred over complex correlations validated and used in other fields (such as two-phase correlations established for other coolants).

The use of more complex correlations should be justified by validation against experimental data. General reviews, like for instance the work performed by Bubelis and Schikorr [305], Chen and Todreas [304] and Kottowski-Dumenil et al. [328] for friction factor correlations, can help in comparing correlations and making a selection among them. However, in some important cases, such as pressure drop prediction, the best available correlations still exhibit a high uncertainty (around $10 - 15\%$). In this case, validation against a dedicated experiment (such as a $1:1$ sub-assembly mockup) seems indispensable for further reducing this uncertainty. For pressure drop measurements in these mockups, sodium could be replaced by other fluids, even water, provided that physical properties at stake (density ratio, surface tension, viscosity, etc.) are similar.

The choice of liquid single-phase heat transfer correlation among those proposed for a circular tube has a small impact on a simulation since the exchange in forced convection liquid sodium is anyway particularly efficient. Moreover, specific correlations for rod bundle are similar to those for a circular tube, so both could give quite similar results. For instance, Borishanskii's correlation for rod bundles gives Nusselt values quite near to that of Skupinski's correlation: at same Peclet number, $Nu(Borishanskii) \approx Nu(Skupinski) + 1$, while minimum value is already as big as 4.82.

For consistency it is recommended to use the turbulent Prandtl number and the heat transfer correlation from the same source.

More specifically, the following issues normally should be taken into account when choosing a correlation:

- Type of fluid: metallic, non-metallic, oils (Prandtl number is discriminator);
- Internal vs. external flow;
- Laminar vs. turbulent vs. transition flow or special flow conditions: e.g. slug flow, etc.;
- Geometry: circular vs. non-circular; bundle; wire wrap, and others;
- Boundary conditions: uniform wall temperature or uniform heat flux;
- Entry region vs. fully developed flow;
- Buoyancy effects;
- Heated or cooled conditions: e.g. in Dittus-Boelter correlations Eq. (49) exponents of Prandtl number are different;
- General vs. local conditions: boiling, special experiments on wetting surfaces, film detachment;
- Check if additional special conditions of applicability are noted, such as surface conditions, etc.;
- For experimentalist it may be relevant to know with which metal experiments were performed and the data obtained in order to arrive at a given correlation (sodium, mercury, etc.). This may be a useful information for reproducibility of the results;
- Also, experimentalist may be interested in a correlation where deviation from actual experimental data is explicitly declared;
- Choose a correlation within the range of its applicability. Avoid extrapolation outside the assigned ranges.

Apart from all that, important information is available also in the reviewing publications where different correlations are discussed and compared. Such reviewing publications are covering different topics: general information on SFRs (Tenchine [307]), heat transfer for turbulent flow in a pipe (Lee [91] and Pacio et al. [382]), heat transfer in rod bundles (Mikityuk [137]), pressure drop in wire-wrapped hexagonal array pin bundles (Bubelis and Schikorr [305] [383] and Chen and Todreas [304]), two-phase friction factors in a single channel (Kottowski-Dumenil et al. [328]), two-phase friction pressure drop and boiling heat transfer coefficient in annulus (Qiu et al. [208]), etc. Important information might be found also in the books covering heat transfer, fluid mechanics, thermal-hydraulics, etc. of liquid metal fast reactors.

As an additional source of data nowadays, also data from high-fidelity simulations (e.g. direct numerical simulations, DNS) can be considered, especially with respect to heat transfer data. Such data is specifically valuable, as it allows a detailed assessment of every location in a liquid metal flow, something which is hardly achievable in a physical experiment. Apart from that, if well performed, the data is at least as accurate as experimental data. But of course, the restriction 'if well performed' is also valid for physical experiments and is crucial. Some examples of the generation of high-fidelity data can be found in Tiselj et al. [384], Bricteux et al. [385], Duponcheel et al. [386] and Errico et al. [387]. On the other hand, Shams et al. [388] shows the application of such high-fidelity data to create new heat transfer models. Finally, Roelofs et al. [389] provides an outlook on new experimental and high-fidelity simulation data to be generated within the coming few years.

