Theoretical considerations of doublet-like configuration in stellarators

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Abstract: In order to investigate the separatrix structure inside the plasma, the magnetohydrodynamics (MHD) equilibrium and the high-energy particle orbit are studied for LHD plasmas with two split axes and an eight-figured separatrix. Magnetic axes are split vertically and horizontally for strongly elongated cross section by the external quadrupole field. MHD equilibria without assuming existence of nested flux surfaces are obtained from HINT code. For the vertical elongation, the eight-figured structure evolves due to the finite beta effect and the structure of magnetic field lines outside the separatrix is ergodized. For the horizontal elongation, since the Shafranov shift is very large, plasma shapes are changed eccentrically. To see MHD equilibrium beta limit, high beta equilibrium (~5%) is studied. The magnetic axis extremely shifts to the outward of the torus and a new eight-figured separatrix is appeared. Singularities of the high-energy particle orbit on the separatrix are studied by solving the guiding-center drift equation on the rectangular grid. If the reflecting point of the trapped particle is near the separatrix, the banana width is very large and some singular orbits are observed.

1. Introduction

The separatrix structure inside the plasma strongly affects to the magnetohydrodynamics (MHD) equilibrium, stability and transport through the shear of the magnetic and electric field and it is an important problem from the viewpoint of the dynamical systems. In recent experiments, singular configurations, which have the separatrix and stochastic field line regions inside the plasma, have been observed as the current-hole discharge in tokamaks [1, 2]. The current-hole discharge has a strongly reversed safety profile q with almost zero toroidal current in the plasma core region. In the way of the physical mechanism of the current-hole, the overlap of two or three magnetic islands by the instability is proposed [3] but it has not clearly understood yet. Since in those discharges not only there are cases in which high performance is obtained but also the concept of those configurations is beyond the conventional theory, the theoretical consideration of those configurations is urgent and critical issue. In tokamaks without vacuum flux surfaces, equilibrium flows may play an essential role to drive the magnetic reconnection and sustain the separatrix structure with the magnetic island. Thus, the generation of the separatrix structure is a dynamical phenomenon. In the conventional theory, analyses of the stability and transport are based on the static MHD equilibrium because many numerical equilibrium calculation codes can calculate the static equilibrium only. In order to see the dynamical effect, the conventional method based on the static equilibrium is unsuitable. On the other hand, in helical systems, the separatrix structure can be created and sustained not by plasma flows but by the perturbed magnetic field and the external coil. However, in many cases, the magnetic island is generated by the high-order perturbed field and the width is small. If it is possible to externally create and sustain large separatrix structure inside the plasma core region, the effect of the separatrix structure to the plasma can be considered.

It is well known that the quadrupole field elongates the cross section of plasma vertically and horizontally. The quadrupole filed is applied to the control of the shape of flux surfaces. In heliotron plasmas, when the shape of flux surfaces is elongated vertically (horizontally), following characteristic are predicted by numerical analyses[4] based on the stellarators expansions[5]: (i) the rotational transform on the axis is decreased, (ii) the Pfirsch-Schlüter (P-S) current is suppressed (enhanced), (iii) the Shafranov shift toward the outside of the torus is small (large). If the elongation produced by the quadrupole field is increased further, the rotational transform crosses zero and the magnetic shear is reversed. Consequently, the magnetic axis is split vertically or horizontally and an eight-figured separatrix appears inside the plasma (see Fig.1). The existence of such a ***doublet-like configuration***, which has the separatrix structure inside the plasma, has been demonstrated theoretically by Wang *et al*. and Pustovitov[6,7]. However, in those theoretical considerations, which are based on the vacuum field, properties of finite beta equilibrium have not been investigated yet. Recently, doublet-like configurations were produced in the LHD experiments [8]. This certifies the possibility of the generation of large separatrix structure in the steady state operation. Thus, in order to understand the basic physics of the separatrix structure inside the plasma, as a first step, the doublet-like configuration of LHD is considered theoretically.

