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Electron Thermal Transport in Tokamak: ETG or TEM Turbulences?

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Abstract

This paper reports progress on numerical and theoretical studies of electron transport in tokamak including: (1) electron temperature gradient turbulence; (2) trapped electron mode turbulence; and (3) a new finite element solver for global electromagnetic simulation. In particular, global gyrokinetic particle simulation and nonlinear gyrokinetic theory find that electron temperature gradient (ETG) instability saturates via nonlinear toroidal couplings, which transfer energy successively from unstable modes to damped modes preferably with longer poloidal wavelengths. The electrostatic ETG turbulence is dominated by nonlinearly generated radial streamers. The length of streamers scales with the device size and is much longer than the distance between mode rational surfaces or electron radial excursions. Both fluctuation intensity and transport level are independent of the streamer size. These simulations with realistic plasma parameters find that the electron heat conductivity is much smaller than the experimental value and in contrast with recent findings of flux-tube simulations that ETG turbulence is responsible for the anomalous electron thermal transport in fusion plasmas. The nonlinear toroidal couplings represent a new paradigm for the spectral cascade in plasma turbulence.

1 Electron temperature gradient turbulence

Electron temperature gradients in magnetically confi ned plasmas provide expansion free energy for driving various drift-wave instabilities, which may induce high level electron heat transport often observed in toroidal experiments. Experimental evidences for the origin of electron transport are not conclusive. Candidate instabilities include the trapped electron mode (TEM) and/or

ion temperature gradient (ITG) mode with a characteristic length of the ion gyroradius [1], the electromagnetic electron temperature gradient (ETG) turbulence with a shorter scale length of the collisionless electron skin depth [2], and the electrostatic ETG mode [3] with the shortest scale length of the electron gyroradius.

The ETG instability has generally been discarded as a realistic driver for the anomalous electron transport since a heuristic mixing length estimate predicts that the electron heat conductivity driven by the ETG turbulence is smaller than the ITG/TEM transport by a factor of the square-root of ion-electron mass ratio. Nonetheless, the ETG nonlinear evolution could be very different. Whereas an $\mathbf{E} \times \mathbf{B}$ nonlinearity associated with zonal fbws [4, 5, 6] dominates in the ITG turbulence, the ETG turbulence is regulated by a much weaker polarization nonlinearity [7].

The renewed interest in the electrostatic ETG instability comes from gyrokinetic continuum simulations using the flux-tube geometry [8], which found that electron transport up to 60 times of the mixing length level is driven by the $\mathbf{E} \times \mathbf{B}$ convection of ETG radial streamers. However, global fluid simulations [9, 10] found that the ETG transport is smaller than the flux-tube result by more than an order of magnitude, and concluded that ETG turbulence is unlikely responsible for the electron anomalous transport. Furthermore, the saturation mechanism and direct relationship between the streamer size and the electron transport have not been established by numerical simulations or by fi rst-principles theories.

In present studies, global gyrokinetic particle simulation and nonlinear gyrokinetic theory find that the ETG instability saturates via nonlinear toroidal couplings, which transfer energy successively from unstable modes to damped modes preferably with longer poloidal wavelengths. The electrostatic ETG turbulence is dominated by nonlinearly generated radial streamers. Both fluctuation intensity and transport level are independent of the streamer size, which scales with the device size. The electron heat conductivity is much smaller than the experimental value or the flux-tube result, and cast doubts on recent claims that ETG turbulence is responsible for the anomalous electron transport in toroidal devices. The nonlinear toroidal coupling found in our studies is a new paradigm for plasma turbulence since it governs the poloidal spectral energy cascade in all toroidal drift wave instabilities.

