Non-Linear Study of Fast Particle Excitation of Global Alfvén Eigenmodes during ICRH

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Abstract. High power ICRH produces fast ions with wide non-standard orbits, which in addition to heating the plasma can enhance the fusion reactivity, drive currents, induce plasma rotation and affect the stability of MHD modes. The latter effects are sensitive to the details of the distribution function. A method to self-consistently include excitation of Alfvén eigenmodes with respect to mode amplitude and particle distribution has been developed and implemented in the SELFO-code, which consists of the Monte Carlo code FIDO for calculating the distribution function of the heated ions and the global wave code LION for calculating the wave field for ICRF. Oscillations of the MHD mode amplitude, consistent with the experimentally observed splitting of the mode frequency, are found. Numerical simulations are also consistent with the experimentally observed fast damping of the mode as the ICRH is switched off.

1. Introduction

ICRH is a versatile heating method that cannot only provide heating but also enhance fusion reactivity, drive currents and induce plasma rotation. Global Alfvén eigenmodes, GAEs, excited by cyclotron heated high-energy ions are frequently seen during ICRH experiments. Excitation of GAEs by ICRH has been proposed as a method to simulate GAE excitation by thermonuclear alpha particles. GAEs flattens the distribution function of the heated ions locally in phase space. Since the performance of ICRH is sensitive to the details of the distribution, the presence of GAEs may affect the performance. In general the effect of a single GAE on the distribution function is small, since the regions in the phase space where the wave particle interactions are resonant are rather narrow. However, the effects on the distribution function and the mode amplitude can be significantly increased when the resonant regions of several unstable GAEs overlap. It can even lead to direct losses of fast ions as they are displaced outwards and their orbits are intercepted by the wall.

There is a significant difference between excitation of GAEs by cyclotron heated fast ions and thermonuclear alpha particles. ICRH does not only produce peaked density profiles of anisotropic high-energy ions with wide trapped or non-standard drift orbits, which destabilise the GAEs, but it also decorrelates the interactions with GAEs and partially restores the distribution function. The decorrelation of MHD resonant ions is an important effect since it leads to an effective broadening of the MHD resonant regions in phase space, and hence increase the energy transport, the saturation level of the MHD mode and the number of regions with overlapping modes. While decorrelation by Coulomb collisions is most important for the low energy ions since the collision frequency decreases with energy, decorrelation by ion cyclotron interactions increases with energy and is therefore most important for the high-energy ions, which in general are also responsible for the excitation of the GAEs.

Splitting of the mode frequency of the excited GAEs are frequently seen during ICRH [1]. A simplified theoretical model of the non-linear dynamics of the interactions between GAEs and fast ions [2, 3, 4] describes the experimental results rather well [1, 5]. The key quantity determining the dynamics in this model is the normalised renewal rate of the distribution function, $\nu = \nu_{eff}/\gamma$, where ν_{eff} is the effective collision rate restoring the distribution function, $\gamma = \gamma_{linear} - \gamma_{damping}$ is the net growth rate, γ_{linear} the linear growth rate and $\gamma_{damping}$ the background damping rate caused by resistivity, electron Landau damping etc [6]. The mode splitting is interpreted as an oscillation of the mode amplitude, which provides symmetric side bands in the frequency spectrum of the Fourier decomposed time evolution of the mode. These side bands are centred at the mode frequency with shifts, corresponding to the oscillation period of the mode amplitude, which are given by an effective collision frequency [4]. A much larger separation of the side bands is seen in experiments than can be explained by Coulomb collisions [4].

When the ICRH is turned off in experiments it is observed that the mode amplitude decreases on a time scale much shorter than the slowing down time [1].

In order to make detailed studies of the GAE dynamics during ICRH the SELFO code, used for calculating the distribution function and the wave field self-consistently during ICRH including finite drift orbit width effects, has been upgraded to treat the interactions with GAEs [7]. Excitation of toroidicity-induced Alfvén eigenmodes, TAEs, by cyclotron heated fast ions studied with the SELFO code demonstrates good agreements with experiments on the separation of the side bands and the fast damping of the TAEs when the ICRH is turned off.