In this study, the MHD equilibrium and the high-energy particle orbit on the doublet-like configuration of LHD are discussed. In section 2, to see the effect of the separatrix structure to the MHD equilibrium, three-dimensional (3D) MHD equilibrium is studied by the HINT code [9]. In this study, since the separatrix inside the plasma core region is studied, 3D MHD equilibrium calculation code without assumption of nested flux surfaces is required as HINT and PIES[10]. However, analyses of the doublet-like configuration by PIES calculation are very difficult. HINT solves time evolution of dissipative MHD equations on the rectangular grid. In the next section, by using MHD equilibrium of HINT, the effect of the separatrix to the high-energy particle orbit is discussed. The particle orbit is obtained from solving the guiding-center drift equation on the cylindrical coordinate. In last section, we summarize the results obtained.

2. MHD equilibrium of doublet-like configurations in LHD plasmas

Figure 1 shows two typical doublet-like configuration; (a) vertically and (b) horizontally elongated configuration for vacuum field. Radial profiles of the rotational transform and the magnetic well are also plotted. For vertically elongated configuration, an eight-figured separatrix clearly exists inside vacuum flux surfaces with the up-down symmetry. In Poincaré plots, bold lines colored with red indicate the last closed flux surface of a normal

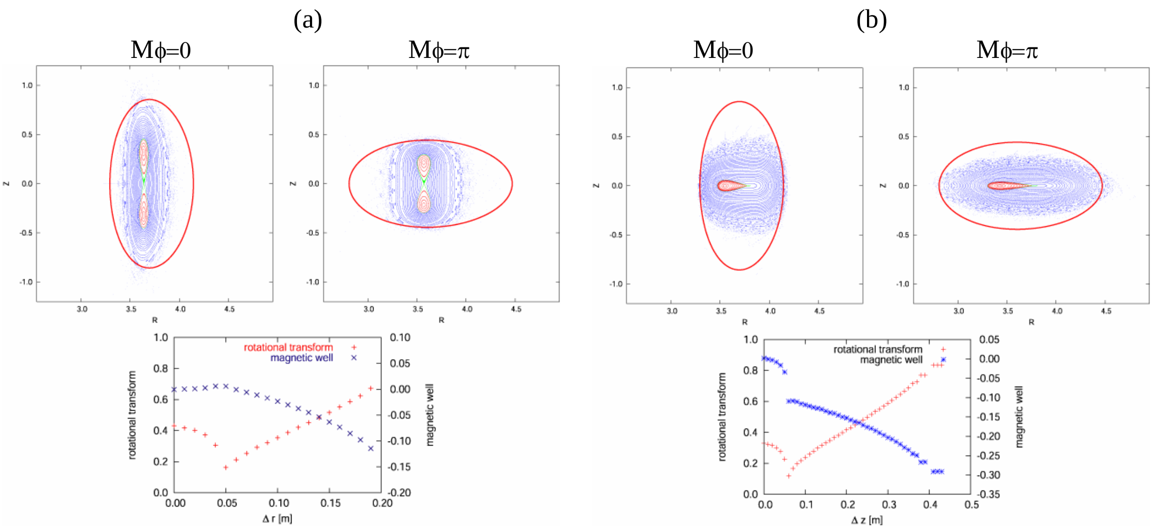

FIG.1 Flux surfaces are (a) vertically and (b) horizontally elongated doublet-like configurations. Radial Profiles of the rotational transform and the magnetic well are also shown. Bold lines colored with red indicate the last closed flux surface of a normal configuration with R_{axis}=3.6m.

configuration. The volume inside the last closed flux surface is very small compared to the normal configurations. Since the quadrupole filed has the axisymmetry, flux surfaces does not rotate along the toroidal direction, the shape of flux surfaces is changed vertically from the equatorial plane. On the other hand, for horizontally elongated configuration, clear eight-figured separatrix does not exist inside flux surfaces and the O-point outside of the torus degenerates to the X-point. In both configurations, the magnetic shear is reversed at the separatrix and the direction of the rotational transform is the same direction. The profile of the rotational transform decreased monotony is similar to one of the tokamak (tokamak-like). The magnetic well exists slightly inside the separatrix for the vertically elongated configuration but does not exist in the whole region for the horizontally elongated configuration.

