Instability Suppression by Sheared Flow in Dense Z-Pinch Devices

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Abstract. The feasibility of stabilizing a Z-pinch by means of an axial shear flow has been studied theoretically, numerically and experimentally. Since the results of the MHD stability analysis can depend on the equilibria that are being considered, we start by analysing families of self-consistent equilibria with sheared flow, which could represent feasible starting points for the stability analysis. The electric field is determined by the generalised Ohm's law. This leads to small deviations from local quasi-neutrality, as is found from Poisson's equation. The self consistency of the flow is related to such deviations, which can be proposed as a function of the plasma density. It is found that for certain flows, self consistent equilibria can only be possible if the plasma is non-neutral. We also study the properties of a nozzle design that ejects a converging supersonic jet on the axis, which could be used as a target for the current sheath in plasma foci. A numerical simulation of this design is carried out, using an adaptive grid code that solves the gas-dynamics equations.

1. Introduction

The theory of magneto-hydrodynamic (MHD) stability with equilibrium flows dates back to the early days of fusion research [1], and its most important results, mainly for toroidal plasmas, have been reviewed in Ref. [2]. The study of local MHD instabilities for cylindrical plasmas, with sheared equilibrium flows, is ideal for the understanding of the problem, since the symmetry allows simpler calculations. The problem was initially studied by Bondeson et al.[3], and the feasibility of stabilizing a Z-pinch by means of an axial sheared flow has been studied theoretically and numerically by Shumlak et al. [4, 5], and Arber and Howell [6]. Although their conclusions differ in the details, they both agree in the general result that sheared flows can have a stabilizing effect. These results are backed, at least qualitatively, by experimental observations [5, 7, 8]. Further theoretical work has been recently done, where the inclusion of finite Larmor radius effects have been considered [9, 10], and Lyapunov stability has been explored [11].

Although there has been little valuable experimental work in this respect, the plasma focus is an ideal test bed for the introduction of this resource for stabilization. On one hand, the density of the filling gas in dense plasma foci is limited by the energy available for breakdown, and ionization of the neutral gas by the plasma sheath, during the rundown phase. On the other, it is desirable to increase the density of the plasma during the compression phase, when the current in the circuit approaches its maximum, in order to improve the neutron yield. The idea of injecting a gas jet at the tip of the inner electrode was originally proposed by Vikhrev [12], and it has been shown both in small [13] and large [14] plasma foci that it is possible to decouple the compression phase from the breakdown and the rundown phases by these means. Not only is the neutron emission significantly increased, but the uniformity of the discharges is also improved. Since the wide variability in the radiation of these kind of devices is strongly related to the m=0 instabilities during the compression phase, it can be speculated that the jet may be able to suppress them, which is consistent with the results separately obtained for the z-pinch. Further experimental work in this line is badly needed, in order to find out whether such speculation can be supported.

Since the results of the MHD stability analysis depends on the equilibria that are being considered, we start by analysing families of self-consistent equilibria with sheared flow, which could represent feasible starting points for the stability analysis. Proposing binomial shapes for the current density radial profiles, we propose a family of equilibria, which are qualitatively different. In the case when only axial sheared flows are considered, the equilibrium pressure profiles are independent of the flow, but when azimuthal flows are included, the pressure surfaces depart from the magnetic field surfaces, as the flow increases. In the present work we shall thus study the axial flow case. The electric field is determined by the generalised Ohm's law. This leads to two different situations. Either a flow that is truncated for small radii is allowed, preserving quasineutrality, or alternatively, small deviations from quasi-neutrality should be allowed, as is found from Poisson's equation. The self consistency of the flow is related to such deviations, which can be proposed as a function of the plasma density. It is found that for certain flows, self consistent equilibria can only be possible if the plasma is non-neutral.

As a contribution to future experiments in plasma foci with gas injection, we also study the properties of a nozzle design that ejects a converging supersonic jet on the axis, which could be used as a target for the current sheath. This converging flow results in the formation of a reflected shock beyond the exit of the nozzle, in which the density is enhanced. A numerical simulation of this design is carried out, using an adaptive grid code that solves the gas-dynamics equations [15]. In section 2 we present our mathematical model and the calculation of the relevant equilibria. The linear stability equations are formulated in section 3, and some discussion of ho this can be applied to the plasma focus is given in section 4.

