On Electron-Cyclotron Waves in Relativistic Non-Thermal Tokamak Plasmas

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Abstract: Waves propagating at an arbitrary angle to the steady magnetic field in fully relativistic plasmas with a small population of superthermal particles are investigated. It is found that for arbitrary $N_{||}$ -values, a noticeable increase of the plasma cut-off density occurs with the increase of the weight and temperature of the superthermal tail. Its presence does not change the features of the dispersion curves of the ordinary EC wave: the increase of the real part of the perpendicular component of the ordinary wave refractive index is its main manifestation. Furthermore, the presence of superthermal particles induces an extended tail in the absorption profile and changes the location of the wave absorption region in large toroidal devices. At high tail temperatures it produces a hump far away in the low frequency wing of the thermal profile so that a double peaked structure of the absorption profile is obtained. Finally, the superthermal tail may effect the mode interaction of the extraordinary and quasilongitudinal modes near the second EC harmonic by decoupling or coupling them.

1. Introduction

A non-thermal feature which is frequently encountered in tokamak-like plasmas is the presence of a low-density population of superthermal particles. This is a common fact in heated plasmas (minority ion heating, lower hybrid heating, electron cyclotron heating, etc.). During electron cyclotron (EC) heating for example, the velocity distribution function of electrons becomes strongly non-Maxwellian, a significant part of the electron energy being in the tail of the distribution. As known, the detailed evolution of the distribution function can be followed by solving the Fokker-Planck equation. By adopting a model distribution function which possess the main envisaged features of the real distribution however, one can obtain an insight into the wave propagation and absorption properties in such media. Recent Fokker-Planck calculations have suggested that a RF-driven distribution function can be accurately modeled as a sum of two isotropic Maxwellian functions. This approach has been adopted in a number of papers in which the influence of a small population of high energy streaming electrons on the wave propagation has been investigated [1]-[3]. The analysis performed in [1] is limited to relatively low temperatures of the superthermal particles ($T\sim$ 20 keV) while [2] and [3] have assumed a nonrelativistic Maxwellian distributions of both, the bulk and tail particles.

Recently, we have examined the propagation and absorption properties of waves propagating at an arbitrary angle to the steady magnetic field [4], [5], by solving the complete fully relativistic dispersion equation for waves in thermal plasmas. In order to investigate properties of ordinary, extraordinary and quasilongitudinal (electron Bernstein) modes in EC frequency range and their coupling for arbitrary propagation angle, taking into account non-thermal features of tokamak plasmas, we have developed a numerical code by using expressions for the fully relativistic dielectric tensor which contains products of Bessel functions instead of an infinite sum over harmonics (generalization of expressions [4]). As an example of the solutions of the fully relativistic dispersion equation, in the present paper we consider waves in a plasma consisting of two relativistic Maxwellian populations simulating the presence of a superthermal tail.

2. Wave Cut-offs

As is known, the EC wave cut-off location depends sensitively on the electron temperature (see for instance [6]). Therefore, it is interesting to examine the influence of the superthermal tail in the electron distribution upon the wave cut-off. The superthermal tail is simulated by adding to a bulk relativistic Maxwellian distribution another one describing the tail particles,

$$f = (1-\eta) f_b + \eta f_t \tag{1}$$

where $f_j = [\mu_j / 4\pi K_2(\mu_j)]\, e^{-\mu_j \gamma}$, $\mu_j = mc^2/kT_j$, $\gamma = (1+v^2/c^2)^{1/2}$, $K_2(\mu_j)$ is the McDonald function. The index j distinguishes the two populations, i.e. $j = b$ refers to the bulk electrons (temperature T_b and density n_b) and $j = t$ to the superthermal electrons having a temperature T_t and density $n_t = [\eta/(1-\eta)]\, n_b$. By changing the free parameters of the tail, namely the weight and the tail temperature we have examined their effect upon the wave cut-off.

