Intermittent Transport and Relaxation Oscillations of Nonlinear Reduced Models for Fusion Plasmas

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Abstract. Generation of sheared flows and their effects on turbulent transport are studied for ion temperature gradient (ITG) driven instability and resistive drift instability. With the use of low degree-of-freedom models as well as the full partial differential equation (PDE) models, the minimum mode structures have been identified that are required for the generation of intermittent transport and relaxation oscillations. Generation of turbulence due to magnetohydrodynamic (MHD) instabilities and their roles in the control of stellarator and tokamak plasmas are also discussed.

1. Introduction

Relaxation oscillations and resulting intermittent transport are widely observed in fusion plasmas. In this work we have studied nonlinear evolutions of drift-wave and magnetohydrodynamic (MHD) instabilities, using various nonlinear reduced models describing them. For example, we have constructed low degree-of-freedom models for nonlinear ion temperature gradient (ITG) modes and examined how "low" the degrees of freedom can be in order for the model to exhibit relaxation oscillations that are at least qualitatively similar to those observed in actual fusion plasmas. Characteristics of anomalous transport obtained from the low degree-of-freedom models are also compared with those obtained from the full-mode model equations. Similarly, we have also examined nonlinear evolution of other nonlinear instabilities such as resistive drift modes, tearing modes, and resistive interchange modes, using the corresponding reduced equations. The goals of the present study are to identify common features of such nonlinear evolutions of various instabilities and also to understand the roles that these instabilities play for the control of fusion plasmas.

2. ITG driven instabilities

First let us discuss anomalous transport of ITG modes.[1–4] ITG modes have two branches, i.e., slab[5–8] and toroidal modes.[9–12] These modes becomes unstable when the ratio of the ion

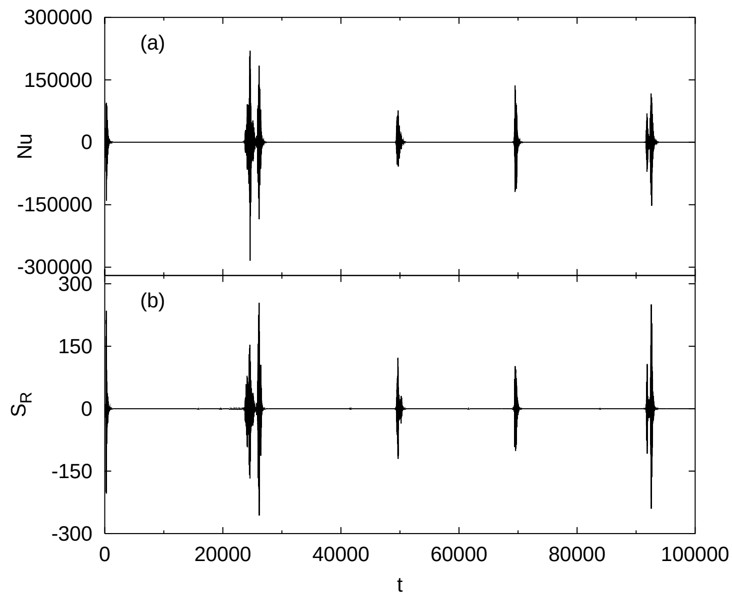


Figure 1: Time evolutions of Nusselt number Nu (a) and Reynolds stress S_R (b) for the 18 ODE model ($L = 3$ and $M = 1$)for ITG modes[2] with $K_i = 4$.

temperature gradient to density gradient, η_i, becomes sufficiently large. The toroidal ITG mode is destabilized by η_i and ∇B-curvature drift. The driving mechanism of this mode is similar to that of interchange instability. The toroidal ITG mode is localized in the outer region of the torus and generally has a higher growth rate than the slab mode.

The nonlinear PDE model of toroidal ITG driven instabilities is given by the following vorticity and ion pressure equations[9]:

$$\frac{\partial}{\partial t}(\nabla_\perp^2\phi - \phi) + [\phi, \nabla_\perp^2\phi] = (1 - g + K_i\nabla_\perp^2)\frac{\partial\phi}{\partial y} - g\frac{\partial p}{\partial y} + \mu\nabla_\perp^2\nabla_\perp^2\phi\ , \tag{1}$$

$$\frac{\partial p}{\partial t} + [\phi, p] = -K_i\frac{\partial\phi}{\partial y} + \kappa\nabla_\perp^2 p\ , \tag{2}$$

where

$$[a, b] \equiv \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x} \tag{3}$$

is the Poisson bracket. Here a constant and uniform (and therefore shearless) magnetic field is assumed to be present in the z direction and the ion pressure gradient is in the x direction. Note that for this mode the perpendicular dynamics and parallel dynamics are decoupled, so it is sufficient to consider a two-dimensional (2D) slab for the underlying geometry.

