Two-fluid limits on stellarator performance: Explanation of three stellarator puzzles and comparison to axisymmetric plasmas[1]

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Abstract. The basic two-fluid processes, related to the nonlinearly self-consistent diamagnetic drifts of the electrons and ions, are shown to have fundamentally different effects on the steady state and beta limits of stellarator configurations, compared to MHD predictions. Nonlinear numerical simulation shows that the ideal MHD ballooning modes and the resistive MHD ballooning and interchange modes at relatively high mode numbers, that set the most severe theoretical limits on beta in stellarators with fixed boundary, are easily stabilized by two-fluid effects at realistic parameters, including finite Larmor radius effects related to the ion diamagnetic drift. Magnetic reconnection at low-order rational magnetic surfaces, on the other hand, is enhanced through the parallel component of the two-fluid electron pressure gradient in Ohm's law. The accelerated reconnection rates may impose the true intrinsic limit on beta in stellarators, as a "soft" or confinement mediated limit due to steady confinement degradation in the presence of large magnetic islands. Study of the corresponding axisymmetric configurations shows that the helical component of the stellarator configuration provides an important amplifying factor for these effects. The two-fluid results may explain several previously puzzling experimental observations on stellarator behavior.

1. Introduction

Experimental studies of the stability and operational limits of stellarator plasmas have generated a number of puzzling observations that resist explanation by the MHD model of a plasma as a simple magnetized fluid, in contrast to its more general success in axisymmetric configurations. Recent numerical studies show that the basic two-fluid effects in a nonlinear, fluid-based plasma model[1], those effects that are required beyond MHD to generate the self-consistent, nonlinear diamagnetic drifts of the electrons and main ions, relax the major MHD limits on stellarator performance in a way that may explain these puzzles. The results also show that the basic properties of helical configurations, in contrast to axisymmetric ones, depend crucially on plasma descriptions that are more comprehensive than MHD.

Numerical simulations were carried out with the initial value code M3D[2], that solves the time-dependent MHD and two-fluid plasma equations in complete stellarator or axisymmetric configurations. The studies reported here concentrate on the most fundamental properties, the major two-fluid constraints on the quasi-steady states and beta limits for high beta stellarators, taking the particular example of the NCSX quasi-axisymmetric stellarator with reversed magnetic shear[3, 4], with some applications to other devices. For NCSX, two-fluid effects at realistic small values of the two-fluid strength parameter $H \equiv c/(\omega_{pi} R) \equiv 1/(\Omega_{ci}\tau_A) \simeq 0.02$, where ω_{pi} is the ion plasma frequency, Ω_{ci} the ion cyclotron frequency and τ_A the Alfvén time, i) stabilize ideal MHD ballooning modes above a certain moderate-to-high mode number,

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ii) stabilize resistive MHD ballooning and interchange modes at their most unstable moderate-to-high mode numbers and
iii) imply the existence of an intrinsic "soft" beta limit for stellarators at high electron beta, where two-fluid magnetic islands grow large enough to seriously reduce thermal and plasma confinement and thereby prevent further plasma heating, as opposed to a "hard" limit caused by large scale plasma instability. This picture of stellarator stability more closely matches observed experimental behavior, where ballooning and interchange modes, although often unstable according to MHD linear theory for a given plasma, are rarely observed. Confinement-limited beta that does not scale with plasma turbulence parameters has been observed in W7-AS. These effects do not occur, or are strongly muted, in axisymmetric configurations.

The MHD and two-fluid results represent the first nonlinear simulations of a plasma in realistic stellarator configurations. While they generally agree with more established stability analyses, in a number of areas they suggest new properties that deserve further investigation.

2. Two-fluid model

The nonlinear "generalized drift" model for a two-fluid plasma[1] describes the electrons and ions as separate fluids with independent motions. The equations consist of the ion (or total) momentum equation with an approximate expression for the ion gyroviscous stress and a collisional ion viscosity, an Ohm's law that describes the momentum balance for massless electrons, including the Hall and $\nabla p_e/en$ terms, the continuity equation, and separate evolution equations for the scalar pressures (temperatures) of the electrons and ions, including perpendicular thermal conductivities and equivalent parallel thermal conductivities (in the form of accelerated smoothing of the temperatures along the magnetic field lines), together with the usual Maxwell's equations of MHD. These equations provide a simple, self-consistent model for the nonlinear diamagnetic drifts, which represent, among other effects, the lowest order ion finite Larmor radius (FLR) effects in a fluid model.

