Toroidal Momentum Confinement in Tokamaks

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Abstract Theories for the toroidal momentum confinement in tokamaks have been developed. It is shown that the logarithmic gradient of the toroidal flow is a linear combination of logarithmic gradients of the plasma pressure and the temperature in neoclassical quasilinear theory. The fluctuation-induced toroidal stress consists of a diffusion flux, a convective flux, and a residual flux. The effects of a variety of magneto-hydrodynamic (MHD) activity, such as magnetic islands, and unstable MHD modes, on toroidal plasma rotation are also addressed. The key mechanism for the toroidal flow damping is the broken toroidal symmetry in $|\boldsymbol{B}|$ that results from MHD activities. Here, $\boldsymbol{B}$ is the magnetic field. The symmetry-breaking-induced toroidal viscosity also provides a mechanism to determine the island rotation frequency.

1. Introduction

The importance of the plasma momentum confinement physics, and its effects on the particle and the energy confinement in tokamaks, is recognized in the theory for the L (low confinement mode) – H (high confinement mode) transition [1]. There, the poloidal momentum transport, usually determined fairly accurately by the neoclassical processes due to the variation of the magnitude of the magnetic field $|\boldsymbol{B}|$ on the equilibrium magnetic surface, seems to be the dominant mechanism in understanding the L-H transition phenomenon. Here, we address the issues related to the toroidal momentum confinement in tokamaks. Besides the importance of the toroidal rotation on the plasma confinement, the new impetus to understand the toroidal momentum confinement iare the experimental observations that the toroidal plasma rotation stabilizes resistive wall modes, and allows high beta, long pulse operation [2]. It is known that the toroidal momentum confinement in tokamaks is anomalous, *i.e.*, the momentum confinement time is two orders of magnitude larger than the neoclassical values, in contrast to the poloidal momentum confinement [3-5]. The inferred toroidal momentum confinement time from the experiments is of the order of the ion energy confinement time. When a variety of magneto-hydrodynamic (MHD) activity is present, the equilibrium toroidal flow profiles evolve in response to these activities. This provides an opportunity to probe the underlying physics mechanism when these MHD activities are present. Here, we present physical mechanisms that exist in realistic tokamak discharges, which can affect toroidal momentum confinement.

2. Neoclassical Quasi-Linear Theory for Toroidal Stress

Turbulent fluctuations are ubiquitous in tokamaks. These fluctuations degrade toroidal momentum confinement. Recent experiments in C-MOD indicate that besides a diffusion term [6-8] in the toroidal momentum equation there should be also a momentum pinch term (*i.e.*, a radially inward convective flux) in Ohmic discharges [9]. The existence of a convective flux in the toroidal momentum transport equation is known in neoclassical theory [10-12]. Its existence in the anomalous toroidal momentum flux is also experimentally inferred [13], and adopted theoretically [14]. A neoclassical quasilinear theory [15] that accounts for the effects of the fluctuations on transport fluxes is extended to calculate the toroidal momentum transport flux. It is found, just like the neoclassical theory [12], besides the diffusive tflux, and a residual flux, there is a convective term in the fluctuation-induced toroidal stress [16]. The salient feature of the theory is that the ratio of the diffusion term to the convective term is not sensitive to the amplitude of the fluctuations and the renormalization procedure. It only depends on the frequency spectrum. It is shown that, in general, the logarithmic gradient of the toroidal flow U is a linear combination of the logarithmic gradients of the plasma pressure P and the temperature T when the fluctuation induced toroidal stress dominates. If the fluctuation frequency is close to the ion diamagnetic frequency, the toroidal momentum flux Γ_ϕ has the form [16]:

$$\Gamma_\phi = - \chi_\phi \partial(NMRU)/\partial\psi - \chi_\phi L_\phi (NMRU) + Re, \qquad (1)$$