In the above equations, $g = 2L_n/R$ is the effective gravity due to curvature of magnetic field, $K_i = T_i/T_e(\eta_i + 1)$ is the equilibrium ion pressure gradient with $\eta_i = \mathrm{d}\ln T_i/\mathrm{d}\ln n$, μ is the viscosity and κ is the thermal conductivity. Here x corresponds to the radial direction and y corresponds to the poloidal direction. The standard drift wave units $x \equiv x/\rho_s$, $y \equiv y/\rho_s$, $t \equiv (L_n/c_s)t$, $p \equiv L_nT_{i0}/(\rho_sP_{i0}T_{e0})p$, $\phi \equiv eL_n/(B_0T_{e0}\rho_s)\phi$, $\mu \equiv eL_n/(B_0T_{e0}\rho_s)\mu$, and $\kappa \equiv eL_n/(B_0T_{e0}\rho_s)\kappa$ are used for normalization, where c_s is the ion sound velocity, $\rho_s = c_s/\Omega_i$, and Ω_i is the ion cyclotron frequency.

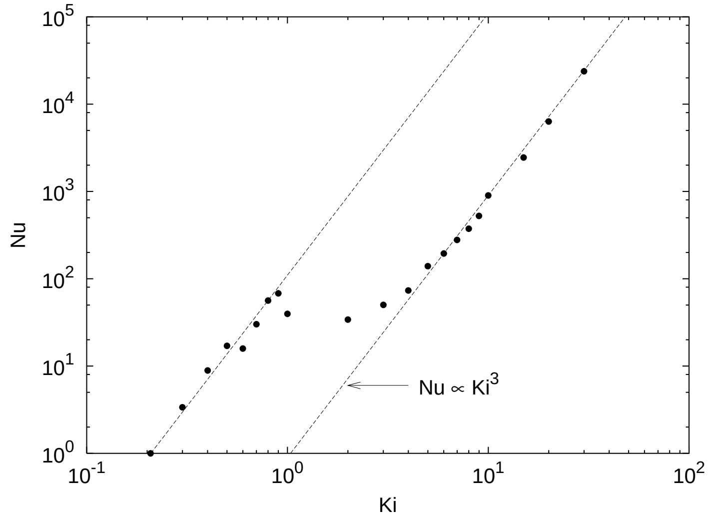


Figure 2: Time averaged Nusselt number Nu as a function of the normalized ion pressure gradient K_i obtained from the 18 ODE model for ITG modes.[2]

By linearizing Eqs. (1) and (2), we obtain the following dispersion relation for the toroidal ITG mode, i.e.,

$$\left[-\mathrm{i}(1+k_\perp^2)\omega + \mathrm{i}k_y(1-g-K_ik_\perp^2) + \mu k_\perp^2\right]\left(-\mathrm{i}\omega + \kappa k_\perp^2\right) - gK_ik_y^2 = 0\ . \tag{4}$$

The dispersion relation (4) gives the linear growth rate as

$$\omega = \frac{1}{2(1+k_\perp^2)}\left\{k_y(1-g-K_ik_\perp^2) - \mathrm{i}\left[\kappa k_\perp^2(1+k_\perp^2) + \mu k_\perp^4\right] \pm \sqrt{D_k}\right\}, \tag{5}$$

where

$$D_k = \left\{k_y(1-g-K_ik_\perp^2) + \mathrm{i}\left[\kappa k_\perp^2(1+k_\perp^2) - \mu k_\perp^4\right]\right\}^2 - 4(1+k_\perp^2)gK_ik_y^2. \tag{6}$$

From Eq. (5), the most unstable mode occurs when

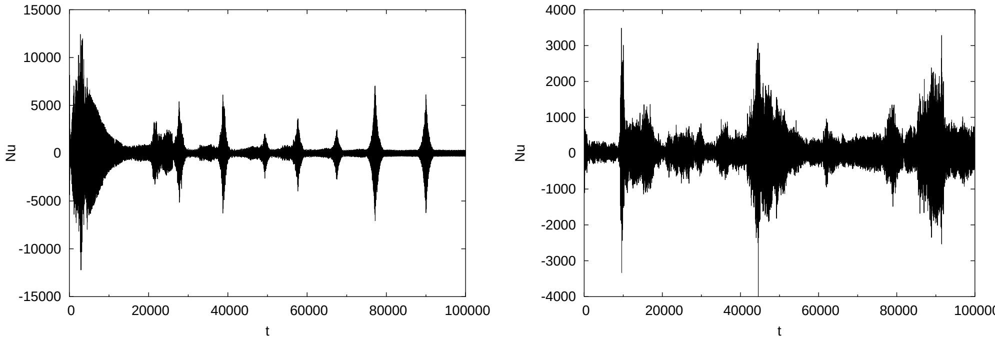
$$k_x^2 \ll k_y^2 \sim k_\perp^2 = \frac{1-g}{K_i}. \tag{7}$$