The two-fluid equations introduce three new parameters compared to MHD, H, p_e/p, and $\nabla_{||}p_e/p_e$. Each has important effects on high beta stellarators, while the MHD β also modulates two-fluid effects. The global parameter $H = c/(\omega_{pi}R) = 1/(\Omega_{ci}\tau_A)$ at at the magnetic axis measures the two-fluid strength corresponding to m_i/e, while p_e/p the ratio of the electron to the total pressure, given an MHD β. The ratio $\nabla_{||}p_e/p_e$ arises from the parallel component of the electron pressure gradient in the two-fluid Ohm's law. It measures the balance between the fast smoothing of the electron pressure gradient along magnetic field lines (physics not contained in fluid plasma models) and the anisotropizing perturbations of the field and pressure caused by the plasma evolution (in this case, fluid-based). Its magnitude is not well established, especially in dynamic situations, since in high temperature plasmas parallel smoothing is poorly described by a collisional thermal conductivity. The electron temperature smoothing used here models particle streaming as waves that progate along field lines at similar velocities, on the order of the electron thermal speed (the "artificial sound" method[2]).

The plasma density was assumed to be uniform and constant, in part because density profiles cannot be predicted from fluid models and were assumed to be relatively broad ("H-mode") in NCSX, and also to eliminate an additional source of variation in $\nabla_{||}p_e$ for the initial studies. The density variation is likely to be important in two-fluids, but this question is left to future work.

The resistivity and ion viscosity were chosen to be relatively large for good numerical stability, reference values $S = 10^5$ (Lundquist number) and $\mu = 10^4$. The MHD and two-fluid processes studied here occur relatively rapidly compared to changes at these levels of dissipation. To limit computational time, most stellarator cases were assumed to be exactly periodic in the toroidal direction, with the periodicity of the equilibrium configuration, $N = 3$ for NCSX. Simulations keeping the full toroidal circumference verified that the major low order internal modes were three-periodic, including the 5/3, 6/3, and 7/3 islands, and that the high mode number ballooning and interchange modes were well described.

For non-axisymmetric configurations, the importance of two-fluid effects can be predicted from the properties of the MHD and two-fluid equations. Many other physical processes that go beyond the MHD model have analogous differences. The MHD equations preserve certain geometrical symmetries relevant to toroidally confined plasmas [1]. The basic plasma variables, as functions of the toroidal and poloidal angles, satisfy $f(\theta, \phi) = \pm f(-\theta, -\phi)$ for either the positive or negative sign (positive or negative symmetry, respectively) if the initial state possesses a definite symmetry. The additional two-fluid terms mix these symmetries. In an axisymmetric MHD equilibrium without rotation, the magnetic field has positive symmetry, as does the plasma pressure, temperature, and density. In a stellarator, the helical component of the equilibrium magnetic field introduces lowest order terms of the opposite symmetry. In the tokamak, the opposite symmetry components are perturbations from the equilibrium, and thus of higher order, at least initially. Amplification of two-fluid effects due to mixing of the two symmetry components can be seen in tokamaks for large enough perturbations. One example is the accelerated growth of large two-fluid magnetic islands in a tokamak.

3. Stellarator Steady States

Initial ideal MHD equilibria was allowed to relax by nonlinear evolution in both the MHD and two-fluid models. The ideal MHD stellarator equilibria used as initial conditions for the M3D simulations were calculated using the VMEC equilibrium code[5], based on the 2001 NCSX reference case[3] at volume averaged $\beta = 4.2\%$. This case has only minor differences from more recent configurations. It was stable or marginally stable to most instabilities according to linear MHD theory. To construct a series of equilibria at increasing β up to 8%, the pressure profile was scaled by a constant factor, keeping the same toroidal current density profile. The reference current density already included a large contribution from the estimated bootstrap current, mostly localized along the outboard side near the plasma boundary. Thus, the bootstrap current is not completely accurate at higher beta, but the general simulation results remain valid.