Therefore the dielectric tensor using the distribution function (1) reads,

$$\hat{\varepsilon} = \hat{1} - i\frac{X}{Y}\left[(1-\eta)\hat{M}_b + \eta\hat{M}_t\right] \tag{2}$$

where $\hat{M}_b$ and $\hat{M}_t$ are the mobility tensor due to the bulk and tail distributions, respectively, $X = \omega_p^2/\omega^2$, $Y = \omega_c/\omega$, ω_p and ω_c are the electron plasma and cyclotron frequencies. The fully relativistic expressions for the mobility tensor can be found elsewhere [4].

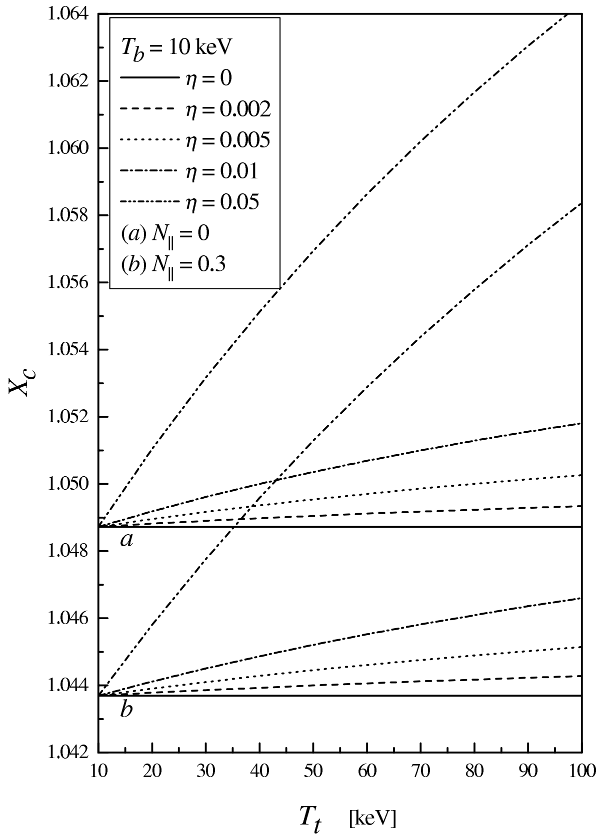

FIG. 1. The wave cut-off X_c against the temperature of the tail T_t expressed in keV's for T_b =10 keV, various values of weight of the tail η and (a) $N_{||}$ = 0 and (b) $N_{||}$ =0.3.

As it can be expected, the location of the true cut-off $N_\perp = N_{||} = 0$ ($N_\perp$and $N_{||}$ are respectively, the perpendicular and parallel component of the wave refractive index) as well as of the cutoff of waves propagating at fixed, real $N_{||}$ -value $N_\perp= 0$, $N_{||} \neq 0$, depend on the weight and tail temperature. In both cases the increase of the weight of the tail displaces the plasma cut-off location to higher density values. Similarly, the increase of the tail temperature leads to higher cut-off values. This observation is illustrated by representing in figure 1 the variation of the plasma cut-off density X_c with the the tail temperature T_t (expressed in keV's) for a bulk temperature T_b = 10 keV, five different values of the weight of the tail and (*a*) $N_{||}$ = 0 and (*b*) $N_{||}$ = 0.3. The discussed modifications of the the cut-off density are relatively small but they may be of practical consequence to a number of microwave experiments on fusion plasmas

3. Ordinary Waves

The following two main features characterize the ordinary wave propagation and absorption in the EC frequency range: the anomaluous wave dispersion near the first and second harmonic of the EC frequency and the strong damping of the wave near these harmonics. Let us examine the effect of the superthermal tail on these features. The performed analysis indicates that in presence of superthermal particles the dispersion curves follow the ones obtained without the tail. Only the real part of the wave refractive index is shifted towards higher values. With increasing the density or more precisely *X*, this shift of $N_{\perp r}$ enlarges. It is usually expected that the superthermal population contributes mainly to the anti-Hermitian part of the dielectric tensor that is, to the wave damping.

