A Possible Mechanism for the Seed Island Formation

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Abstract. The evolution of neoclassical tearing modes (NTMs) is usually described by the generalized Rutherford equation allowing the mode growth from a finite level, which is referred to as seed island. It is generally accepted that the seed island is induced by some MHD event, but sometimes the NTMs start without visible triggers. Here we discuss a possible role of the error fields in producing the seeds. The analysis is based on Maxwell equations and Ohm's law for magnetic perturbation outside the plasma. The plasma enters the problem via boundary conditions. Its contribution is described by the decay/growth rate and the toroidal rotation frequency of perturbation. The model also assumes a resonant harmonic in the spectrum of the error field. It is shown that the resonant field amplification near the stability boundary of the mode may be a mechanism resulting in the "spontaneous" formation of the seed island. In contrast to NTM seeding due to the sawteeth, fishbones, or ELMs, the considered mechanism needs some longer time. However, all the estimates give realistic values consistent with typical experimental conditions.

1. Introduction

The theory of neoclassical tearing modes (NTM), supported by numerous experimental evidences, requires existence of large enough seed island for the NTM onset [1-6]. The seed island must be induced by some MHD event. Usually the sawteeth, fishbones, or edge localized modes (ELMs) are considered as possible candidates, however, the exact physical mechanism of the seed island formation has not been resolved experimentally and still remains unclear [1-6].

It is true that in many cases there is clear experimental correlation between the sawtooth crash and the NTM onset [1-4]. On the other hand, it is not well understood why a particular sawtooth crash seeds a NTM after several preceding sawteeth did not [6]. Also, the concept of forced reconnection leading to tearing mode onset from a sawtooth crash ($m=1$, $n=1$) cannot explain a formation of a seed island of different helicity ($m=3$, $n=2$) far from the $q=1$ surface [5, 7]. In the case of ELMs, the difficulty in interpretation comes from the fact that ELMs amplitudes are too weak at the rational surfaces in the core of the plasma [7].

Finally, there are experiments where NTMs start without any detectable trigger [6-14]. That happened rarely in ASDEX Upgrade [9, 10]. In JT 60U and T-10 tokamaks, however, such cases were rather frequent [11, 12]. Also, in DIII-D, the tearing modes often appear suddenly and grow quickly without an obvious ideal mode causing a seed island [7]. In JET, at low field (1.7 T) NTMs are nearly always present without preceding sawteeth or fishbones [13]. This even led to the conclusion that seed events are not always required for NTM onset [13]. Maybe, it would be better to say instead that no *typical* seed events are observed before the NTM onset in those experiments. Anyway, one can consider the statement [13] as a confident confirmation of the fact that spontaneous (without visible triggers) start of NTMs is a reality for some tokamak regimes.

The problem was analyzed theoretically, and possible explanations were proposed in [5-7]. In particular, it was shown that the tearing mode may be "classically" destabilized as the ideal kink stability limit is approached [7], because of the strong increase in Δ' near this limit [6, 7].

The proximity to a stability limit also greatly increases the plasma sensitivity to the error field. Theoretically, near the stability threshold of weakly rotating or nonrotating mode the plasma becomes an "amplifier" of the error field (see for details [15, 16]). This effect called 'resonant field amplification', RFA, has recently attracted attention as playing an important role in destabilization of the resistive wall modes (RWM) [13, 17]. The analysis [15, 16] shows that the RFA effect must develop near the stability boundary of any unstable mode, not necessarily "conventional" RWM. Therefore, amplification of the resonant error field should be considered as one of the mechanisms of the seed island formation.

It has been already shown in experiments in COMPASS-D [18] and JET [19] that the error field modes can lead to NTMs. In these cases, NTMs were excited when a resonant magnetic perturbation (RMP) was applied. Similar results have been recently obtained in TEXTOR [20]. The RMP is the field intentionally created by the saddle coils (sometimes called correction coils). The mode was excited when the amplitude of RMP exceeded some critical level [18-20]. Without RMP the plasma was stable, which means that the amplitude of the naturally existing error field was not sufficient for the NTM onset. It is shown below that the level of the resonant field necessary for the NTM seeding decreases when the stability boundary is approached. If so, at some conditions the intrinsic error field may be a reason of the "spontaneous" onset of NTMs.