Based on this observation, we select the fundamental mode of our numerical simulations to be the mode satisfying

$$k_x = k_y/2 \tag{8}$$

with $k_x = [(1-g)/5K_i]^{1/2}$. Note that this mode is close to the most unstable mode.

For numerical simulations of the model above, we typically employ the following parameters unless otherwise indicated; $g = 0.05$, $\mu = 0.04$, and $\kappa = 0.01$. The equations are solved in a rectangular region $0 \leq x \leq L_x$ and $0 \leq y \leq L_y$ with perfectly conducting walls being located at $x = 0$ and $x = L_x$. Periodic boundary conditions are imposed in the y direction and $\phi(x = 0, L_x) = 0$ and $p(x = 0, L_x) = 0$ are assumed at the walls. For these parameters, the stability threshold for parameter K_i is given by $K_{ic} \simeq 0.21$. We typically vary the parameter K_i from K_{ic} to 100.



(a) (b)

Figure 3: Time evolutions of Nusselt numbers Nu for the one dimensional model ($L = 64$ and $M = 1$)(a) and the full PDE model ($L = 32$ and $M = 15$) with $K_i = 6.0$ (b).

In order to construct low-degree-of-freedom models, variables $\phi(t, x, y)$ and $p(t, x, y)$ are Fourier expanded as

$$\phi(t,x,y) = \sum_{\ell=1}^{L}\sum_{m=0}^{M}\Big[\hat{\phi}^c_{\ell,m}(t)\sin(\ell k_x x)\cos(m k_y y) + \hat{\phi}^s_{\ell,m}(t)\sin(\ell k_x x)\sin(m k_y y)\Big], \tag{9}$$

$$p(t,x,y) = \sum_{\ell=1}^{L}\sum_{m=0}^{M}\Big[\hat{p}^c_{\ell,m}(t)\sin(\ell k_x x)\cos(m k_y y) + \hat{p}^s_{\ell,m}(t)\sin(\ell k_x x)\sin(m k_y y)\Big], \tag{10}$$

where L and M are the maximum mode numbers in the x and y directions, respectively. By substituting Eqs. (9) and (10) into Eqs. (1) and (2), we obtain coupled ordinary differential equations (ODEs) for the harmonics.

The 11 ODE model proposed by Hu and Horton[12] corresponds to the case of $L = 2$ and $M = 1$. Hu and Horton have demonstrated that the 11 ODE model reproduce an L-H like transition and oscillations of anomalous heat transport when the system is sufficiently unstable. However it has been also shown that sheared flows generated by turbulence in the 11 ODE model are not strong enough to suppress the instabilities. Base on such observations, Takeda *et al.*[1,2] included more modes in the x direction for the same set of equations and increased the degrees of freedom from 11 to 18 (i.e., $L = 3$ and $M = 1$) to analyze the anomalous transport. They have found that the 18 ODE model generates relaxation oscillations and intermittent bursts when the system is highly unstable. This indicates that, under the conditions that Takeda *et al.* employed, at least four modes (i.e., $0 \leq \ell \leq 3$) in the x direction are needed to cause relaxation oscillations of ITG driven instabilities.

Anomalous thermal transport may be represented by the Nusselt number, which is the ratio of the total heat transport including convective (i.e., anomalous) transport to the conductive heat transport defined as

$$Nu = 1 + \langle p v_x \rangle_V \Big/ (\kappa \frac{\mathrm{d}P_0}{\mathrm{d}x}) = 1 + \frac{\langle p v_x \rangle_V}{\kappa K_i}, \tag{11}$$

where $\langle \ \rangle_V$ denotes the spatial average.