The VMEC algorithm enforces nested magnetic flux surfaces without magnetic islands. Such helical equilibria in ideal MHD generally have parallel current densities that become locally singular (infinite) at one or more interior rational flux surfaces, due to an unavoidable mismatch between the internally and externally generated magnetic fields. The parallel current taken indirectly from the VMEC magnetic field, $\mathbf{J}_\parallel = (\nabla \times \mathbf{B})_\parallel$, smoothes the singularities and yields initial magnetic islands on the singular surfaces (*FIG. 1a*). Nonlinear relaxation under MHD then rapidly reduced the initial islands at all betas, on a time scale faster than the resistive/viscous dissipation. Strong reduction in island sizes agrees with the solution of $\mathbf{J} \times \mathbf{B} = \nabla p$ by the PIES[7] code. Some major islands, eg the 5/3 or 6/3 in NCSX, did not vanish completely, also in agreement with PIES[4, 8] (*FIG. 1b*).

Unlike MHD, two-fluid configurations possess a global electrostatic potential Φ and radial elec-

a) $t = 0$ b) $t = 25\tau_A$

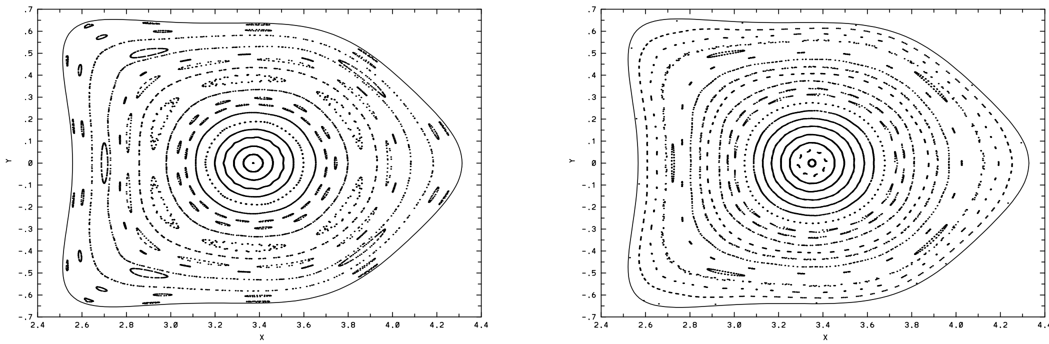

FIG. 1. NCSX ideal MHD initial condition and dissipative island healing at $\beta = 7\%$.

tric field E_r in steady state, to balance the opposing diamagnetic drifts of electrons and ions. In the generalized drift model, Ohm's law provides major constraints on Φ. NCSX and other stellarators have smooth Φ profiles, peaked at the magnetic axis. At very large electron pressure $p_e/p \sim 1$, the perpendicular component of Ohm's law and the ion ∇p_i no longer provide the main influence on E_r and the sign of Φ and E_r reverse. The corresponding NCSX axisymmetric cases, derived from the axisymmetric $n = 0$ component with the same pressure and q profiles, have comparable radial electric fields at moderate and large p_e/p. The helical geometry influences E_r strongly at large p_e/p as it tries to reverse sign. A small plasma poloidal rotation, less than the total diamagnetic drift, may also be driven, but depends sensitively on the configuration.

4. MHD Ballooning Modes and Two-Fluid Stabilization

At $\beta = 7$ and 8%, after the initial rapid island healing, the resistive MHD simulations rapidly developed fast growing, small wavelength instabilities very near the plasma edge. These modes possess many of the expected characteristics of ballooning modes. They have ballooning structure and parity in velocity, pressure, and current density. They first appeared and then developed most rapidly in the regions with the most unfavorable magnetic field curvature, as identified in linear stability analyses [4, 6, 11] (*see FIG. 2a*). They initially formed in the narrow region of low magnetic shear near the plasma edge, where ballooning modes should be most unstable. Using finer spatial resolution in the simulation increases the mode number and growth rate of the modes and further localizes them to the regions of bad curvature. Their growth rate also increased with increasing beta, rapidly at high beta.

Linear MHD stability results from the Terpsichore code[12] show that the 7% beta case should be marginally unstable to ideal MHD ballooning at the reference spatial resolution of the simulations. Growth rates vary rapidly with mode number and although it is marginally ideal MHD unstable at infinite toroidal mode number n, the 4.2% beta case is marginally stable to resistive ballooning at $S = 10^5$ in the standard M3D resolution, in agreement with Terpsichore results.

Nonlinearly, the simulations show that the modes expand inward from the plasma edge as they grow in amplitude, coupling to more interior resistive (interchange-type) modes that mix the field lines. The nonlinear modes rapidly lead to loss of good magnetic flux surfaces over a substantial fraction of the plasma radius, even if the plasma does not disrupt (*FIG. 2a*).

