Synergistic effects of magnetic and velocity shear on electromagnetic drift modes in tokamaks

A.K.Wang, Y.X.Long, Z.Z.Mou, and J.Q.Dong

Southwestern Institute of Physics, P.O.Box 432, Chengdu 610041, P.R.China
e-mail contact of main author: akwang@swip.ac.cn

Abstract: In the fluid model, electromagnetic drift mode in low β tokamak plasmas is studied from a set of new-derived eigenequations, including magnetic shear, perpendicular and parallel velocity shears, perpendicular and parallel current density and their shears, and finite β. It is found that there exists a threshold of perpendicular velocity shear, at which the growth rate tends to zero. The threshold increases with decrease of magnetic shear. On the other hand, the increase of β reduces the growth rate but increases the velocity shear threshold. In addition, we study the effects of parallel velocity shear on the instability and find that it enhances the instability. Furthermore, the preliminary calculations show that the perpendicular current density shear suppresses the instability while the parallel current density shear strengthens it.

It is commonly believed that the anomalous transports in tokamak plasmas are attributed to drift or drift-like instabilities driven by the gradients of plasma parameters. e.g. pressure gradient, magnetic curvature, and parallel velocity shear. Thus, it is important to understand the stabilizing mechanisms on the instabilities. The magnetic shear, perpendicular velocity shear, and finite β, which could suppress the instabilities, have widely been studied both experimentally [1] and theoretically [2-4]. But, understanding on them might be still incomplete. Therefore, it is necessary to study their properties and effects on the instabilities further.

We now study an electromagnetic perturbation of low β plasma in the usual circular cross-section tokamak with $B_z = B_0/(1+\varepsilon\cos\theta)$ and $B_\theta = (\varepsilon/q)B_z$, which varies as

$$f = \tilde{f}(r)\exp i[k_\theta r\theta + k_z z - \omega t], \tag{1}$$

where $k_\theta = m/r$, $k_z = -n/R$; m and n are the poloidal and toroidal mode number, and r and R ($r << R$) are the minor and major radius of plasma, respectively.

In the hydrodynamic regime of ions and electrons, our model consists of the continuity equations of ions and electrons, energy equation of ions, and the parallel motion equation of electrons, together with the quasineutrality condition ($n = n_i = n_e$),

$$(\mathbf{v}_{i//}-\mathbf{v}_{e//})\cdot\nabla n_e+\nabla\cdot[n_e(\mathbf{v}_{*pi}-\mathbf{v}_{*pe})]+\nabla\cdot(n_e\mathbf{v}_{pi}) \\ +n_e\nabla\cdot(\mathbf{v}_{i//}-\mathbf{v}_{e//})=0\,, \tag{2}$$

$$\frac{\partial}{\partial t}p_i+(\mathbf{v}_{i//}+\mathbf{v}_E)\cdot\nabla p_i+\frac{5}{3}p_i\nabla\cdot(\mathbf{v}_{i//}+\mathbf{v}_E)+ \\ \frac{5}{3}\mathbf{v}_{Di}\cdot\nabla p_i+\frac{5}{3}n_i\mathbf{v}_{Di}\cdot\nabla T_i+\frac{5}{3}p_i\nabla\cdot\mathbf{v}_{pi}=0 \,, \tag{3}$$

$$e(\nabla_{//}\phi+\frac{1}{c}\frac{\partial\mathbf{A}_{//}}{\partial t}-\frac{1}{c}\mathbf{v}_e\times\mathbf{B}\cdot\hat{b})-\frac{1}{n_e}\nabla_{//}p_e=0\,, \tag{4}$$

where

$$\mathbf{v}_{i//}-\mathbf{v}_{e//}=\mathbf{j}_{//}/en_e\,, \tag{5}$$

$$\mathbf{j}_{//}=-\frac{c}{4\pi}(\nabla_r{}^2+\frac{\hat{r}}{r}\cdot\nabla_r+\nabla_\perp{}^2)\mathbf{A}_{//}\,, \tag{6}$$