Plasma flows generated by turbulence alter the characteristics of anomalous transport since they tend to reduce the growth rates of or even completely suppress instabilities. Flows are typically generated through the Reynolds stress S_R, which is defined as

$$S_R = \langle v_x v_y \rangle_V \, . \tag{12}$$

Figure 1 shows time evolutions of the Nusselt number Nu and Reynolds stress S_R, obtained from numerical simulation of the 18 ODE model for ITG turbulence.[2] The normalized ion pressure gradient used for the simulation is $K_i = 4.0$. Figure 2 shows the time averaged Nusselt number Nu as a function of K_i obtained from the 18 ODE model.[2] The system exhibits relaxation oscillations when K_i is approximately larger than unity. The scaling $Nu \propto K_i^3$ (when $K_i \gg K_{ic}$) is clearly seen although its proportional coefficient is reduced for $K_i \gtrsim 1$, where intermittent bursts of transport (i.e., relaxation oscillations) are observed.

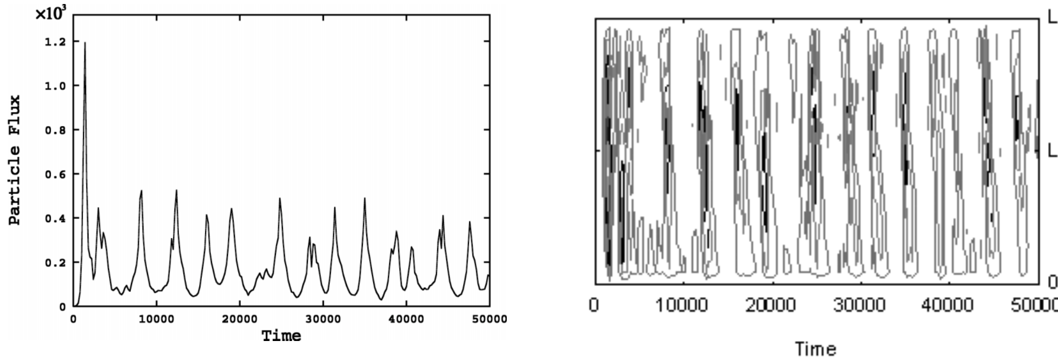
When the system is slightly above the stability threshold, i.e., $K_i \gtrsim K_{ic}$, ITG driven convections reach a steady state. As we increase K_i slightly more, periodic oscillations in the kinetic energy and turbulent (i.e., convective) heat transport are observed. If K_i is further increased, the system bifurcates to a turbulent state. When the turbulence is sufficiently strong, intermittent bursts appear. We have found that the intermittency we observed in the 18 ODE model is repetition of the following 3 steps; (1) generation of sheared flows by ITG driven turbulence and resulting suppression of the turbulence, (2) gradual reduction of the sheared flows due to viscosity, and (3) rapid re-growth of ITG modes and turbulence due to the reduction of the sheared flows.

We have also examined the case of $L = \infty$ and $M = 1$ (which we call one-dimensional model) as well as the full PDE model (i.e., $L = M = \infty$) in Eqs. 9 and 10.[3,4] In practice, it is sufficient to use $L = 64$ and $M = 1$ for the one-dimensional model and $L = 32$ and $M = 15$ for the full PDE model in our numerical simulations under the conditions considered here. Figure 3 compares time evolutions of Nusselt numbers Nu for the one-dimensional model(a) and the full PDE model with $K_i = 6.0$ (b).

3. Resistive drift instability

Another nonlinear instability we examined is the resistive drift instability, which is generally considered to account for particle losses in the edge plasma region.[13] For this problem, we solved time evolution of Hasegawa-Wakatani equations,[13] given below, in the three dimensional space.

$$\left(\frac{\partial}{\partial t} - \nabla_\perp \phi \times \hat{\mathbf{z}} \cdot \nabla_\perp \right) \omega = -\frac{1}{\eta} \frac{\partial^2}{\partial z^2} (\phi - n) + \mu \nabla_\perp^2 \omega \, , \tag{13}$$



(a)

(b)

Figure 4: (a) Time evolution of the convective density flux normalized by the diffusive density flux, i.e., $\bar{\Gamma}_n(t)/Dn_0''$, due to resistive drift turbulence.[14] Time is normalized by ion cyclotron frequency. $L_n = 15$. (b)Temporal-special contours of the convective density flux normalized by the diffusive density flux, i.e., $\Gamma_n(x,t)/Dn_0''$. The vertical axis represent the x coordinate. Positive $\Gamma_n(x,t)$ indicates that the mass flow is in the direction of x. Note that the equilibrium mass density is highest at $x = 0$.