2. Formulation of the problem

In experiments, the MHD activity of the plasma is monitored by measuring the perturbation of the magnetic field outside the plasma. This field $\mathbf{b} = \nabla\varphi$ can be found by solving $\nabla^2\varphi = 0$ in the vacuum with proper boundary conditions. Mathematically, only $\mathbf{n}\cdot\nabla\varphi$ at the plasma surface and the vessel wall is needed to find the solution ($\mathbf{n}$ is the unit normal). The boundary conditions at the wall are therefore a necessary part of the problem.

Formally, both surfaces, the plasma boundary and the wall, are equally important, which implies a possible strong effect of the wall on the solution. Indeed, the wall position affects the plasma stability. A stabilizing role of the ideal wall on the plasma stability was understood long ago. It was found later that the wall must be considered as a real conductor with finite conductivity [1]. This fact was emphasized in the name 'RWM' which was attributed to the kink modes. However, the boundary conditions at the wall are the same for any magnetic perturbation $\mathbf{b}$, irrespective of its origin. This means, in particular, that a correct description of tearing modes and NTMs must also include the same treatment of the wall as needed for RWMs.

Penetration of the magnetic perturbation $\mathbf{b}$ through the wall is described by the equation

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla^2 \frac{\mathbf{b}}{\mu_0 \sigma}, \tag{1}$$

which is a direct consequence of the Maxwell equations and Ohm's law for a conducting wall. Here σ is the wall conductivity, and $\mu_0 = 4\pi\times10^{-7}$ H/m is the permeability of vacuum. For a thin wall, integration of the normal projection of (1) over the wall with a weight function f yields

$$\tau_w \frac{\partial}{\partial t}\int f\mathbf{b}\cdot d\mathbf{S}_w = -\int\left[(\mathbf{b}_+ - \mathbf{b}_-)\times\mathbf{c}\right]\cdot d\mathbf{S}_w\,, \tag{2}$$

where both integrals are over the wall surface,

$$\tau_w = \mu_0 \sigma_0 r_w \delta_w \tag{3}$$

is the 'wall time', σ_0, r_w, and δ_w are, respectively, the averaged conductivity, minor radius, and thickness of the wall, $\mathbf{b}_+$ and $\mathbf{b}_-$ are the values of $\mathbf{b}$ at the outer and inner sides of the wall, and

$$\mathbf{c} \equiv r_w \frac{\sigma_0 \delta_w}{\sigma \delta} (\nabla f \times \mathbf{n}), \tag{4}$$

δ is the wall thickness, and $\mathbf{n}$ is the normal to the wall.

In the cylindrical approximation, assuming $f = \exp i(m\theta - n\zeta)$ (θ is the poloidal angle, and $\zeta = z/R$ is equivalent to the toroidal angle) we obtain from (2) the equation for the amplitude B_m of the (m,n) harmonic of the radial magnetic field at the wall [15, 16]:

$$\tau_w \frac{\partial B_m}{\partial t} = \Gamma_m B_m - \Gamma_m^0 B_m^{ext}. \tag{5}$$

Here

$$\Gamma_m = \Gamma_m^0 (1 - B_m^{pl} / B_m), \tag{6}$$

$$\Gamma_m^0 = -2\mu, \tag{7}$$

$\mu = |m|$, B_m^{pl} is the contribution to $B_m(t)$ from the plasma, B_m^{ext} is that part of B_m which is created by all the sources outside the wall (in the region $r > r_w$),

$$B_m = B_m^{pl} + B_m^w + B_m^{ext}, \tag{8}$$

and B_m^w is the field created by the currents in the wall.

The parameter Γ_m must be determined from the boundary conditions for $\mathbf{b}$ at the plasma surface as described in [16]. This implies knowledge of $\mathbf{b}$ inside the plasma. On the other hand, according to (5), Γ_m can be expressed through the natural (in the absence of an external magnetic perturbation, $B_m^{ext} = 0$) decay/growth rate γ_0 of the mode and the frequency Ω_0 of the mode toroidal rotation,

$$\Gamma_m = \tau_w (\gamma_0 + in\Omega_0). \tag{9}$$

Both γ_0 and Ω_0 can be found from magnetic measurements outside the plasma [16]. A possibility of using experimental data for Γ_m gives us two advantages: the problem is simplified because Eq. (5) can be used without solving for unknown $\mathbf{b}$ in the plasma, which makes the predictions more reliable since no assumptions on the plasma model and unknown distributions are involved.