It should be realized that over time new experimental data and high-fidelity numerical data will be added to the reference database. It is obvious that for all data, the reliability is very important. However, very often it is hard to judge on the reliability of the data, while the reviewer cannot be aware of all the details of the work, especially if the work has been performed in the far past. This makes comparison of data from different sources complicated, if not impossible at all. Nevertheless, in this Handbook an attempt was made to collect the most important available data and to put forward some recommendations.

Assessment of the single-phase friction factor for turbulent pipe flow and evaluation of existing single-phase friction factor correlations can be found in Fang et al. [233]. As for the two-phase friction pressure drop correlations an extended evaluation can be found in Xu et al. [234] [347].

As for the experimental studies on the air–water frictional pressure drop characteristics in helically coiled tubes an extended review can be found in Fsadni et al. [258].

The following items have been identified as conditions of phenomena which should be further investigated experimentally:

- Sodium boiling under reactor accidental conditions (both pool and flow boiling regimes and characteristics)
- Impact of the presence of oxide in sodium on friction and heat transfer
- Friction under highly transient conditions. In particular, there is the need for information on how friction may differ when sodium in a channel is suddenly accelerated compared with the use of steady state frictional correlations. Similar for heat transfer.
- Transient wetting of stainless-steels, Inconel and ferritic steels by sodium. How do the wetting phenomena differ with respect to adherence of sodium to the structure, electrical continuity/resistance, acoustic effects? How is wetting affected by temperature, oxides, dissolved gas, other contaminants?
- Sodium Jet Dynamics: Liquid Splashing on Solid; Liquid into Pool; Liquid Jets; Jet Breakup

6 CONCLUSIONS

The key objective of the Sodium Coolant Handbooks on “Thermal-Hydraulic Correlations” is to provide an organized collection of thermal hydraulic correlations, such as heat transfer coefficients and friction factors for the typical design geometries utilized in sodium cooled fast reactors. The Handbook is one of the outputs of the IAEA NAPRO coordinated research project (CRP). More than 400 original publications have been examined and more than 600 experimental correlations obtained at the liquid metal experimental facilities are presented in the Handbook. Approximately half of correlations is used for calculation of the friction factors and pressure drops, and another half for the heat transfer coefficients. Different geometries have been examined including circular pipes, flat plates, rectangular ducts, concentric annuli, hexagonal and squared rod bundles, helical pipes, and others, as well as different flow conditions, such as laminar and turbulent flow, natural circulation and forced convection, fuel assemblies with smooth and wire-wrapped pins, purified and non-purified sodium, single and two-phase flow, etc.

Since the full assessment of the empirical correlations against experimental data cannot be performed in the framework of the NAPRO CRP, each correlation has been briefly presented as proposed in the original publication including the range of validity and the uncertainty, when available. In addition, heat transfer and pressure drop correlations used in the system codes for safety analysis of the sodium cooled fast reactors are introduced in Sections 3.6 and 4.3. As a final contribution, Section 5 provides useful information concerning important aspects to consider when choosing a correlation and introduces reviewing publications that assess different correlations.

Despite of the fact that thermal-hydraulics correlations for sodium facilities are commonly considered as ‘established’, inconsistencies and gaps have been identified by the participants of the NAPRO CRP. Several items have been identified as conditions of phenomena which should be further investigated experimentally.

Participants have advised the IAEA on further actions aimed toward the preservation of knowledge on thermal hydraulic correlations for sodium facilities and the reactors. Integration of the empirical correlations in the online calculation tool would add a new valuable dimension to the data presented in the Handbook.

The exchange of data and information among international partners have demonstrated high efficiency and it is anticipated that spirit of collaboration will continue beyond the completion of the NAPRO CRP.