Two-fluid effects robustly stabilize the ballooning modes at higher mode numbers in the M3D

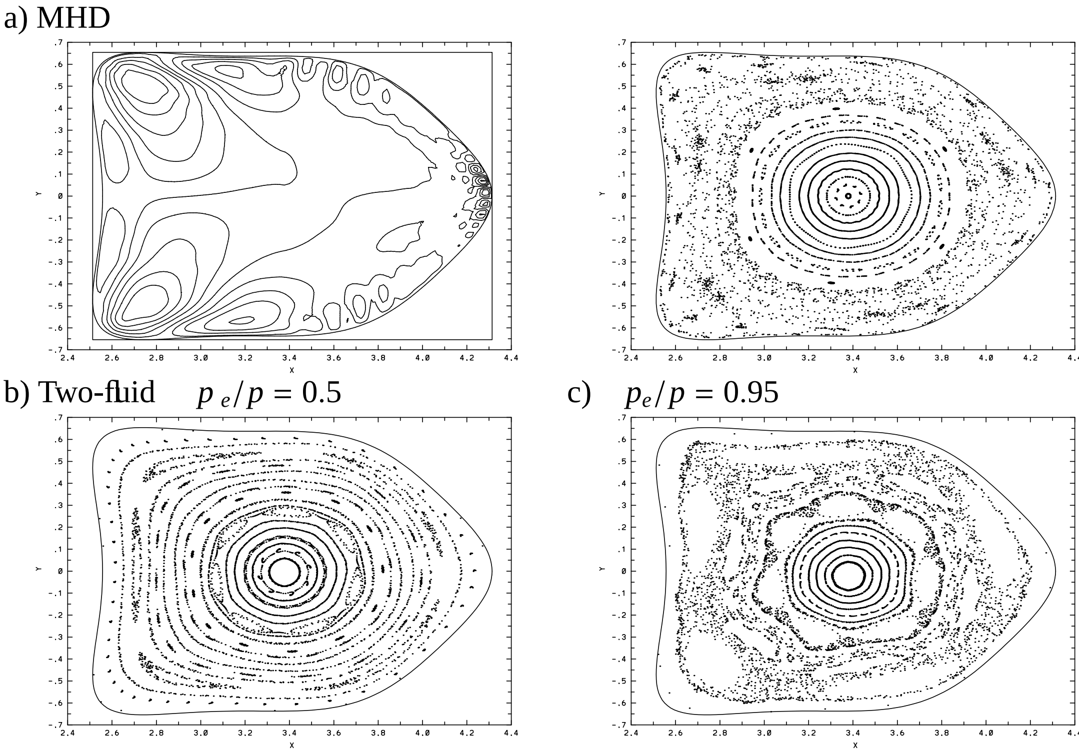

FIG. 2. NCSX at $\beta = 7\%$. a) Resistive MHD ballooning mode at late time $t = 77\tau_A$. Left frame shows ballooning vortices in the velocity stream function in regions of unfavorable magnetic curvature, right the destruction of magnetic surfaces in the puncture plot. Two-fluid terms easily stabilize the ballooning mode at realistic $H = 0.02$, but magnetic islands grow large at large p_e/p. b) $p_e/p = 0.5$, $t = 154$, c) $p_e/p = 0.95$, $t = 89$ (islands still growing).

simulations. For NCSX, complete nonlinear stability was seen at volume-averaged betas up to 8% and perhaps beyond, even for unrealistically large values of the plasma resistivity that should enhance the MHD growth. These values of beta are well above the reference value of 4.2% determined by the MHD linear stability limits on ballooning[3]. Realistic values of the two-fluid parameter, although small, $H \simeq 0.01$–0.02 for NCSX, more than suffice for simple ion finite Larmor radius (FLR) stabilization according to the dispersion relation with the ion diamagnetic frequency ω_{*i}, $\omega(\omega - \omega_{*i}) = -\gamma^2_{MHD}$. At the observed M3D poloidal mode numbers $\gtrsim$ 25–30, the ratio $\omega_{*i}/\gamma_{MHD} \sim$ 10–15 is well above the value $\omega_{*i}/\gamma_{MHD} \gtrsim 2$ required for strong stabilization.