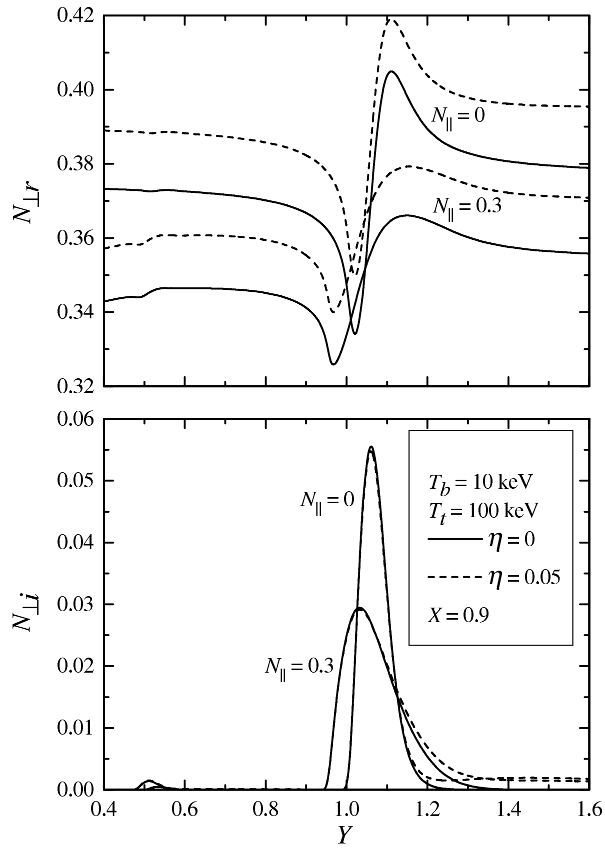

FIG. 2. The real $N_{\perp r}$ and imaginary part $N_{\perp i}$ part of the ordinary wave refractive index against Y for X = 0.9, T_b = 10 keV and T_t = 100 keV, $N_{||}$ = 0 and $N_{||}$ = 0.3, η =0 and η = 0.05.

The obtained absorption profiles show a harmonic structure with purely relativistic or oblique wave propagation features. However, the presence of superthermal particles induces an extended tail in the absorption profile. At high tail temperatures it produces a hump far away in the low frequency wing of the thermal profile so that a double peaked structure of the absorption profile is obtained. The thermal feature of the absorption profile of the ordinary wave dominates over the non-thermal (tail) one.

The variation of the real $N_{\perp r}$ and imaginary $N_{\perp i}$ part of the perpendicular component of the wave refractive index with Y in a broad frequency range $\omega = (0.625\text{-}2.5)\ \omega_c$, represented on figure 2 illustrates the discussed general effects of the superthermal tail. Here, the bulk and tail temperatures are taken to be $T_b = 10$ keV and $T_t = 100$ keV, respectively, $N_{||} = 0$ and $N_{||} = 0.3$, and $X = 0.9$. The weight of the tail η is taken both $\eta = 0$ and $\eta = 0.05$ to exhibit the dependence of the the wave refractive index on η itself.

In order to take an insight into the influence of the superthermal tail on the location of the absorption layer in real toroidal discharges, we have examined the solutions of the complete fully relativistic dispersion equation along the equatorial plane of these devices. The following plasma parameters and radial profiles are assumed: parabolic density and temperature profiles with $X(0) = 0.9$, $T_b(0) = 10$ keV and $T_t(0) = 100$ keV, $Y(x) = Y(0)/(1+x/3)$ with $Y(0) = 1$, and the weight of the tail $\eta = 0$ and $\eta = 0.05$. The obtained variation of the real $N_{\perp r}$ and imaginary $N_{\perp i}$ part of the perpendicular component of the wave refractive index with the dimensionless coordinate x is shown on figure 3.