2. Wave-particle interaction

The guiding centre orbit of a charged particle can be defined by the invariants of the equation of motion. The changes in these invariants due to interactions with GAEs are obtained by integrating the equation of motion along the orbit. In absence of decorrelations by collisions or interactions with other waves the guiding centre orbit invariants will execute a non-linear superadiabatic oscillation in phase space. In the space (E, P_ϕ, μ) for axisymmetric plasmas, the superadiabatic oscillation of an orbit will take place near its resonance along a characteristic defined by

$$\Delta P_\phi = \frac{n}{\omega}\Delta E \tag{1}$$

and

$$\Delta\mu = 0, \tag{2}$$

where E is the energy, $P_\phi = mRv_\phi + eZ\Psi$ the canonical toroidal angular momentum, Ψ the poloidal flux and μ the magnetic moment. The maximum and minimum values of the energy of a resonant ion during these superadiabatic oscillations along the MHD characteristics depend on the amplitude, A, of the GAE and on the non-linear bounce frequency ω_{NLB}, with $\omega_{NLB} \propto A^{1/2}$ [8]. Decorrelation of the MHD interactions results in a diffusion process of the distribution function along the MHD characteristics. When the decorrelation time is much shorter than the non-linear bounce time, the boundaries in energy, E_{min} and E_{max}, of the resonant regions in phase space are determined by the resonance condition $(n\omega_\phi - m\omega_\theta - \omega t \pm 2\pi n_0/\tau_b)\tau_d \leq 2\pi$, where ω_ϕ is the toroidal angular frequency, ω_θ is the poloidal angular frequency, ω is the frequency of the mode, n and m are toroidal and poloidal mode numbers, τ_b is the poloidal bounce time, n_0 an integer representing higher harmonics and τ_d the decorrelation time. Phase decorrelation, as the guiding centre moves along its orbit, occurs due to changes of the invariants by collisions or interactions with other waves, such as magnetosonic waves used for cyclotron heating. These changes of the guiding centre move the invariants to a new orbit, on a neighbouring GAE characteristic in the phase space, with a different orbit time and hence with a different frequency. As the time passes the phase between the guiding centre and the GAE starts to differ from that it would have had if it had continued along its original characteristic. After a number of such interactions the phase of the guiding centre orbit relative the wave phase will change but the scattering away from the original characteristic in phase space for the GAE interactions can still be small. The phase decorrelation time by ion cyclotron interactions and Coulomb collisions is given by

$$\tau_d^3 = \frac{3 \cdot 2\pi}{\dot{\sigma}_{IC}^{EE} G_I^2 + \dot{\sigma}_C^{EE} G_E^2 + \dot{\sigma}_C^{\Lambda\Lambda} G_\Lambda^2}, \tag{3}$$

where $\dot{\sigma}^{J_1 J_2}$ is the time derivative of the covariance of the invariants J_1 and J_2 caused by the operator denoted by the subscript, where IC and C denote ion cyclotron interactions and Coulomb collisions respectively. G is the derivative of the change in phase angle in the direction denoted by the subscript, where I is along the characteristic of the ion cyclotron interaction and $\Lambda = \mu B_0 / E$.

When the resonant regions of GAEs with different toroidal mode numbers or frequencies overlap the diffusion in phase space becomes two dimensional, resulting in larger regions where the distribution function is flattened.

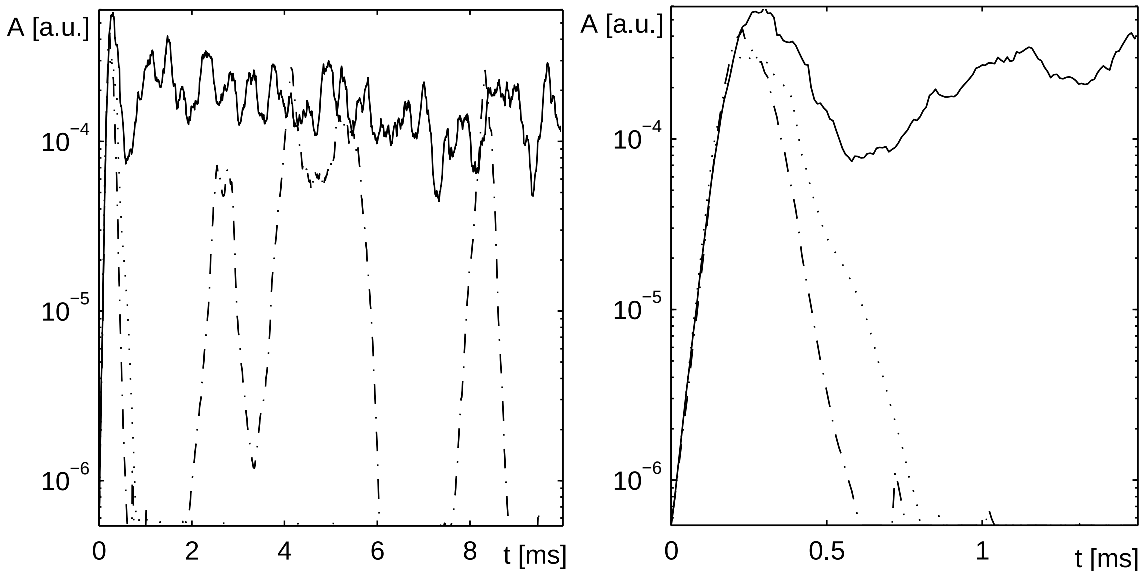
The change in energy due to interactions with an MHD mode for which the magnetic moment is conserved is given by [9].