The simulations actually show much stronger ballooning mode stabilization in the two-fluid model, effectively eliminating ballooning down to very small values of H that are well below any possible ω_{*i} stabilization from the dispersion relation. The results support the validity of a theoretical analysis of ideal MHD ballooning modes in stellarators[13], that predicts that in helical plasmas the ballooning mode structure couples to very high, essentially infinite, mode numbers n (or m). The very high n or m part of such a structure is easily removed or distorted by very small ω_{*i} or other two-fluid effects. This hypothesis is difficult to test numerically in realistic plasma configurations because of the limited spatial resolution of numerical methods.

M3D simulations show that other stellarators also develop high mode number, MHD ballooning instabilities with a fixed plasma boundary. In W7-AS, near-edge MHD modes grow very rapidly at moderate beta, but they have small wavelengths and two-fluid stabilization is very effective at realistic values of H. This agrees with experimental observations, where such modes did not appear.

5. Two-Fluid Magnetic Reconnection and Soft Beta Limits

Not all two-fluid effects are stabilizing. The simulations show that the parallel electron pressure gradient term $\nabla_{\|} p_e/(en)$ in Ohm's law increases the nonlinear growth rate and saturated size of stellarator magnetic islands at interior low-order rational surfaces. The effect scales with $\nabla_{\|} p_e/(en)$. For NCSX, it can be large at high beta or large ratios of p_e/p. It is reduced by small H.

Nonlinear acceleration of magnetic reconnection due to $\nabla_{\|} p_e/en$ has been seen previously with other numerical plasma models in different configurations[14, 15]. The effect is seen for large, low order islands in tokamaks with M3D, for the corresponding axisymmetric NCSX cases in this study and for a different case using an earlier version of M3D.

The $\nabla_{\|} p_e$ term introduces additional sources of uncertainty in two-fluid simulations, because the rate of equilibration of the electron pressure (temperature and density) along magnetic field lines is not well described by fluid processes and is poorly understood for dynamic situations. Furthermore, the degree of anisotropy introduced by field line reconnection in actual plasma configurations is also poorly quantified, since analytical studies use approximate mode structures and limited physics models. The simulations presume a physical picture of electron particle streaming along field lines to accelerate the smoothing of temperature, that is modelled as wave propagation at the same effective speed[2]. The plasma density equilibrates more slowly than temperature, due to ion sound waves in the fluid continuity equation. In the NCSX cases, density evolution was neglected for simplicity. Thus, the actual case may be closer to the large islands seen with large p_e/p or no acceleration of parallel equilibration. Thermal losses due to the large islands may then impose a serious limit on plasma confinement. For NCSX, large island size could then limit the achievable beta below 7%, even at moderate $p_e \simeq p_i$.

The two-fluid islands form over longer time scales than MHD ballooning modes or the two-fluid radial electric field. Unlike in axisymmetric configurations, it affects the islands even at small size. The relatively large value of resistivity used in the simulations should not distort the two-fluid island results excessively. For NCSX at $S = 10^5$ and $\beta = 7\%$, The magnitude of the $\nabla_{\|} p_e/en$ term is large compared to the maximum equilibrium value of $\eta_{\|} J_{\|}$ over the plasma and the maxima occur in different places. At larger resistivity $S = 10^4$, the island growth rate and final sizes are larger, but in this case $|\eta_{\|} J_{\|}|_{max} > |\nabla_{\|} p_e/en|$. Studies with $S = 10^6$, although noisy, suggest that the island sizes are similar those of $S = 10^5$.

To check that the initial ideal MHD islands did not affect the results, two-fluid simulations were started from the mostly healed MHD islands at $t \simeq 30\tau_A$. The resulting islands were similar to those of the original cases, showing that the final two-fluid quasi-steady states are not strongly dependent on the initial conditions.

Many of these effects are only partly explained by existing theoretical analyses, which make strong simplifying assumptions and typically neglect the two-fluid steady state fields and flows. Polarization drift theories (eg, [16]) do not adequately explain the observed effects in the stellarator or the corresponding axisymmetric configuration.