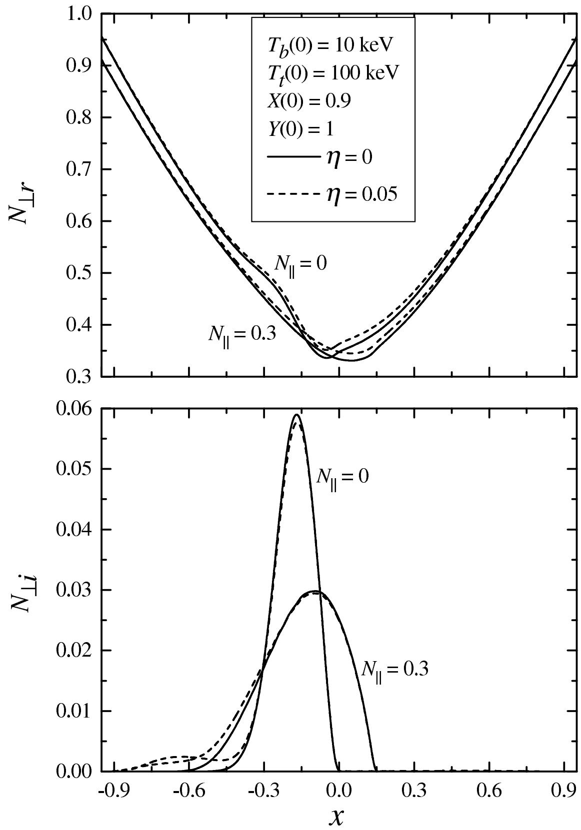

FIG. 3. The variation of the real $N_{\perp r}$ and imaginary part $N_{\perp i}$ part of the ordinary wave refractive index with the dimensionless coordinate x. It is assumed that the density and temperatures have parabolic radial profiles with $X(0) = 0.9$, $T_b(0) = 10$ keV and $T_t(0) = 100$ keV, $Y(x) = Y(0)/(1 + x/3)$ with $Y(0) = 1$ and the weights of the tail are $\eta = 0$ and $\eta = 0.05$.

We see that here also, the real parts of the wave refractive indices are close. The main effect of superthermal electrons is the production of a tail of $N_{\perp i}$ -values which extends up to the periphery of the plasma column. The corresponding $N_{\perp i}$ -values in this region are however small as compared to the ones obtained in the plasma core (at least for the assumed parameters). High reduced optical thickness values $A = \tau / 2k_0 a = \int_{-1}^{1} N_{\perp i}(x)dx$ are obtained for both $N_{||} = 0$ and $N_{||} = 0.3$ wave launching from the high magnetic field side: $A \approx 1.18\times10^{-2}$ and about 6-7% larger A-values when a tail with η = 0.05 is present. This indicates that in the considered physical situation the single pass power absorption is total. It should be noted however, that in large toroidal devices ($k_0 a > 10^3$) where k_0 is the wave number in vacuum and a is the minor radius) the presence of the superthermal tail influences the location of the wave absorption region. Taking as an example the parameters of the stellarator LHD (a = 60 cm, f = 168 GHz), one has $k_0 a$ = 2110 and the presence of a tail with η = 0.05 shifts the e-folding power distance from x = -0.361 to x = -0.676 for $N_{||} = 0$, and from x = -0.463 to x = -0.640 for $N_{||} = 0.3$. This shift may be of consequence in experiments in which localized absorption or current drive is important, such as the stabilization of MHD islands.

4. Extraordinary and Quasilongitudinal Waves

The phenomenon of mode interaction marks the extraordinary and quasilongitudinal wave propagation in the EC frequency range. It is found [5] that there are three points at which the solutions of the fully relativistic dispersion equation describing the extraordinary and the least damped quasilongitudinal modes coincide. The first branch point (b_1) occurs at low plasma densities, near the EC frequency. It describes the mode coupling near the upper hybrid resonance. The remaining two branch points (b_2) and (b_3) form a closed region in the plasma parameter space in which mode interaction occurs. At low electron temperatures, the range of frequency variation of this region is very narrow and centered very close to the second EC harmonic. Mode coupling between the extraordinary and quasilongitudinal waves occurs in the discussed plasma parameter space.

