

The Stability of Internal Transport Barriers to MHD Ballooning Modes and Drift Waves: a Formalism for Low Magnetic Shear and for Velocity Shear

J. W. Connor, R. J. Hastie, A. J. Webster and H. R. Wilson

EURATOM-UKAEA Fusion Association, Culham, United Kingdom

Email contact of main author: jack.connor@ukaea.org.uk

Abstract. Tokamak discharges with internal transport barriers (ITBs) provide improved confinement, so it is important to understand their stability properties. The stability to an important class of modes with high wave-numbers perpendicular to the magnetic field, is usually studied with the standard ballooning transformation and eikonal approach. However, ITBs are often characterised by radial q profiles that have regions of negative or low magnetic shear and by radially sheared electric fields. Both these features affect the validity of the standard method. A new approach to calculating stability in these circumstances is developed and applied to ideal MHD ballooning modes and to micro-instabilities responsible for anomalous transport.

1. Introduction

An important class of instabilities in a tokamak has short wavelength across the magnetic field $\mathbf{B}$, characterised by a high toroidal mode number, n , but long wavelength parallel to it. The stability of such modes is normally addressed using the ballooning representation followed by an eikonal ansatz, which reduces an apparently two-dimensional problem in radius, x , and poloidal angle, θ , to two simpler, consecutive one-dimensional calculations. Physically this depends on the toroidal coupling between adjacent resonant surfaces exceeding the effect of the non-degeneracy between such surfaces due to radial profile variation. The first of these calculations corresponds to the lowest order theory (in an expansion in $1/n$), and determines the mode structure along $\mathbf{B}$; it is not radically affected by low shear, $s = (r/q)(dq/dr)$. However, the higher order WKB theory that determines the radial width, Δ_{bal} , of the ballooning mode, soon fails for physically reasonable values of n , as s becomes small. This can be seen from a two-scale analysis of the lowest order ballooning equations, followed by using a variational principle with the two-scale solutions as trial functions. This allows one to recover a dependence on the radial wave number, k , required for the WKB analysis [1], but which is lost in the simple two-scale analysis. One finds Δ_{bal} contains an exponentially small factor at low s due to the weak toroidal coupling, and unless n is very large, $\Delta_{\text{bal}} \leq \Delta_{\text{mrs}} \equiv 1/nq_s$, the separation between mode resonant surfaces (mrs), and the standard ballooning method fails.

This problem has been overcome by a procedure that complements the ballooning transformation at low s . We solve a three-term recurrence relation satisfied by the amplitudes of ‘modelets’ localised about each mrs (the modelets differ from simple Fourier modes by the presence of toroidally-induced side-bands localised about the same resonant surface). The modelets are coupled by toroidal effects to form a radially extended ballooning mode [2]. The coupling coefficients in this recurrence relation involve radial overlap integrals, I , of the basic modelets. These overlap integrals are exponentially weak at low s since the modelets have a width much smaller than Δ_{mrs} ; consequently one need only consider the coupling of adjacent harmonics. Rather than calculate these coupling coefficients directly in x -space, they can be deduced directly from the dispersion relation obtained from a ballooning-space calculation. Solutions of the recurrence relation span the range from extended ballooning modes to

uncoupled modelets localised at separate m 's, depending on the competition between the toroidal coupling of modelets and the detuning of modelets arising from radial profile variations. In the presence of a $q_{\min}$ surface where $s \rightarrow 0$, the toroidal coupling vanishes and ballooning mode structures tend not to penetrate too near $q_{\min}$. Furthermore, the recurrence relation recaptures effects from the discreteness of the m 's. We apply this formalism to two types of mode near an ITB: ideal MHD ballooning modes and electron and ion drift modes.

The presence of a sheared plasma rotation, $\Omega(x)$, as often occurs near an ITB, also breaks the degeneracy between resonant surfaces since it introduces a fast x -dependence through the Doppler-shift: $\omega \rightarrow \omega + n\Omega(x)$. In the limit $\Omega' \rightarrow \infty$, this effectively averages the instability drive over the periodic variation on the wave-number k . However, it is not clear how this mode is related to the standard ballooning mode in the absence of rotation. We address this question using both two-dimensional calculations and the recurrence relation approach.