$$\frac{dE}{dt} = eZ\mathbf{E}_1 \cdot \mathbf{v}_{d0} + \mu \frac{\partial B_{1\parallel}}{\partial t}, \tag{4}$$

where $\mathbf{E}_1 = -\boldsymbol{\xi}_1 \times \mathbf{B}_0$ is the first order electric field, $\boldsymbol{\xi}_1$ the first order plasma displacement and $\mathbf{B}_0$ the zeroth order magnetic field. The zeroth order drift velocity, $\mathbf{v}_{d0}$, is caused by gradients and curvature of $\mathbf{B}_0$. In general there are several resonant regions in phase space in which the variation of the change in energy, ΔE, of the particle due to interaction with the MHD mode has a rather complicated behaviour, sometimes separated by a surface where ΔE vanishes causing boundaries even in the resonant regions. Mode excitation can appear when the distribution function increases with energy along the GAE characteristics in some part of the resonant region in phase space. For excitation it is necessary that the net growth of all resonant interactions exceed the damping of the mode by other effects such as resistivity, electron Landau damping etc, here referred to as background damping. In general the distribution function will be decreasing with energy along a set of characteristics in one or several resonant regions, providing an intrinsic damping stabilising the mode, and along other characteristics increasing with energy and hence destabilising the mode. An unstable mode will then first grow while flattening the distribution function in the unstable regions and thereby reducing the free energy and possible growth of the mode. In absence of the background damping and the intrinsic damping the mode amplitude will saturate at a finite level. As the energy distribution flattens in the most unstable regions the drive weakens and the stabilising effects in other parts of phase space start to dominate, resulting in a damping of the mode while locally flattening the distribution function along the characteristics also in the stable regions. Thus even in the absence of background damping the mode is expected to be damped. It is therefore important to include the interactions in the whole phase space when studying the dynamics of GAE mode excitation. To allow self-consistent studies of the GAE mode dynamics during ICRH the ICRH code SELFO has been upgraded to also include interactions with GAEs [7]. This has been done by including the changes in the orbit invariants caused by the MHD interactions in the Monte Carlo code FIDO [10], which describes the evolution of the distribution function of the cyclotron heated ions and at the same time calculating the corresponding change of the amplitude of the GAEs. The invariants (E, P_ϕ, Λ) are used in the SELFO code. The displacement $\boldsymbol{\xi}_1$ in Eq. 4 is obtained either by solving the linearised MHD equation with the LION [9, 11] code or from a model [2]. The increment for the change in energy, ΔE, is obtained by integrating Eq. 4 over a decorrelation time and depends on the phase between the guiding centre of the resonant ion and the wave. The change in Λ is related to the changes in energy by $\Delta\Lambda = \Lambda \Delta E/E$.

3. Results

The effects of the decorrelation of the wave-particle interactions by ICRH on the dynamics of the GAE excitation is studied with the SELFO code for a JET-like H-minority heating scenario with $5\ MW$ of ICRH power at $51\ MHz$ with $+90^o$ phasing between the currents in the antenna straps in a plasma with circular cross section, $r_0 = 0.9\ m$, $R_0 = 2.97\ m$, $n_H/n_D = 0.04$, $n_D = 2 \times 10^{19}\ m^{-3}$, $Z_{eff} = 2.2$, $T_e(0) = T_D(0) = 10\ keV$, $B_0 = 3.45\ T$ and $I_p = 2.6\ MA$. The distribution function of H ions in absence of GAEs is first computed. The dynamics of an unstable TAE, illustrated in Fig. 1, is then studied for three cases using the simplified model of the mode given in Ref. [2]: in absence of collisions and ICRH; with collisions only; and with collisions and ICRH. The simulations give for the three cases an almost identical initial growth rate $\gamma = 3.8 \times 10^4\ s^{-1}$. For a mode frequency of $\omega = 1.45 \times 10^6\ s^{-1}$ we obtain $\gamma/\omega = 2.6\ \%$.

