Effects of Steep Gradients and Stochasticity on the Rotation Dynamics of Collisional Tokamak Edge Layer

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Abstract. Toroidal and poloidal rotations of a collisional tokamak edge plasma with steep temperature and density gradients are investigated via the fluid equations. Stability of the solutions over time and space coordinates is considered. In the stationary case, trajectory bifurcations in phase space of toroidal and poloidal speeds are discussed.

1. Introduction

The onset of rotation, or spinning-up of plasma in toroidal and/or poloidal directions, and the concomitant rise of a radial electric field are among the most common phenomena in toroidal plasmas observed during the L-H transition. The basic assumption in explaining these facts is that the shear created by the plasma rotation may serve to suppress the micro turbulences and thus lead to the reduction of radial transport and to the enhancement of confinement. It is, therefore, important to explain the rotation in tokamak plasmas, as one of the major problems (see, for example, Refs. [1, 2]). At the high collisionality regime of the toroidal edge plasma with steep thermal and density gradients poloidal and toroidal rotations were investigated in Refs.[3-5] by using the modified stress tensor due to Mikhailowskii and Tsypin [6]. It was pointed out in Ref.[4] that, when parameter $\Lambda_1 \equiv (v_i / \Omega_i)(q^2 R^2 / r L_w)$ is larger than 1/3, then the parallel momentum equation, which is one of the basic equations for determining plasma rotation, needs modification. Here, v_i is the ion collision frequency, Ω_i the ion Larmor frequency, q the safety factor, L_{Ψ} is the scale length of radial gradients, r and R are the minor and major radii, respectively. In order to investigate the rotation phenomena in this study we use modified fluid equations for collision dominated toroidal plasma with circular cross section, large aspect ratio and steep gradients near the tokamak edge including some mass and momentum sources [3-5]. In these equations rotation velocities are expanded in a small parameter μ ~0.1, which was defined in the modified neoclassical theory [3-5] as the ratio $\mu \sim B_{\theta}/B_{\omega} \sim L_{w}/r \sim r/(qR) \sim c_{i}/(qRv_{i})$, where c_{i} is the ion thermal speed, and B_{θ} and B_{ω} are the poloidal and toroidal magnetic fields, respectively; [3]. By assuming this ordering scheme, the toroidal momentum equation was found as [5, 7]

$$m_{i}N_{i}\frac{\partial U_{\varphi,i}}{\partial t} = \frac{\partial}{\partial r} \left[\eta_{2,i} \left(\frac{\partial U_{\varphi,i}}{\partial r} - \frac{0.107 q^{2}}{1 + Q^{2}/S^{2}} \frac{\partial \ln T_{i}}{\partial r} \frac{B_{\varphi}}{B_{\theta}} U_{\theta,i} \right) \right] + J_{r}B_{\theta}$$

$$-m_{i} \oint \frac{d\vartheta}{2\pi} h^{2}S_{i}^{N}U_{i\varphi} + \oint \frac{d\vartheta}{2\pi} h^{2}\vec{S}_{i}^{M} \cdot \vec{e}_{\varphi} .$$

$$(1)$$

A toroidal momentum source is also included in the toroidal momentum balance (ambipolarity constraint) equation above, where h=1+(r/R₀) cos ϑ , and J_r is a possible radial polarization current. The classical perpendicular viscosity coefficient is denoted by $\eta_{2,i} = 1.2N_iT_iv_i/\Omega_i^2$.

An equation for the poloidal velocity in a collisional plasma edge with steep thermal gradient from the parallel momentum balance equation was also derived in Ref. [4], where the term involving the time derivative of the poloidal velocity was dropped as this term was multiplied by a small factor. Hence, in that derivation the time evolution of the poloidal velocity was strongly synchronized with that of the toroidal velocity. However, the presence of time derivatives with factors of different orders is indicative of the toroidal and poloidal velocities having different damping times. Indeed, in Ref. [8] poloidal and toroidal damping times were also calculated to be of the following orders

$$\tau_{\rm P} < O[(a_i/r)^2, (v_{\rm cx}/v_i)]\tau_{\rm T} << \tau_{\rm T}.$$

After a flux surface averaging of the term including the time derivative of the poloidal velocity, the parallel momentum equation reads as follows [9]:

$$\begin{split} & m_{i}N_{i}^{(0)}(1+2q^{2})\partial_{t}U_{\theta,i} = q^{2}\varepsilon^{-1}m_{i} < \widetilde{Z}\cos\theta > U_{\theta,i} + < S_{\theta,i}^{M} > \\ & -\frac{3\eta_{0,i}}{2R^{2}} \left(U_{\theta,i} + \frac{1.833}{e_{i}B_{\phi}}\frac{\partial T_{i}}{\partial r} \right) + 0.54 \frac{\eta_{2,i}q^{2}}{1+Q^{2}/S^{2}} \frac{e_{i}B_{\phi}}{T_{i}}\frac{\partial \ln T_{i}}{\partial r} \left[\frac{T_{i}}{e_{i}B_{\theta}}\frac{\partial U_{\phi,i}}{\partial r} + \frac{1}{2}U_{\phi,i}^{2} \right] \\ & - U_{\phi,i}\frac{B_{\phi}}{B_{\theta}} \left(U_{\theta,i} - \frac{T_{i}}{e_{i}B_{\phi}}\frac{\partial \ln N_{i}^{2}T_{i}}{\partial r} \right) + 1.90 \frac{B_{\phi}^{2}}{B_{\theta}^{2}} \left(U_{\theta,i} - 0.8 \frac{T_{i}}{e_{i}B_{\phi}}\frac{\partial \ln N_{i}^{1.6}T_{i}}{\partial r} \right)^{2} \right] - J_{r}B_{\phi}. \end{split}$$

where $Q = [4B_{\phi}U_{\theta,i} - 2.5(T_i/e_i)\partial \ln N_i^2T_i/\partial r]B^{-1}$, and $S = 2r\chi_{\parallel,i}N_i^{-1}/(q^2R^2)$, parallel heat diffusion coefficient is $\chi_{\parallel,i} = 3.9P_i/m_i\nu_i$. The first term on the right which is proportional to U_{θ} describes damping or growth of the poloidal velocity depending on its sign. A similar homogeneous term was derived in Ref. [10, 11] and identified as the likely cause of the subsonic Stringer spin-up. However, the inhomogeneous term due to a momentum source is seen to be a direct drive for the poloidal flow.

2. Two-time-scale analysis

If the equations are normalized by the toroidal damping time τ_T , a small number $\hat{\epsilon}=O(\mu^2)$ will appear as a factor of the time derivative on the l.h.s. of Eq.(2). This small factor can be used to define a new fast time variable by $\tau=t/\hat{\epsilon}$, which is characteristic for the poloidal relaxation. Now, the term including the time derivative w.r.t. this new variable will become of zeroth order and its effect on the transient evolution of poloidal velocity in the short time scale will become evident. We note that, this term, also called the Pfirsch-Schlüter factor, which expresses an increase of inertia, depends on the collisionality and is altered in the plateau regime [12]. Enhancement of the poloidal inertia arises from our basic assumptions used in the calculations, such as the incompressibility of flow and the zero radial velocity at the order considered [8, 9, 13].

The poloidal, U_{θ} , and toroidal, U_{ϕ} , speeds in the modified toroidal and parallel momentum equations, Eq.(1) and Eq.(2), can be normalized by $\mu^2 c_i$, and by μc_i , respectively. Assuming the following two time scale expansions for these velocities [14]

$$U_{\varphi}(\xi,\tau,t) = \sum_{n=0}^{N} \hat{\epsilon}^{n} U_{\varphi}^{(n)}(\xi,\tau,t) + O(\hat{\epsilon}^{N}), U_{\theta}(\xi,\tau,t) = \sum_{n=0}^{N} \hat{\epsilon}^{n} U_{\theta}^{(n)}(\xi,\tau,t) + O(\hat{\epsilon}^{N}), \qquad (3)$$

we obtain the following set of equations

$$\frac{\partial U_{\varphi}}{\partial \tau} + \hat{\epsilon} \frac{\partial U_{\varphi}}{\partial t} = \hat{\epsilon} \Big[Y(\xi, U_{\varphi}) + Z(\xi, U_{\theta}) \Big]$$
(4)

$$\frac{\partial U_{\theta}}{\partial \tau} + \hat{\epsilon} \frac{\partial U_{\theta}}{\partial t} = H(\xi, U_{\phi}, \partial U_{\phi} / \partial \xi, U_{\theta})$$
(5)

and substituting Eq.(3) in Eq.(4) we find

$$\frac{\partial U_{\varphi}^{(0)}}{\partial \tau} = 0.$$
 (6)