2. Model Equations for the Equilibria

The equilibria are obtained in terms of the standard ideal magneto-hydrodynamic model given by the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad , \tag{1}$$

the momentum equation

$$\rho(\partial \mathbf{v} / \partial t + \mathbf{v} \cdot \nabla \mathbf{v}) = \mathbf{j} \times \mathbf{B} - \nabla p \quad , \tag{2}$$

conservation of specific entropy

$$(\partial / \partial t + \mathbf{v} \cdot \nabla)(p / \rho^{\gamma}) = 0 \quad , \tag{3}$$

generalised Ohm's law

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \quad , \tag{4}$$

Faraday's law

$$\nabla \times \mathbf{E} = -\partial B / \partial t \quad , \tag{5}$$

and Ampere´s law

$$\nabla \times \mathbf{B} = \mu_o \mathbf{j} \quad , \tag{6}$$

subject to the initial conditions

$$\nabla \cdot \mathbf{E} = \frac{\rho_c}{\varepsilon_o} \quad , \tag{7}$$

$$\nabla \cdot \mathbf{B} = 0 \quad . \tag{8}$$

Here ρ and ρ_c are the mass and charge densities, p is pressure, **v, E, B** and **j** are respectively the plasma velocity, the electric and magnetic fields, and the current density. Although equation (7) is usually ignored, assuming deviations from quasineutrality are negligible for magnetohydrodynamic time scales, in our case it might be important, so we shall keep it, still assuming that it is small enough to ignore it in the momentum equation (2).

Thus, the corresponding equilibrium equations will be:

$$\nabla \cdot (\rho_o \mathbf{v}_o) = 0 \quad , \tag{9}$$

$$\rho_o (\mathbf{v}_o \cdot \nabla \mathbf{v}_o) = \mathbf{j}_o \times \mathbf{B}_o - \nabla p_o \quad , \tag{10}$$

$$(\mathbf{v}_o \cdot \nabla) p_o + \gamma p_o \nabla \cdot \mathbf{v}_o = 0 \quad , \tag{11}$$

$$\mathbf{E}_o + \mathbf{v}_o \times \mathbf{B}_o = 0 \quad , \tag{12}$$

$$\nabla \times \mathbf{B}_o = \mu_o \mathbf{j}_o \quad , \tag{13}$$

$$\nabla \cdot \mathbf{E}_o = \rho_{co} / \varepsilon_o \quad , \tag{14}$$

where the equilibrium variables are tagged by subindex o.

Assuming cylindrical and axial symmetry, so the equilibrium variables depend only on the radius *r*, and an axial flow $\mathbf{v} = v_z(r)\mathbf{e}_z$, (10-14) can be rewritten as:

$$\frac{dp_o}{dr} = -j_{oz} B_{o\vartheta} \quad , \tag{15}$$

$$E_{or} = v_{oz} B_{o\vartheta} \quad , \tag{16}$$

$$j_{oz} = \frac{1}{\mu_o r} \frac{d}{dr}(r B_{o\vartheta}) \quad , \tag{17}$$

$$\nabla \cdot \mathbf{E}_o = \frac{1}{r}\frac{d}{dr}(r v_{oz} B_{o\vartheta}) = \frac{\rho_{co}}{\varepsilon_o} \; . \tag{18}$$

If finite Larmor radius effects were considered in (12), then (16) and (18) should be modified accordingly. However, for current experimental situations, in which the axial flow can be significant enough they are negligible, so in this work we shall ignore them.

Under this symmetry assumptions, $\rho_o (\mathbf{v}_o \cdot \nabla \mathbf{v}_o) = 0$ in (7), and therefore equilibria are the same as those when there is no flow. The problem is now to find the flows that are self-consistent with them.

Let us now rewrite each variable as he product of a shape factor, which depends on the radius *r*, and a parameter that can be varied: $p_o(r) = P_o p_o(r)$, $j_{oz}(r) = J_o j_o(r)$, $B_{o\vartheta}(r) = B_o b(r)$, and $v_{oz}(r) = v_o u_o(r)$. The charge density will be expressed as

$$\rho_{co}(r) = e(Zn_i - n_e) \equiv e\lambda n_o(r) = e\lambda N_o n_o(r) \, , \tag{19}$$

where n_i and n_e are the ion and electron densities, and $|\lambda|$ <<1 so that it can still be ignored in (2) and (15).