2. Ideal MHD ballooning modes at low magnetic shear

First we consider high- n MHD ballooning modes which could potentially limit pressure gradients at an ITB where s is low. We use the $s - \alpha$ equilibrium, including the effects of favourable average curvature (of order $\varepsilon_t = r/R$) for $q_{\min} > 1$. An analytic dispersion relation near marginal stability has been obtained by Pogutse and Yurchenko [3]. They use a variational form of the familiar $s - \alpha$ ballooning equation with a trial function for the radial displacement ξ_r . This is based on a two-scale (i.e., η and $u = \varepsilon(\eta - k)$) where η is the periodic, equilibrium scale, poloidal co-ordinate in ballooning space, $nq'k$ is the radial wave number and the 'stretching parameter', $\varepsilon = s$), low s , asymptotic solution of the ballooning equation:

$$\xi_r(\eta, u) = G^{-1/2} \left[1 + \frac{\alpha \cos \eta}{(1 + u^2)} \right], \quad (1)$$

where $G = 1 + (u - \alpha \sin \eta)^2$, leading, for unstable modes, to the local dispersion relation (LDE)

$$s\gamma = h(\alpha, s) \cos k - g(\alpha, s) \quad (2)$$

where $g(\alpha, s) = (19\alpha^2 - 96s\alpha + 64s^2)/128 + \varepsilon_t\alpha(1 - q^2)$ and $h(\alpha, s) = (5\alpha/4)\exp(-1/|s|)$. (The inertial term is linear in γ near marginal stability because the poloidal displacement, ξ_θ , scales as $\gamma^{1/2}\xi_r$.) In x -space the trial function (1) corresponds to a set of modelets (each consisting of a main poloidal harmonic and two weaker, $O(\alpha)$, side-bands), each highly localised about its own m . The variational approach is essential to recover the exponentially weak toroidal coupling of these modelets ($\sim \exp(-1/|\varepsilon|)$) where $\varepsilon = s$ is the two-scale parameter).

In the ballooning limit the modelets have equal amplitudes, $c_m = \exp(imk)$, and eqn. (2) follows, with the $\cos k$ term representing their coupling through toroidicity. More generally, when the ballooning limit is no longer valid due to low s or rapidly varying profiles, the c_m will no longer have equal amplitudes, but they continue to be coupled by toroidicity. This coupling can be computed directly from a radial overlap integral of adjacent modelets, but it

is simpler to use eqn. (2) to deduce the appropriate recurrence relation for the c_m at high n . Compatibility with eqn. (2) in the ballooning limit, implies it must have the form [4]

$$(s\gamma + g(\alpha, s))c_m = h(\alpha, s)(c_{m+1} + c_{m-1})/2 \quad (3)$$

The solution of eqn. (3) is a global one, spanning a trajectory in $s - \alpha$ space. Since α and s are functions of x we replace this dependence by one on m/nq' (or $\pm(2m/nq'')^{1/2}$ near a minimum in q) in the recurrence relation (3).

Solutions at low s with $\varepsilon \rightarrow 0$ show that the c_m spectrum narrows as $|s|$ decreases and that the standard ballooning theory exaggerates the finite- n corrections; nevertheless the marginal stability curve approaches that of the $n \rightarrow \infty$, $s - \alpha$ stability diagram as $s \rightarrow 0$ (see Fig. 1 in [4]). Turning to the ITB case, we represent it by a localised peak, of width L_* , in the profile of α and assume the q profile has a minimum, $q_{\min}$, there. In particular we take

$$\begin{aligned} \alpha(x) &= \alpha_{\max} \operatorname{sech}^2(x/L_*) \\ q(x) &= q_{\min} + q''x^2/2 \Rightarrow s = (r^2q''/q)x \equiv \mu x \end{aligned} \quad (4)$$

where we now measure x from the location of the ITB. This introduces a further parameter, $\Delta q = (m/n - q_{\min})$, which recognises the discreteness of the mrs.

