Non-conventional Fishbone Instabilities

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Abstract. New instabilities of fishbone type are predicted. First, a trapped-particle-induced $m = n = 1$ instability with the mode structure having nothing to do with the conventional rigid kink displacement. This instability takes place when the magnetic field is weak, so that the precession frequency of the energetic ions is not small as compared to the frequency of the corresponding Alfvén continuum at r=0 and the magnetic shear is small inside the $q = 1$ radius [the case relevant to spherical tori]. Second, an Energertic Particle Mode fishbone instability driven by circulating particles. Third, a double-kink-mode instability driven by the circulating energetic ions. In particular, the latter can have two frequencies simultaneously: we refer to it as "doublet" fishbones. This instability can occur when the radial profile of the energetic ions has an off-axis maximum inside the region of the mode localization.

1. Introduction.

The well-known fishbone instability is an $m = n = 1$ (m and n are the mode numbers) rigid kink displacement of the plasma core inside the q=1 radius, its frequency being either the precessional frequency of the energetic ions (ω_{pr}) or the bulk ion diamagnetic frequency (ω_{di}) [1–3]. However, recent experiments have shown that other types of fishbones are possible, too. In particular, fishbones with $n = 2-4$ were observed in experiments in the NSTX spherical torus [4]; fishbones with doublet frequency ($f_1 \sim 15$kHz and $f_2 \sim 20$kHz) but with the same mode numbers ($m = n = 1$) occurred in the ASDEX-U tokamak [5]. Thus, further development of the theory is required. One can see that there are, at least, two factors, which can lead to non-conventional fishbones. They are the weak magnetic field [in Spherical Tori (ST)] and non-monotonic profile of the safety factor, $q(r)$ (which can take place in both STs and tokamaks). In the present work, we study fishbone instabilities taking into account the mentioned factors. In particular, we will show that even the $m = n = 1$ fishbone mode in STs differs from that in CTs.

2. An $\mathbf{m = n = 1}$ fishbone mode driven by trapped particles in spherical tori.

In STs the magnetic field is lower whereas β (the ratio of the plasma pressure to the magnetic field pressure) is higher than in tokamaks. When β is very high, so that a considerable magnetic valley arises in the equilibrium magnetic field and Shafranov shift becomes very large, fishbone instabilities tend to be stabilized [6,7]. On the other hand, in many current experiments β is not so high, and therefore, the results of Refs. [6,7] are not applicable to them. Regimes with moderate (in the mentioned sense) β are subject to study in the present work. We restrict ourselves to the precession (high-frequency) branch of fishbones associated with the trapped energetic particles [1]. This instability has bursting character and can strongly affect the energetic ions.

The precession fishbones are an Energetic Particle Mode (EPM) associated with the perturbations of Alfvén type. Therefore, restricting ourselves to the case of the $m = n = 1$ perturbation, we can write the following equation:

$$\frac{d}{dr} r^3 \left(\frac{\omega^2}{v_A^2} - k_\parallel^2 \right) \frac{d\xi}{dr} = \frac{r^2}{Rr_s} \frac{d\beta_\alpha}{dr} \Omega \ln\left(1 - \Omega^{-1}\right) \xi \,, \tag{1}$$

where ξ is the plasma displacement, ω is the mode frequency, $k_\parallel = \left(q^{-1} - 1\right)/R$ is the longitudinal wavenumber, $q(r)$ is the safety factor, R is the large radius of the torus, $\Omega = \omega / \omega_D$, $\omega_D = \omega_{pr}(\varepsilon_\alpha, r_s)$, ε_α is the birth energy, r_s is the radius where $q = m/n$, v_A(r) is the Alfvén velocity, $\beta_\alpha = 8\pi p_\alpha / B_0^2$, p_α is the energetic ion pressure. The RHS of Eq. (1) describes the response of the energetic ions. It is obtained from Ref. [8].

Let us first recover the results of the conventional fishbone theory assuming that the system is on the margin of stability. In this case, the real and imaginary parts of Eq. (1) are

$$\frac{d}{dr} r^3 \left(\frac{\Omega^2}{\Omega_A^2} - (1 - q^{-1})^2 \right) \frac{d\xi_1}{dr} = \frac{Rr^2}{r_s} \frac{d\beta_\alpha}{dr} \Omega\left(\xi_1 \ln|1 - \Omega^{-1}| - \pi\xi_2\right), \tag{2}$$

$$\frac{d}{dr} r^3 \left(\frac{\Omega^2}{\Omega_A^2} - \left(1 - q^{-1}\right)^2 \right) \frac{d\xi_2}{dr} = \frac{Rr^2}{r_s} \frac{d\beta_\alpha}{dr} \Omega\left(\pi\xi_1 + \xi_2 \ln|1 - \Omega^{-1}|\right), \tag{3}$$