Our global ETG simulations and associated gyrokinetic theory have important implications on plasma turbulence studies. First, particle dynamics must be treated on the same footing as fluid nonlinearity. While wave-wave couplings determine fluctuation characteristics, transport is driven by wave-particle interactions. The heuristic mixing length rule, which underlies most of transport models, do not correctly describe transport processes in collisionless plasmas. Secondly, toroidal geometry must be treated rigorously in turbulence simulations. The radial variations of the safety factor q(r) need to be retained to properly account for nonlinear waveparticle interactions. All eigenmodes participate in nonlinear toroidal couplings and thus must be included in simulations. Finally, the contradictory results from ETG turbulence simulations between flux-tube and global codes are consequences of differences in the respective geometry representations. While toroidicity is treated rigorously in global codes, flux-tube codes make key approximations, the validity regime of which remains dubious for nonlinear simulations involving fluctuations with low toroidal mode numbers and nonlinear particle dynamics. Therefore, the flux-tube simulation is a reduced model, and its validity rests on the ability to recover results of the more general global simulation in appropriate asymptotic regimes.

ETG turbulent transport – The massively parallel gyrokinetic toroidal code (GTC) [4] employs billions of spatial grids and particles to provide adequate resolutions for global ETG simulations. Toroidal geometry is treated rigorously and radial variations of safety factor q, magnetic shear \hat{s} , and trapped particle fraction are retained. An efficient global field-aligned

mesh [11], which reduces computational requirements by three orders of magnitude for global ETG simulations, provides the maximal efficiency without any simplification in the geometry or physics models. In contrast, flux-tube codes remove important radial variations of all equilibrium quantities and use a periodic boundary condition, the validity of which is questionable in the presence of radial streamers.

GTC simulations use representative tokamak plasmas with the following local parameters at r = 0.5a identical to flux-tube simulations: $R_0/L_T = 6.9$, $R_0/L_n = 2.2$, q = 1.4, $\hat{s} \equiv (r/q)(dq/dr) = 0.78$, $\tau = T_e/T_i = 1$, and $a/R_0 = 0.36$. Here R_0 is the major radius, a is the minor radius, L_T and L_n are electron temperature and density gradient scale lengths, respectively, T_i and T_e are the ion and electron temperatures, and q is the safety factor. The electrostatic potential $\delta \phi = 0$ is enforced at r < 0.25a and r > 0.75a. Simplified physics models include: a parabolic profile of $q = 0.854 + 2.184(r/a)^2$, a temperature gradient profile of $\exp\{-[(r-0.5a)/0.2a]^6\}$, a circular cross section, and electrostatic fluctuations with an adiabatic ion response. A collision operator modeling a heat bath [11] prevents the temperature profile relaxation. The computational mesh consists of 64 toroidal grids, and a set of unstructured radial and poloidal grids with a perpendicular grid size of $1.5\rho_e$. Numerical convergence studies use 5 - 20 particles per cell. The simulation device size ranges from NSTX to DIIID tokamaks.

The linear ETG dispersion relation for these plasma parameters shows that the most unstable mode has a poloidal wavevector $k_{\theta}\rho_e = 0.33$ with a linear growth rate $\gamma_0 = 0.038 v_e/L_T$ and a real frequency $\omega_r \simeq 3\gamma_0$, where $v_e =$ $\sqrt{T_e/m_e}$ and m_e is the electron mass. In nonlinear simulations, random fluctuations with a very small amplitude first grow exponentially, then saturate, and eventually reach a quasi-steady state. _500 The poloidal spectrum down-shifts gradually from a linear phase to a nonlinear phase, and peaks around $h_{\rm OOO}$ $k_{\theta}\rho_e \simeq 0.12$ at $t \simeq 10/\gamma_0$ after the mode saturation for a device size $a = 2000 \rho_e$. Each mode is represented by the amplitude of the harmonics (n, m)with a toroidal mode number n

and a poloidal mode number m



Figure 1: Poloidal contour plots of electrostatic potential at $t = 20/\gamma_0$ after saturation. The poloidal projection of a typical electron orbit is plotted from saturation to t. The length unit is ρ_e .