In other words, the zero order toroidal velocity, $U_{\phi}^{(0)}$, can only depend on ξ , and the slow time variable t, and therefore it will not change in the fast time scale, τ [9]. We use this result and, similarly substitute Eq.(3) in Eq.(5) to find an equation for $U_{\theta}^{(0)}(\xi, \tau, t)$ by expanding the r.h.s. of Eq.(5)

$$\frac{\partial U_{\theta}^{(0)}(\xi,\tau,t)}{\partial \tau} = H^{(0)}\left(\xi, U_{\phi}^{(0)}(\xi,t), \partial U_{\phi}^{(0)}(\xi,t) / \partial \xi ; U_{\theta}^{(0)}(\xi,\tau,t)\right)$$
(7)

where ξ , $U_{\phi}^{(0)}(\xi,t)$, $\partial U_{\phi}^{(0)}(\xi,t)/\partial \xi$ can be considered as mere parameters for the fast time variation of the poloidal velocity [9]. A solution of Eq.(7) for τ in terms of $U_{\theta}^{(0)}$ can be given as

$$\tau = \int dX \left[\frac{1 + K(X + L)^2}{\alpha X^3 + \beta X^2 + \gamma X + \delta} \right] + \text{Const.} \equiv \int dX \frac{g_2(X)}{f_3(X)} + \text{Const.}$$
(8)

Here K, L, α , β , γ , and δ are functions of the parameters ξ , $U_{\phi}^{(0)}(\xi,t)$, $\partial U_{\phi}^{(0)}(\xi,t)/\partial \xi$. g_2 and f_3 are the second and third degree polynomials of $X \equiv U_{\theta}^{(0)}$. Evaluating this simple integral for three real roots for $f_3(X)$ we find an implicit function of the zero order poloidal velocity $U_{\theta}^{(0)}$ in the following form

$$\tau = A \ln \left| \frac{U_{\theta}^{(0)} - a}{U_{\theta}^{(0)}(0) - a} \right| + B \ln \left| \frac{U_{\theta}^{(0)} - b}{U_{\theta}^{(0)}(0) - b} \right| + C \ln \left| \frac{U_{\theta}^{(0)} - c}{U_{\theta}^{(0)}(0) - c} \right|,$$
(9)

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where a, b, c are three real zeros of f_3 , and coefficients $A = g_2(a)/f'_3(a)$, $B = g_2(b)/f'_3(b)$ and $C = g_2(c)/f'_3(c)$ are functions of the slow time t, since they are functions of $U_{\phi}^{(0)}(\xi,t)$, $\partial U_{\phi}^{(0)}(\xi,t)/\partial \xi$. Depending on the values of its coefficients, the polynomial f_3 admits one real, say a, and two conjugate complex roots which we denote by $b=b_{re} \pm ib_{im}$. Then the integral in Eq.(8) yields the following solution

$$\tau = A \ln \left| \frac{U_{\theta}^{(0)} - a}{U_{\theta}^{(0)}(0) - a} \right| + B_{re} \ln \left| \frac{(U_{\theta}^{(0)} - b_{re})^2 + b_{im}^2}{(U_{\theta}^{(0)}(0) - b_{re})^2 + b_{im}^2} \right| -2B_{im} \left[\tan^{-1} \left(\frac{U_{\theta}^{(0)} - b_{re}}{b_{im}} \right) - \tan^{-1} \left(\frac{U_{\theta}^{(0)}(0) - b_{re}}{b_{im}} \right) \right].$$
(10)

The implicit solution for $U_{\theta}^{(0)}$ given by either Eq.(9) or Eq.(10) determines the stability of the poloidal velocity in the short time scale, τ , by the local temperature and density distributions together with the initial toroidal velocity profile [9]. At a particular radial point, a topological criterion defined by these equations can be given to decide whether an initial poloidal velocity will relax to a finite value or it will spin-up in the short time scale. Clearly, the assumed subsonic flow conditions are not necessarily violated by such a growth of poloidal velocity.