where χ_ϕ is the anomalous momentum diffusion coefficient, N is plasma density, M is the ion mass, R is the major radius, $L_\phi \approx -(5/2)(T'/T)$, prime denotes $\partial/\partial\psi$, ψ is the poloidal flux function, and Re represents the residual term which is basically the spectrum average of the parallel wave vector. The equilibrium toroidal rotation is determined by balancing these three fluxes. This mechanism, similar to that in the neoclassical theory [10-12], is different from the earlier theory advanced in Ref.[17] which is based on the mechanism that the toroidal momentum is deposited in the edge region, and moves inward through the momentum pinch. In the theory developed in Ref.[16], the diffusion term and the convective term approximately determine the toroidal rotation profile, and the Re term determines the magnitude and the direction of the equilibrium toroidal rotation. In this case, the equilibrium toroidal flow profile is related to the temperature profile through the relation $U/U_0 = (T/T_0)^{5/2}$, where U_0 and T_0 are values of U and T on the magnetic axis respectively, if we neglect the residual term. This particular functional relation between U, and T is observed in C-MOD, although the corresponding fluctuation frequency spectrum is not measured [18]. When the frequency spectrum differs from the ion diamagnetic frequency, the functional relation between U, P, and T will change accordingly, and the corresponding convective flux can be either radially inward, or outward, or vanish.

3. Symmetry-Breaking-Induced Toroidal Viscosity due to MHD Acticity

Besides the turbulent fluctuations, a variety of coherent MHD activity, such as magnetic islands, or resistive wall modes, also degrades toroidal momentum confinement. A magnetic island almost always exists in high temperature tokamak discharges. The magnetic perturbation due to the presence of the island is of the resonant type in the sense that $m - nq \approx 0$. Here, m is the poloidal mode number, n is the toroidal mode number, and q is the safety factor. It is shown that because of the distortion of the magnetic surface due to the presence of a magnetic island, the $|\boldsymbol{B}|$ on the distorted island magnetic surface is not toroidally symmetric [19]. This leads to an enhanced (over toroidally symmetric tokamaks) toroidal momentum loss that results from the symmetry-breaking-induced toroidal viscosity. Because the magnitude of the symmetry-breaking-induced toroidal viscosity usually increases with plasma temperature, the corresponding magnitude of the ion heat conductivity in the vicinity of the islands can become much larger than the standard neoclassical ion heat conductivity, and become comparable to the anomalous ion heat conductivity for the typical size of the islands observed in experiments. Thus, the toroidal momentum confinement can be influenced by magnetic islands. It is also interesting to note that because the typical size of the island is comparable to the radial correlation length of the turbulence fluctuations, the anomalous toroidal stress is not applicable for describing the fine radial structures around the island separatrix.

The symmetry-breaking-induced toroidal viscosity also provides a natural mechanism to determine the island rotation frequency [20,21]. The island rotation is determined by the $\sin x$ component of the relevant Ampere's law [22]. Here, $x = m(\theta - \phi/q) + \omega t$ is the helical angle of the island, ω is the island rotation frequency, θ is the poloidal angle, and ϕ is the toroidal angle. The parallel current density in the Ampere's law is determined from $\nabla \bullet \boldsymbol{J} = 0$ with the perpendicular current density driven by the frequency (*i.e.*, time) dependent symmetry-breaking-induced local (in ξ) toroidal viscosity. The island rotation frequency in both the collisional and the collisionless regimes are calculated [20,21]. The more relevant regime in a neutral beam heated tokamak discharge is the plateau regime associated with the helical variation in $|\boldsymbol{B}|$. The calculated Doppler-shifted island frequency,

$$\omega - \omega_{E0} = - \omega_{*pi} - 0.5\omega_{*Ti}, \tag{2}$$

seems to be consistent with that observed in beam-heated discharges [23]. Here, ω_{E0}, ω_{*pi}, and ω_{*Ti} are the equilibrium $\boldsymbol{E}\times\boldsymbol{B}$ frequency, diamagnetic frequency, and diamagnetic frequency due to the ion temperature gradient respectively. When the plasma collisionality decreases, electron viscous force can become dominant, and the island rotation frequency can reverse to the electron diamagnetic drift frequency.

In contrast to the magnetic islands, resistive wall modes induce magnetic perturbations that are non-resonant, *i.e.*, $m - nq \neq 0$ over most of the plasma radius. For the non-resonant perturbed magnetic fields, the direct modification on $|\boldsymbol{B}|$ due to the perturbation [24] is of the same order as that due to the surface distortion [25]. Both of these modifications on $|\boldsymbol{B}|$ result in broken toroidal symmetry. It is shown that in this case the toroidal momentum loss is also enhanced, and the physics of the system is very similar to that in stellarators. To complete the description of the collision frequency dependence of the symmetry-breaking-induced toroidal viscosity, a theory for the banana oscillation under the influence of the broken symmetry in $|\boldsymbol{B}|$, without the assumption that $|m - nq| >> 1$ employed in stellarator viscosity calculations, is developed [25]. In the $1/\nu$ regime, the toroidal momentum damping rate scales like $\varepsilon^{3/2}(\delta B)^2/\nu$, where ν is the ion-ion collision frequency, $\varepsilon < 1$ is the inverse aspect ratio, and δB is the perturbation of the MHD activity on $|\boldsymbol{B}|$. The results can be compared quantitatively with the toroidal momentum damping rate observed in tokamak experiments when resistive wall modes are present.