The influence of the superthermal tail on the mode interaction is illustrated by representing in figure 4 the variation of the real $N_{\perp r}$ and imaginary $N_{\perp i}$ part of the perpendicular component of the wave refractive index with Y in a frequency range which is close to the branch point. The bulk and tail temperatures are taken to be T_b = 10 keV and T_t = 100 keV, respectively, $N_{||} = 0.06$ and $X = 0.3$. For η = 0.05 the considered plasma parameters lie out of the (b_2, b_3) mode interaction region (see [5]) so that in the presence of a superthermal population the solution trajectories of the extraordinary and quasilongitudinal modes are close but decoupled (lower case in figure 4). On the contrary, for η = 0.0 the extraordinary and quasilongitudinal mode interchange their solution trajectories. Namely, for $Y > Y_{b2}$ the extraordinary mode ($N_{\perp r1}$, $N_{\perp i1}$) matches smoothly with the highly damped quasilongitudinal plasma mode ($N_{\perp r2}$, $N_{\perp i2}$) and *vice versa*, the high-field quasilongitudinal mode links with the solution trajectory of the extraordinary mode. Therefore, the superthermal tail may effect the mode interaction by decoupling or coupling the extraordinary and quasilongitudinal modes near the second EC harmonic.

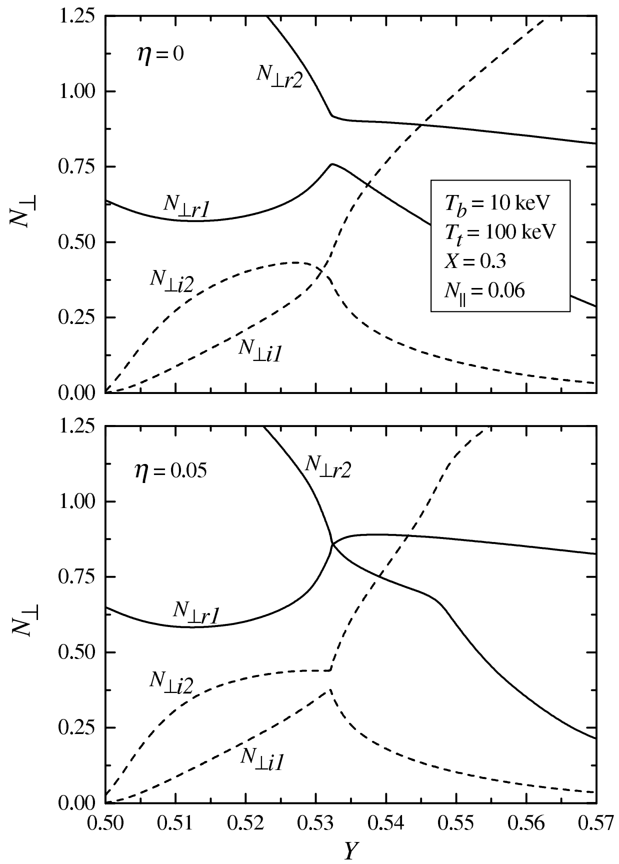

FIG. 4. The real $N_{\perp r}$ and imaginary part $N_{\perp i}$ part of the extraordinary wave refractive index against Y for X = 0.3, T_b = 10 keV and T_t = 100 keV, and $N_\parallel$ = 0.06. The weights of the tail are η = 0 (top) and η = 0.05 (bottom).

5. Conclusions

We have investigated waves propagating at an arbitrary angle to the steady magnetic field in fully relativistic plasmas with a small population of superthermal particles. The fully relativistic dispersion equation describing waves in relativistic two-Maxwellian plasma is solved numerically in a wide range of bulk and tail plasma parameters and wave propagation conditions. It is found that for arbitrary $N_\parallel$ -values, a noticeable increase of the plasma cut-off density occurs with the increase of the weight and temperature of the superthermal tail. The presence of the superthermal tail does not change the features of the dispersion curves of the ordinary EC wave: the main manifestation of the superthermals is the increase of the real part of the perpendicular component of the ordinary wave refractive index. Furthermore, the presence of superthermal particles induces an extended tail in the absorption profile and changes the location of the wave absorption region in large toroidal devices. This shift may be of consequence in experiments in which localized absorption or current drive is important. At high tail temperatures it produces a hump far away in the low frequency wing of the thermal profile so that a double peaked structure of the absorption profile is obtained. Finally, the superthermal tail may effect the mode interaction by decoupling or coupling the extraordinary and quasilongitudinal modes near the second EC harmonic.

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