In Fig. 2, finite β equilibria of the vertically elongated configuration are shown for various β values. Poincaré plots of magnetic filed lines are plotted at $M\phi=0$ and vacuum flux surfaces are also plotted for references. The beta β_0 is defined on the axis. The initial pressure distribution is prescribed to $p=p_0(1\text{-}s)(1\text{-}s^4)$, where s is the normalized toroidal flux. Red lines in figs indicate the position of two axes at $M\phi=0$. In Fig. 2(b), for medium β equilibria, though the horizontal shift of two axes toward the outside of the torus is very small, the vertical shift from the equatorial plane is very large and the X-point of the separatrix moves slightly outward. The volume inside the eight-figured separatrix is evolved due to the increased β, but closed flux surfaces still exist on the outside of the separatrix up to β_0~2%. For high β equilibrium (see Fig. 2(c)), the shift of two axes from the equatorial plane becomes further large but the horizontal shift is small. In the finite β equilibrium, two flows of P-S current are located near two axes and one flow in the reverse direction appears near the X-point. Consequently, a secondary quadrupole field generated by the plasma equilibrium itself is superimposed on the vacuum quadrupole field. Through this effect, it is found that in the higher-β equilibrium the eight-figured separatrix expands further and the closed surfaces on its outside gradually disappear as the separatrix evolves. In Fig. 2(b) and (b), Poincaré plots of the magnetic field lines are ergodized by the finite β effect. The plasma pressure diffuses in

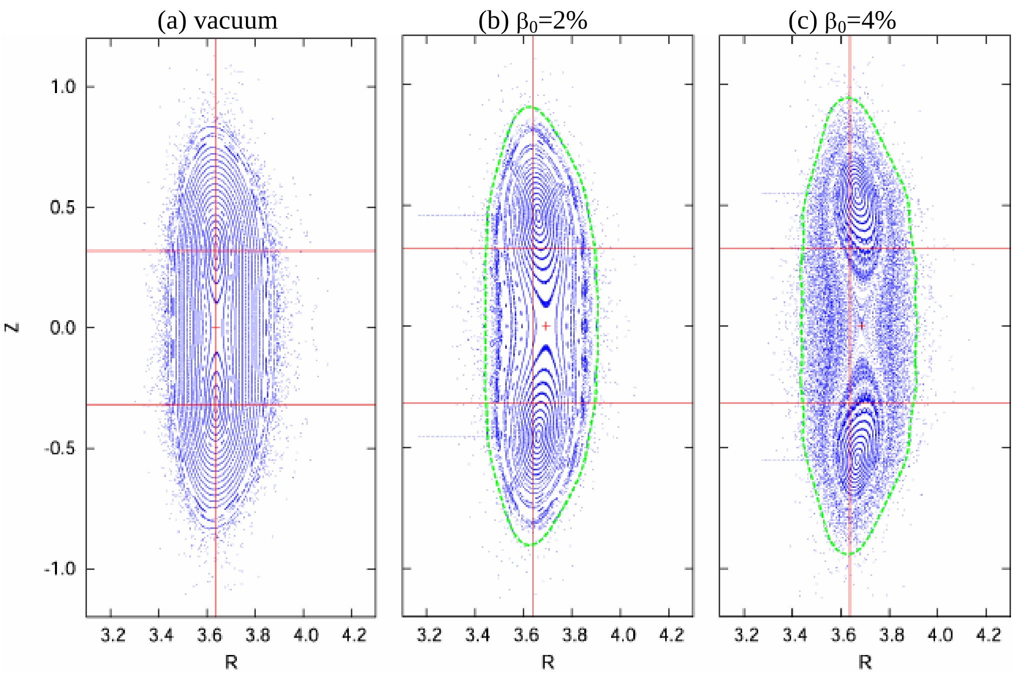

FIG. 2 Poincaré plots of magnetic field lines of vertically elongated configuration are shown for various beta values. Vacuum flux surfaces are also plotted for references. All figures are plotted at $M\phi=0$. Red lines in Poincaré plots indicate positions of magnetic axes for vacuum field. Symbols in plots indicate the X-point. Green lines indicate the plasma region defined by the plasma pressure.