$$\frac{\partial\mathbf{v}_{i//}}{\partial t}+\mathbf{v}_i\cdot\nabla\mathbf{v}_{i//}=-\frac{e}{m_i}[\nabla_{//}\phi+\frac{1}{c}(\frac{\partial\mathbf{A}_{//}}{\partial t}-\mathbf{v}_i\times\mathbf{B}\cdot\hat{b})]-\frac{1}{m_in_i}\nabla_{//}p_i\,, \tag{7}$$

in addition, the ion polarization drift is

$$\mathbf{v}_{pi}=-c(B\omega_{ci})^{-1}[\frac{\partial}{\partial t}(\nabla_r+\nabla_\perp)\phi+(\mathbf{v}_i\cdot\nabla)(\nabla_r+\nabla_\perp)\phi]\quad. \tag{8}$$

Here Eq.(2) is obtained by subtracting the continuity equation of electrons from that of ions with the quasi-neutrality condition $n=n_e=n_i$. In Eq.(3), the cancellation relation between the convective diamagnetic drift and diamagnetic heat flow of ions has been considered. Then, we obtain, respectively, from linearizing Eqs.(2)-(8),

$$(\mathbf{V}_{i//}-\mathbf{V}_{e//})\cdot\nabla\tilde{n}+\mathbf{V}_{Di}\cdot\nabla\tilde{p}_i-\mathbf{V}_{De}\cdot\nabla\tilde{p}_e+\delta(\nabla\cdot\mathbf{v}_{pi}) \\ +\nabla_{//}\cdot(\delta\mathbf{v}_{i//}-\delta\mathbf{v}_{e//})=0\,, \tag{9}$$

$$\frac{\partial \tilde{p}_i}{\partial t} + (\mathbf{V}_{i//} + \mathbf{V}_E) \cdot \nabla \tilde{p}_i + \tau^{-1}(\frac{5}{3}\mathbf{V}_{Di} - \mathbf{V}_{*pi}) \cdot \nabla \tilde{\phi} + \\ \frac{10}{3}\mathbf{V}_{Di} \cdot \nabla \tilde{p}_i - \frac{5}{3}\mathbf{V}_{Di} \cdot \nabla \tilde{n} + \frac{5}{3}\delta(\nabla \cdot \mathbf{v}_{pi}) = 0\,, \quad , \tag{10}$$

$$\tilde{n}_e = \tilde{p}_e = \tilde{\phi} - \frac{\Omega_x + \omega_{J\perp} + \mathbf{k}_{//} \cdot \mathbf{V}_i}{k_{//} c_s} \tilde{A}_{//}\,, \tag{11}$$

$$\nabla_{//} \cdot (\delta \mathbf{v}_{i//} - \delta \mathbf{v}_{e//}) = \nabla_{//} \cdot [\delta \mathbf{j}/(en)] - (\mathbf{V}_{i//} - \mathbf{V}_{e//}) \cdot \nabla_{//} \tilde{n}\,, \tag{12}$$

$$\delta \mathbf{j}_{//} / en_e = -\frac{c_s(1+\tau)}{4\pi\beta}(\frac{\partial^2}{\partial x^2} + \frac{1}{r_s}\frac{\partial}{\partial x} + \nabla_{\perp}{}^2)\tilde{\mathbf{A}}_{//} \quad , \tag{13}$$

$$\mathbf{k} \cdot \delta \mathbf{v}_{i//} = \frac{\tau k_{//}{}^2 c_s{}^2}{\Omega_x} \tilde{p}_i + \frac{k_{//}{}^2 c_s{}^2}{\Omega_x} \tilde{\phi} - k_{//} c_s (1 + \frac{k_{//} V_{i//}}{\Omega_x}) \tilde{A}_{//}\,, \tag{14}$$

$$\delta(\nabla \cdot \mathbf{v}_{pi}) = i\Omega_x (\frac{\partial^2}{\partial x^2} + \frac{1}{r_s}\frac{\partial}{\partial x} + \nabla_{\perp}{}^2)\tilde{\phi} \\ - i(k_{//}\frac{\partial V_{i//}}{\partial x} + k_{\perp}\frac{\partial V_{i\perp}}{\partial x})\frac{\partial}{\partial x}\tilde{\phi} \quad , \tag{15}$$