$$\left(\frac{\partial}{\partial t} - \nabla_\perp \phi \times \hat{\mathbf{z}} \cdot \nabla_\perp\right)(n + \ln n_0) = -\frac{1}{\eta}\frac{\partial^2}{\partial z^2}(\phi - n) + D\nabla_\perp^2 n\,, \tag{14}$$

$$\omega = \nabla_\perp^2 \phi \tag{15}$$

$$\Gamma_{\rm n}(x,t) = \frac{1}{L_y L_z}\iint n v_x \, dy dz$$

$$\bar{\Gamma}_n(t) = \frac{1}{L_x}\int \Gamma_n(x)\, dx$$

In the presence of a sufficiently large density gradient, we have observed relaxation oscillations, which result in bursty density fluxes,[14] as shown in Fig. 4. In this figure, the density (i.e., particle) flux due to plasma convection is normalized by the collisional diffusion flux. As in the case of ITG driven turbulence, resistive drift turbulence generates sheared flows, which suppress the fluctuations and are slowly weakened by viscosity.

4. Summary

We have examined relaxation oscillations and associated intermittent anomalous transport arising from ITG driven turbulence and resistive drift turbulence. As to ITG turbulence, in the small K_i region of $K_i \leq 3$, a bifurcation process similar to that observed in the 11 ODE model also appears in the 18 ODE model. The noteworthy difference between the 11 and 18 ODEs is that, only in the case of 18 (or larger) ODEs, intermittent bursts (so called avalanches) are observed when K_i is large. In other words, we have found that there is a minimum number of modes in the radial direction (i.e., x direction) that would be required to form sufficiently large sheared flows that can suppress the instabilities.

In strongly turbulent states, intermittent bursts appear through the following processes: (1) The drift instabilities generate plasma turbulence, which induces anomalous (i.e., turbulent) transport. (2) Sheared flows are generated via Reynolds stress of plasma turbulence, which stabilize the turbulence. (3) The flows are gradually reduced by viscous damping and eventually become unable to keep the turbulence level sufficient low, which results in the re-growth of instabilities and the system returns to the process (1).

For practical fusion applications, controlling nonlinear evolution of instabilities and relaxation oscillations by some external means is of great interest. Recent three dimensional numerical simulations based on reduced magnetohydrodynamics (RMHD) equations by Unemura *et al.* has successfully demonstrated that the growth of magnetic islands in a stellarator plasma can indeed be controlled by an external perturbational magnetic field that generates vacuum magnetic islands.[15] Recent numerical simulations using multiple kink-tearing modes by Bierwage *et al.* also indicate that current-driven electromagnetic turbulence forming in an annular region around the $q = 1$ surfaces (with q being the safety factor) affects the dynamics of sawtooth collapse.[16]

References

[1] K. Takeda, S. Hamaguchi, and M. Wakatani, Plasma Phys. Control. Fusion **44**, A487 (2002).

[2] K. Takeda, S. Benkadda, S. Hamaguchi, and M. Wakatani, Phys. Plasmas **11**, 3561 (2004).

[3] K. Takeda, S. Benkadda, S. Hamaguchi, and M. Wakatani, J. Plasma Fusion Res. *to appear* (2004)

[4] K. Takeda, S. Benkadda, S. Hamaguchi, and M. Wakatani, *in preparation* (2004).

[5] W. Horton and R. D. Estes and C. Biskamp, Plasma Phys., **22**, 663 (1980).

[6] S. Hamaguchi and W. Horton, Phys. Fluids B**2**, 1834 (1990).

[7] S. Hamaguchi and W. Horton, Phys. Fluids B**2**, 3040 (1990).

[8] S. Hamaguchi and W. Horton, Phys. Fluids **B4**, 319 (1992).

[9] W. Horton, D.-I. Choi, and W. Tang, Phys. Fluids **24**, 1077 (1981).

[10] A. Jarmen and P. Anderson and J. Weiland, Nuclear Fusion **27**, 941 (1987).

[11] H. Mordman and J. Weiland, Nuclear Fusion **27** 941 (1987).

[12] G. Hu and W. Horton, Phys. Plasmas **3**, 3262 (1997).

[13] A. Hasegawa and M. Wakatani, Phys. Rev. Lett. **50**, 682 (1983).

[14] H. Sato, *Master Thesis, Graduate School of Energy Science, Kyoto University* (2004).

[15] T. Unemura, S. Hamaguchi, and M. Wakatani, Phys. Plasmas **11** 1545 (2004).

[16] A. Bierwage, S. Hamaguchi, M. Wakatani, X. Leoncini, and S. Benkadda, *in preparation* (2004).