Effect of Sheared Flows on Neoclassical Tearing Modes

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Abstract. The influence of toroidal sheared equilibrium flows on the nonlinear evolution of classical and neoclassical tearing modes (NTMs) is studied through numerical solutions of a set of reduced generalized MHD equations that include viscous force effects based on neoclassical closures. In general, differential flow is found to have a strong stabilizing influence leading to lower saturated island widths for the classical ($m/n = 2/1$) mode and reduced growth rates for the ($m/n = 3/1$) neoclassical mode. Velocity shear on the other hand is seen to make a destabilizing contribution. An analytic model calculation, consisting of a generalized Rutherford island evolution equation that includes shear flow effects is also presented and the numerical results are discussed in the context of this model.

1. Introduction

It is now widely recognized that the β limit of advanced tokamaks is determined by the nonlinear instabilities associated with neoclassical tearing modes (NTMs) and not by the linearized theory of ideal MHD instabilities [1–3]. In recent years a great deal of work has been carried out on the Rutherford theory of neoclassical tearing modes and many important results on critical β values and their improvement by the use of RF current drive and heating methods, stabilization by the use of external helical current coils etc. have been obtained [4–6]. There are, nevertheless, a number of issues related to the origin of excitation of the mode, its excitation threshold, its nonlinear behaviour and its interaction with error fields and equilibrium shear flows that have not been satisfactorily resolved and need to be better understood [7]. The influence of shear flows is a particularly important issue since sheared velocity flows are known to be widely prevalent in tokamak devices and can be generated by neutral beams, ion cyclotron heating and self-consistent drift turbulence. A number of past studies have examined the effect of flows on tearing modes, particularly in the linear regime and for simplified geometries [8]. There have also been a few nonlinear studies [9, 10] but the problem is quite complex, particularly in realistic toroidal geometries, and is an important area of present and future study for major numerical initiatives such as NIMROD. In this paper we report on numerical studies that we have begun on this problem with the help of a finite difference code NEAR that solves a set of generalized reduced MHD equations [11] and that includes viscous force effects based on neoclassical closures. While not as comprehensive or sophisticated as the NIMROD initiative, the present numerical model and the code is a lot simpler to implement and has been tested in the past for the effect of flows on linear resistive tearing modes. Our emphasis in this work is to extend the investigation to the nonlinear regime for both classical tearing modes and NTMs. To complement the numerical work and to gain some physical insight we also present an analytic calculation that describes the evolution of a single helicity NTM within the framework of an extended Rutherford model. The model incorporates flow effects coming through the inertial contributions and also includes a simplified mockup of the pressure curvature contribution - the so called GGJ effect.

2. Model Equations

Our numerical simulations are based on the solutions of a set of reduced MHD equations originally proposed by Kruger *et al* in 1998 [11]. These equations, which are valid at any aspect ratio, are derived using $k_\perp/k_{||}$ as a small expansion parameter and by employing a multiple scale analysis that respects equilibrium constraints and also permits elimination of fast time scales associated with perpendicular wave motion. The model equations thus evolve scalar potential quantities on a time scale associated with the parallel wave vector (shear-Alfven wave time scale), which is the time scale of interest for resistive MHD instabilities like tearing modes. In addition the model equations also permit incorporation of sub-Alfvenic equilibrium shear flows and neoclassical closures in a straightforward manner and are therefore well suited for studying the nonlinear evolution of neoclassical tearing modes in the presence of flows. In the limit of $\beta \sim \delta^{1/2}(\delta \ll 1)$, a simplified set of the evolution equations are as follows,

$$\frac{\partial \Psi}{\partial t} - (\boldsymbol{b}_0 + \boldsymbol{b}_1)\cdot\nabla\phi_1 - \boldsymbol{b}_1\cdot\nabla\phi_0 = \eta\tilde{J}_{||} - \frac{1}{ne}\boldsymbol{b}_0\cdot\nabla\cdot\boldsymbol{\Pi}_e \tag{1}$$