the stochastic region. For β_0~4%, since the pressure near the X-point is diffused, the pressure is peaked to two axes and the saddle-backed distribution is obtained: the averaged beta is not increased. The averaged beta is about 0.07% for β_0~4%. However, though the structure of field lines is stochastic, the connection length of stochastic field lines is very long. Therefore, the plasma pressure is slightly remained in the stochastic region. Green lines in figs indicate p=const. line (p/p_0=0.01). The region with the pressure is spread to the edge region.

In Fig. 3, radial profiles of the rotational transform and the magnetic well of the vertically elongated configuration are shown for various β values. Profiles are plotted from the upper axis along the line *Z=const.* and Δr is the grid size for the field line tracing. The rotational transform on axes is increased by the finite β effect and it is greater than one for a high-equilibrium (β_0~4%). On the other hand, the magnetic well exists inside the separatrix for finite β equilibria and the depth of the magnetic well becomes deeply due to the increased β. This is desirable from the viewpoint of MHD stability.

On the other hand, finite β equilibria for the horizontally elongated configuration are shown in Fig. 4. Vacuum flux surfaces are also shown for references. Though the clear eight-figured structure does not exist for the vacuum fields (see Fig. 4(a)), the clear eight-figured structure is appeared due to slightly β increased (see Fig. 4(b). According to the increased β, the eight-figured structure is disappeared (see Fig. 4(c)) but the surface where the magnetic shear is reversed exists. For β_0~2%, the magnetic axis shifts very large toward the outside of the torus and the shape of flux surfaces is changed eccentrically. Poincaré

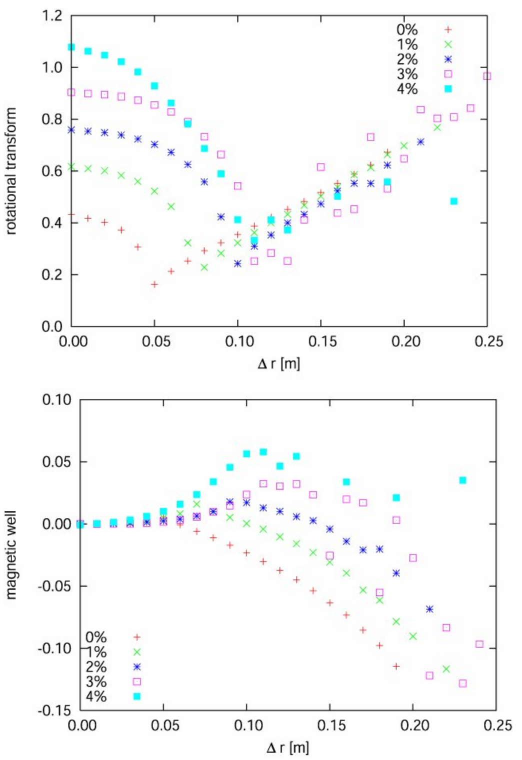

FIG. 3 Radial profiles of the rotational transform and the magnetic well of the vertically elongated configuration are shown for various β values. The profile for a vacuum configuration is also plotted for references.