We can also find the slow time evolution of the zero order toroidal velocity, $U_{\phi}^{(0)}(\xi,t)$. The first order terms the Eq.(4) have the following relation

$$\frac{\partial U_{\phi}^{(1)}(\xi,\tau,t)}{\partial \tau} + \frac{\partial U_{\phi}^{(0)}(\xi,t)}{\partial t} = Y^{(0)} \Big[\xi, U_{\phi}^{(0)}(\xi,t) \Big] + Z^{(0)} \Big[\xi, U_{\theta}^{(0)}(\xi,\tau,t) \Big].$$
(11)

We note that the requirement for a self-consistent solution for $U_{\omega}^{(l)}(\xi,\tau,t)$ leads to

$$\frac{\partial U_{\phi}^{(0)}(\xi,t)}{\partial t} = Y^{(0)} \Big[\xi, U_{\phi}^{(0)}(\xi,t) \Big] + \left\langle \left\langle Z^{(0)} \Big[\xi, U_{\theta}^{(0)}(\xi,\tau,t) \Big] \right\rangle \right\rangle$$
(12)

where the symbol $<<\cdots>>$ denotes an averaging over the fast time scale, τ . Or writing explicitly, we find the slow time evolution of the toroidal velocity in the zeroth order

$$\frac{\partial U_{\phi}^{(0)}(\xi,t)}{\partial t} = \frac{1}{m_{i}N_{i}} \frac{\partial}{\partial \xi} \left[\eta_{2,i} \left(\frac{\partial U_{\phi}^{(0)}(\xi,t)}{\partial \xi} - \left\langle \left\langle \frac{0.107q^{2}}{1+Q^{2}/S^{2}} \frac{\partial \ln T_{i}}{\partial \xi} \frac{B_{\phi}}{B_{\theta}} U_{\theta}^{(0)}(\xi,\tau,t) \right\rangle \right\rangle \right) \right]$$

$$- \nu_{cx} U_{\phi}^{(0)}(\xi,t) + \dot{m} + J_{r}B_{\theta}/m_{i}N_{i}, \qquad \text{for } t > 0, \quad -\infty < \xi < +\infty.$$
(13)

In Eq.(13), the averaged $U_{\theta}^{(0)}(\xi, \tau, t)$ over τ , can be replaced by the limiting value $U_{\theta}^{(0)}(\xi, \infty, t)$ that is obtained in the fast time scale.

The parabolic equation for $U_{\theta}^{(0)}(\xi, t)$, Eq.(13), can be solved by numerical methods for arbitrary profiles of $T_i(\xi)$, $N_i(\xi)$, and a given initial velocity distribution along the infinite ξ -

axis, $U_{\phi}^{(0)}(\xi,0) = \Phi(\xi)$ ($-\infty < \xi < +\infty$). Even for a zero initial velocity distribution along ξ , it is seen that $U_{\phi}^{(0)}(\xi,t)$ can be driven in time by the inhomogeneous terms in Eq.(13) due to neutral beam injection etc. by overcoming the diminishing effect of the charge exchange collisions with neutrals on the toroidal velocity profile. It is noted that, to the zero order, $U_{\theta}^{(0)}(\xi,t)$ is coupled only to the limiting values of the poloidal velocity which is calculated in the fast time scale [9].

3. Study of equilibrium

Now, we look for the equilibrium solutions of Eq.(7) and Eq.(13) for U_{θ} and U_{ϕ} , using their lowest order equations in time independent forms. These equations can be written as

$$\frac{\mathrm{d}\,\mathrm{U}_{\varphi}}{\mathrm{d}\,\xi} = \mathrm{F}(\mathrm{U}_{\varphi},\mathrm{U}_{\theta},\xi) \tag{16}$$

$$\frac{d}{d\xi} \left[\eta_2 \frac{dU_{\phi}}{d\xi} \right] = K U_{\phi} + \frac{d}{d\xi} \left[G U_{\phi} \right] \quad , \text{ where } \quad K \equiv -m_i N_i v_{cx}$$
(17)