4. Conclusions

We have developed theories for the toroidal momentum confinement in realistic tokamak discharges. We find that in general the logarithmic gradient of the toroidal flow is a linear combination of the logarithmic gradients of the plasma pressure, and the temperature when the fluctuation-induced toroidal stress dominates. The toroidal momentum flux consists of a diffusion term, a convective term, and a residual term. The equilibrium toroidal flow is determined by balancing these three fluxes. We also illustrate that MHD activity, such as magnetic islands, and unstable MHD modes, breaks the toroidal symmetry in $|\boldsymbol{B}|$, and results in enhanced toroidal momentum transport. The frequency-dependent symmetry-breaking-induced toroidal plasma viscosity provides a natural mechanism to determine the island rotation frequency as well. In the more relevant plateau regime, the Doppler-shifted island rotation frequency is of the order of the ion diamagnetic drift frequency. These theories can be tested when fluctuations, resonant, or non-resonant MHD activity are present. They can all be included in NCLSS [26] to model general transport phenomena with and without MHD activities present in tokamaks.

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Transport of Thermal Energy and its Relation to Magnetic Reconnection and to the Spontaneous Rotation Phenomenon[1]

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Abstract. The high-temperature theory of the collisional drift-tearing mode is presented. In the regimes relevant to present day experiments the parallel electron thermal conductivity plays a key role and the novel analysis that is presented shows that the structure of the mode as well as the characteristics of the region where reconnection takes place differ significantly from the ones described in the original work [1] where the regime with relatively high collisionality was considered. A brief description is given of the "accretion" theory of the "spontaneous" rotation phenomenon and of the associated toroidal plasma collective modes that produce an inflow of angular momentum towards the center of the plasma column.

1. Introduction

The so-called drift-tearing mode that was described first in Ref. [1] couples the effects of magnetic reconnection, driven primarily by the plasma current density gradient, with those of the gradient of the longitudinal electron pressure. This mode has found a renewed appreciation recently in view of its relevance to current experiments. Consequently, the linearized theory of the drift-tearing mode has been reformulated for the high-temperature regimes in the important limit where the electron thermal conductivity along the field is significant. New characteristics for the mode amplitude radial profile and for the width of the reconnection layer have been found. We have pointed out originally that for modes involving singular perturbations the effects of nonlinearities become important at very small amplitudes. Thus we have considered a model equation relating the plasma displacement to the reconnecting component of the magnetic field where the ad hoc non-linear terms are included to simulate the effects of a local steepening of the local electron pressure gradient. This leads to a broadening of the reconnection layer and to an increase of the growth rate, relative to the linearized theory.

We consider a simple magnetic configuration, which can simulate more realistic and complex ones and is represented by $\mathbf{B}=B_z(x)\mathbf{e}_z+B_y(x)\mathbf{e}_y$ with $B_y^2 << B_z^2$. The normal modes that involve magnetic reconnection are of the form $\hat{B}_x=\tilde{B}_x(x)\exp(-i\omega t+ik_y y+ik_z z)$ with $k_yB_y+k_zB_z=0$ for $x=x_0$, and $0<|x_0|<a$, where a represents the (macroscopic) width of the plasma layer. Thus $k_y^2 >> k_z^2$, $k_\perp\approx k_y\sim 1/a$ and $k_\parallel=\mathbf{k}\cdot\mathbf{B}/B=k_\parallel(x)$. The electron thermal energy balance equation and the longitudinal electron momentum balance equation that we adopt include all the components that are relevant to rather weakly collisional regimes. In particular,

$$0\approx -en\mathbf{E}_\parallel-\nabla_\parallel p_e-\alpha_T n\nabla_\parallel T_e+en\eta_\parallel\mathbf{J}_\parallel, \tag{1}$$

where α_T is the thermal force coefficient and the other terms are easily identifiable. The effects of longitudinal electron pressure gradient terms are represented by the frequencies

