Propagation and stability of perturbations in radiative plasmas.

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Abstract. Perturbations in multi-component collisional impurity seeded plasmas are studied. The influences of the thermal force, heavy ion inertia and finite time of the impurity relaxation over ionization states are taken into account. In the present paper it is shown that the relative species motion and the internal friction caused by the heavy ion inertia together with the thermal force increase the sound wave damping significantly. The damping mentioned above may exceed the viscous damping, sometimes, by factor ten or more. Hence, the sound waves in the multi-component radiative plasmas are significantly more stable than it has been supposed earlier. In the present paper it is shown that for the marginal stability the thermal force action on the impurity ions transforms the aperiodic character of the radiative-condensation mode into almost periodic one. The similar effect is connected with the finite relaxation time of the impurity distribution over ionization states. The ion inertia is not important for the radiation-condensation mode. The finite relaxation time of heavy ions over ionization states may also produce the nonlinear self-sustained oscillations. These oscillations are examined in compressible plasmas.

1. Introduction

Many phenomena in the edge tokamak plasmas like Multi-faceted Asymmetric Radiation From the Edge (MARFE), detached plasma regimes, propagation of the perturbations caused by ELMs etc. are associated with the radiative-condensation mode and the sound waves in the impurity seeded plasmas (See, for instance, review papers [1-3].) The radiation effects in plasmas have been studied for a long time; however, thermal forces and the finite relaxation time of the impurity distributions over ionization states have been investigated only in a few papers. In particular, the authors of Ref. 4 have found the impurity relaxation time to affect significantly the growth rate of the radiative-condensation instability as well as the shock wave propagation under the conditions typical for many experiments. As shown in [5] the finite relaxation time transforms drastically the spectra of thermal-drift modes. As shown in [6] one can find new slow thermal waves taking into account thermal forces.

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The simultaneous influence of the thermal force and the heavy ion inertia as well as of the finite relaxation time of the impurity distribution over ionization states, on the perturbation propagation and on stability conditions is studied in the present paper. The heavy ion inertia is also taken into consideration. First, the thermal force and heavy ion inertia cause the relative motion of species, internal friction and consequently additional damping. Despite of the small impurity concentration, the effect may be strong for the sound waves because the thermal force is proportional to the parameter z^2 , and the ion inertia is proportional to the heavy ion mass M. (Here z is the typical impurity ion charge for the equilibrium temperature.) It is shown in the present paper that the new type of damping may exceed the viscous damping by factor 10 or more even if the relative impurity concentration is of only a few percent. Second, the finite relaxation time in many cases may be comparable with the mode frequency or even higher. Together with the thermal force, finite relaxation time shifts the phases of the temperature oscillations and radiation respectively. Two effects are the consequences of this fact.

i. The frequency of the radiative-condensation instability acquires a real part.. Hence, the radiative-condensation mode is not purely aperiodic, especially for the marginal stability. The

effect was predicted in Ref. [6], but the finite relaxation time mentioned above was ignored. It is shown in the present paper that sometimes it may exceed the thermal force effect.

ii. The phase shift between the radiation and the temperature oscillations increases the order of the differential equation describing the process. Hence, one can expect the existence of some nonlinear self-sustained oscillations. The effect has been predicted in Ref. [8] and examined for incompressible plasmas. In the present paper the effect is examined for more realistic compressible plasmas.

2. Equations

Using the two most representative ion approximation, developed earlier [7], one may reduce the full set of equations describing the carbon seeded plasmas to the set of 9 equations:

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x} n_e v_e = n_e (J_z n_z - R_{z+1} n_{z+1}), \qquad (1)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} n_i v_i = 0, \qquad (2)$$

$$\frac{\partial P_e}{\partial t} = \frac{1}{2} \sum_{i=1}^{n_i} \frac$$

$$\frac{\partial e}{\partial x} = -en_e E + m_e n_e V_{ei} (v_i - v_e) + m_e n_e V_{ez} (v_z - v_e) + m_e n_e V_{e(z+1)} (v_{z+1} - v_e) + R_T^e, \quad (3)$$

$$m_i n_i \left(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x} \right) v_i = -\frac{\partial P_i}{\partial x} + en_i E + m_i n_i V_{ie} (v_i - v_e) + m_i n_i V_{iz} (v_z - v_i) + m_i n_i V_{i(z+1)} (v_{z+1} - v_i) + R_T^i + \nabla \eta \nabla v_i, \quad (4)$$

$$\frac{\partial n_z}{\partial t} + div(n_z v_z) = n_e(R_{z+1}n_{z+1} - J_z n_z), \qquad (5)$$

$$\frac{\partial n_{z+1}}{\partial t} + div(n_{z+1}v_{z+1}) = -n_e(R_{z+1}n_{z+1} - J_z n_z), \qquad (6)$$

$$Mn_{z}\left(\frac{\partial}{\partial t} + v_{z}\frac{\partial}{\partial x}\right)v_{z} = \frac{\partial P_{z}}{\partial x} + ezn_{z}E + Mn_{z}v_{za}(v_{a} - v_{z}) + Mn_{z}v_{zi}(v_{i} - v_{z})) + R_{T}^{z},$$
(7)

$$\partial x = V_{z} + V_{z}$$

The impurity distribution over ionization states is represented by two species with the electric charge numbers *z*, and *z*+1. Here $\Lambda = S - n_e (n_z L_z + n_{z+1} L_{z+1}),$ (10)

S is the thermal source, L_z and L_{z+1} are the radiation functions of the ions with the charges z and z+1 respectively, R_T^j are the thermal forces. M, m_i , and m_e are the impurity ion mass, the hydrogen ion mass and the electron mass, respectively, v_{ij} are the collisional frequencies. Other definitions are standard. The ion viscosity η is taken into account. The impurity and electron viscosities are neglected. Also, the equality

$$T_e = T_i = T_z = T_{z+1} \equiv T \tag{11}$$

is assumed. The plasma is assumed to be optically transparent.

The impurity density is assumed to be small. Hence, the impurity contributions to the left hand side of the temperature equation (9) are ignored. On the other hand, the impurity ion mass is significantly higher than the hydrogen mass, and therefore hydrogen-impurity friction may be important even if the impurity concentration is small.

It is interesting to note that the equilibrium with a non-zero temperature gradient and velocities of all species equal to zero is impossible. Equation (1) determines the relation for the densities n_z and n_{z+1} :

$$n_z = n_{z+1} \frac{R_{z+1}}{J_z} \,. \tag{12}$$

On the other hand, equations (7) and (8) also determine n_z and n_{z+1} as the functions of the temperature:

$$z\frac{\partial P_{z+1}}{\partial x} - (z+1)\frac{\partial P_z}{\partial x} + (z+1)R_T^z - zR_T^{z+1} = 0,$$
(13)

Note that, that the set of equations (12-13) together with (9) and (11) is over-determined.

Thermal forces R_T^z and R_T^{z+1} are expressed in terms of electron and background plasma ion temperature gradients (see, for instance, [9]):

$$R_T^z = n_z (0.71 z^2 \nabla T_e + A \nabla T_i), \qquad (14)$$

where the factor A depends on z, M and m_i . If $M >> m_i$ one can set $A \approx 2.2z^2$. Hence, taking into account (11):

$$R_T^z \approx 3z^2 \nabla T.$$
(15)
Substituting (15) into (7-8) one can find for $z \gg 1[10]$:

$$v_{z} \approx v_{z+1}.$$
(16)

Hence, impurity dynamics may be described with the following equation:

$$Mn_{I}\left(\frac{\partial}{\partial t} + v_{I}\frac{\partial}{\partial x}\right)v_{I} = \frac{\partial P_{I}}{\partial x} + e < z > n_{I}E + Mn_{I}v_{ze}(v_{e} - v_{I}) + Mn_{I}v_{Ii}(v_{i} - v_{z})) + R_{T}^{I}, \qquad (17)$$

Here $n_I = n_z + n_{z+1}$, $v_I = v_z = v_{z+1}$, $P_I = P_z + P_{z+1}$, $R_T^I = R_T^z + R_T^{z+1}$, and $< z >= \frac{zn_z + (z+1)n_{z+1}}{n_I}$.