where $\xi_1 = \mathrm{Re}\,\xi$, $\xi_2 = \mathrm{Im}\,\xi$, $\Omega_A = v_A / (R\omega_D)$. If ω were vanishing, $\xi\,(r)$ would be constant everywhere except for a region close to r_s, which provides a possibility to satisfy the condition $\xi(a) = 0$, with a the plasma radius, by taking a step function $\xi(r) = \xi_0 \eta(r_s - r)$, with $\eta(x) = \int_{-\infty}^{x} \delta(x) dx$. On the other hand, for this $\xi(r)$, it is possible to satisfy Eq. (2) at $r < r_s$ for finite Ω by taking $\ln|1 - \Omega^{-1}| = 0$ and $\xi_2 \approx 0$, which leads to $\Omega = 0.5$, in agreement with Refs. [1,8]. However, finite Ω changes the structure of the mode because two local Alfvén resonances determined by the equation $\omega = \omega_{AC}(r) \equiv k_\parallel v_A$ (the subscript "AC" means "Alfvén Continuum") appear in the vicinity of r_s, $r_1 < r_s < r_2$, see Fig. 1. The resonances are $q_{1,2}^{-1} = 1 \pm \omega R / v_{As}$, where $v_{As} \cong v_A(r_s)$, from which it follows that the distance between the resonances is $r_2 - r_1 \cong 2\omega R / (v_{As} \hat{s})$, with $\hat{s}$ the magnetic shear at the $q = 1$ surface. In fact, $r_2 - r_1$ is the width of the double resonance layer in the case when the plasma is close to the margin of stability [the singularities at r_1 and r_2 will be removed when the terms proportional to $\gamma = \mathrm{Im}\,\omega$ will be added to Eqs. (2), (3), the spread of each resonance being negligible compared to $\left(r_2 - r_1\right)$ for small γ]. Note that when the longitudinal wavenumber at $r = 0$ can be approximated as $k_\parallel(0) \cong \hat{s}/R$ and $v_{As} \cong v_A(0)$, the half-width of the resonance layer is $\delta_{res} \equiv r_s - r_1 \cong r_s \omega / \omega_{AC}(0)$. The continuum damping arising from Alfvén resonances leads to a threshold beta of the energetic ions, $\beta_{\alpha c}$. The latter can be evaluated by considering Eq. (3) in the region $r_1 < r < r_2$. We assume that ξ_1 and ξ_2 are of the same order in this region and take $d\xi_2/dr \sim \xi_2/\delta$, $d^2\xi_2/dr^2 \approx 0$ [$\xi_2(r)$ can be approximated by a linear function because $\delta/r_s << 1$ and, in addition, the signs of $(\omega^2 - k_\parallel^2 v_A^2)$ at $r < r_1$ and $r > r_2$ are different and, thus, the signs of the peaks $\xi_2(r_1)$ and $\xi_2(r_2)$ are different, too]. Then we obtain $\pi\beta_{\alpha c} \approx (3L_\alpha/R)\omega_D(R\hat{s}/v_A)$, which agrees (qualitatively) with Refs. [1,8]. Numerical solution of Eq. (1) confirms this qualitative consideration, although it gives somewhat smaller mode frequency, see Fig. 2.

It is clear that Alfvén resonances are located close to r_s provided that $\omega^2/\omega_{AC}^2(0) << 1$. However, this condition is difficult to satisfy when the magnetic field is weak because $\omega^2/\omega_{AC}^2(0) \propto 1/B^4$ (we used $\omega \sim \omega_D$). For instance, for NSTX we can take $q_0 = 0.9$, $r_s = 30\,\text{cm}$, $\rho_\alpha = v_\alpha/\omega_B = 20\,\text{cm}$, $v_\alpha/v_A = 3$, which leads to $\omega/\omega_{AC}(0) \le \omega_D/\omega_{AC}(0) \approx 0.5(\rho_\alpha/r_s)(v_\alpha/v_{A0})\left(q_0^{-1}-1\right)^{-1} = 1$. Therefore, in NSTX $r_1 << r_s$, see Fig.3, or even one of the resonances (the left resonance) can be absent. We conclude from here that the weak magnetic field may prevent the conventional precession fishbone mode. Another factor which tends to prevent the conventional fishbones in STs is a very small magnetic shear at $r < r_s$.

An example of the calculated mode structure for an NSTX plasma with $q_0 < 1$ is shown in Fig. 3. This structure has nothing to do with the rigid shift; in addition, only one (very small) peak is seen at $r \ge r_s$, although ω is a little bit less than $\omega_{AC}(0)$ and, thus, there are two local Alfvén resonances. Note that $\gamma/\omega_D = 0.01$ in both Fig. 2 and Fig. 3, but β_0 is higher in Fig . 3.