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LIST OF ABBREVIATIONS

ANL	Argonne National Laboratory, USA
CEA	French Alternative Energies and Atomic Energy Commission (Commissariat à l'énergie Atomique et aux Énergies Alternatives)
CFD	Computational fluid dynamics
CFX	High-performance CFD software by ANSYS company
CIAE	China Institute of Atomic Energy
CNEA	National Atomic Energy Commission, Argentina
CRP	Coordinated research project
DHX	Decay heat exchanger
DT	Developing thermal region
DTV	Developing thermal and velocity region
ENEA	Italian National Agency for New Technologies, Energy and Sustainable Economic Development
EoS	Equation of State
FBR	Fast breeder reactor
HZDR	Helmholtz Zentrum Dresden Rossendorf
IAEA	International Atomic Energy Agency
IBRAE	Nuclear Safety Institute, Russia
IGCAR	Indira Gandhi Centre for Atomic Research, India
IPPE	Institute of Physics and Power Engineering -State Science Centre of the Russian Federation
IHX	Intermediate heat exchanger
JAEA	Japan Atomic Energy Agency
KAERI	Korea Atomic Energy Research Institute
KIT	Karlsruhe Institute of Technology, Germany
LBE	Lead bismuth eutectic

LIST OF ABBREVIATIONS

LMFR	Liquid metal cooled fast reactor
LWR	Light water reactor
NAPRO	IAEA Coordinated research project on “Sodium properties and safe operation of experimental facilities in support of the development and deployment of sodium cooled fast reactors”
NRG	Nuclear Research and Consultancy Group, Netherlands
RCM	Research coordination meeting
SA, S/A	Sub-assembly
SFR	Sodium cooled fast reactor
TWG-FR	IAEA technical working group on fast reactors
WP	Work package

CONTRIBUTORS TO DRAFTING AND REVIEW

Anderhuber, M.	Centre d'études nucléaires de Cadarache (CEA), France
Athmalingam, S.	Indira Gandhi Centre for Atomic Research (IGCAR) , India
Batra, Ch.	International Atomic Energy Agency (IAEA)
Bubelis, E.	Karlsruhe Institute of Technology (KIT), Germany
Cachon, L.	Centre d'études nucléaires de Cadarache (CEA), France
Gerardi, C.	Argonne National Laboratory (ANL), USA
Gerschenfeld, A.	Centre d'études nucléaires de Cadarache (CEA), France
Hu, R.	Argonne National Laboratory (ANL), USA
Jayaraju, S.T.	Nuclear Research and Consultancy Group (NRG), The Netherlands
Khoroshev, M.	International Atomic Energy Agency (IAEA)
Kuzina, J.	Institute for Physics and Power Engineering (IPPE), Russian Federation
Kriventsev, V.	International Atomic Energy Agency (IAEA)
Lee, Y-B.	Korea Atomic Energy Research Institute (KAERI), Republic of Korea
Li, L.	International Atomic Energy Agency (IAEA)
Matteo. L.	Centre d'études nucléaires de Cadarache (CEA), France
Mahanes, J.	International Atomic Energy Agency (IAEA)
Monti, S.	International Atomic Energy Agency (IAEA)
Mui, T.	Argonne National Laboratory (ANL), USA
Morelova, N.	International Atomic Energy Agency (IAEA)
Ohira, H.	Japan Atomic Energy Agency (JAEA), Japan
Otlik, K.	International Atomic Energy Agency (IAEA)
Passerini, S.	Argonne National Laboratory (ANL), USA
Perez-Martin, S.	Karlsruhe Institute of Technology (KIT), Germany
Perumal, S.	Indira Gandhi Centre for Atomic Research (IGCAR) , India
Roelofs, F.	Nuclear Research and Consultancy Group (NRG), The Netherlands
Selvaraj, A.	Indira Gandhi Centre for Atomic Research (IGCAR) , India
Selvaraj, P.	Indira Gandhi Centre for Atomic Research (IGCAR) , India
Slobodchouk, V.	Moscow Engineering Physics Institute (MEPhI), Russian Federation
Sorokin, A.	Institute for Physics and Power Engineering (IPPE), Russian Federation
Zeman, M.	International Atomic Energy Agency (IAEA)

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