$$\begin{aligned}\nabla\cdot\left(\frac{\rho}{B_0}\frac{d}{dt}\frac{\nabla\phi_1}{B_0}\right) + (\boldsymbol{V}_1\cdot\nabla)\left(\nabla\cdot\left(\frac{\rho}{B_0}\frac{\nabla\phi_0}{B_0}\right)\right) &= (\boldsymbol{B}_0\cdot\nabla)\frac{\tilde{J}_{||}}{B_0} + (\boldsymbol{B}_1\cdot\nabla)\frac{J_{T||}}{B_0} \\ +\nabla\cdot\frac{\boldsymbol{B}_0\times\nabla p_1}{B_0^2} \qquad &+\nabla\cdot\frac{\boldsymbol{B}_0}{B_0^2}\times\nabla\cdot\boldsymbol{\Pi}\end{aligned} \tag{2}$$

$$\frac{dp_1}{dt} + (\boldsymbol{V}_1\cdot\nabla)p_0 + \Gamma p_T\nabla\cdot\boldsymbol{V}_1 = (\Gamma-1)\left[\eta J_{T||}^2 - \boldsymbol{\Pi}:\nabla\boldsymbol{V} + \boldsymbol{\Pi_e}:\nabla\frac{\boldsymbol{J}}{ne} - \nabla\cdot\boldsymbol{q}\right] \tag{3}$$

$$\rho\frac{d\widetilde{V}_{||}}{dt} + (\boldsymbol{V}_1\cdot\nabla)V_{||0} = -\boldsymbol{b}_0\cdot\nabla p_1 - \boldsymbol{b}_1\cdot\nabla p_T - \boldsymbol{b}_0\cdot\nabla\cdot\boldsymbol{\Pi} \tag{4}$$

$$\begin{aligned}\nabla\cdot\boldsymbol{q} \;=\; & -\chi_\perp\nabla^2 p_1 - (\chi_{||}-\chi_\perp)[\boldsymbol{b}_1\cdot\nabla(\boldsymbol{b}_0\cdot\nabla p_0) + \boldsymbol{b}_0\cdot\nabla(\boldsymbol{b}_0\cdot\nabla p_1 + \boldsymbol{b}_1\cdot\nabla p_0) \\ & +\boldsymbol{b}_0\cdot\nabla(\boldsymbol{b}_1\cdot\nabla p_1) + \boldsymbol{b}_1\cdot\nabla(\boldsymbol{b}_1\cdot\nabla p_0) + \boldsymbol{b}_1\cdot\nabla(\boldsymbol{b}_0\cdot\nabla p_1)]\end{aligned} \tag{5}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{V}\cdot\nabla \;\;;\; \Phi_T = \Phi_0 + \Phi_1 \;\;;\; p_T = p_0 + p_1 \;\;;\; \boldsymbol{b}_T = \boldsymbol{b}_0 + \boldsymbol{b}_1 = \frac{\boldsymbol{B}_0}{B_0} + \frac{\boldsymbol{B}_1}{B_0}$$

$$J_{T||} = J_{0||} + \tilde{J}_{||} = \boldsymbol{b}_0\cdot\nabla\times\boldsymbol{B}_0 + \nabla^2\Psi$$

$$\boldsymbol{V} = \Omega(\psi)R^2\boldsymbol{\nabla}\zeta + \boldsymbol{V}_1 = \frac{\boldsymbol{B}_0\times\nabla\Phi_0}{B_0^2} + V_{0||}\boldsymbol{b}_0 + \frac{\boldsymbol{B}_0\times\nabla\Phi_1}{B_0^2} + \tilde{V}_{||}\boldsymbol{b}_T$$

where the notations are standard (for a more detailed discussion see [11]). The equilibrium toroidal velocity which is conveniently expressed in terms of a function $\Omega(\psi)$ is ordered such that $V_0/V_A \sim \epsilon \ll 1$ so that the flows are restricted to the sub-Alfvenic range. The above equations have been programmed into an initial value code, called NEAR, which is a derivative of an older code called FAR. An early benchmarking of this code was carried out in [12], where terms proportional to $\mathbf{\Pi}_e$, $\mathbf{\Pi}$ and $\nabla \cdot \boldsymbol{q}$ were dropped and the tests were restricted to the linear growth regime of classical tearing modes. Our emphasis in the present work is to explore the effects of shear flow in the nonlinear regime and in particular to examine its influence on the evolution of neoclassical tearing modes.