If the current density profile can be experimentally tailored, the magnetic field can be obtained,

$$B_{o\vartheta}(r) = \frac{\mu_o J_o}{r}\int_0^r s j_o(s)ds \equiv \frac{\mu_o J_o}{r} F(r) \quad , \tag{20}$$

and the equilibrium pressure profile can be determined by integrating (15).

$$p_o(r) = \mu_o J_o^2 \int_r^a \frac{1}{s} j_o(s)F(s)ds \quad , \tag{21}$$

where a is the radius of the plasma column.

In order to determine the self-consistent axial sheared flow there are two possibilities: From (18), if $\rho_{co}(r)$, or its integral over the radius, taking the scale factor into account, are zero,

$$v_{oz}(r) = \frac{K}{rB_{o\vartheta}(r)} = \frac{K}{\mu_o J_o}\frac{1}{F(r)} \quad , \tag{22}$$

where K is constant. Since $B_{o\vartheta}(0) = 0$, this means that there should be no flow on the axis, and thus K must be treated as a step function which is null close to the axis, and finite for radii r such that $0 < r_o \le r \le a$. Alternatively, one may allow a small deviation from quasineutrality, and from (18) and (19),

$$v_{oz}(r) = \frac{eN_o c^2}{J_o}\lambda \frac{\int_0^r s n_o(s)ds}{F(r)} \quad , \tag{23}$$

where the magnitude of the flow is limited by need to keep $|\lambda| \ll 1$. In this work we shall limit ourselves to analyse the second case.

3. Sample Equilibria

A family of equilibria with a wide range of qualitative behaviours, with a maximum current at the axis, can be generated by choosing a density current profile of the form [16]

$$j_{oz}(r) = J_o\left[1 - \left(\frac{r}{a}\right)^n\right] \equiv J_o j_o(r); \qquad n>0 \quad . \tag{24}$$

The magnetic field is found to be

$$B_{o\vartheta}(r) = \frac{\mu_o a J_o}{2}\frac{r}{a}\left[1 - \frac{2}{n+2}\left(\frac{r}{a}\right)^n\right] \quad , \tag{25}$$

and the pressure profile

$$p(r) = \mu_o \frac{J_o^2 a^2}{4}\left[1 - \frac{2(n+4)}{(n+2)^2} + \frac{2}{(n+2)(n+1)}\right] -$$

$$-\mu_o \frac{J_o^2 a^2}{4}\left[\left(\frac{r}{a}\right)^2 - \frac{2(n+4)}{(n+2)^2}\left(\frac{r}{a}\right)^{n+2} - \frac{2}{(n+2)(n+1)}\left(\frac{r}{a}\right)^{2n+2}\right] \tag{26}$$

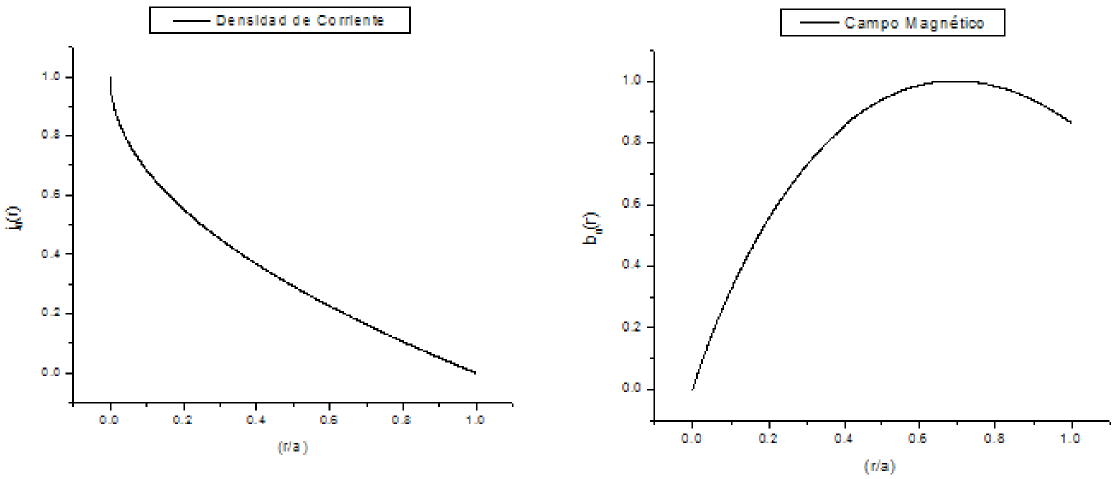


Fig. 1. Current density and magnetic field profiles for the case n=1/2.