The averaged impurity charge is determined by the equation [7]:

$$\frac{\partial \langle z \rangle}{\partial t} + v_I \nabla \langle z \rangle = -v_z (\langle z \rangle - z_0).$$
⁽¹⁸⁾

Here $v_z = n_e (R_{z+1} + J_z)$, and $z_0 = z + 1 - \frac{R_{z+1}}{R_{z+1} + J_z}$.

3. Linear analysis

To the linear approximation, one can look for a solution in the form $e^{ikx-i\omega t}$.

Under the condition $zn_z \ll n_i$ the electron density n_e is approximately equal to the hydrogen ion one, $n_i \approx n_e \equiv n$. Combination of eqs. (2-4) with (15) yields:

$$\left(\omega^{2} + i\omega\left(v_{il} + \frac{4}{3}\frac{k^{2}c_{s}^{2}}{v_{ii}}\right) - 2k^{2}c_{s}^{2}\right)\frac{\tilde{n}}{n} - iv_{il}\omega\frac{\tilde{n}_{l}}{n_{l}} = k^{2}c_{s}^{2}\left(2 + 3z_{0}^{2}\xi\right)\frac{\tilde{T}}{T}.$$
(19)

Here \tilde{n} , \tilde{n}_{I} and \tilde{T} are the perturbed concentrations and temperature respectively, $c_{s}^{2} = T/m$, and $\xi = n_{I}/n$. The Braginsky viscosity $\eta \approx \frac{4nT}{3v_{ii}}$ has been substituted. Also, the approximate

relation $R_T^z \approx R_T^{z+1} \approx 3 < z >^2 \nabla T$ is used.

Usually, the electrostatic energy does not exceed the thermal one, $e\varphi \leq T$. Using the assumption $3z^2 >> 1$, one can neglect the electric field for the impurity description. Hence, equations (5, 6, 17) yield:

$$\left(\omega^{2} + i\omega v_{Ii} - k^{2}c_{I}^{2}\right)\frac{\tilde{n}_{I}}{n} - iv_{Ii}\omega\frac{\tilde{n}}{n} = -3k^{2}c_{I}^{2}z^{2}\xi\frac{\tilde{T}}{T}.$$
(20)

Here $c_I^2 = T / M$.

In the linear approximation, eq. (18) yields:

$$\widetilde{z} = \frac{V_z}{V_z - i\omega} \frac{dz_0}{d\ln T} \frac{\widetilde{T}}{T}.$$
(21)

Here \tilde{z} is the perturbed part of $\langle z \rangle$, $z_0 = z + 1 - \frac{R_{z+1}}{R_{z+1} + J_z}$, and $v_z = n_e (R_{z+1} + J_z)$.

Performing the linearization procedure for temperature eq. (9) one has to take into account that the function (10) of thermal sources and sinks depends on T, n, n_I , and < z >. Taking into account (21),

$$\frac{\tilde{n}}{n} = \frac{1}{2\omega + i\nu_n} \left(\left((3\omega + i\beta)\frac{\tilde{T}}{T} \right) - i\nu_I \frac{\tilde{n}_I}{n_I} \right).$$
(22)

Here
$$\beta = \chi_e k^2 - v_T + v_d \frac{dz_0}{d \ln T} \frac{v_z}{v_z - i\omega}$$
, $v_T = \frac{1}{n} \frac{\partial \Lambda}{\partial T}$, $v_n = \frac{1}{T} \frac{\partial \Lambda}{\partial n}$, $v_d = \frac{1}{nT} \frac{\partial \Lambda}{\partial < z}$, and