3. Numerical simulation results

3.1. Classical tearing modes

As a preliminary to our study of the neoclassical tearing modes and for proper benchmarking of the code in the nonlinear regime we have first studied the nonlinear evolution of the classical tearing mode and the effect of sheared flow on its evolution. For this we have adopted the simplified approach of [12] and dropped the stress tensor contributions as well as the $\nabla \cdot \boldsymbol{q}$ term in the model. In the absence of flow we generate an equilibrium profile by numerically solving the Grad-Shafranov equation with the help of an equilibrium code called TOQ [13]. Using this equilibrium in NEAR we study the evolution of the $(m/n = 2/1)$ tearing mode from an initial arbitrary perturbation. In the linear regime the growth rate of the (2/1) mode displays all the characteristic scalings of the resistive tearing instability (*viz.* $\gamma \sim S^{-3/5}\Delta'^{4/3}$ etc.) in agreement with earlier benchmark results [12]. Fig. 1(a) illustrates one such benchmark test showing the scaling of the linear growth rate with Δ'. In the nonlinear regime the growth rate slows down to the algebraic rate of the Rutherford regime and eventually saturates with an island width W_{sat} that is in good agreement with theoretical estimates based on the nonlinear modification of $\Delta' \rightarrow \Delta'(1 - W/W_{sat})$ [14]. This is shown in Fig. 1(b). We next incorporate sheared flow in the system by generating an appropriate flow modified equilibrium from TOQ and using it in NEAR. For our numerical studies we have used two kinds of flow profiles to identify effects arising from differential flow and flow shear. The flow profiles are shown in Fig.2(a) where profile 1 has only differential flow between two adjacent mode rational surfaces ((2, 1) and (3, 1)in this instance) and no shear whereas profile 2 has both differential flow and shear. In the linear regime differential flow is found to have a stabilizing effect. This is illustrated in Fig.2(b) where the growth rate is seen to decrease with increasing differential flow.

In the nonlinear regime the differential flow effect continues to have a stabilizing influence and leads to a lower level of mode saturation as shown by the lowest curve of Fig. 3(a). The oscillations in the amplitude of the mode energy are due to the mode rotation frequency induced by the toroidal flow. The mode rotation frequency is close to the flow frequency but shows some nonlinear increase close to the saturation region. When we use profile 2 we find a decrease in the stabilization effect (the intermediate curve in Fig 3(a)) indicating that velocity shear has a destabilizing trend.

We do not yet have a complete quantitative understanding of the source of the stabilizing and destabilizing terms but some qualitative insights have been obtained by examining the effect of certain individual terms in the code. In particular we have seen the enhancement of the pressure curvature term due to equilibrium modification of the pressure surfaces due to flow. However there are other effects such as parallel velocity effects and global changes affecting Δ' that are more difficult to isolate. These could also be playing an important contributory role.