such that $k_{\theta} = nq/r$, m = nq, and q = 1.4 at r = 0.5a. The dominant modes with $k_{\theta}\rho_e \sim 0.12$ are obviously nonlinearly driven since the actual growth rates, $\geq \gamma_0$, are much larger than their linear growth rates. This suggests that the most unstable modes with $k_{\theta}\rho_e \simeq 0.33$ saturates via a nonlinear mode coupling to longer wavelength modes, and that structures in the fully developed turbulence are nonlinearly generated. Long wavelength modes down to $k_{\theta}\rho_e \sim 0$ are all excited, and grow before shorter wavelength modes of $k_{\theta}\rho_e \simeq 0.12$, suggesting that the downshift of the poloidal spectrum is not the conventional inverse cascade [7]. Similar nonlinear down-shift of the poloidal spectrum also occurs during a period of ~ 10 growth times in the ITG turbulence [11].

The poloidal contour plots of the electrostatic potential (or density) in the fully developed turbulence is shown in Fig. 1. The seemingly coherent structures actually contain hundreds of toroidal modes. Clearly the ETG turbulence is dominated by nonlinearly generated radial streamers. The nonlinear decorrelation rate γ_{nl} can be estimated from the streamer eddy turnover time associated with the $\mathbf{E} \times \mathbf{B}$ drift: $\gamma_{nl} \sim \Omega_e k_r \rho_e k_\theta \rho_e e \delta \phi / T_e$. We find that $\gamma_0 / \gamma_{nl} \sim 17$ for the case of $a = 2000\rho_e$, and that the value of γ_0 / γ_{nl} increases linearly with a / ρ_e . The fact that $\gamma_0 / \gamma_{nl} \gg 1$ contradicts the mixing length rule of γ_{nl} balancing γ_0 , and invalidates a common practice, where γ_{nl} is replaced by γ_0 for the condition of the turbulence suppression by sheared fbws, i.e., $\omega_{\mathbf{E} \times \mathbf{B}} \sim \gamma_0$.

A typical electron orbit during a period of $20/\gamma_0$ is shown in Fig. 1. ETG streamers only exert a small perturbation to the electron free streaming motion, so the electron does not rotate around streamers. Streamers are simply amplitude contours of electrostatic potential (or density), or equivalently, $\mathbf{E} \times \mathbf{B}$ velocity fields. Obviously, particles execute not only this $\mathbf{E} \times \mathbf{B}$ motion, but also free streaming motion along the magnetic field line in collisionless plasmas. In fact, the radial excursion averaged over all electrons is diffusive, and is roughly $80\rho_e$ over a period of $20/\gamma_0$, during which streamers should have completed a full rotation as estimated by the eddy turn over time. Again, this supports the thesis that transport are due to the overlapping of phase-space islands of resonant electrons, and further invalidates the transport scaling obtained from the mixing length estimate, which assumes that particles rotate around turbulence eddies. The key here is that resonant electrons, which contribute to the transport, can decorrelate with streamers because of a nonlinear loss of the parallel resonant condition ($\omega - k_{\parallel}v_{\parallel} = 0$) due to radial variations of q(r).

We find that the streamer size scales with the device size. However, both fluctuation intensity and heat conductivity are approximately independent of the device size for $a/\rho_e = 1000 - 8000$. The electron transport level, $\chi_e \simeq 3.2cT_e\rho_e/eBL_T$, is well below the typical experimental values, and is about an order of magnitude smaller than the flux-tube result [8]. Additional effects, e.g., space charge effects, externally driven fbws [12], and couplings to ITG/TEM turbulence [13], should further reduce the electron transport, but would not affect qualitatively the key physics discussed in this paper, i.e., nonlinear toroidal couplings and nonlinear waveparticle interactions.

Nonlinear Toroidal Couplings – Nonlinear interactions of toroidal ETG eigenmodes take three forms: a nonlinear mode coupling between two toroidal *n*-modes, a modification of the parallel mode structure determined by the radial width of poloidal *m*-harmonics, and a modulation of the radial envelope represented by a ballooning angle θ_0 . Although all interactions are formally of the same order, we find that dominant saturation mechanism is the two *n*-modes coupling. These linear streamers can interact nonlinearly because of the unique ballooning mode structure, i.e., radial localization of linearly coupled *m* harmonics near mode rational surfaces. This nonlinear toroidal coupling is strictly geometry-specific since two parallel streamers in a slab geometry cannot interact.