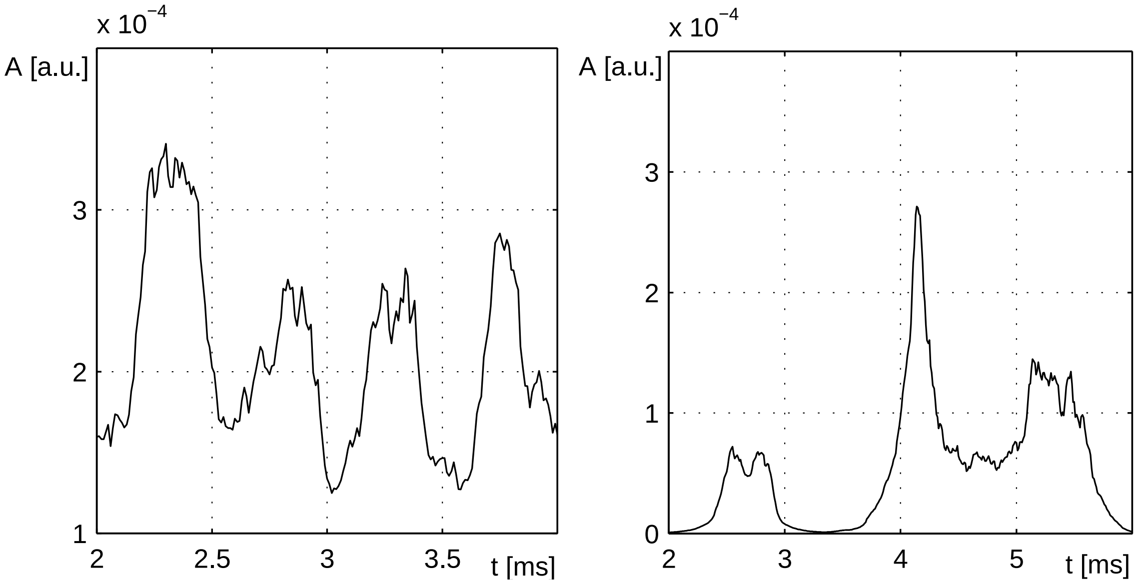


(a) $0 < t < 10\ ms$ (b) $0 < t < 1.5\ ms$

Figure 1: *Evolution of the TAE mode amplitude, dotted line without collisions and ICRH interactions, dotted dashed line collisions only and full line ICRH interactions and collisions.*

When the mode grows the distribution function in the high-energy regions in phase space is flattened along the MHD characteristics, resulting in a weaker drive. The interactions in the regions in phase space where the resonant ions have lower energy become more important. If the local distribution function decreases with energy along the MHD characteristics these interactions will then damp the mode on longer time scales. The intrinsic damping rate in absence of collisions and ion cyclotron interactions becomes $\gamma_d = 1.4 \times 10^4\ s^{-1}$, $\gamma_d/\omega = 1\ \%$. In absence of mechanisms restoring the distribution function, such as collisions or ion cyclotron interactions, the mode will be damped out by the intrinsic damping even in the absence of a background damping. The presence of ion cyclotron interactions and Coulomb collisions will restore the unstable regions in phase space with new ions entering the resonant regions and by removing ions from them. The new or lost resonant ions can either damp or excite the mode depending on where in phase space they enter or leave the resonant regions. Whether the mode is damped or excited depends on the gradients along the characteristics outside the resonant regions. The simulations demonstrate that the initial growth rate is not significantly affected by Coulomb collisions or by ion cyclotron interactions. However, the damping after the initial excitation depends strongly on the decorrelations. When collisions are included the damping rate increases to $\gamma_d = 2.1 \times 10^4\ s^{-1}$ while collisions and ion cyclotron interactions together decrease the damping rate to $\gamma_d = 0.7 \times 10^4\ s^{-1}$.

As the particles are heated by ICRH and enter the resonant region they are in general displaced outwards with respect to minor radius while transferring energy to the mode. This may lead to an oscillation of the mode amplitude, or rather bursts of GAE mode activity, since the flattening and restoration of the distribution function take place on different time scales. The initial condition of starting the GAE simulation with a preheated distribution function affect essentially only the first GAE burst. In the following bursts the distribution function is partially restored by collisions and ion cyclotron interactions. That the growth and damping of the GAE mode take place on the same time scale hints that the unstable GAE mode takes off when the linear growth rate just exceeds the background damping as assumed in the model by Berk et al [2]. If we exclude ICRH interactions in the simulation the distribution function becomes more slowly restored by collisions alone resulting in less frequent bursts of GAE mode activity. The infrequent bursts give rise to a frequency splitting of the Fourier decomposed time dependent wave field. In absence of ion cyclotron interactions the typical period of the fluctuations of the mode amplitude becomes $1.5\ ms$, corresponding to $\Delta\omega = 2\pi \times 6.7 \times 10^2\ s^{-1}$. When both ion cyclotron interactions and Coulomb collisions are included the resulting period between the bursts decreases to $0.5\ ms$, corresponding to $\gamma = 2\pi \times 2 \times 10^3\ s^{-1}$. The variation of the mode amplitude in the time interval 2 to $4\ ms$ is shown in Figs. 2(a) and 2(b) for the two cases and the power spectrum of the side bands in Fig. 3.