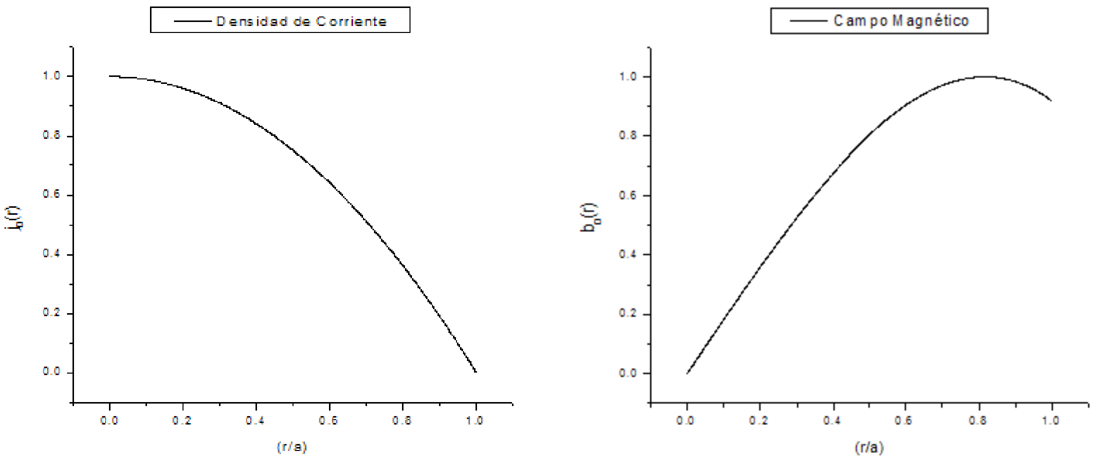


Fig. 2. Current density and magnetic field profiles for the case n=2.

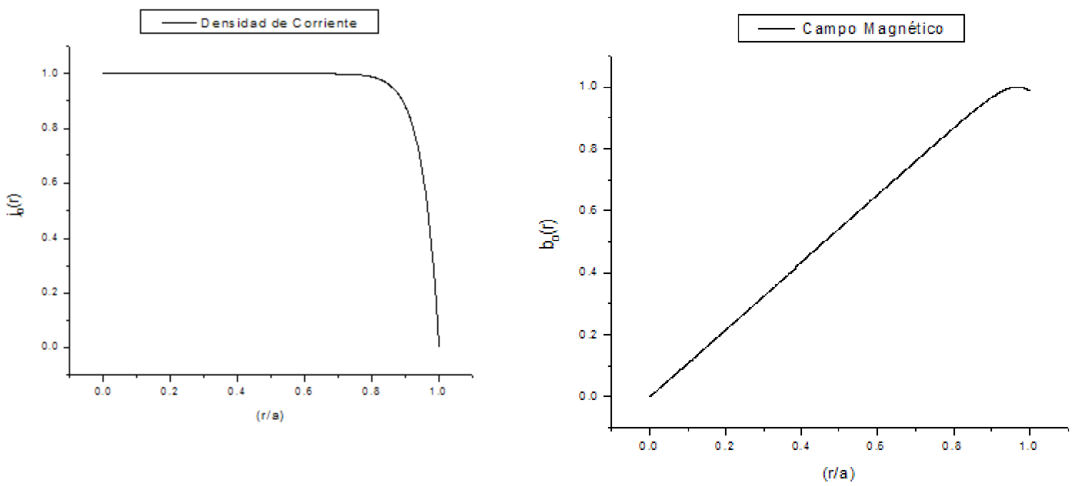


Fig. 3. Current density and magnetic field profiles for the case n=20.