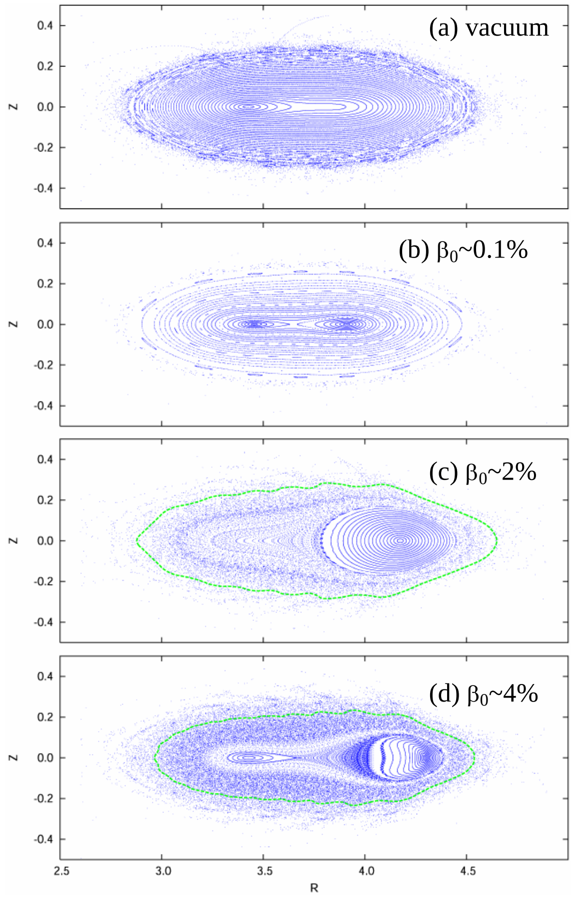

FIG. 4 Poincaré plots of the horizontally elongated configuration is shown for various β values. Vacuum flux surfaces are also plotted for references. All figures are plotted at Mϕ=π. Green lines indicate the plasma region defined by the plasma pressure.

plots of magnetic filed lines are ergodized in the inside of the torus. By the stochastically behavior of magnetic field lines, the plasma pressure is diffused in the stochastic region and the distribution of the plasma pressure is peaked to the magnetic axis. For higher-β equilibrium (β_0~4%), the magnetic axis extremely shifts. In the stochastic region, since the plasma pressure is almost zero, the finite β effect is very small. Therefore, an eight-figured separatrix due to the vacuum quadrupole field appears again (see Fig. 4(d)).

In Fig. 5, radial profiles of the rotational transform and the magnetic well are shown for the horizontally elongated configuration. Profiles are plotted from the outer axis along the line *R*=*const*., where Δz is the grid size for the field line tracing. The rotational transform on the axis is increased due to the increased. For β_0~2%, the rotational transform on the axis is greater than one. However, for higher-β (β_0>4%), the rotational transform is decreased. The magnetic well is increased because of the large Shafranov shift.

In above mentioned, the horizontally elongated configuration has the large Shafranov shift and eccentric flux surfaces at the low β equilibrium. From the viewpoint of the study of MHD equilibrium beta limit, this characteristic is desirable. MHD equilibrium beta limit is defined by the generation of the separatrix inside the plasma, because the poloidal field becomes zero to increase the longitude field due to the finite β effect [11]. However, in many cases, since the beta limit is defined by the MHD instability, the theoretical and experimental produce of MHD equilibrium beta limit in actual configurations is very difficult. In order to see MHD equilibrium beta limit, a high β equilibrium (β_0~5%) of the horizontal elongated configuration