 $v_I = \frac{1}{nT} \frac{\partial \Lambda}{\partial \ln n_I}$ The Braginsky form for the electron heat conductivity is used:

$$\chi_e = \frac{\kappa}{n} \approx 3 \frac{T}{m_e V_{ee}} \,.$$

The dispersion relation takes the form:

$$X\left(\left(\mu-\omega^{2}\right)\frac{\xi}{\mu}+i\frac{v_{I}C}{2\omega+iv_{n}}\right)-Y\left(\frac{3\omega+i\beta}{2\omega+iv_{n}}C-2\right)+\left(\mu-\omega^{2}\left(\frac{3\omega+i\beta}{2\omega+iv_{n}}C-3z_{0}^{2}\xi\right)\right)=i3z_{0}^{2}\xi\frac{v_{I}C}{2\omega+iv_{n}}.$$

$$(23)$$

Here $X = iv_{Ii}\omega \frac{3\omega + i\beta}{2\omega + iv_n}$, $Y = iv_{Ii}\omega(1 + i\delta)$, $C = \omega^2 - 2 + i\frac{4\omega}{3v_{Ii}}$, $\delta = \frac{1}{3\omega} \left(\beta - \frac{3}{2}v_n\right)$, and

 $\mu = \frac{m_i}{M}$. The dimensionless variables are introduced, $\omega/kc_s \rightarrow \omega$ etc.

4. Sound waves

In order to investigate the sound branch one must assume $\omega \sim 1$, $1 \gg \beta$, and $v_{ij} \gg \omega$. The last condition coincides with the validity condition of hydrodynamics. To the first order, one can find from (23): $\omega_0^2 = \frac{10}{3(1+\xi/\mu)}$.

It coincides with usual sound frequency in multi-component plasmas.

In second order, (23) yields the imaginary part of the frequency:

$$\operatorname{Im}\Delta\omega = -\frac{1}{3(1+\xi/\mu)} \left(\left(\frac{2}{v_{ii}} + \xi \left(\frac{5}{1+\mu} + 3z_0^2 \right) \right) - \frac{v_I}{\omega_0^2} + 3\frac{\operatorname{Im}\delta}{\omega_0} \right).$$
(24)

The last two terms in the right hand of (24) (in brackets) describe the radiation influence on the mode stability. The finite relaxation time of the impurity distribution over ionization states is taken into account. The effect was discussed briefly in Ref. [4]. The two first terms correspond to the collisional damping. The second term corresponds to the usual ion viscosity. The last damping term appears as a sequence of the internal friction between the hydrogen ions and the impurity ones. The first part in the internal brackets appears due to the large mass ion inertia, and the second corresponds to the thermal force. Despite of the inequality $\xi_{z_0} \ll 1$; the inequalities $5M\xi/m \gg 1$, $3z_0\xi \gg 1$ may be valid. Hence, the collisional sound damping in the impurity seeded plasmas may be significantly higher than in the pure hydrogen ones even if the impurity concentration is small. It is possible if

$$F(\xi) = \frac{z_0^2 m_i}{\sqrt{2}M} \xi \left(\frac{5M / m_i}{1 + \xi M / m_i} + 3z_0^2 \right) >> 1.$$
(25)

Braginsky form for the Coulomb collisional frequencies is used here. The ratio of the new damping rate to the usual viscous one v/s the impurity concentration is shown in Fig.1 for the carbon seeded (solid line) and nitrogen seeded (dotted line) plasmas. T = 60eV, $z_0 = 4.5$ have been chosen for carbon, and T = 100eV, $z_0 = 5.5$ have been chosen for nitrogen.



Fig. 1. The ratio of the new sound damping to the viscous one for carbon (red line, T=60 eV) and for nitrogen (blue line, T=100 eV)

The two ion approximation is valid with a good accuracy for these temperatures.

One can see that the stability region for the sound waves is significantly wider than it was assumed earlier.

5. Radiation-condensation mode.

The opposite limit, $\omega \ll 1$, corresponds to the radiation-condensation mode. The viscosity may be omitted. Assuming $\omega \sim \beta \sim v_n \sim v_1$ etc., $\omega/v_{li} \gg 1$, $C \approx 2$ and ignoring the third term in the right hand of (32) one can get:

$$5\omega + i \left(\lambda - v_d \frac{dz_0}{d \ln T} \frac{v_z}{v_z - i\omega} \right) + 3z_0^2 \frac{v_I}{v_I \omega} = 0.$$
⁽²⁶⁾