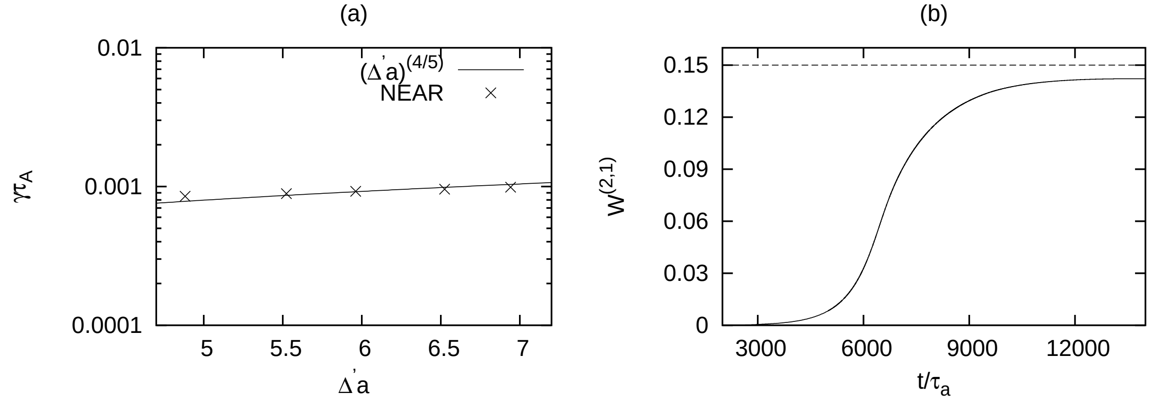

Figure 1. (a)Benchmark results confirming the $\Delta'^{4/3}$ scaling of the linear growth rate of the $(m/n = 2/1)$ classical tearing mode. (b) Nonlinear saturation of the (m/n=2/1) mode.

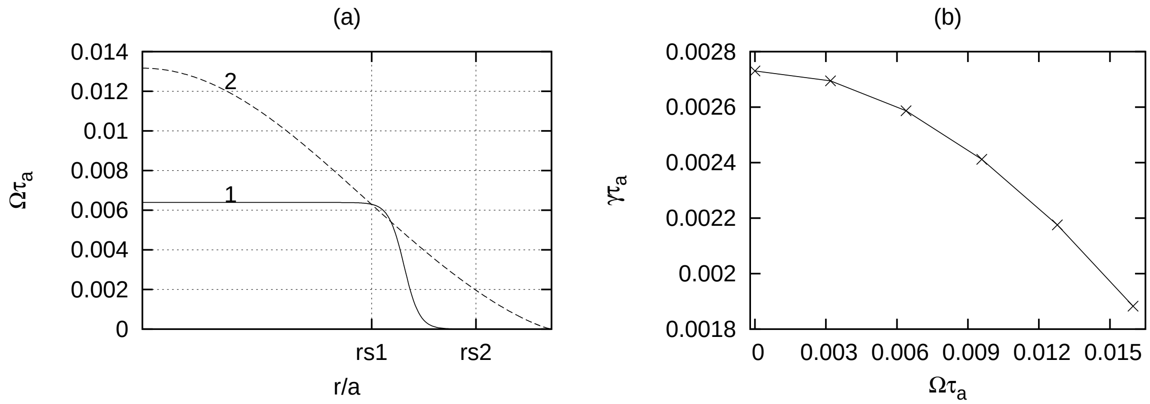

Figure 2. (a) Equilibrium toroidal flow profiles - 1. differential flow with no shear at the mode rational surfaces 2. differential flow with finite shear at the mode rational surfaces. (b) The reduction in growth rate of the $(m/n = 2/1)$ mode as a function of the differential flow amount.