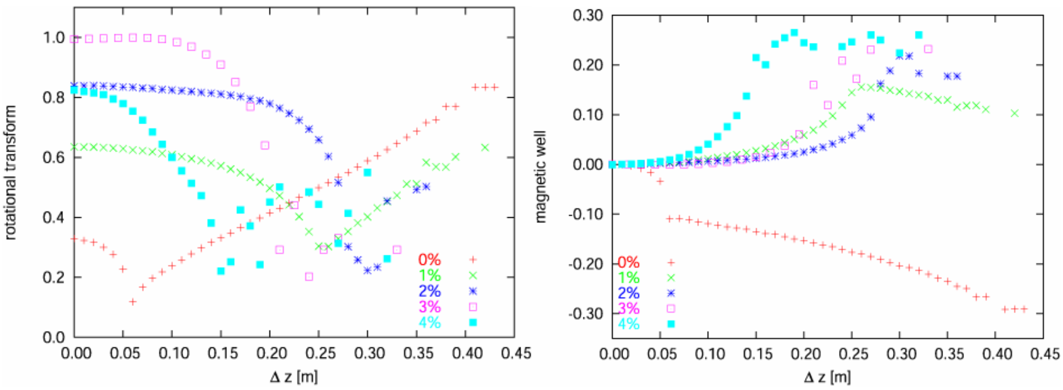

FIG. 5 Radial profiles of the rotational transform and the magnetic well of the vertically elongated configuration are shown for various β values. The profile for a vacuum configuration is also plotted for references.

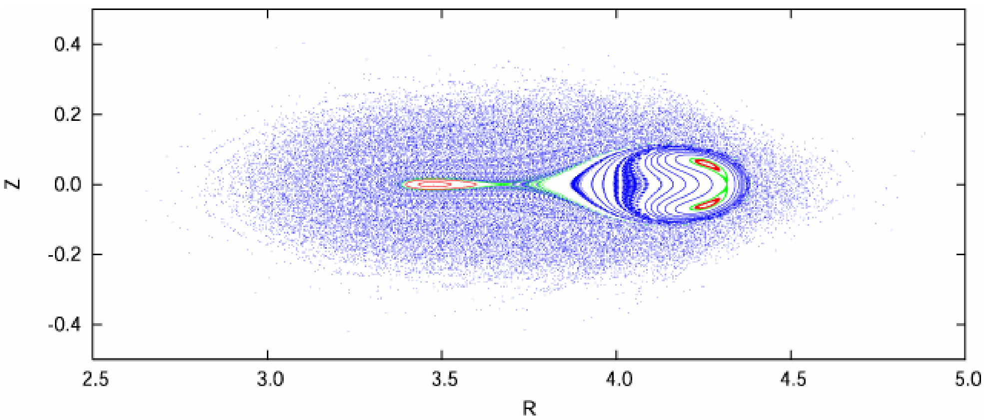

FIG. 6 Poincaré plots of the horizontally elongated configuration are shown for β_0=5%. The cross section corresponds to Fig. 4 (at $M\phi=\pi$). Points colored with green indicate the eight-figured separatrix and points colored with red indicate flux surfaces inside the separatrix.

is studied. In Fig. 6, Poincaré plot is shown corresponding to Fig. (4). In Fig. 5, the rotational transform on the axis is decreased for high β equilibria above β_0~4%. For β_0~5%, the rotational transform on the axis crosses 0.5, a new eight-figured separatrix is appeared. Since magnetic field lines become stochastic in edge region, the plasma pressure is diffused. Thus, the averaged beta is very small (about 0.21%). Medium β equilibria are already reproduced in the experiment. More extensive study between the numerical simulation and experiments is performed now in progress.

3. High-energy particle orbits in doublet-like configurations

The separatrix structure inside the plasma strongly affects the confinement of high-energy particles. In the separatrix, since the adiabatic invariant is not conserved, the collisionless pitch-angle scattering is important [13]. In addition, the poloidal drift width of the particle orbit is proportional to the Lamor radius ρ and the safety factor q. In order to investigate the particle orbit near the separatrix, the collisionless drift orbit of high-energy ion (proton: 500eV) is studied by KGCR code. KGCR code solves the guiding center drift equation on the cylindrical coordinates. The magnetic field for finite β is obtained from the HINT code.