Solutions of the recurrence relation for $\Delta q = 0$ show that the ITB can sustain a stable, large step in pressure, $\Delta p \propto \alpha_{\max} L_*$, provided it is sufficiently narrow (Fig. 1). The dashed lines in Fig. 1 are analytic, asymptotic expressions: $\alpha_{\max} = 1.96\varepsilon_t^{1/3}$ for $\alpha_{\max} \ll 1$, while for $sr/L_* \gg 1$, $\alpha_{\max} = 0.37s_c^{1/2} \exp(2s_c r/\mu L_*)$ where $0.29s_c^{3/2} + 1.25 \exp(-1/s_c) = \varepsilon_t$ (thus, for $\varepsilon_t \ll 1$, $s_c = 2.28\varepsilon_t^{2/3}$, but for $s_c > 0.1$, $s_c \cong 0.23 + \varepsilon_t$). Furthermore stability is enhanced when $q_{\min}$ takes low-order rational values since then Δq cannot take its most unstable value (physically the mrs tend to fall outside the unstable region of the $s - \alpha$ diagram). Inclusion of the self-consistent bootstrap current modifies the shear: $s \rightarrow s - (\alpha/\varepsilon_t)^{1/2} f(\tau, \eta_e, \eta_i)$ where $\eta_j = d(\ln T_j)/d(\ln n_j)$, $\tau = T_e/T_i$ and f is given in [4] (e.g., for $\tau = 1$ and $\eta_e = \eta_i \gg 1$, $f = 0.07$, while for $\eta_e, \eta_i \approx 0$, $f = 0.61$). This implies that stable ITBs can exist even when the background shear is not so small.

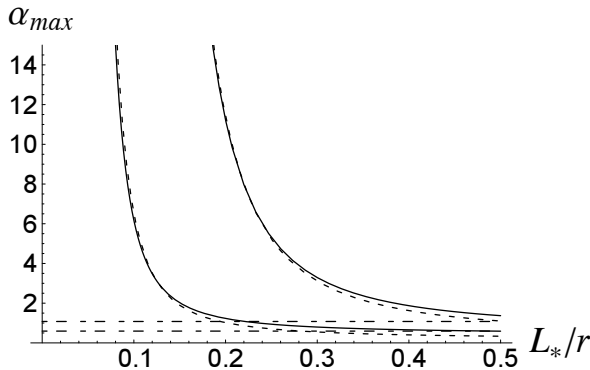


FIG. 1. Maximum stable α , $\alpha_{\max}$, as a function of the barrier width L_* for $\varepsilon_t = r/R = 0.05$ (lower curve), 0.3 (upper curve). The dashed lines are the approximations for large and small L_*

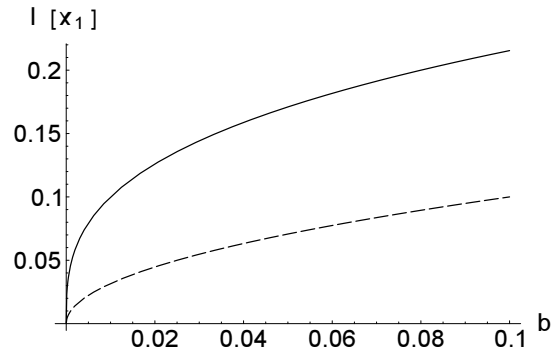


FIG. 2. Toroidal coupling coefficient, $I(x_1)$ evaluated at x_1 , the nearest resonant surface to $q_{\min}$, for ITG modes as a function of $b = (k_{\perp} \rho_s)^2$; toroidal branch (mixed parity): full line; slab branch: dashed line)

3. Electron and ion drift modes at low magnetic shear

The second application is to longer wavelength ($b_s = (k_\perp \rho_s)^2 \ll 1$, $\rho_s^2 = (2m_i T_e / e^2 B^2)$), electron drift waves and ion temperature gradient (ITG) modes, which could contribute to anomalous transport near an ITB. We first consider the electron drift wave to illustrate the approach; the ballooning space eigenmode equation is [2]

$$\left\{ \frac{1}{\Omega_e^2} \left(\frac{\partial}{\partial \eta} + \varepsilon_1 \frac{\partial}{\partial u} \right)^2 + \varepsilon_1^2 \left(\frac{\varepsilon_1^2}{s^2} + u^2 \right) + \frac{2q\varepsilon_1}{\Omega_e} \left(\frac{\varepsilon_1}{|s|} \cos \eta + \frac{s}{|s|} u \sin \eta \right) + \hat{D}_e \right\} \zeta = 0 \quad (5)$$