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$\omega_{\|e} = -k_y(c/enB)dp_e/dx$, $\omega_{*e} = -k_y(cT_{e\|}/enB)dn/dx$ and $\omega_e^T = -k_y(c/eB)dT_e/dx$. Thus $\omega_{\|e} = \omega_{*e} + \omega_e^T$ and we define $\omega_{\|e}^T \equiv \omega_{\|e} + \alpha_T\,\omega_e^T$. To describe the transverse electron guiding center motion, we take $\hat{\mathbf{E}}_\perp + \hat{\mathbf{v}}_E \times \mathbf{B}/c \approx 0$. Consequently Eq.(1) becomes

$$i(\omega - \omega_{\|e}^T)\tilde{B}_x \approx i\,c\,k_y\eta_\|\,\tilde{J}_\| - \frac{c}{eB}i(\mathbf{k}\cdot\mathbf{B})\left[i\,k_y\tilde{T}_e(1+\alpha_T) + i\,k_yT_e\frac{\tilde{n}_e}{n}\right] - i(\mathbf{k}\cdot\mathbf{B})(-i\,\omega\tilde{\xi}_{Ex}) \qquad (2)$$

where $\tilde{\xi}_{Ex} = \tilde{v}_{Ex}/(\gamma - i\omega)$, $\tilde{v}_{Ex} \approx c\tilde{E}_y/B$. In addition,

$$(\gamma - i\omega)(\gamma - i\,\omega + i\,\omega_{di})\frac{d^2\tilde{\xi}_{Ex}}{dx^2} = i\frac{(\mathbf{k}\cdot\mathbf{B}')}{4\pi\rho}(x - x_0)\frac{d^2\tilde{B}_x}{dx^2}, \qquad (3)$$

where $\omega_{di} = k_y(c/eBn)dp_{i\perp}/dx$ is the ion diamagnetic frequency. This equation, derived from the total momentum conservation equation [1], and Eq. (2) are valid in the "reconnection" layer of width $\delta_L \ll a$, around $x=x_0$, where k_y^2 can be neglected relative to the operator $\partial^2/\partial x^2$ when applied to the perturbed quantities. Thus $\tilde{J}_\| \approx (c/4\pi)\partial\tilde{B}_y/\partial x \approx (c/4\pi)/(-ik_y)\partial^2\tilde{B}_x/\partial x^2$. Considering the magnetic diffusion coefficient $D_m \equiv \eta_\| c^2/4\pi$ and the frequency ω_A defined by $\omega_A^2 \equiv (\mathbf{k}\cdot\mathbf{B}')^2/4\pi\rho k^2$, we define $\varepsilon_\eta \equiv D_m k^2/\omega_A \ll 1$. Within the δ_L-layer the effects of finite ε_η, electron thermal conductivity $\kappa_{e\|}$ and ion gyroradius, represented by ω_{di}, are important as will be shown.

2. Influence of the Finite Electron Thermal Conductivity and Electron Compressibility

The relevant linear theory, that includes the effects of electron thermal conductivity and electron compressibility, is considerably more complex than the original one given in Ref. [1], but is necessary for its application to high-temperature regimes. In particular, we refer to the form of the electron thermal energy balance equation

$$\frac{3}{2}n\left(\frac{\partial\hat{T}_e}{\partial t} + \hat{v}_{Ex}\frac{dT_e}{dx}\right) + nT_e\nabla_\|\hat{u}_{e\|} \approx \nabla_\|\kappa_{e\|}\nabla_\|\hat{T}_e, \qquad (4)$$

and consider the case $\gamma < |\omega_R| \sim \nu_{\|e} \equiv k_\|^2\,2\kappa_{e\|}/(3n) \equiv k_\|^2 D_{e\|}$ for $\omega = \omega_R + i\gamma \approx \omega_{\|e}^T + \delta\omega_R + i\gamma$ and $\delta\omega_R \sim \gamma$. The appropriate theory requires the consideration of more than 2 asymptotic regions, the "outer" region corresponding to $|k(x - x_0) \sim 1|$. For this we define a transition distance δ_L^T by $\omega_{\|e}^T \equiv (k_\|'\delta_L^T)^2 D_{e\|}$. Consequently $\delta_L^T \sim (\nu_e/\Omega_{ce})^{1/2}L_s$ where $L_s \equiv B/B_y'$. Then for $|x - x_0|^2 > (\delta_L^T)^2$ the electrons can be treated as isothermal, for $|x - x_0|^2 < (\delta_L^T)^2$ as adiabatic, and for $|x - x_0|^2 \sim (\delta_L^T)^2$ the complete expression for $\tilde{T}_e/T_e$ has to be considered.