In a controlled simulation with $a = 1000\rho_e$, two pump eigenmodes are allowed to grow first with $n_0 = 110$ and $n_1 = 95$, which correspond to $k_{\theta}\rho_e = 0.31$ and 0.27, respectively, at r = 0.5a. When their amplitudes are much higher than any other mode, all eigenmodes are allowed to grow. Each m_0 harmonic of the n_0 mode interacts most strongly with one m_1 harmonic of the n_1 mode, where m_0 and m_1 are the harmonics whose mode rational surfaces sit close to each other. The coupling proceeds as:

$$(n_0, m_0) + (n_1, m_1) \Rightarrow (n_0 \pm n_1, m_0 \pm m_1).$$

This coupling transfers energy to a low-*n* quasi-mode, $n_l = 15$ more efficiently than to a very high-*n* mode, $n_h = 205$. The n_l mode is a forced oscillation since its intrinsic frequency is much smaller than the frequency difference between n_0 and n_1 . Each m_l harmonic of the n_l quasimode is localized near its own mode rational surface with a very long parallel wavelength. The radial coherent length of m_l harmonics is similar to the distance between pump mode rational surfaces, which is the radial width of interactions between m_0 and m_1 harmonics. Therefore, the n_l quasi-mode does not possess the ballooning mode structure.

The n_l quasi-mode now couples back to two pump modes and generates secondary modes $n_2 =$ 80 and 125 (upper panel of Fig. 2, before saturation). In turn, each secondary n_2 mode couples with the far-side pump mode to generate another quasi-mode $n_l = 30$. These successive couplings proceed until all n-modes that satisfy the *n*-matching condition are populated with either a quasi-mode or a secondary mode (middle panel of Fig. 2, after saturation). The amplitudes of longer wavelength n_2 modes, $80, 65, \cdots$, are much higher than those of shorter wavelength n_2 modes, $125, 140, \cdots$, indicating that the energy cascades preferably to lower-*n* secondary modes. The n_l quasi-modes do not contain much energy, or drive much transport. Rather, they act as mediators for the energy transfer from pump modes to secondary modes with longer wavelengths. Therefore, the nonlinear toroidal coupling is similar to the Compton Scattering [14] with the quasi-modes playing the role of quasi-particles. In short, we ² find that the ETG instability saturates via nonlinear toroidal couplings, rather than a Kelvin-Helmholtz secondary instability suggested by flux-tube simulations [8]. Parallel wavevector also increases through coupling to the (0,1) mode, which is a weaker nonlinear interaction due to Landau damping of the (0, 1) mode. The generation of the zonal fbw is the weakest nonlinear interaction because



the amplitude of the sidebands with $\theta_0 \neq 0$ is much smaller than the pump modes. Steady state is achieved both via energy transfer to damped modes and via enhanced Landau damping due to modification by the (0, 1) mode of the parallel structure of linearly unstable modes.

Nonlinear Gyrokinetic Theory – Nonlinear coupling requires spatial overlap between poloidal harmonics of n_0 and n_1 pump modes. This can be expressed as a *selection rule* via the ballooning-mode representation, i.e., $\delta\phi_{0,1}(\vec{r},t) = e^{-i(n_{0,1}\zeta - m_{s0,1}\theta)}A_{0,1}(t)\sum_j e^{ij\theta}\Phi(z_{0,1}-j)$, where $m_{s0,1} = n_{0,1}q(r_s)$, $z_{0,1} = (r - r_s)/\Delta_{0,1}$, $\Delta_{0,1} = 1/n_{0,1}q'(r_s)$, $\Phi(z)$ is the normalized linear eigenmode with $\int |\Phi|^2 dz = 1$, and r_s is a reference low-order mode rational surface with $q(r_s) = m_l/n_l$. The low-*n* quasi-mode is then written as $\delta\phi_l(\vec{r},t) = e^{-i(n_l\zeta - m_l\theta)}A_l(t)\Phi_{l0}(\vec{r})$, ignoring both radial variations on a long scale of $|1/n_lq'|$ and envelope modulations due to either equilibrium variations or zonal fbws.