where $\Omega_e = \omega/\omega_{*e}$ (i.e., $\Omega_e \cong 1$), $\varepsilon_1 = (qb_s |s| / \varepsilon_n)^{1/2}$ and $\hat{D}_e = (q^2 b_s / \varepsilon_n) [(\tau \delta \Omega_e) / (1 + \tau + \eta_i) - i \delta_e]$ with $\delta \Omega_e = \delta \omega / \omega_{*e}$, $\delta \omega = (\omega - \omega_{*e})$. This corresponds to a variational quantity

$$H = \int d\eta \left\{ \left(\frac{\partial \zeta}{\partial \eta} \right)^2 - \left[\varepsilon_1^2 \left(\frac{\varepsilon_1^2}{s^2} + u^2 \right) + 2q\varepsilon_1 \left(\frac{\varepsilon_1}{|s|} \cos \eta + \frac{s}{|s|} u \sin \eta \right) + \hat{D}_e \right] \zeta^2 \right\} \quad (6)$$

We introduce a two-scale trial function

$$\zeta = \left[1 + 2q\Omega_e \left(\frac{\varepsilon_1}{|s|} \cos \eta + \frac{s}{|s|} \sin \eta \right) \right] \exp \left(i\sigma \frac{u^2}{2} \right) \quad (7)$$

where $u = \varepsilon_1(\eta - k)$ and σ is a variational parameter, determined in leading order as $\sigma^2 = (1 + 2q^2) \Omega_e$. In this case the LDE takes the form

$$\hat{D}_e + \frac{\varepsilon_1^4}{s^2} (1 + 2q^2) + i\sigma \varepsilon_1^2 + \frac{2q\varepsilon_1^2}{|s|} \exp \left(-\frac{i}{4\sigma \varepsilon_1^2} \right) \cos k = 0 \quad (8)$$

This leads to a recurrence relation between modelets of a similar form to eqn. (3), where any x-dependence of the coefficients again translates into an m-dependence. There is a transition from extended ballooning structures to isolated modelets as $4\sigma \varepsilon_1^2 \text{Im} \Omega_e$ ($\text{Im} \Omega_e \sim \delta_e$) falls below unity, i.e. for low shear or long wavelength, $b_s \ll 1$. The stability of the special case of an isolated mode near $q_{\min}$ can be analysed as a function of $\Delta q = (m/n - q_{\min})$; we find that there are values of m and n for which shear damping is lost [2].

However it is more interesting to investigate the long wavelength ITG mode, which has slab-like and toroidal branches. In the flat density limit the eigenmode equation is [1]

$$\frac{d^2 \zeta}{d\eta^2} + q^2 \Omega_i^2 b_T \left[\frac{\tau \Omega_i}{1 + \varepsilon_T^{1/2} \tau \Omega_i} + b_T [1 + s^2 (\eta - k)^2] + \frac{2}{\Omega_i} (\cos \eta + s(\eta - k) \sin \eta) \right] \zeta = 0 \quad (9)$$

where $\Omega_i = \omega / \tau (\omega_T \omega_D)^{1/2}$, positive Ω_i corresponding to the electron drift direction, with $\omega_{Ti^*} = - (nq\rho_s/r) c_s / L_{Ti}$, and $b_T = (nq\rho_s/r)^2 / \varepsilon_T^{1/2}$ (i.e. $b_T = b_s / \varepsilon_T^{1/2}$), where $\varepsilon_T = L_{Ti} / R \ll 1$. This leads to a variational form

$$H = \int_{-\infty}^{\infty} d\eta \left\{ \left(\frac{d\zeta_0}{d\eta} \right)^2 - q^2 b_T \Omega_i^2 \left[\frac{\tau \Omega_i}{1 + \varepsilon_T^{1/2} \tau \Omega_i} + b_T [1 + s^2 (\eta - k)^2] \right. \right. \\ \left. \left. + \frac{2}{\Omega_i} [\cos \eta + s(\eta - k) \sin \eta] \right] \zeta_0^2 \right\} \quad (10)$$