The low thermal conductivity region is defined by $\gamma/\left(k_\|'^2 D_{e\|}\right) < (x - x_0)^2 < (\delta_L^T)^2$ and is the most important of all the asymptotic regions considered as the mode is, basically, localized within it. An approximate analytic solution of Eqs. (2)-(4) that we have found within this region shows that it is characterized by two length-scales. In particular, the characteristic distance for variation of $\tilde{\xi}_{Ex}$ is $\delta_{osc} \sim L_s^{2/3}\rho_s^{1/3}(\nu_e/\Omega_{ce})^{1/3}$ and the decay scale-length, which could be taken as the characteristic width of the reconnection layer is

$\delta_L \sim \rho_s^{1/5} L_s^{4/5} (\nu_e/\Omega_{ce})^{2/5}$. The condition $\delta_{osc} > \rho_i$ implies that $(\nu_e/\Omega_{ce})^{1/2} > \rho_i / L_s$, which also ensures that $\delta_{osc} < \delta_L < \delta_L^T$. We may argue that these conditions can be satisfied realistically for relatively high temperatures. It is evident that the growth rate of this mode is rather weak, $\gamma \sim D_m/(\delta_L k) \propto \nu_e^{3/5}$ and the amplitude of the plasma displacement is of the order of $\tilde{\xi}_{Ex} \sim (\tilde{B}_{x0} k / \mathbf{k}\cdot\mathbf{B}') \delta_L^{T\,2} D_m / \delta_L^4 \omega_{e\|}^T$. These results are valid for sufficiently small $\varepsilon_T \equiv \delta_{osc}/\delta_L^T$. Our numerical analysis shows that the asymptotic theory gives satisfactory description of the eigenmode structure for $\varepsilon_T \le 0.3$ In a typical Ohmic discharge by the Alcator C-Mod machine, for example, ε_T varies between 0.2 and 0.8. In fact, for the sake of completeness, the equation for $\tilde{\xi}_{Ex}$ that covers the low and the finite conductivity regions and extends into the isothermal region has been integrated numerically.

The finite thermal conductivity region corresponds to $(x - x_0)^2 \sim (\delta_L^T)^2$. Within this region, the appropriate dmensionless form of Eqs. (2) and (3) has to be solved numerically. Our solution demonstrates that for a wide range of relevant parameters the peak of the eigenfunction occurs around $\bar{x} \sim 1$ where $\bar{x} \equiv (x - x_0)/(\delta_L^T)$ and that the asymptotic solution $\frac{\mathbf{k}\cdot\mathbf{B}'}{\tilde{B}_{x0} k}\tilde{\xi}_{Ex} \approx \frac{\gamma - i\delta\omega_R}{\omega_{e\|}^T - \omega_{*e}} \frac{1}{k(x-x_0)} \sim \frac{D_m k}{\delta_L\, \omega_{e\|}^T} \frac{1}{k(x-x_0)}$ is recovered for $\bar{x}$ between 2 and 4. Therefore we may consider δ_L^T as the basic scale distance for the reconnection layer. We note that the quantity $(\mathbf{k}\cdot\mathbf{B}')\tilde{\xi}_{Ex}/(\tilde{B}_{x0}k)$ remains well below unity in the "outer" region, a situation that is quite different from that of the purely resistive theory where this quantity is of order unity in this region.