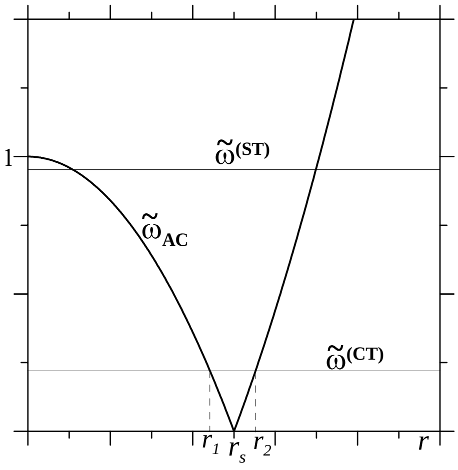

Fig. 1.

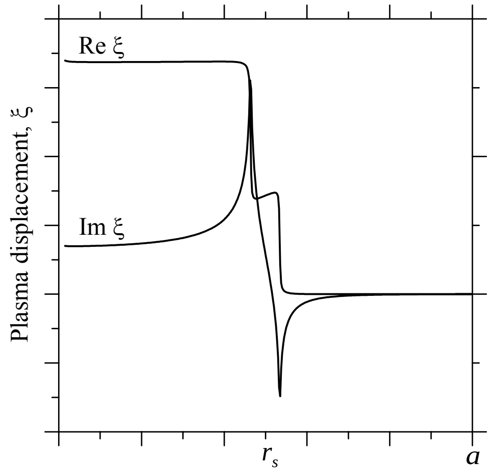

Fig. 2.

Fig. 1. Qualitative plot of the normalized Alfvén continuum, $\tilde{\omega}_{AC} \equiv \omega_{AC}/\omega_{AC}(0)$, and the normalized mode frequency, $\tilde{\omega} \equiv \omega/\omega_{AC}(0) \le \omega_D/\omega_{AC}(0)$, in CTs and STs. Notations: $\omega_{AC} = |k_\parallel| v_A$, r_1 and r_2 are the points of the local Alfvén resonance.

Fig. 2. Radial structure of a fishbone mode in a conventional tokamak with $\beta_\alpha(r) = \beta_0\left(1 - r^2/a^2\right)^2$, $\beta_0 = 2.22\cdot 10^{-3}$. The calculated mode frequency is $\omega/\omega_D = 0.28$, which corresponds to $\omega/\omega_{AC}(0) = 0.14$, and the growth rate is $\gamma/\omega_D = 0.01$. The used parameters: the aspect ratio A=5, $q^{-1} = 1 + 0.2\cdot[1-(r/r_s)^2]$, $r_s = 0.5a$.

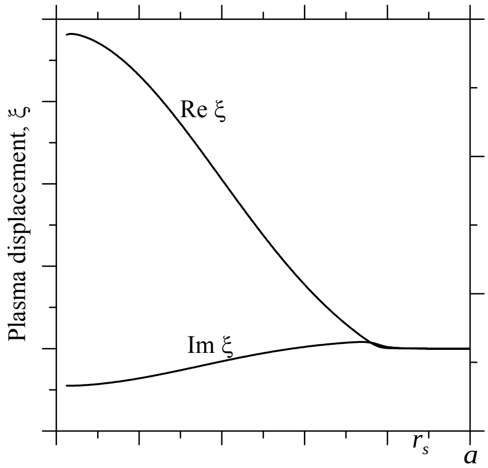

Fig. 3. Radial structure of a fishbone mode in the NSTX spherical torus for $\beta_\alpha(r)=\beta_0\left(1-r^2/a^2\right)^2$, $\beta_0=8.82\cdot10^{-3}$. *The calculated mode frequency is* $\omega/\omega_D=0.485$, *which corresponds to* $\omega/\omega_{AC}=0.998$, *and the growth rate is* $\gamma/\omega_D=0.01$. *The used parameters: A=1.27,* $r_s/a=0.87$, $q=0.8$ *for* $r<r_c$, *and* $q=0.8+[1.9(r-r_c)/r_c]^{10}$ *for* $r>r_c$ *with* $r_c=0.6a$.

3. Circulating-particle-induced fishbones with arbitrary m/n .

As in the previous section**,** here we study the EPM fishbones. However, in contrast to the previous section, now we consider circulating-ion-induced fishbone instability, and moreover, we consider the mode with conventional radial structure. We assume that $q(r)$ has an off-axis minimum, $q_{\min}$, although our results in the case of $\Delta_b<<\Delta_m<<r_s$ (Δ_b is the orbit width of the energetic ions, Δ_m is the mode width) will be applicable to a plasma with a monotonic $q(r)$. Note that the circulating-ion-induced EPM fishbones were not considered yet (only the diamagnetic branch was considered [3]).