Again we introduce two-scale trial functions. For the slab-like branch the analysis is similar to that for the electron drift mode, with the stretching parameter being given by $\varepsilon_2 = b_T^{1/2} |s|^{3/4}$. However the toroidal branch has a novel, strongly ballooning, mode structure reminiscent of Toroidal Alfvén Eigenmodes. We take trial functions of the form [1]

$$\zeta_0 = \left[\cos\left(\frac{\eta}{2}\right) + A \sin\left(\frac{\eta}{2}\right) \right] \exp\left(-\frac{\hat{\sigma} u^2}{2}\right) \quad (11)$$

where $u = \varepsilon_3(\eta - k)$ with $\varepsilon_3 = b_T^{1/3} |s|^{1/2}$, for the even parity mode (we also need to consider odd parity ones for completeness). Here A and $\hat{\sigma}$ are variational parameters determined by $\delta H = 0$. The lowest order eigenvalue is given by

$$\Lambda(\Omega_i) = \frac{\tau q^2 \Omega_i^3 b_T}{1 + \varepsilon_T^{1/2} \tau \Omega_i} - \frac{1}{4} = 0 \quad (12)$$

which has an unstable root with $\text{Re} \Omega_i < 0$. The requirements that $\text{Im} \Omega_i > 0$ and the leading order variational solution for $\hat{\sigma}$ satisfies $\text{Re} \hat{\sigma} > 0$ lead to different mode structures and corrections to the eigenvalue on either side of $s = 0$. Thus we find a pure parity (in η) solution (11) with $A = 0$ for $s > 0$ and a pure odd parity mode for $s < 0$, both satisfying the LDE

$$D_+ - \frac{s}{|s|} \left(\frac{1}{\varepsilon_3} \right) \exp\left(-\frac{1}{4\hat{\sigma}\varepsilon_3^2}\right) [A_0 + \varepsilon_3 A_1 + \varepsilon_3^2 A_2] \cos k = 0 \quad (13)$$

while there are different mixed parity modes on either side of $s = 0$, both satisfying the LDE

$$D_+ D_- + 2b_T^{1/3} \Omega_i q^2 + \frac{s}{|s|} \frac{\exp\left(-\frac{1}{4\hat{\sigma}\varepsilon_3^2}\right)}{\varepsilon_3^2 D_+} [B_0 + \varepsilon_3 B_1 + \varepsilon_3^2 B_2 + \varepsilon_3^3 B_3 + \varepsilon_3^4 B_4] \cos k = 0 \quad (14)$$

Here the leading coefficients are $A_0 = B_0 = (3q^2 + 1)/4q^2$, $D_{\pm} = (d\Lambda/d\Omega_i) \delta\Omega_i \pm (\varepsilon_3/s) b_T^{1/3} \Omega_i q^2$ and the variational parameter $\hat{\sigma} = -b_T^{1/3} \Omega_i q^2$. Equations (13) and (14) are the relevant LDEs for ITG modes and generalise those in Ref. [1].

Again one can deduce recurrence relations with the structure of eqn. (3). The strength of the coupling is again largely determined by the exponents in eqns. (13, 14), but their pre-factors indicate that the mixed parity mode (14) is the more strongly coupled; again we see how the coupling is weakened at long wavelength and low shear. However, to be quantitative we calculate the coupling coefficient evaluated at the nearest resonant surface to $q_{\min}$ as a function of b_s in Fig. 2 for the most strongly coupled mode, the mixed parity mode, and the weakest, the slab mode. Clearly one needs to go to very long wavelength, say $n < 6$ for $\rho_s/r =$

1/256, to lose the coupling. This result is consistent with Ref. [5]. Solutions of the recurrence relation for the case of $q_{\min}$ assuming a quadratic dependence of $L_T^{-1} = L_T^{-1}(0)(1 - (x/L_{T*})^2)$ show a set of eigenmodes which can be characterised by the dominant m in their spectrum. As Δm ($\Delta m = m - m_0$, $m_0 = nq_{\min}$), approaches zero the spectrum widths, δm , narrow sharply: thus for the mixed parity mode (assuming $n = 40$, $b_s = 0.01$, $\varepsilon_T = 0.1$, $L_{T*}/r = 0.2$, $\mu = 1$, $q_{\min} = 2$), with $\Delta m = 15$, $\delta m = 5$, with $\Delta m = 5$, $\delta m = 3$, and with $\Delta m = 2$, $\delta m = 1$ [2]. As we have seen, the toroidal branch of the ITG mode has different poloidal mode structures on either side of $q_{\min}$, which therefore acts as a 'barrier' to the modes, perhaps reducing the transport across the ITB.