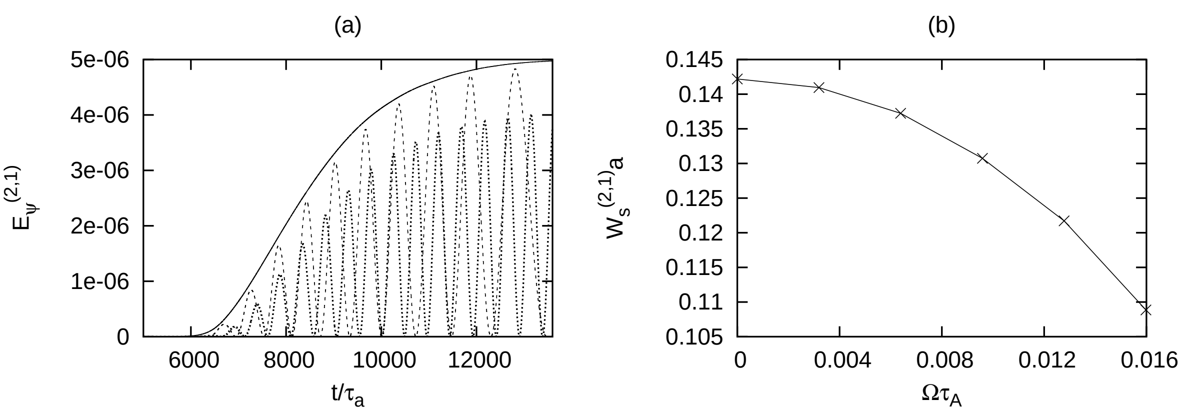

Figure 3. (a) Evolution of mode energy for the $(m/n = 2/1)$ mode for the no flow case (solid curve), profile-1 (dark dotted curve) and profile-2 (dashed curve). (b)Decrease of the saturated island width for the $(2, 1)$ mode as a function of the differential flow amount.

3.2. Neoclassical tearing modes

As is well known, the neoclassical tearing mode is driven unstable by a perturbation of the bootstrap current which arises from the viscous damping of the poloidal electron flow and which is proportional to the cross-field pressure gradient. In the presence of a seed magnetic island, the equilibrium pressure gradient flattens locally, thereby switching

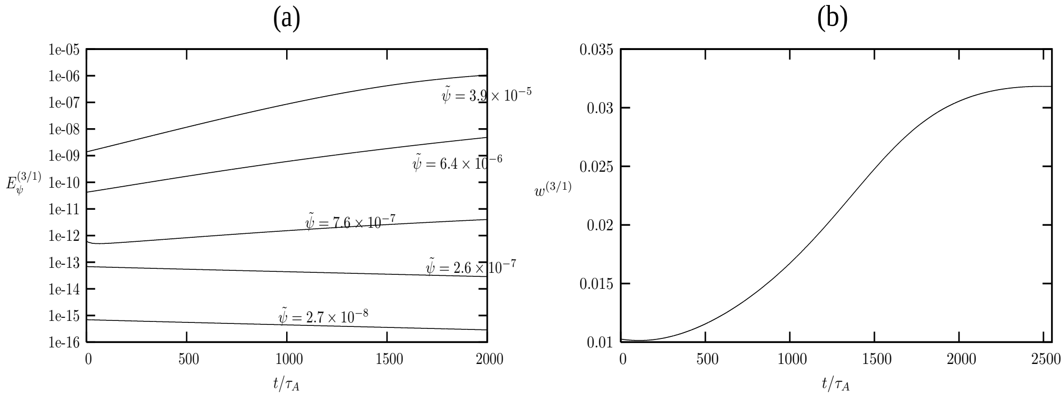

Figure 4. (a) Benchmark results showing the existence of a threshold amplitude for the $(3, 1)$ NTM. (b) Nonlinear evolution of the $(3, 1)$ NTM.

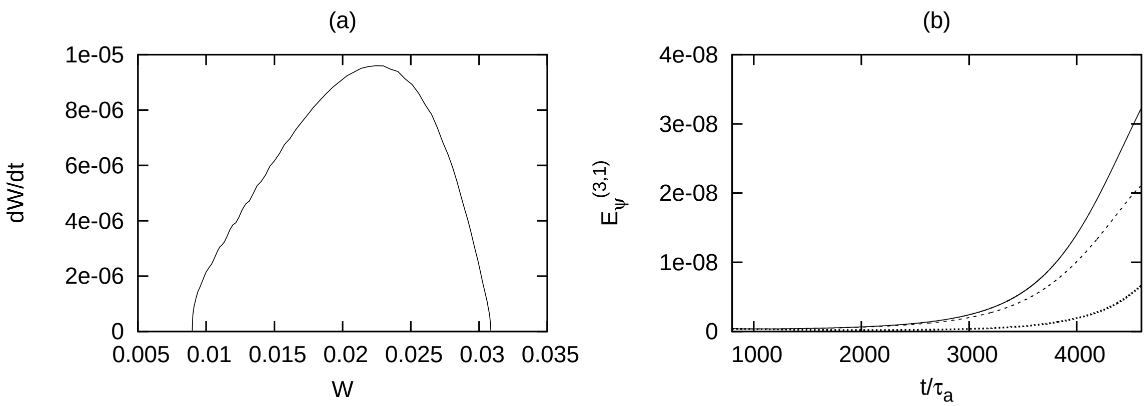

Figure 5. (a) Phase diagram of the $(3, 1)$ NTM in the absence of flow. (b) Nonlinear evolution of the $(3, 1)$ NTM for no flow (solid curve), flow profile-1 (heavy dotted curve), flow profile-2 (dashed curve).