Figures 1-3 show the cases when n=1/2, 2 and 20. The first is strongly peaked, while the latter is practically that of a constant current. If current profiles with a minimum on the axis are required, they can be obtained by taking $j_{oz} = J_o(r/a)^n$. By increasing n, it is possible to have density current profiles which range from those with a mild slope, to others which are essentially constant, with a strong slope close to the border.

In order to obtain self consistent flows, we propose $n_o(r) = \left[1-(r/a)^l\right]$, l > 0. It is found that the self consistent flows are given by

$$u_o(r) = \frac{n(l+2)}{l(n+2)} \frac{\left[1-\frac{2}{l+2}\left(\frac{r}{a}\right)^l\right]}{\left[1-\frac{2}{n+2}\left(\frac{r}{a}\right)^n\right]} \quad ; \quad n < l \quad , \tag{27a}$$

$$u_o(r) = \frac{\left[1-\frac{2}{l+2}\left(\frac{r}{a}\right)^l\right]}{\left[1-\frac{2}{n+2}\left(\frac{r}{a}\right)^n\right]} \quad ; \quad n > l \quad . \tag{27b}$$

A couple of examples are given in Fig. (4a) for n =1/2, l = 30, and Fig. (4b) for for n =2, l = 30.

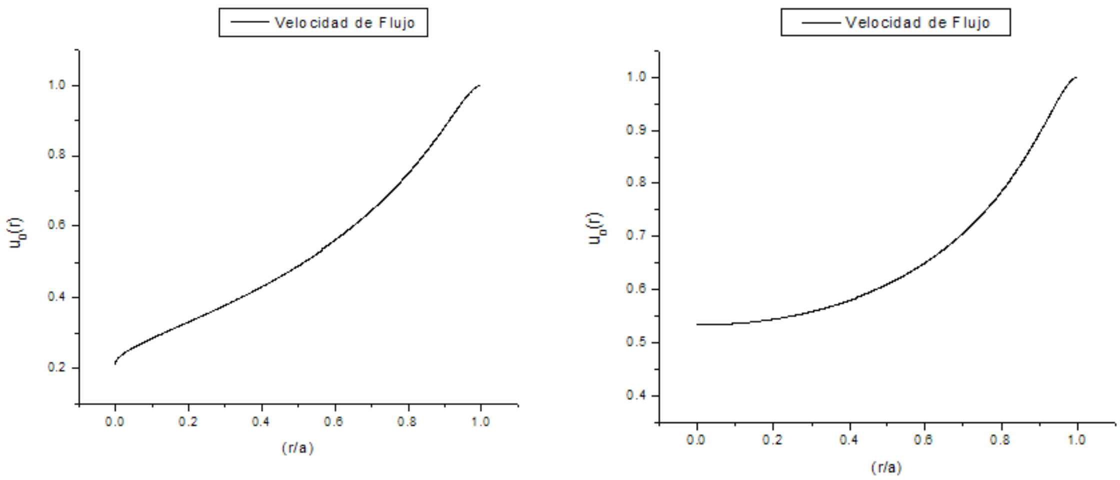


Figure 4. Examples of self-consistent flows with deviations from quasi-neutrality. (a) n =1/2, l = 30, (b) n =2, l = 30.

4. Stability Analysis

Following Refs. [3] and [6], it is now possible to make a linear stability analysis based on the Lagrangian displacement $\xi = \xi(r)\exp[i(\omega t + m\vartheta - kz)]$. The problem is reduced as usual, to solve for the radial component ($r\xi_r$) and $p_* = -\gamma p\nabla\cdot\xi - \xi\cdot\nabla p + [\nabla\times(\xi\times B)]\cdot B$:

$$AS(1/r)(r\xi_r)' = C_{11} r\xi_r + C_{12} p_* \quad , \tag{28a}$$

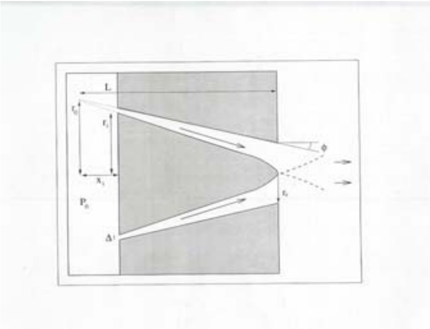
$$ASp_*' = C_{21} r\xi_r + C_{22} p_* \quad , \tag{28b}$$

where A,S,C_{ij}, with i = 1,2; j = 1,2, are functions of the equilibrium quantities, and ω, k, and z.