Figure 7 shows Poincaré plots of two orbits for horizontally elongated configuration (β_0~0.1% see Fig.4(b)). The starting point is near the X-point and the drift orbit is traced for 5ms. The pitch angle λ is assumed to $\sin\lambda = v_\perp/v$. Figure 7(a) shows the orbit of a passing particle for λ=0.3π. Poincaré plots show an eight-figured orbit but the passing orbit is transited chaotically between two O-points. On the other hand, in Fig. 7(b), the orbit of a reflected particle for λ=0.6π is shown. The orbit is combined the banana-like and potato-like and transited between both orbits. Initially (<1ms), the orbit is the banana orbit. After that, the potato-like orbit is appeared. In this case, since the reflecting point locates near the separatrix, the adiabatic invariant is changed slightly and the reflecting point is also changed. For finite β equilibria, the structure of magnetic field lines is stochastic; the particle orbit is also chaotically. In this study, only collisionless orbit are studied. However, the existence of those

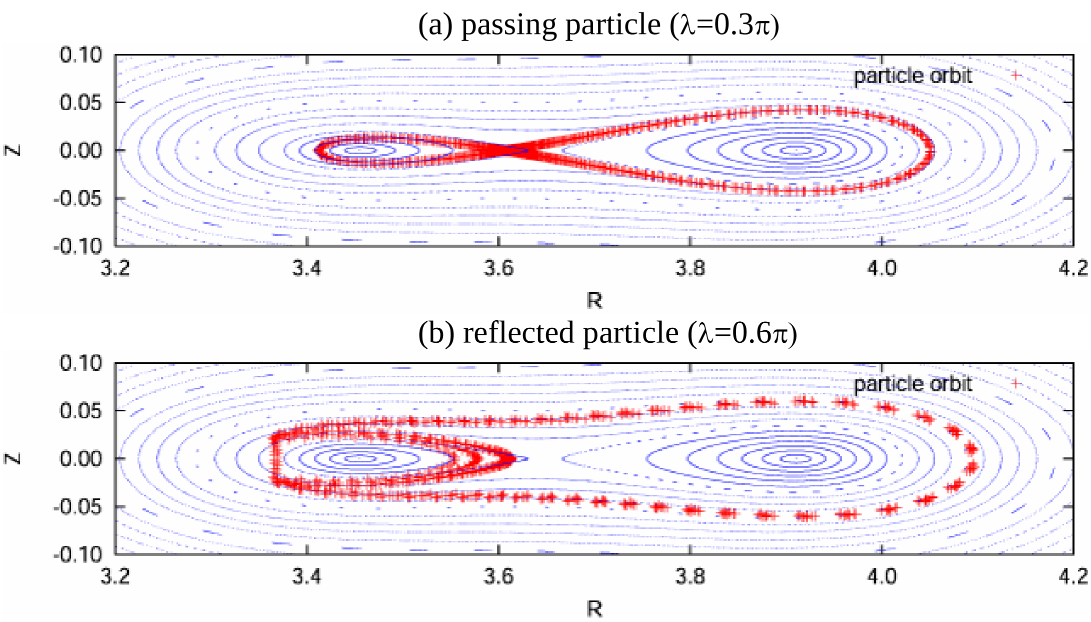

FIG. 7 Poincaré plots of high energy ion for horizontal elongated configuration (β_0=0.1%). Figures are plotted at $M\phi$=0. Symbols colored with red indicate particle orbits and dots colored with blue indicate flux surfaces.

singular orbits affects to the local transport theory with the collision.

4. Summary

The MHD equilibrium of the doublet-like configuration is investigated. For vertically elongated configuration, the eight-figured separatrix inside the plasma evolves at finite β equilibrium and the stochastic behavior of magnetic field lines is appeared in the outside of the separatrix. For the horizontally elongated configuration, eccentric flux surfaces by the large Shafranov shift are obtained. When β_0~5%, a new eight-figured separatrix by the finite β effect is appeared because the rotational transform crosses 0.5. This reproduces the MHD equilibrium beta limit.

In the analysis of the high-energy particle orbit near the eight-figured separatrix, some characteristic orbits are obtained. If the reflecting point locates near the separatrix, the transition of the orbit without the collision and singular orbits are appeared.

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