off the bootstrap current; this results in a negative current perturbation on the given rational surface which drives up the amplitude of the magnetic island by the Rutherford non-linear growth mechanism. The pressure flattening relies on the assumption that the parallel transport is much faster than the perpendicular transport. To study the evolution of neoclassical tearing modes it is therefore necessary to retain the stress tensor terms in the reduced MHD equations in order to provide the drive term and to keep the heat flow terms in the pressure evolution equation. For the neoclassical viscous stress tensor we have used the following closure ansatz [11],

$$\nabla \cdot \vec{\Pi}_j = \rho_j \mu_j \left\langle B^2 \right\rangle \frac{\boldsymbol{v}_s . \nabla\theta}{(\boldsymbol{B} \cdot \nabla\theta)^2} \nabla\theta, \tag{6}$$

where $j = i, e$ and μ_j is the viscous damping frequency of each species j. This closure ansatz is appropriate for the long mean free path (low collisional) limit and reproduces poloidal flow damping as well as gives an appropriate bootstrap current perturbation.
The NTM instability is essentially a nonlinear (or subcritical) instability that can develop even in the limit of resistive MHD stability (i.e. $\Delta' < 0$) and requires a threshold amplitude of the magnetic perturbation for its excitation. The evolution of a single helicity NTM is well described by the Rutherford nonlinear theory suitably modified to include neoclassical effects. Some of the characteristic signatures of the mode include scaling of the growth rate with plasma β, sensitivity of the threshold to the ratio $(\chi_\perp/\chi_{||})$ and to the relative

signs of the pressure gradient and magnetic shear. Before investigating the effect of flows on NTMs we have benchmarked the NEAR code by reproducing these characteristic features of the NTMs and paid particular attention to pressure equilibration [15]. The typical ratio of $\chi_{\|}/\chi_{\perp}$ in most of our runs has been of the order of 10^6or more and the Reynolds number S has been kept at 10^5 or higher. We have studied the evolution of the $(m/n = 3/1)$ mode using equilibrium configurations for which the mode is linearly stable. In Fig. 4(a), we show the dependence of the mode evolution on the initial amplitude and the existence of a threshold amplitude for the destabilization of the mode. Fig. 4(b), shows a typical nonlinear evolution of the island width and its eventual saturation. Fig. 5(a), is a reproduction of the characteristic 'phase diagram' of an NTM obtained in this case for the $(m/n = 3/1)$ mode from numerical runs on NEAR. We next introduce toroidal flow in the system using equilibrium flow profiles 1 and 2 (of Fig.2(a)) generated through TOQ. The basic trend in this case is quite similar to what we observe for classical tearing modes, namely destabilizing effects of flow shear and a more dominant stabilizing effect from differential flow. In Fig.5(b) , we show the island width evolution for three different cases - the top curve is without any flow, the bottom curve is for flow profile 1 (pure differential flow) and the intermediate curve is for flow profile 2 (differential flow + shear). Unlike as in the case of the classical tearing mode we have been unable to follow these evolutions to the nonlinear saturation levels due to numerical problems. Finite flow appears to place more severe constraints on pressure equilibration requirements and may be a reason for this difficulty.