On the left hand side of Eq.(17) one could also include a term like $m_i N_i \dot{m}_{\phi,i}$ for the neutral beam injection, but this was dropped for simplicity. Substituting Eq.(16) in Eq.(17), and rearranging, we obtain a new nonlinear ordinary differential equation:

$$\frac{dU_{\theta}}{d\xi} = \frac{KU_{\varphi} + U_{\theta}\frac{\partial G}{\partial\xi} - (\eta_2' + \eta_2\frac{\partial F}{\partial U_{\varphi}})F - \eta_2\frac{\partial F}{\partial\xi}}{\eta_2\frac{\partial F}{\partial U_{\theta}} - U_{\theta}\frac{\partial G}{\partial U_{\theta}} - G}$$
(18)

where the prime sign denotes the derivative w.r.t. ξ . After normalization at the separatrix, the functions F and G can be obtained as

$$F = -\frac{U_{\phi}^{2}}{2T} + U_{\phi} \left(\frac{U_{\theta}}{T} - \frac{d\ln N^{2}T}{d\xi} \right) - 1.9T \left(\frac{U_{\theta}}{T} - \frac{d\ln N^{1.6}T}{d\xi} \right)^{2} + \frac{T^{3}}{N^{2}} \frac{U_{\theta} + 1.833T' + \Delta}{0.45(d\ln T/d\xi)} \left(1 + \frac{Q^{2}}{S^{2}} \right)$$
(19a)
$$G = \eta_{2} \frac{0.107q^{2}(d\ln T/d\xi)}{(1 + Q^{2}/S^{2})}.$$
(19b)

Here, Δ denotes a term due to a possible radial current. For simplicity, in the calculations we shall consider the term K, representing charge exchange effects, as a constant. In Eq.(18) the individual terms indicating partial derivatives of F and G will not be given here as they are lengthy. To this system of two first order nonlinear ordinary differential equations, Eqs.(16,18), also an equation for the temperature variation along ξ must be added. This can be taken conveniently as a model equation:

$$T(\xi) = \frac{1}{2}T_{c}\left[1 - \tanh\left(a + \frac{\xi}{\mu}\right)\right], \text{ where } a = \tanh^{-1}\left(1 - 2\frac{T_{s}}{T_{c}}\right).$$
(20)

 μ is a scale length for the temperature gradient, and T_c , T_s are values of temperature at the core and separatrix, respectively. Similarly, the density profile is assumed to be given by a relationship like $N{=}T^{1/\gamma}$, where γ can be taken as a parameter, specifically $\gamma{=}1.6$. Hence we obtain a set of autonomous first order differential equation system:

$$\frac{dU_{\varphi}}{d\xi} = f_1(U_{\varphi}, U_{\theta}, T; M), \quad \frac{dU_{\theta}}{d\xi} = f_2(U_{\varphi}, U_{\theta}, T; M), \quad \frac{dT}{d\xi} = f_3(U_{\varphi}, U_{\theta}, T; M)$$
(21)

where M is a vector of control parameters (μ , γ , Tc/Ts, K, \dot{m} , Δ ,..). The particular model chosen for T in Eq.(20) allows us to write T' and T'' also as functions of T. For example, $T' = -(2/\mu)T[1-(T_s/T_c)T]$, $T'' = (2/\mu)^2T[1-(T_s/T_c)T][1-2(T_s/T_c)T]$.

4. Fixed points of the system

The fixed points of the autonomous system Eq.(21) are defined by the vanishing of the vector field, $\mathbf{f}_i(\mathbf{u};\mathbf{M})=0$. The stability of the solutions $\mathbf{u}(\xi,\mathbf{M})$, where $\mathbf{u}=(U_{\phi}, U_{\theta}, T)$ depend on the values of the eigenvalues of the Jacobian matrix at the fixed points. At the fixed points, we must have T=0. Hence, we take the limit of **f** for this value and find:

$$F = \frac{1}{T} \left[-\frac{1}{2} U_{\phi}^{2} + U_{\phi} U_{\theta} - 1.9 U_{\theta}^{2} - \frac{\mu (U_{\theta} + \Delta) U_{\theta}^{2}}{0.9 \times 1.95^{2}} \right],$$
(22)