Assuming an adiabatic ion response and a fluid limit for the nonlinear electron response in

the gyrokinetic equations [15], we have a Hasegawa-Mima-like mode coupling equation,

$$\frac{\partial}{\partial t}L_k\delta\phi_k = \alpha_e \frac{c}{2B}\rho_e^2(\vec{k}_\perp^{\prime\prime} \times \vec{k}_\perp) \cdot \vec{e}_{\parallel}(k_\perp^{\prime\prime\,2} - k_\perp^{\prime\,2})\delta\phi_{k'}\delta\phi_{k''}$$

with $\vec{k} = \vec{k}' + \vec{k}''$, $\alpha_e \equiv [\delta P_{\perp,ek}/(eN_{0e}\delta\phi_k) - 1] = \{\tau(1+\eta_e)/[(3\tau-1)L_n/R + 1/2] + 1\}$, and L_k the linear eigenmode operator, i.e., $L_0\Phi(z_0) = L_1\Phi(z_1) = 0$. \vec{k}' and \vec{k}'' should be strictly interpreted as operators, i.e., $i\vec{k}'\delta\phi_{k'} = \vec{\nabla}\delta\phi_{k'}$.

The nonlinear evolution equation for the low-*n* quasi-mode, using $k = k_{n_l}$, $k'_{\perp} = (k_{n_0})_{\perp}$ and $k''_{\perp} = -(k_{n_1})_{\perp}$, is

$$\frac{\partial}{\partial t}L_l a_l(t)\Phi_{l0}(r) = ia_0 a_1^* \hat{\alpha}_e(\frac{k_{\theta 0}}{\Delta_0}) \frac{\partial}{\partial z_0} [\delta k_{\perp}^2]_0 \Psi_0$$

with $a_k = eA_k/T_e$, $\hat{\alpha}_e = \alpha_e |\Omega_e| \rho_e^4$, $\Psi_0 = \sum_j |\Phi_{0j}|^2$, $\Phi_{0j} = \Phi(z_0 - j)$, and $[\delta k_{\perp}^2]_0 \Psi_0 = \frac{2n_l}{n_0} k_{\theta 0}^2 (1 + \hat{s}^2/W_0^2) \Psi_0 \equiv \frac{2n_l}{n_0} k_{\theta 0}^2 \Psi_0 + {q'_s}^2 \sum_j (n_1^2 \Phi_{0j} \frac{\partial^2}{\partial z_0^2} \Phi_{0j}^* - n_0^2 \Phi_{0j}^* \frac{\partial^2}{\partial z_0^2} \Phi_{0j})$. Here, we have considered $k_{\theta 1} \simeq k_{\theta 0}$ and $k_{\theta l} \equiv k_{\theta 0} - k_{\theta 1} = k_{\theta 0} (n_l/n_0)$. Furthermore, $\hat{s} = r_s q'_s/q_s$, and $W_0 \sim O(1)$ denotes the typical width of $\Phi(z_0)$. Noting $L_l \simeq \tau$, and defining $\Phi_{l0}(r) = i\partial_{z_0}(1 + \hat{s}^2/W_0^2)\Psi_0$, we have

$$\frac{\partial a_l(t)}{\partial t} = \frac{\hat{\alpha}_e}{\tau} \frac{2n_l}{n_0} \frac{k_{\theta 0}^3}{\Delta_0} a_0 a_1^*. \tag{1}$$