3. Nonlinear Model

The nonlinear effects which are included in the simple model equation that we have analyzed are related to i) the (quasilinear) decrease of $dp_{e\|}/dx$ due to the effects of pre-excited modes, of the same kind, that couple with the considered mode and ii) the fact that $\hat{B}_x \partial \hat{p}_{e\|}/\partial x$ becomes important relative to $\mathbf{B}\cdot\nabla\hat{p}_{e\|} = i(\mathbf{k}\cdot\mathbf{B})\hat{p}_{e\|}$ as $\mathbf{k}\cdot\mathbf{B}$ tends to vanish within δ_L while $|\partial\hat{p}_{e\|}/\partial x|/|\hat{p}_{e\|}| \sim 1/\delta_L$ tends to become singular. Consequently, the width of the layer where these two terms are comparable is of the order of $\delta_{NL} \sim |\tilde{B}_{x0}/(\mathbf{k}\cdot\mathbf{B}')|^{1/2}$ and this width can exceed easily that obtained form the linearized theory for quite small amplitudes of the reconnected field $\tilde{B}_{x0}$ at $x=x_0$. As an illustrative example, we consider a model equation for a "large thermal conductivity limit" $k_\|^2 T_{e\|}/m_e > \nu_e |\omega|$. The model consists of Eq. (3) and the following nonlinear replacement of Eq. (2)

$$\left[-i(\omega_R - \omega_{\|e}^T + \omega_{\|e}^T f_{NL}) + \gamma\left(1 - \frac{\omega_{\|e}^T}{\omega_{\|e}^T - \omega_R}\alpha_N f_{NL}\right)\right]\tilde{B}_{x0}$$
$$\approx i(\mathbf{k}\cdot\mathbf{B}')(x - x_0)\tilde{\xi}_{Ex}(-i\omega_R + \gamma + i\omega_{*e}) + D_m \frac{d^2\tilde{B}_x}{dx^2} \tag{3}$$

Here f_{NL} represents the nonlinear effects described earlier, α_N is a constant parameter and we have taken $f_{NL} = |d\tilde{\xi}_{Ex}/dx|^2$. We note that within the δ_{NL}-layer $\tilde{B}_x \approx \tilde{B}_{x0}[1 + \varepsilon_\delta \varphi(\bar{x})]$, where

$\varepsilon_\delta \equiv k\delta_{NL}$ and $\bar{x} \equiv (x - x_0)/\delta_{NL}$, and the solution for $\xi_{Ex}(x)$ of these equations develops a singularity in the curvature ($d^2\xi_{Ex}(x)/dx^2$) that is removed by the effects of finite resistivity. The growth rate of the mode is enhanced relative to that found by the linearized resistive theory and can reach values of the order of $\omega_A \varepsilon_\delta^{3/2}$ for $\omega_R < \omega_{||e}^T < \gamma$.

We note that tridimensional stability analyses concerning both cylindrical [2] and toroidal [3] configurations have indicated that when the effects of the longitudinal pressure gradient are included, the rate of reconnection produced by the excited modes increases considerably, and the width of the reconnection region is definitely broader. In fact we consider that nonlinear drift-tearing modes can provide the explanation for modes involving magnetic reconnection that have been observed experimentally [4], and do not appear to correspond to neoclassical tearing modes.

4. Accre tion Theory of "Spontaneous Toroidal Rotation"

Another important process that is intrinsically connected to the transport of the plasma thermal energy is the "spontaneous toroidal rotation" of axisymmetric plasmas. This connection was pointed out first in the formulation of the so called accretion theory [5,6] and has been confirmed consistently by the most recent series of experiments [7] on this phenomenon. According to this theory, angular momentum in one direction is "accreted" on the material wall surrounding the plasma column [8] while angular momentum of the opposite direction (e.g. in the direction of the ion diamagnetic velocity in the case of the H-confinement regime) is carried from the edge toward the center of the plasma column by modes that are driven primarily by the plasma pressure gradient. A transport equation for angular momentum that included an inflow term in the direction of increasing angular momentum, like that adopted earlier to describe the particle transport, was in fact given in Ref. [9]. We have verified that toroidal ballooning modes do not provide significant transport of net angular momentum. Thus toroidal "travelling modes", along the magnetic field, which instead can carry net angular momentum in the radial direction, have to be present and have significant amplitudes. Two forms of the relevant quasilinear theory are derived identifying the ion pressure gradient as the driving factor for the angular momentum inflow and associating the sign of the ratio of the relevant poloidal to the toroidal mode numbers to that of the plasma toroidal velocity gradient. According to this analysis the source of the excitation of travelling modes is near the edge of the plasma column, from which the rotation velocity has been observed to enter, while the source of excitation of the ballooning modes is well within the plasma column where the ratio (η_i=d ln T_i/ d ln n) of the ion temperature gradient to the density gradient is maximum.

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