5. Nozzle Design for Stabilization in Dense Plasma Focii

The idea of introducing a gas-puffed target at the tip of the inner electrode, just before the compression phase, was proposed by Vikhrev several years ago [12], and tested experimentally by Schmidt et al. in the Poseidon plasma focus, which is a large device, that can operate up to 500 kJ at 80 kV [14], and Milanese et al. in the PACO device, which is a small device, operating at 2 kJ, 31 kV [13]. In the former case, the inner electrode is 460 mm long, 131 mm dia., while in the latter it is only 4 mm long, with 4 mm dia. In POSEIDON the target gas jet was formed with a system of nozzles arranged on a circle. In Ref. [14], 16 nozzles parallel to the electrode axis, with 5 mm dia. were used, arranged on a r=28 mm circle. The conclusion was that the target gas allowed a better reproducibility of the pinch, although the effect of the neutron yield varied widely. While in some cases it was reduced, under optimal conditions it could be increased by 30\%. They later improved their results substantially with an arrangement of 24 nozzles (2mm dia.) slanted 15 degrees towards the axis, on a r=24 mm circle. In this study, the device was operated at two different energy levels: 135kJ, 41.6kV and 210kJ, 51.5kV. While for the lower energy mode of operation, some turbulence appeared in the plasma column, induced by the gas streams, it disappeared for the higher energy one. Furthermore, the first neutron pulse was increased with a consequent increase in the neutron yield, while the second neutron pulse was suppressed, along with its associated electron pulse. Apparently, this is due to a suppression of plasma instabilities by the gas flow, although further investigatin is neeed. For best conditions, an 80\% increase in the neutron yield was found. Since PACO's electrode is smaller, a single straight on axis nozzle (2 mm dia.) was used [13]. This work confirms the findings that the target jet allows a better uniformity of the neutron yield on a shot to shot basis, and optimising the operation they were able to produce an almost 200\% increase in the neutron yield.

It is therefore desirable to design nozzles that produce better targets for compression, and to study their interaction with the current sheath. The nozzles should maintain a narrow beam of high density gas in the focus zone during the compression. Figure 5 shows a nozzle design we have in process, for which only the gas dynamics have been studied so far. The two main features we look for are the convergence of the cylindrical beam towards the symmetry axis, and a high Mach number. We have performed 2D simulations with an axisymmetric adaptive grid code developed by Raga [15], which solves the gas-dynamics equations together with a system of rate equations for atomic/ionic and molecular species. This code has been previously validated by comparing its results with experiments of a shock wave driven in a gas by a Nd-YAG laser [16]. We propose a nozzle in which the outer wall is conical, with a half-opening angle ϕ (see Figure 1). This opening angle is chosen so as to fix the required convergence for the emerging jet beam. The nozzle also has an internal obstacle, which has a conical base. The opening angle of the internal cone has of course to be larger than the opening angle of the outer cone, in order to ensure that the cross section available for the flow (which is limited to the gap between the two cones grows with distance along the axis of the nozzle. The prolongation of the two cones in the upstream direction would join at a cylindrical radius r_o, and we choose the position of this point along the symmetry axis as the zero-point for the x-(axial) coordinate. In order to have a finite gap of width Δ (in order to allow the flow to enter the nozzle) it is of course necessary to truncate the two cones at a distance x_i (along the symmetry axis) from the point at which the two cones would intersect.



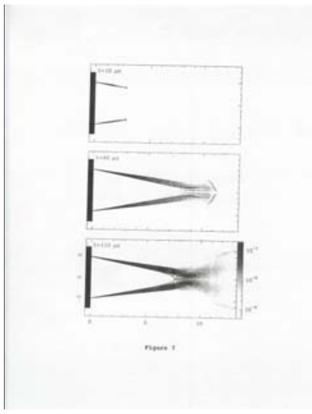


Figure 5. Nozzle design for a target jet in a dense plasma focus, for which convergence of the beam and high a Mach number are sought.

Acknowledgements

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