The feedback equation for the pump mode $a_0(t)$, using $k = k_{n_0}$, $k' = k_{n_1}$, and $k'' = k_{n_l}$, is

$$(\partial_t - \gamma_0)a_0(t) = -(\hat{\alpha}_e/\tau)a_1a_l(k_{\theta 1}/\Delta_1)(k_{\perp 1}^2/W_l^2)$$
(2)

with $(k_{\perp 1}^2/W_l^2) \equiv -\int dz_0 \Phi^*[k_{\perp 1}^2] \Phi \frac{\partial^2}{\partial z_0^2} (1 + \frac{\hat{s}^2}{W_0^2}) \Psi_0$, $[k_{\perp 1}^2] f(z_0) g(z_0) \equiv -k_{\theta 1}^2 [g(1 - \hat{s}^2 \partial_{z_0}^2)f + \hat{s}^2 f \partial_{z_0}^2 g]$, and γ_0 being the linear growth rate. W_l represents the typical radial scale of Φ_{l0} or $\delta \phi_l$. For symmetry reasons, the evolution equation of another pump mode $a_1(t)$ is:

$$(\partial_t - \gamma_1)a_1(t) = (\hat{\alpha}_e/\tau)a_0a_l^*(k_{\theta 0}/\Delta_0)(k_{\perp 0}^2/W_l^2).$$
(3)

We extend the results to the case of multi-*n* pump modes interacting with a single n_l quasimode. Assuming $n \gg n_l$, we take a continuum limit and obtain a spectral-cascading equation for the pump wave energy density $I_n = |a_n|^2/2$,

$$\left(\frac{\partial}{\partial t} - 2\gamma_n\right)I_n + v_n\frac{\partial}{\partial_n}I_n = 0,\tag{4}$$

where $v_n(t) = -[(2\hat{\alpha}_e/\tau)\hat{s}k_{\theta n}^2(k_{\perp n}^2/W_l^2)n_l]|a_l(t)|$, and,

$$\left(\frac{\partial}{\partial t} + \gamma_l\right)|a_l(t)| = 4(\hat{\alpha}_e/\tau)q'_s \int k^3_{\theta n} I_n dn \tag{5}$$

with γ_l the Landau damping rate of the low-*n* quasi-mode.

Equation 4 indicates cascades toward lower *n* secondary modes if $v_n < 0$. Meanwhile, $sgn(v_n) = -sgn(k_{\perp n}^2)$, and, approximately, $k_{\perp n}^2 \approx k_{\theta n}^2 [\hat{s}^2/W_l^2 - (1 + \hat{s}^2/W_n^2))]$. Noting $W_n \leq 1$ and $\hat{s} \sim 1$, we have $\Phi_{l0}(r) \sim |\Phi_n(z)|^2$, and hence, $W_l \sim W_n/2$ and $k_{\perp n}^2 > 0$, *i.e.*, $v_n < 0$. So wave energy cascades from high-*n* pump to lower-*n* secondary modes. We emphasize that rapid radial variations of the low-*n* quasi-mode are crucial in determining, not only the direction of energy cascade, but also the cascading rate, i.e., $|v_n| \sim W_l^{-4}$.

2 **Trapped Electron Mode Turbulence**

We further find that the ETG transport level is much smaller than that driven by the trapped electron mode (TEM) turbulence for the same plasma parameters. Kinetic electrons have been implemented in GTC code using the fluid-kinetic hybrid electron model. The linear frequencies and grow rates of electrostatic TEM/ITG modes from GTC simulations are found to be in good agreement with a comprehensive linear eigenvalue code (FULL) of PPPL and a global gyrokinetic particle-in-cell code (GT3D) developed in JAERI of Japan. Global nonlinear simulations of trapped electron modes have been carried out with contribution of kinetic electrons to zonal fbws properly retained. The TEM driven electron thermal conductivity is found to be at a level of experimental relevance. The nonlinear electron dynamics in the TEM turbulence is constrained by the conservation of the second invariant, which implies that energy is not conserved. The consequences of simultaneous diffusions of electron banana orbits in both energy and real space, which have not been studied in theories or flux-tube simulations, remained to be further explored. Zonal flows with short radial wavelength are found to be generated in the TEM turbulence, and the electron contribution to the zonal fbw generation is found to be larger than the ion contribution.