The neoclassical parallel collisional stresses are hypothesized to act to stabilize magnetic island growth in the presence of negative magnetic shear just as they enhance it with positive shear. NCSX was designed to have negative magnetic shear ($dq/d\psi < 0$) over the plasma radius, except at the plasma edge. The simulations suggest that neoclassical stabilization may be less effective than originally estimated for MHD plasmas. Two-fluid terms enhance island growth starting from very small widths in stellarators. The pressure cannot equilibrate instantaneously over an island, particularly a fast growing island, which slows the removal of the bootstrap current that leads to stabilization. Also, in NCSX the neoclassical pressure tensor terms are fairly small over much of the plasma interior, since pressure profiles are broad and the background pressure gradient is relatively small. This is verified by M3D estimates of the neoclassical coefficients, using the flux-surface averaged tokamak expressions[1] as an approximation. Complete study of these points requires knowledge of the time evolution of the plasma from low to high beta, and is beyond the scope of this study.

Two-fluid island growth accelerates rapidly with increasing β. The enhanced two-fluid magnetic reconnection rates, combined with robust suppression of the most unstable, smaller wavelength MHD modes, imply that high beta stellarators should be more sensitive to "soft" beta limits caused by deteriorating confinement in the presence of large magnetic islands, compared to axisymmetric plasmas. Since the growth rates depend on the electron β_e rather than total beta, a second implication is that hot-ion stellarator plasmas may have better intrinsic confinement than hot-electron ones, assuming that the turbulent transport losses can be controlled. These properties do not apply to tokamak plasmas.

6. Axisymmetry

Axisymmetric ideal MHD equilibrium were computed for the 7% beta NCSX case, keeping only the toroidal $n = 0$ component and using the same pressure and q profiles. The same low order rational q-surfaces were retained to study reconnection. The resulting equilibrium had a slightly indented, D-shaped cross section with a vertical elongation of 1.6. No MHD ballooning modes appear in the axisymmetric configuration at 7% beta, in agreement with its generally favorable curvature, compared to the stellarator.

Enhanced magnetic reconnection due to the gradient $\nabla_{\|} p_e$ in Ohm's law has been observed in tokamaks with pre-existing large magnetic islands earlier, in simulations with a predecessor to the M3D code. Saturated island size for a 2/1 magnetic island was found to be larger with the $\nabla_{\|} p_e$ term, even in an almost-MHD simulation when this was the only two-fluid term kept and the p_e was defined to be $p/2$. Similar behavior was observed in the equivalent axisymmetric cases for NCSX. These case had no initial islands, unlike the stellarator. Island growth at the main 5/3 and 6/3 surfaces initially proceeded slowly and apparenty independent of p_e/p. As the islands grew to significant size, differences in growth rate with p_e/p appeared, but differently from the stellarator. The intermediate case with $p_e/p = 0.5$ grew fastest. Growth accelerated even more once the islands became large enough to approach a more circular shape and the opening angle of the separatrix at the X-points expanded significantly. Island growth was slower at the two extremes of p_e/p, slowest at small $p_e \simeq 0$.

7. Conclusions

MHD and initial two-fluid studies of the nonlinear quasi-steady states and beta limits of a high beta stellarator configuration have been carried out with the M3D simulation code. The M3D simulations, including earlier studies[9, 10], also represent the first nonlinear calculations of MHD ballooning modes in stellarators. The two-fluid steady states and beta limits for NCSX differ significantly from MHD and explain a number of previously puzzling aspects of stellarator behavior.

Two-fluid effects can stabilize the near-edge MHD ballooning modes with relatively high mode numbers that give the main theoretical limit on beta in stellarators with fixed boundary. Both the ideal and resistive MHD modes are stabilized above a certain critical mode number by FLR (ω_{*i}) effects at realistic two-fluid parameters. Kink modes in free boundary plasmas would also tend to be stabilized, if poloidal mode numbers are large enough. The two-fluid ballooning stabilization is considerably stronger than the simple ω_{*i} dispersion relation predicts, and may be related to the essential role of very high mode-number components in ballooning modes in helical, as opposed to axisymmetric, magnetic configurations, as suggested by theoretical analyses.

Electron two-fluid effects, due to the parallel component of the electron pressure gradient $\nabla_{\|} p_e$ in Ohm's law, can strongly enhance magnetic reconnection in helical configurations, in stellarators and tokamaks with large magnetic islands. Important theoretical uncertainties remain regarding its behavior, since neither the anisotropy introduced by a growing helical island nor the rate of parallel thermal equilibration are well understood. The simulated islands fit poorly to existing analytical theories of nonlinear island growth, which make many simplifying assumptions.

For NCSX, the two-fluid results leave open the possibility for good quasi-steady states at betas significantly above the volume-averaged value of 4% set by linear MHD stability limits. The practical beta limit may be set by magnetic reconnnection.

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