3. Discharge evolution and RFA

A general solution to (5) is

$$B_m = B_m^0 e^{\alpha} - \Gamma_m^0 e^{\alpha} \int_0^{\tau} e^{-\alpha} B_m^{ext} d\tau. \tag{10}$$

where $\tau = t / \tau_w$, $B_m^0 = B_m(t = 0)$, and

$$\alpha = \int_0^\tau \Gamma_m d\tau \,. \tag{11}$$

By definition,

$$B_m^{ext} = B_m^{er} + B_m^{CC} \,, \tag{12}$$

with B_m^{er} representing the error field, and B_m^{CC} the field produced by the correction coils (RMP, in terms of [18]). A static error field can be modeled by $B_m^{er} = \text{const}$.

Consider the case when $B_m^{CC} = 0$ and only intrinsic error field is present. The perturbation B_m resulting from interaction of the plasma with error field depends on the time history of the discharge through $\Gamma_m(t)$. At the start of the discharge, $t = 0$, when the plasma does not contribute to B_m, we have $\Gamma_m = \Gamma_m^0$ and $B_m^0 = B_m^{er}$. Then for $t > 0$ we obtain from (10)

$$B_m = B_m^{er} A(t) \,, \tag{13}$$

with

$$A = e^{\alpha} - \Gamma_m^0 e^{\alpha} \int_0^\tau e^{-\alpha} d\tau \,. \tag{14}$$

Starting from unity at $t = 0$, the "amplification coefficient" A can grow to large values. Assume, for example, that

$$\Gamma_m = \Gamma_m^0 (1 - t/T) \,, \tag{15}$$

which keeps $\Gamma_m < 0$ till $t = T$. The time evolution of A for this case is shown in Fig. 1. At $t = T$, when the stability boundary is reached,

$$A(T) = \exp(-z^2) + \sqrt{\pi} z \Phi(z) \,, \tag{16}$$

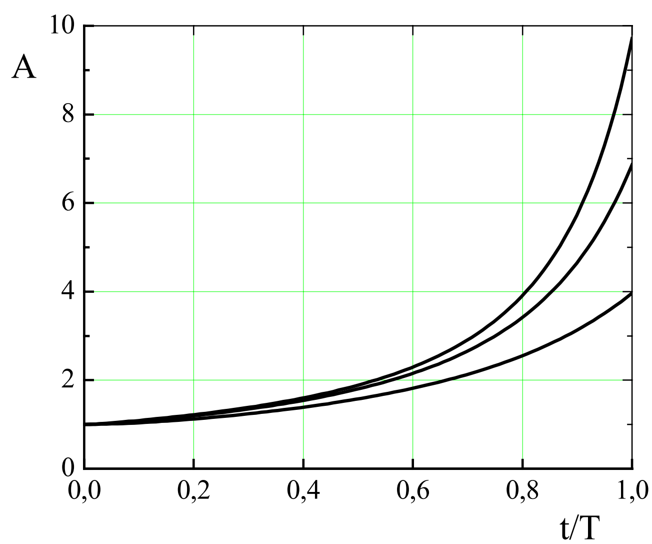

FIG. 1. Time dependence of the amplification factor for $\Gamma_m = \Gamma_m^0(1 - t/T)$ *at different* $\mu T/\tau_w$*: 5 (lower curve), 15 (middle), and 30 (upper curve).*