Here $\lambda = \chi_e k^2 - v_T + v_n + v_I$. The limit case $v_z \gg \omega$ was examined in Ref. [6]:

$$10\omega = -i\left(\lambda - v_d \frac{dz_0}{d\ln T} v_z\right) \pm \sqrt{-\left(\lambda - v_d \frac{dz_0}{d\ln T} v_z\right)^2 - 300z_0^2 \mu \frac{v_I}{v_{Ii}}}.$$
(27)

For many practically important conditions the opposite case $v_z \ll \omega$ is more interesting. It yields:

$$10\omega = -i\lambda \pm \sqrt{-\lambda^2 - 100} \left(v_d \frac{dz_0}{d\ln T} v_z + 3z_0^2 \mu \frac{v_I}{v_{Ii}} \right).$$
(28)

One can see from (9) that the function v_I is negative for any temperature. For the marginal stability the term $3z_0^2 \mu \frac{v_I}{v_{Ii}}$ provides the real part of ω . For example, it can be estimated for carbon at the temperature $T \approx 10 \ eV$. Using the data from Ref. [7] one can get:

$$3z_0^2 \mu \frac{V_I}{V_{Ii}} \approx -(2 \div 3)\xi \,. \tag{29}$$

Under the similar conditions the estimation for v_z yields $v_z \approx \frac{n}{k} 10^{-18}$. This value is able to exceed the value in (29) only for extremely small wave numbers or extremely high densities. The sign of the other term determining the real part of ω depends on the temperature. For the example chosen, v_d is negative. Radiation ability L_{z+1} of the helium-like ion is significantly smaller than that of the lithium-like ion L_z . The radiation decreases rapidly with the average charge increase. Hence, two terms in brackets of (28) have the opposite signs for the temperature interval chosen. The first for $T = 10 \ eV$ may be estimated as follows:

$$v_d \frac{dz_0}{d\ln T} v_z \approx +10^{-30} \frac{n^2}{k^2} \xi \,. \tag{30}$$

One can see that this term can exceed (29) for high densities and small k. Under these conditions the mode becomes aperiodic again. It is useful to remark that the mode (28) may be significantly more stable than the mode (27) due to the large value of v_d .

6. Self-sustained oscillations

As shown in Ref. [8] the nonlinear self-sustained oscillations may appear in incompressible radiative plasmas as a consequence of the phase shift between temperature oscillations and

the distribution of impurities over ionization states. In the present paper the more realistic case of the compressible plasmas is considered. The pure carbon plasmas are examined. In contrast to the Section 2, the electron density is determined by the carbon density:

$$n_e = zn_z + (z+1)n_{z+1}. ag{31}$$

The impurity species are described separately. Eqs. (7) and (8) are used for the impurity dynamics description. The process is supposed to be slow. Hence, the ion inertia is omitted: $P_z + P_{z+1} + P_e = const = P_0$. (32)

Using the quasi-neutrality condition (31) one can find n_{z+1} and n_e in terms of n_z :

$$n_{z+1} = \left(\frac{P_0}{T} - (z+1)n_z\right) \frac{1}{z+2}, n_e = \left(\frac{P}{T}(z+1) - n_z\right) \frac{1}{z+2}.$$
(33)

One must take into account the ionization energy I of the ion with the charge z adding the corresponding term to the energy equation (9).

The set of equations may be reduced significantly with the assumption that the perturbations of relative concentrations $y_z = n_z/N_0$, $y_e = n_e/N_0$, dimensionless temperature $\tau = T/T_0$ and velocities are small, and the main nonlinearity is connected with the processes of ionization, recombination and radiation. Here N_0 and T_0 are the unperturbed carbon concentration and the temperature respectively. Long-wavelength oscillations are examined below, and the space derivatives are ignored. After cumbersome but simple transformations one can get the set of two ordinary differential equations:

$$\frac{dy_z}{dt} = N_0 y_e \frac{\left(z + 2 - y_z^0 \left(1 + \frac{2I}{5T_0}\right)\right) \left(R_{z+1} y_{z+1} - J_z y_z\right) - \frac{2}{5} y_z^0 \Lambda}{2 + z - y_z^0},$$
(34)

$$\frac{d\tau}{dt} = N_0 y_e \frac{\left(z + 2 - y_z^0 \left(1 + \frac{2I}{5T_0}\right)\right) \left(R_{z+1} y_{z+1} - J_z y_z\right) + \frac{2}{5} y_z^0 \Lambda}{2 + z - y_z^0}.$$
(35)