where $x = (r - r_s)/\rho$, $\tilde{A}_{//} = (c_s / c)(e\delta A_{//} / T_e)$, $\tilde{\phi} = e\delta\phi / T_e$, $\tilde{p}_i = \delta p_i / P_i$, $\tilde{p}_e = \delta p_e / P_e$, $\tilde{n} = \delta n / n$, $\tau = T_i / T_e$, $\rho = c(T_e m_i)^{1/2} / eB$, $\beta = n_e(T_i + T_e)/B^2$, and $\Omega_x = \omega - \mathbf{k} \cdot \mathbf{V}_i$. Here Eq.(11) is the adiabatic electron response and it reduces to the Eq.(10) in Ref.(5) when V_E and $V_{i//}$ are taken to zero. Subsequently, we expand the equilibrium quantity at the rational surface (r_s), e.g.,

$$\Omega_x = \Omega + \frac{k_\theta}{2mq} s s_{V//} V_{i//} x^2 - \frac{R k_\theta}{2m} s_{V\perp} V_{i\perp} x\,, \tag{16}$$

where $\Omega = \omega - \mathbf{k} \cdot \mathbf{V}_i(r_s)$, $V_{i//}(r_s)$ and $V_{i\perp}(r_s)$ are respectively the equilibrium parallel and perpendicular fluid velocity of ions, $s = (r_s / q)(\partial q / \partial r)$, $s_{V//} = (r_s / V_{i//})(\partial V_{i//} / \partial x)$, and $s_{V\perp} = (r_s / V_{i\perp})(\partial V_{i\perp} / \partial x)$. Here the frequency, speed, and spatial scale are normalized to $\omega_{De} = 2k_\perp T_e / eBR$, $c_s = \rho\omega_{ci}$, and ρ, respectively. Then, using the local approximation ($\theta = 0$) we obtain a set of eigenequations from Eqs.(9)-(15),

$$A_2 \frac{\partial^2 \tilde{\varphi}}{\partial x^2} + A_1 \frac{\partial \tilde{\varphi}}{\partial x} + A_0 \tilde{\varphi} + B_2 \frac{\partial^2 \tilde{A}_{//}}{\partial x^2} + B_1 \frac{\partial \tilde{A}_{//}}{\partial x} + B_0 \tilde{A}_{//} = 0 , \quad (17a)$$

$$C_2 \frac{\partial^2 \tilde{\varphi}}{\partial x^2} + C_1 \frac{\partial \tilde{\varphi}}{\partial x} + C_0 \tilde{\varphi} + D_2 \frac{\partial^2 \tilde{A}_{//}}{\partial x^2} + D_1 \frac{\partial \tilde{A}_{//}}{\partial x} + D_0 \tilde{A}_{//} = 0 . \quad (17b)$$

where

$$A_2 = -\frac{s^2}{4q^2} x^2 \quad (17c)$$

$$A_1 = -\frac{s^2 k_\theta}{4mq^2}[x^2 + \frac{(s_{V\perp} V_{i\perp} R - s s_{V//} V_{i//} x / q)}{2\Omega_x} x^2] \quad (17d)$$

$$A_0 = \Omega_x + \omega_{J\perp} + R/2L_{T_e} + \frac{s^2 k_\theta{}^2}{4q^2} x^2 \quad (17e)$$

$$B_2 = \frac{s(1+\tau)}{8\pi q\beta}(x - \frac{s^2}{4q^2\Omega_x} x^3) \quad (17f)$$