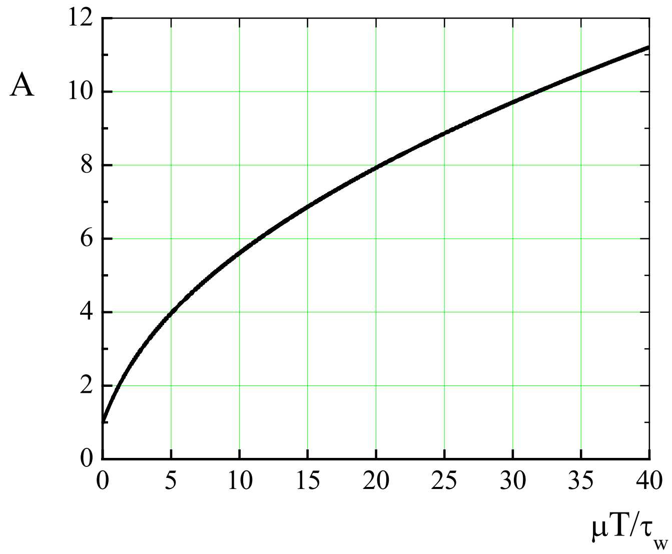

FIG. 2. Amplification factor at $t = T$ *for* $\Gamma_m = \Gamma_m^0(1 - t/T)$.

where

$$z = \sqrt{\mu T / \tau_w}\,, \tag{17}$$

and $\Phi(z)$ is the normal error integral,

$$\Phi(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-x^2)dx\,. \tag{18}$$

Dependence (16) is shown in Fig. 2. For large z (with good accuracy, for $z > 2.5$) the expression (16) can be approximated by

$$A(T) \cong \sqrt{\pi} z = \sqrt{\pi \mu T / \tau_w}\,. \tag{19}$$

This shows that quite a short time T is needed to reach large amplification values. For example, Eq. (19) gives $A > 10$ for $T > 64\tau_w/(2\mu)$, or $T > 160 \div 320$ ms for standard tokamak parameters. Typically τ_w can be estimated as 10-20 milliseconds: according to [21], the wall time for $n = 1$ in DIII-D is $\tau_w = 2\mu \times 2.5$ ms, and in JET $\tau_w = 2\mu \times 5$ ms (recall that $\mu = |m|$). With amplification coefficient $A > 10$ and the error field of 2 G, Eq. (13) yields $B_m > 20$ G, the perturbation certainly above the level of NTM seeding. Here, the results are obtained for real Γ_m in (15) which corresponds to the locked modes. The difference between locked and rotating modes is discussed below.

4. RFA near the stability boundary

In the above example, larger T makes larger $A(T)$. This means that the tokamak discharge stronger reacts on the error field if the stability boundary is approached slowly. The effect comes from the integration in (14) over the time interval where Γ_m becomes small, as can be seen in Fig. 1. The conclusion is not related to the particular choice (15).

The importance of small Γ_m for RFA can be illustrated by another example. The RFA coefficient (14) is sensitive to Γ_m time dependence which is determined by the discharge evolution. However, in any case, it must come to the stationary level

$$A_{st} = \Gamma_m^0 / \Gamma_m \tag{20}$$

in a stable state ($\gamma_0 < 0$), if this state is sustained long enough, for $\delta t >> 1/|\gamma_0|$. The value $|A_{st}|$ increases with decreasing $|\Gamma_m|$. This makes smaller the threshold value of B_m^{er} which is needed for growth of B_m up to the seed level B_m^{seed}:

$$\min B_m^{er} = \frac{B_m^{seed}}{A_{st}} = \frac{\Gamma_m}{\Gamma_m^0} B_m^{seed}. \tag{21}$$

Even for very low amplitude of the error field (let say, 1 G) $A_{st} = 10$ would be large enough for considering the resulting perturbation as exceeding the seed threshold. For $A_{st} = 10$ we need $\Gamma_m = -0.4$ for $m = 2$ mode, and $\delta t >> 2.5\tau_w$. Thus, just 100 ms in a state with $\gamma_0 \tau_w = -0.4$ might be sufficient to expect a strong destabilizing effect of the error field on NTMs.

The steady state amplification coefficient A_{st}, defined by (20), becomes infinite when $\Gamma_m \to 0$. In [22], where RFA was analyzed as a steady state effect only, this was interpreted as an infinite RFA at marginal stability. However, a steady state with $B_m^{er} \neq 0$ is impossible in the vicinity of $\Gamma_m = 0$. Indeed, the first term on the right hand side of (5) vanishes when $\Gamma_m = 0$, and one cannot assume zero $\partial B_m / \partial t$ on the left. For $\Gamma_m = 0$ and $B_m^{ext} = B_m^{er} = \text{const}$, equation (5) has a solution with a slow linear growth of perturbation, without saturation:

$$B_m = B_m(t_0) - \Gamma_m^0 B_m^{er} (t - t_0) / \tau_w . \tag{22}$$

The perturbation can grow large, but always remains finite. In this case, the seed level B_m^{seed} is reached in a time

$$\Delta t = \frac{\tau_w}{2\mu} \frac{\delta B_m}{B_m^{er}}, \tag{23}$$

where $\delta B_m = B_m^{seed} - B_m(t_0)$. For $\delta B_m / B_m^{er} = 10$, which is rather optimistic assumption, Eq. (23) yields $\Delta t = 2.5\tau_w$ for the $m = 2$ mode. This is a small value on a time scale of typical tokamak discharges: $\Delta t < 50$ ms for $\tau_w < 20$ ms. Thus, a short time is needed for "spontaneous" growth of the seed island in a state near marginal stability of the mode with $\Omega_0 = 0$.

5. RFA and mode rotation

The presented estimates are obtained assuming real Γ_m, which means a nonrotating mode, $\Omega_0 = 0$. In experiments on COMPASS-D and TEXTOR, a locked mode has been induced by a nonrotating resonant magnetic perturbation [18, 20]. However, when the perturbation was switched off, the mode then rapidly 'spanned up' to some natural rotation frequency. In our model this corresponds to $\Omega_0 \neq 0$ or complex Γ_m. Note that real Γ_m was used in several expressions only starting from Eq. (15).

According to (20), which is valid for any Γ_m, the mode rotation always makes the RFA weaker:

$$\left|A_{st}\right| = \frac{2\mu}{\tau_w\sqrt{\gamma_0^2 + n^2\Omega_0^2}}. \tag{24}$$

This means that the plasma response to the error field must decrease when the mode rotation starts. The decay of the mode after its unlocking and spinning up was observed, for example, in COMPASS-D and JET [18, 19]. Eq. (24) gives also a natural explanation to the fact that only locked or weakly rotating modes are induced by the error field, while the modes with $\Omega_0\tau_w >> 1$ are weakly affected by the static error field. In experiments, the latter is observed as additional resistance to mode penetration when the additional plasma momentum is introduced [19]. In HBT-EP device, the amplitude of the slowly growing external kink mode was observed to decrease as the mode toroidal rotation accelerated [23].

6. Discussion

The error fields are an inherent property of any device, therefore, they can serve as a trigger for NTMs when all other destabilizing mechanisms are absent or inefficient. The error field itself may be much smaller than the seed level B_m^{seed}, but can be strongly "amplified" by the plasma. The resonant error field amplification (RFA) increases when plasma approaches the marginal stability. Accordingly, smaller error field is needed for the seed island formation and "spontaneous" NTM onset. When $\Gamma_m \to 0$ in (21), the admissible level of $\left|B_m^{er}\right|$ goes to zero, which makes the stability boundary of the locked modes the most dangerous point.

The RFA is essentially dynamic process, so that the resulting "amplification" depends on the discharge evolution. Slower approach to the marginal stability results in larger RFA, as shown in Figs. 1 and 2. One can see that strong integral effect can be gained after $(5 \div 10)\tau_w$. Even smaller time is needed for perturbation to reach the seed level in a state with $\Gamma_m = 0$, where RFA is strongest. On the other hand, the perturbation developing on the time scale of several τ_w may remain undetected, and the NTM onset may then seem "spontaneous" or without a trigger event. If our analysis is correct, the problem can be simply resolved by increasing the sensitivity of the magnetic diagnostics. Note that with improved diagnostics a slowly growing perturbation similar to that described by (22) was observed in DIII-D experiments before the RWM onset [17].

Our conclusions are based on the theory prediction that, near the marginal stability, the RFA can result in a sufficiently large magnetic perturbation, even if the error field is small. Approaching the stability boundary is accompanied also by reduction of the seed island size [6, 9]. Both effects reduce the level of error field necessary for NTM seeding. This is consistent with other observations that thresholds for error field driven modes fall as the high β is approached [19].

The proposed model explains the main features of the phenomena observed in a tokamak plasmas near the marginal stability and allows experimental verification by intentional destabilization of NTMs, tearing modes, or RWMs with externally applied error field pulses with properly designed amplitude and pulse length. Also, within the model, the correction of the intrinsic error field must lead to the enhanced stability of NTMs.

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