4. Rutherford model equations in the presence of sheared flows

To gain some analytic understanding of the nature of the sheared flow contributions we have tried to construct a Rutherford model description of the island evolution in the presence of flows. For this we have restricted ourselves to a single helicity magnetic perturbation, such that the total magnetic field in the vicinity of the island can be expressed in terms of the flux function ψ as,

$$\psi = -\frac{B_0}{L_s}\frac{x^2}{2} + \tilde{\psi} cos\xi \tag{7}$$

where B_0 is the average equillibrium toroidal magnetic field, $x = r - r_s$ is the distance from the rational surface, $L_s = qR/s$ is the shear length, $s = r_s q'/q$, $\tilde{\psi}$ is the perturbed magnetic flux, $\xi = m\hat{\theta} - \int \omega(t')dt'$, $\hat{\theta} = \theta - \zeta/q_s$ is the helical coordinate with θ denoting the poloidal angle and ζ the toroidal angle. For $m \geq 2$, when the constant $\tilde{\psi}$ approximation holds, the magnetic island halfwidth is given by, $W = \left(\frac{4L_s\tilde{\psi}}{B_0}\right)^{1/2}$. The nonlinear evolution equation of the magnetic island is derived from the matching conditions obtained by integrating Ampere's equation across the nonlinear region.

$$\int_{-\pi}^{\pi} d\xi cos\xi \int_{-\infty}^{\infty} dx J_{\|} = \frac{c}{4\pi}\Delta'_c \pi\tilde{\psi} \quad ; \quad \int_{-\pi}^{\pi} d\xi sin\xi \int_{-\infty}^{\infty} dx J_{\|} = \frac{c}{4\pi}\Delta'_s \pi\tilde{\psi}$$

where the matching parameters $\Delta'_{c,s}$ are determined from the outer (linear) region and are assumed to be given by ideal MHD equations. The longitudinal current can be obtained, following standard procedures by perturbative solutions of the parallel Ohm's law (to determine Φ), the pressure evolution equation, eq.(3), and finally the vorticity equation, eq.(2). In carrying out this calculation we have ignored $\tilde{V}_{\|}$ contributions in the convective derivative term (thereby decoupling eq.(4)) and also omitted stress tensor contributions in the pressure evolution and vorticity equations. We have also used the ansatz (6) for the electron viscosity term in Ohm's law, so that the perturbed bootstrap current has the form, $\tilde{J}_b = \frac{\mu_e}{\nu_{ei}}\frac{c}{B_\theta}\frac{dp}{dx}$, where μ_e is the viscosity coefficient, ν_{ei} is the electron-ion collision frequency, B_θ is the poloidal magnetic field and p is the plasma pressure. A further simplification is made in the vorticity equation where the pressure curvature term is modeled in terms of a simple centrifugal force term that is proportional to the equilibrium pressure gradient and the curvature. The polarization drift term, which is proportional to the plasma inertia incorporates the flow effects through the potential Φ which can be written in terms of an equilibrium and perturbed part, as $\Phi = \Phi_0(x) + \tilde{\phi}(x)$. Using the lowest order solution of the parallel Ohm's law, $\tilde{\phi}$ can eventually be obtained in the form,

$$\tilde{\phi} = \frac{B_0}{ck_\theta}(\omega - \omega_E)(x - \lambda) - \frac{B_0}{ck_\theta}\frac{\omega_E'}{2}(x^2 - \lambda^2) \qquad (8)$$

where $\omega_E = k_\theta c \Phi_0'(r = r_s)/B_0$ is the drift frequency due to the equilibrium electric field and is the flow contribution, and $\omega_E' = k_\theta c \Phi_0''(r = r_s)/B_0$ is the flow shear contribution. The function $\lambda(\psi)$ is an integration constant which is chosen to provide the correct asymptotic behaviour for $\tilde{\phi}$. A convenient form that has been used frequently in the past, and is the one that we have employed, is to take it to be zero inside the magnetic separatrix and $\lambda(\psi) = \frac{W}{\sqrt{2}}\left[\left(-\frac{\psi}{\tilde{\psi}}\right)^{1/2} - 1\right]$ in the outer region. Our final island evolution equations are as follows,