(a) *Including ICRH interactions and Coulomb collisions,* $2 < t < 4\ ms$.

(b) *Including only Coulomb collisions,* $2 < t < 6\ ms$.

Figure 2: *Oscillation of the mode amplitude.*

When the ICRH is turned off at $t = 10\ ms$ the mode amplitude damps rapidly, as can be seen in Fig. 4 with a damping rate $\gamma_d = 0.9 \times 10^4\ s^{-1}$. Decorrelations by collisions will still cause a flow of particles through the resonant regions.

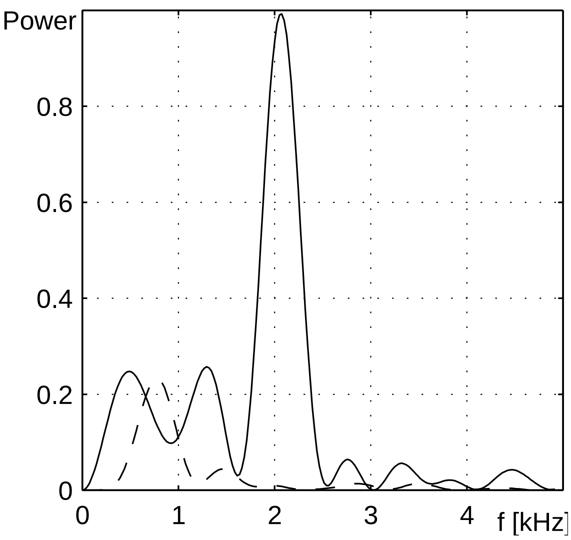


Figure 3: *Power spectrum of side bands with collisions dashed line, both collisions and ICRH full line.*

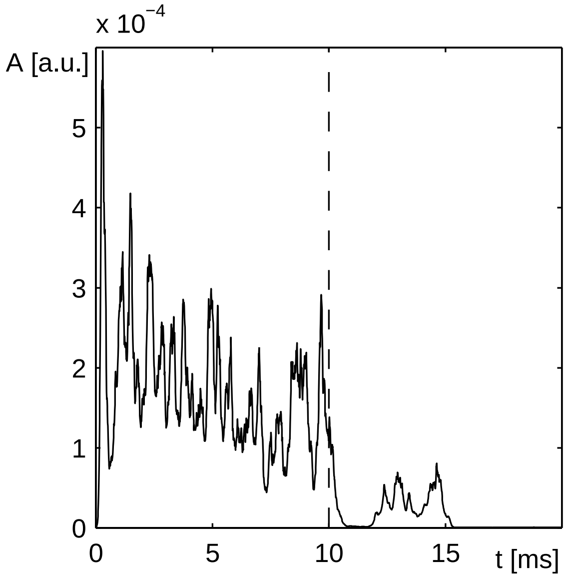


Figure 4: *The evolution of the TAE mode amplitude when ICRH and collisions are included. ICRH is turned off at 10ms.*

The excitation of a single TAE mode resulted in a change of the fast particle energy content with only $1\ \%$ after $10\ ms$ of mode activity. The results are within the noise level and for interaction with a single mode no significant effect can be seen on the heating.

4. Conclusions

A model allowing self-consistent studies of the effects of decorrelations by ICRH and Coulomb collisions of ions interacting with GAE modes has been developed and implemented in the SELFO code taking into account the complex structure of the resonant regions in phase space [7]. The variation of the distribution function produces regions destabilising and stabilising GAEs. A typical intrinsic damping rate of about $1\ \%$ is found, comparable with the damping by resistivity and ELD and the growth rate of the GAEs. The decorrelation of fast particles and the restoration of the distribution function by ICRH have a strong effect on the dynamics of the modes. Particles are constantly pushed in and out of the resonant regions, leading to a dynamic course of events of the mode amplitude. The typical oscillation period, which is of the order $1\ ms$, of the mode amplitude is seen to decrease with increasing decorrelation by ICRH, which is in agreement with experimental observations [4] and numerical simulations [12, 13]. The fast decay of the TAE-mode, which is observed in experiments as the ICRH is turned off, is also reproduced in the simulations.

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