$$\frac{\partial F}{\partial U_{\phi}} = \frac{1}{T} \left(-U_{\phi} + U_{\theta} \right), \qquad (23)$$

$$\frac{\partial F}{\partial U_{\theta}} = \frac{1}{T} \left[U_{\phi} - 3.8 U_{\theta} + \frac{\mu U_{\theta} (3 U_{\theta} + 2\Delta)}{0.9 \times 1.95^2} \right], \tag{24}$$

$$\frac{\partial F}{\partial \xi} = \frac{1}{T} \left[-\frac{2}{\mu} \left(\frac{1}{2} U_{\phi}^{2} - U_{\phi} U_{\theta} + 1.9 U_{\theta}^{2} \right) + \left(3 - 2\gamma - \frac{4 - 2\gamma}{1.95^{2}} \right) \frac{(U_{\theta} + \Delta) U_{\theta}^{2}}{0.45} \right],$$
(25)

$$G = -0.98 \frac{q^2}{\mu} \frac{T^{7/2}}{U_{\theta}^2}; \qquad \frac{\partial G}{\partial U_{\theta}} = -2 \frac{G}{U_{\theta}}; \qquad \frac{\partial G}{\partial \xi} = -\frac{7}{\mu}G.$$
(26)

At the fixed points of the system (21), T must vanish. Hence, at this limit Eqs.(21) can be considerably simplified:

$$\frac{dU_{\varphi}}{ds} = F, \quad \frac{dU_{\theta}}{ds} = -\frac{F_{U_{\varphi}}}{F_{U_{\theta}}}F$$
(27)

From these equations we find the fixed points of the system (21), as a curve, and a "node-saddle" pair in the $(U_{\phi}, U_{\theta}, T_0, \mathbf{M})$ plane.

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5. Results

In many cases while integrating the system of Eqs.(21), one observes chaotic oscillations of the solutions near the magnetic separatrix, as seen in Fig.1(a). This effect becomes more pronounced by the increasing steepness of the temperature gradient, i.e., for decreasing μ . This observation is due to the bifurcations of the solutions near the fixed points of the vector field \mathbf{f}_i at the chosen parameter values. Study of the solutions near the fixed points in the phase space indicates the likelyhood of bifurcations near a saddle-node pair as seen on Figs 2 (a,b). Integration of Eq.(27) for various initial value pairs ($U_{0}(0)$, $U_{0}(0)$) is shown on Fig. 2(a). Such a picture in the phase space determines the stability behaviour of the solutions depending on the parameters describing steepness of gradients, composition, and the likely external disturbances on the system. In this figure we show that the chosen parameter values has led to a particular bifurcation in our problem, namely to the the formation of a chaotic ring system around the node. Among the external disturbances we mention that a weak periodic excitation of temperature over the radial coordinate would be a likely trigger for another homoclinic orbit bifurcation in our saddle-node system. Indeed, the stability of temperature profile itself is sensitive to many physical factors, as shown by Bachmann et al. in Refs.[16, 17]. For example, in radiative edge plasmas, temperature bifurcations and chaos can be driven by a given time-modulated impurity density. In the light of that study, it is not unrealistic to assume a weak periodic temperature disturbance superimposed on the monotonous model profile Eq.(20). In that case, a further homoclinic bifurcation of the saddle-node-fixed-point system would be the outcome.



Figure 1. Depending on the parameters velocities in the steady state can be chaotic (a), or not (b).

Above, we have not considered the time dependent stability of the full partial differential system, Eqn.(7) and Eq.(13), in the slow-time scale. Due to the coupling of governing equations, however, transient U_{θ} becomes important in driving U_{ϕ} as well. One can further look into the effects of random temporal and spatial perturbations in radial temperature and density profiles in above equations, as they are likely to exist in tokamaks. Perturbation of the regular profiles by such random components can, under specific circumstances, also lead to chaotic behavior of the rotation speeds, as above coupled equations would act like stochasticity amplifiers as well as stochasticity generators.



Figure 2. Bifurcations of homoclinic orbits obtained by numerical integration of Eqs.(27): (a) Orbits near the "saddle-node" fixed points. (b) Blow-up of the same picture showing the details of chaotic tangles inside a ring system around the node.

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