$$0.4\frac{\partial W}{\partial t} = D_R^{neo}\left[\frac{\Delta_c'}{4} - \frac{19.5}{W}\frac{\epsilon L_s^2}{B_0^2}\frac{\partial p(0)}{\partial \psi} + 0.6\frac{\sqrt{\epsilon}\beta_\theta \frac{L_q}{L_p}}{W}\frac{W^2}{W^2 + W_\chi^2} + \frac{L_s^2}{k_\theta^2 v_A^2}\left(2.3\frac{(\omega - \omega_E)(\omega - \omega_E - \omega_*)}{W^3} + \frac{{\omega_E'}^2}{W}\right)\right] \qquad (9)$$

$$0.82\frac{\partial}{\partial t}\left[W(\omega - \omega_E) + \frac{\omega_E'}{2}W^2\right] = -12.6\frac{\mu_e}{W}(\omega - \omega_E) - \frac{1}{4\sqrt{2}}\left(\frac{nsV_A}{R^2 q}\right)^2 W^4 \Delta_s' \qquad (10)$$

where, $D_R^{neo} = c^2/4\pi\sigma_{neo}$ is the magnetic diffusion coefficient calculated using the neoclassical resistivity, $\beta_\theta = 8\pi p_e/B_\theta^2$, $L_p = -(dlnp/dr)^{-1}$, $L_q = (dlnq/dr)^{-1}$. In eqn.(9) the second termm on the RHS is the perturbed bootstrap current contribution which drives the mode unstable when it is larger than the ($\Delta_c' < 0$) term. For evaluating the neoclassical contribution we have adopted the standard procedure outlined in [16] where $\mu_e \simeq \sqrt{\epsilon}\nu_{ei}$ for the long mean-free-path regime has been used and $\omega_* = \omega_{*pi} + k\omega_{*T}$ [16], with $\omega_{*T} = k_\theta c T_i'/eB_0$. The factor $W^2/(W^2 + W_\chi^2)$ in the neoclassical term is the usual effect associated with finite radial thermal diffusion and sets a critical island width $W_\chi = \sqrt{\frac{RqL_q}{m}}\left(\frac{\chi_\perp}{\chi_\parallel}\right)^{1/4}$, below which radial transport becomes significant and the pressure is no longer flattened across the island. The third term is the simplified Glasser term arising from the pressure curvature term and has a stabilizing nature. The next term is the usual polarization current term modified by the flow contribution. It can switch signs depending on the rotation frequency of the mode (in the rotating plasma frame) relative to the diamagnetic frequency. However since the NEAR code does not have two fluid effects the diamagnetic frquency can be set to zero in which case this term has zero contribution in the rotating plasma frame. The fifth term is the contribution from velocity shear and is seen to be of a destabilizing nature. Eqn.(10) is the evolution equation for the mode frequency obtained with the help of the second matching condition and containing the flow shear correction.

5. Summary and Discussion

Our present set of numerical results, using two different profiles of toroidal equilibrium flow, indicate that differential flow has a strong stabilizing influence on the nonlinear evolution of both classical and neoclassical tearing modes whereas velocity shear has a destabilizing effect. While a quantitative comparison with any existing analytic model is not possible some qualitative features of the results can be understood on the basis of past theoretical work on shear flows as well as nonlinear evolutionary studies of tearing modes in the absence of flows. The destabilization effect of weak shear flows is consistent with the findings of earlier linear studies as well as the simple Rutherford model derived in the previous section. The source of the stabilizing influence of differential flow is somewhat difficult to pin down. The pressure-curvature term has a stabilizing influence (when the overall curvature is favourable) even in the absence of flow. We have confirmed this by artificially turning this term off in the code. We have also numerically confirmed that the stabilizing effect of this term is enhanced by the presence of differential flow possibly due to equilibrium modifications of the pressure profile caused by the centrifugal effects of flow. However the measured enhancement is quite small, for the weak flows we have investigated, and cannot account for the total amount of stabilization observed. One strong possibility is flow induced modification of Δ' - a global effect that would be a sensitive function of the overall flow profile. We are currently examining this question both analytically and by further numerical runs with other profiles.

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