Eqs. (34, 35) have been solved numerically for carbon. The equilibrium values $T_0 = 5 \ eV$, $N_0 = 10^{12} \ cm^{-3}$ were chosen. The heating and cooling function is approximated by the expression:

$$y_e(S-Q) = a_\tau(\tau-1) + a_y(y-y_z^0) + a_{\tau\tau}(\tau-1)^2 + a_{y\tau}(y-y_z^0)(\tau-1) + a_{yy}(y-y_z^0)^2$$

with $a_\tau = 4946.8$, $a_y = 6578.5$, $a_{\tau\tau} = -270$, $a_{y\tau} = a_{yy} = 0$.



1.4 1.2 1 0.8 0.6 0.4 0.2 0 1002 10.04 10.05 10.01 10.03 10.06 10 t (a.u.)

Fig. 3. The relative concentration oscillations.

Fig. 4. The temperature oscillations.

One can see that in contrast to the incompressible plasmas [8], the phase shift between y_z and y_{z+1} is significantly smaller than $\pi/2$. The oscillation frequency is proportional to the total concentration and may be varied significantly, from zero to the upper limit. The latter is determined by the condition of the plasma transparency. For instance, if the carbon plasma has the temperature of about 5 eV and the transverse size of 10 cm, the upper limit is about 10^5 c^{-1} .

7. Summary

The influence of the thermal force, the heavy ion inertia and the finite relaxation time of the impurity distribution over ionization states on the propagation and stability of the sound waves, radiative-condensation mode and nonlinear self-sustained oscillations in radiative plasmas is examined.

1. It is shown that the thermal force together with the inertia of heavy ions in the impurity seeded collisional radiative plasmas causes the additional damping of the sound waves exceeding the viscous damping significantly.

2. The thermal force as well as the finite relaxation time of the impurity distribution over ionization states is shown to transform the purely aperiodic radiative-condensation mode into an oscillating one near the marginal stability.

3. It was found that the self-sustained, non-linear oscillations in the compressible radiative plasmas may exist as a consequence of the finite relaxation time of the impurity distribution over ionization states. The frequency is proportional to the plasma density and may be varied from zero to the upper limit determined by the plasma optical transparency.

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9. References

[1] Lipshultz, B. J. Nucl. Mater., 145-147, (1987) 15.

[2] Meerson, B., Rev. Mod. Phys., 68, p. 215 (1996).

[3] Morozov, D.Kh., Herrera, J.J.E., Plasma Phys. Rep, 24, (1998) 347.

[4] Krasheninnikov, S.I., Morozov, D.Kh., Sigmar, D.J., Herrera, J.J.E., and Soboleva, T.K., Contributions to Plasma Phys., **36**, (1996) 271.

[5] Morozov, D.Kh., Krasheninnikov, S.I. Proc. of 1996 International Conference on Plasma Physics, Magoya, Japan, Sept. 1996, ed. by H. Sugai, T. Hayashi, Vol. 1, (1997) 626-629.

[6] Morozov, D.Kh., and Herrera, J.J.E., Phys. Rev. Lett., 76, (1996) 760.

[7] Gervids, V.I., Kogan, V.I., and. Morozov, D.Kh, Plasma Phys. Rep, 27, (2001) 994.

[8] Morozov, D.Kh., Proc. of 29th EPS. Conf. on Plasma Phys. And Contr. Fusion, Montreux (2002), Vol. 26B, CDROM, file P2_002.

[9] Yushmanov, P.N., Nucl. Fusion 23, (1983) 1599.

[10] Morozov, D.Kh., Rozhansky, V.A., Herrera, J.J.E., and Soboleva, T.K., Physics of Plasmas, **7**, (2000) 1184.