Thus, we consider a double kink mode characterized by a "top-hat" radial displacement, $\xi(r)$, localized between two rational surfaces, $r_{s1}<r<r_{s2}$, r_{s1} and r_{s2} being defined by $q(r_{s1})=q(r_{s2})=m/n$. Then we can use the dispersion relation in a generic form similar to that in the case of the monotonic $q(r)$:

$$i\frac{\omega}{\omega_A}+\lambda_c+\lambda_h=0, \tag{4}$$

where λ_c and λ_h are the normalized negatives of the MHD potential energy and the energy associated with the energetic ions, respectively, $\omega_A=|m|(|\hat{s}_1|+|\hat{s}_2|)v_A/(q_sR)$ or, $\omega_A=|m|(|\hat{s}|)v_A/(q_sR)$ when $q(r)$ is monotonic.

Let us consider the case of $\Delta_b<<\Delta_m<<r_s$. Because of the assumption $\Delta_b<<\Delta_m$, the particles crossing r_{s1} do not reach r_{s2} and vice versa. Therefore, we can easily generalize the energetic particle response calculated for the monotonic safety factor in Ref. [3] to the case of non-monotonic $q(r)$ (only particles crossing r_{s1} and r_{s2} mainly contribute to λ_k). On the other hand, due to the assumption $\Delta_m<<r_s$ we can take $|\hat{s}_1|=|\hat{s}_2|$, $d\beta_\alpha/dr|_{r_{s1}}=d\beta_\alpha/dr|_{r_{s2}}$, $v_A(r_{s1})=v_A(r_{s2})$ and write Eq. (4) as follows:

$$0=D(\Omega^{cir})\equiv-i\Omega^{cir}-\widetilde{\lambda}_c-\pi_\alpha F(\Omega^{cir}), \tag{5}$$

where $\Omega^{cir}=\omega/\omega_{s1}$, $\widetilde{\lambda}_c=\lambda_c\omega_A/\omega_{s1}$,

$$\pi_\alpha=-(2/3)m(m/n)^2(v_A/v_\alpha)(R/|\hat{s}|^3)d\beta_\alpha/dr|_{rs}, \tag{6}$$

$$\pi F(x) = 10x - 8x^{3/2}\left(\tan^{-1} x^{-1/2} + \tanh^{-1} x^{-1/2}\right) + (1+3x^2)\ln[(1+x)/(x-1)]. \quad (7)$$

Note that $\omega_s \approx \omega_D$ for $\hat{s} \approx q/2$, and $\Omega^{cir} \approx \Omega$ in this case. Equation (5) was analyzed by a Nyquist approach. An unstable solution was found for π_α exceeding a certain threshold magnitude.

The found fishbone mode exists due to the resonance $\omega = k_\| v_\|$, which leads to a characteristic frequency $\omega_s \approx |k_\|'(r_s)|\Delta_b v_\alpha$ when $|k_\|'(r_{s1})| = |k_\|'(r_{s2})|$. The latter is justified for $k_\|(r)$ symmetric with respect to $r_{\min}$ in the region $r_{s1} < r < r_{s2}$. However, in general, $|\hat{s}_1| \neq |\hat{s}_2|$ for a double kink mode with the finite width. For this reason, there are two, rather than one, characteristic frequencies, and one can expect that an instability with two frequencies with given m, n can exist. A corresponding Nyquist analysis confirmed this possibility.

4. Summary and conclusions.

(i) The weak magnetic field and low shear inside the $q=1$ radius are the factors which can lead to the $m=n=1$ fishbone mode with the interchange-like radial structure. This will be the case when $q(0)<1$ and $\omega_D \sim \omega_{AC}(0)$. The latter condition can be satisfied, in particular, in NSTX.

(ii) It is shown that the circulating energetic ions can lead to an EPM fishbone instability (i.e., not only the low-frequency fishbones considered in Ref. [3] are possible in the presence of the circulating fast ions).

(iii) It is shown that when the profile of $q(r)$ is non-monotonic, a double-kink mode with $m=n=1$ in the case of $q(0)<1$ and $m/n \neq 1$ in the case of $q(0)>1$ can be destabilized by the circulating energetic ions.

(iv) A new kind of the instability, which we refer to as “doublet” fishbones, is predicted. This instability is characterized by two frequencies and two growth rates, although it is relevant to the same double kink mode. It seems possible that the predicted “doublet” instability was observed in an ASDEX-U experiment reported in Ref. [5].

Details concerning fishbones in plasmas with a non-monotonic $q(r)$ can be found in the recent publication [9].

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