$$B_1 = B_2 \frac{k_\theta}{m} \quad (17g)$$

$$B_0 = -\frac{s}{2q}[x - \frac{sV_{i//}}{2q\Omega_x}(x^2 + \frac{k_\theta}{m} s_{V//} x^3)] - \frac{s(1+\tau)k_\theta{}^2}{8\pi q\beta} x + [\Omega_x + \omega_{J\perp} - \frac{sV_{i//}}{2q}(1 + s_{V//} x)] \times$$
$$[\frac{2q(\Omega_x + \omega_{J\perp} + R/2L_{T_e} + R/2L_n - 1)}{sx} + \frac{s}{2q\Omega_x} x] + \frac{s^3 k_\theta{}^2 (1+\tau)}{32\pi q^3 \beta\Omega_x} x^3 \quad (17h)$$

$$C_2 = \Omega_x[\tau^{-1}\Omega_x - (R/L_{T_i} + R/L_n)/2 + 5/3] - \frac{5s^2}{12q^2} x^2 \quad (17i)$$

$$C_1 = \frac{k_\theta}{m}[C_2 - \tau^{-1}\Omega_x + (R/L_{T_i} + R/L_n)/2 - 5/3 + \frac{5s^2}{12q^2\Omega_x} x^2] \quad (17j)$$

$$C_0 = [\frac{5s^2}{12q^2\Omega_x} - \frac{5}{3}\Omega_x k_\theta{}^2 + \frac{5}{3}(\tau - 1) + (\frac{R}{L_n} + \frac{R}{L_{T_i}})/2] \\ -[\tau^{-1}\Omega_x - (\frac{R}{L_n} + \frac{R}{L_{T_i}})/2 + \frac{10}{3}\tau - \frac{5s^2}{12q^2\Omega_x}](1 + \Omega_x k_\theta{}^2) \tag{17k}$$

$$D_2 = \frac{s(1+\tau)}{8\pi q\beta}[\tau^{-1}\Omega_x - (\frac{R}{L_n} + \frac{R}{L_{T_i}})/2 + \frac{10}{3} - \frac{5s^2}{12q^2\Omega_x}x^2] \tag{17l}$$

$$D_1 = D_2 \frac{k_\theta}{m} \tag{17m}$$

$$D_0 = -\frac{5s}{8q}x[1 - \frac{sV_{i//}(x + s_{V//}k_\theta x^2/m)}{2q\Omega_x}] + \\ \frac{2q(\Omega_x + \omega_{J\perp} + R/2L_n + R/2L_{T_e} - 1)[\Omega_x + \omega_{J\perp} - sV_{i//}(x + s_{V//}k_\theta x^2/m)/2q]}{sx} \\ -[\tau^{-1}\Omega_x - (\frac{R}{2L_n} + \frac{R}{2L_{T_I}}) + \frac{10}{3} - \frac{5s^2}{12q^2\Omega_x}x^2]\times \\ [\frac{2q(\Omega_x + \omega_{J\perp} - sV_{i//}(x + s_{V//}k_\theta x^2/m)/2q)}{sx} + \frac{s(1+\tau)k_\theta{}^2}{8\pi\beta}x] \tag{17n}$$

In Eq.(17), the drift-like frequency associated with equilibrium current density $\mathbf{J}$, $\omega_J = \omega_{J//} + \omega_{J\perp} = \mathbf{k}_{//}\cdot\mathbf{J}/en_e + \mathbf{k}_\perp\cdot\mathbf{J}/en_e$, is included [6], which, after being normalized to ω_{De}, can be written as,

$$\omega_{J//} = -[\tau(\frac{R}{L_n} + \frac{R}{L_{Ti}})/2 + (\frac{R}{L_n} + \frac{R}{L_{Te}})/2](sx + ss_{J//}x^2), \tag{18a}$$

$$\omega_{J\perp} = -[\tau(\frac{R}{L_n} + \frac{R}{L_{Ti}})/2 + (\frac{R}{L_n} + \frac{R}{L_{Te}})/2](1 + s_{J\perp}x) \quad . \tag{18b}$$

Here $s_{J//}$ and $s_{J\perp}$ are defined as $s_{J//} = (r_s/J_{//})(\partial J_{//}/\partial x)$ and $s_{J\perp} = (r_s/J_\perp)(\partial J_\perp/\partial x)$, respectively. The other quantities in Eq.(17) are conventional. The set of equations (17a) and (17b) is a singular one due to the resonance at rational surface ($x = 0$ or r_s). It is solved numerically by a standard shooting code procedure under the boundary conditions $\tilde{\varphi}(x \to \pm\infty) \to 0$ and $\tilde{A}_{//}(x \to \pm\infty) \to 0$. The singular point ($x = 0$) and its vicinity are carefully treated with the singular initial value method [7].