We have estimated [16] the role of the trapped electron nonlinearity in zonal fbw generation quantitatively in the context of a modulational instability theory in toroidal geometry[6]. We follow the usual weak turbulence expansion for fluctuations with a single non-zero toroidal mode number n involving the pump TEM ϕ_0 , the side band TEM's ϕ_+ and ϕ_- , and the zonal flow mode ϕ_Z . The Hasegawa-Mima type nonlinear coupling of ϕ_0 and $\phi_{+,-}$ is balanced by the neoclassically enhanced polarization shielding of the zonal flow potential ϕ_Z as described in Eq. (3) of the Ref.[6]. Because the trapped electron banana width is much smaller than the trapped ion banana width, and $\nu_{ee}\rho_e^2/(\nu_{ii}\rho_i^2) \simeq \sqrt{(m_e/M_i)}$, the shielding of ϕ_Z on the left hand side of Eq. (3) is not changed; therefore the presence of trapped electrons can modify the balance of the nonlinear polarization current and the neoclassical polar-

linear susceptibility α_i of the pump TEM in that



ization current only through modification of the Figure 3: A trapped electron orbit scattered by TEM turbulence.

equation. On the other hand, the nonlinear excitation of the linearly damped side bands $\phi_{+,-}$ through $\mathbf{E} \times \mathbf{B}$ nonlinear coupling between ϕ_0 and ϕ_Z is described by the quasi-neutrality condition for the density responses obtained from the nonlinear ion gyrokinetic equation (Eq. (4) of Ref. [6]) and the trapped electron nonlinear bounce kinetic equation. We find that the trapped electrons merely reduce the $\mathbf{E} \times \mathbf{B}$ nonlinearity (which has no explicit mass dependence), which produces side band fluctuations via the zonal flow modulation, by a factor of $\sqrt{8\epsilon/\pi}$, i.e., the surface averaged fraction of trapped electron population (The right hand side of Eq. (4) in Ref.[6] should be multiplied by $(1 - \sqrt{8\epsilon/\pi})$. Therefore, most of the zonal flow growth rate change due to trapped electrons may occur via changes in linear properties such as the side band damping rate and the linear susceptibility (dispersion relation) of the pump TEM.

3 A Finite Element Solver for Global Electromagnetic Simulation

Conventionally GTC uses an iterative Poisson solver [17] which is efficient for the adiabatic electron response. With the inclusion of the non-adiabatic electron response (using either the split-weight schemes [18] or the hybrid model [19] for finite- β plasmas), the resulting gyrokinetic Poisson's equation requires a new algorithm. In the new solver, we use Padé approximation to cast the gyrokinetic Poisson equation in a differential form to properly treat the response of the short wave length mode. Normalizing with ρ_s , n_0 , and $e\Phi/T_e$ for the potential, we obtain $(\tau = T_e/T_i)$

$$\nabla_{\perp}^{2} \Phi = -\left(1 - \frac{1}{\tau} \nabla_{\perp}^{2}\right) \left(\bar{\delta n_{i}} - \delta n_{e}\right),\tag{6}$$

which is in a differential form compared to the integral form of the original solver.

The global code GTC has unique grid structures due to the global field aligned mesh we employed. The main task in FEM is the bookkeeping in relating the labels of the vertices and the labels of the triangle elements. Then the rest is reduced to solving a sparse matrix equation $A \cdot \Phi = b$. We employ **V** the the state of the art PETSc code from Argonne National Laboratory [20]. The timing of PETSc is quite promising. The CPU time versus the number of grids scales almost linear and MPI (in PETSc) speeds up the computation proportional to the number of processors. One approach to further speed up the elliptic solver is to employ the algebraic multigrid (AMG) method, which is a multilevel method where geometry in-



Figure 4: An illustration of GTC grid for fi nite element solver.

formation is not required (as compared to the geometrical multigrid method). Conveniently enough, the PETSc code is interfaced with *hypre* (high performance preconditioners) [21] and *Prometheus* [22] which employ AMG as a preconditioner. We plan to employ AMG to further accelerate the elliptic solve. Taking the ITG mode as an example, the solution from the new fi nite element solver has been successfully benchmarked with the solution from the original GTC's iterative solver.

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