In the following, we mainly study the perpendicular velocity shear $s_{V\perp}$, magnetic shear s, and finite β effects on the instability. Figs.1 and 2 show the real frequency and growth rate as the function of perpendicular velocity shear for different magnetic shear. β is taken to be 0.005 in Fig.1 but be 0.012 in Fig.2. Here the dashed and solid lines represent the real frequency and growth rate while the curves 1 and 2 do the cases $s = 1.1$ and $s = 0.9$, respectively. It is found that there exists a threshold of $s_{V\perp}$, at which the growth rate tends to zero. Qualitatively, the conclusion is in agreement with the experiments [1] and simulations [8]. It seems that the perpendicular velocity shear threshold, obtained here, increases with decrease of magnetic shear. Experimentally, core transport barriers exist in the region where the perpendicular velocity shear is considerably large even if the magnetic shear is much small. Thus, the present result is not incompatible with experiments. Comparing Fig.1 with Fig.2, we can note that the increase of β reduces the growth rate but increases the velocity shear threshold.

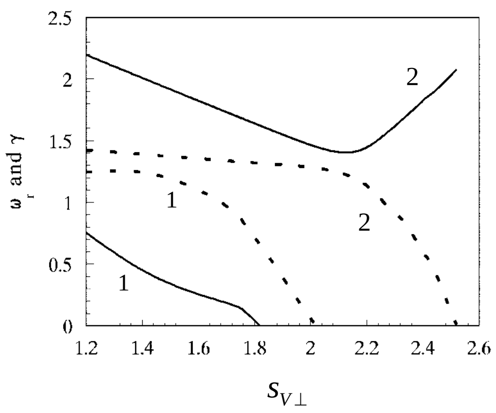

Fig.1 Real frequency (solid line) and growth rate (dashed line) vs perpendicular velocity shear for different magnetic shear s=1.1(curve 1) and s=0.9 (curve 2). q=1.5, $k_0\rho$=0.036, $V_{i//}/c_s$=0.3, $s_{V//}$=-3, $V_{i\perp}/c_s$=0.1, $s_{J//}$=0.0, $s_{J\perp}$=0.0, R/L_n=10, R/L_{Ti}=10, R/L_{Te}=10, τ=1.0, and β=0.005.

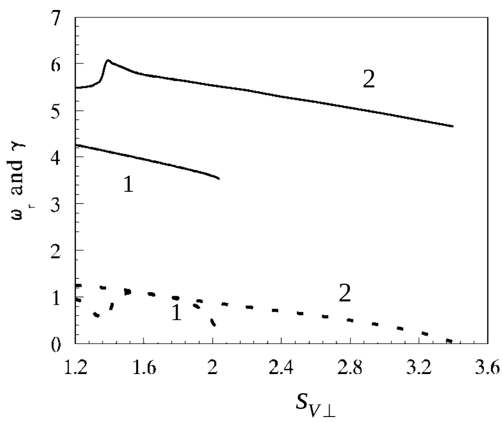

Fig.2 Real frequency (solid line) and growth rate (dashed line) vs perpendicular velocity shear for different magnetic shear s=1.1(curve 1) and s=0.9 (curve 2). The other parameters are the same as Fig.1 except that β=0.012.

In addition, we study the parallel velocity shear effects. It is found that the parallel velocity shear enhances the instability, which has been shown in the previous work [9]. Furthermore, the preliminary calculations show that the perpendicular current density shear $s_{J\perp}$ suppresses the instability while the parallel current density shear $s_{J//}$ strengthens it.

ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China under Grant Nos. 10375019 and 10135020.

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