4. Sheared Plasma Rotation

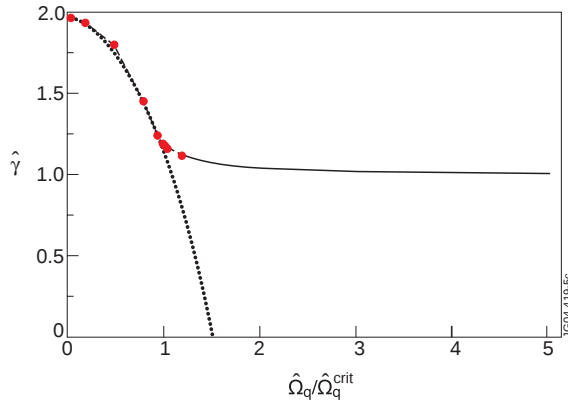


FIG. 3. Normalised growth rate as function of normalised rotation shear ($\Lambda^2 = 400$, $\hat{\varepsilon} = 1$): full line is from eqn. (17); red full circles, eqn. (19); dashed line, standard high- n ballooning result

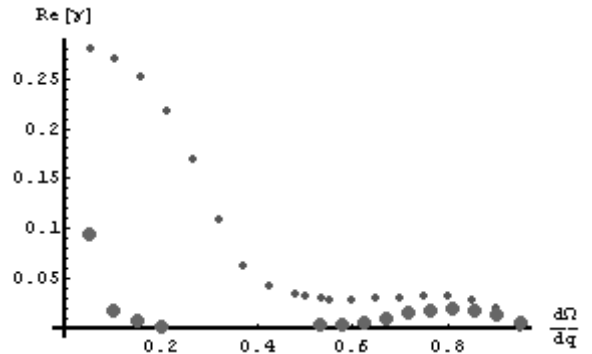


FIG. 4. Growth rates γ as a function of rotation shear $d\Omega/dq$. $\alpha = 1$, $s = 1$; $\Lambda^2 = 1.82$ (small circles), $\Lambda^2 \rightarrow \infty$ (large circles).

Radial $\mathbf{E} \times \mathbf{B}$ shear, which leads to a rotation shear, $d\Omega/dq$, at an ITB can also disrupt ballooning mode structures. In the infinite- n limit, i.e. finite- n effects due to the pressure profile are neglected, an eigenmode approach has shown that the growth rate, γ , is given by $\langle \gamma(k) \rangle_k$, the average over the ballooning angle, k [6]. We first examine the transition from the zero rotation theory, when $\gamma = \gamma_{\max}$, the maximum with respect to both k and x , to this result.

An essential effect of rotation shear is to introduce a Doppler shift, $\gamma \rightarrow \gamma + i n \Omega(x)$ into the zero flow theory. Considering the $s - \alpha$ model at low, but constant, s , we can introduce this modification into the recurrence relation (3). Although this can be readily solved numerically for a general pressure profile [7], it is instructive to obtain an analytic solution, which is possible in the case of a quadratic variation of the instability drive $\sim (r/L_*)^2$ and constant flow shear. The finite- n profile effects are captured by the parameter $\Lambda^2 = (nqs)(L_*/r)$ [8]. Introducing a generating function $G(k) = \sum c_m \exp(imk)$, periodic in k , and writing $G(k) = \exp(-\Lambda^2 \hat{\Omega}_q k/2) H(k)$, where $\hat{\Omega}_q = (d\Omega/dq)/\gamma_{\max}$ ($\gamma_{\max} = g(\alpha_{\max})/s$), we find $H(k)$ satisfies:

$$\frac{1}{\Lambda^4} \frac{d^2 H}{dk^2} + Q(k)H = 0, Q = 1 - \hat{\gamma} - \left(\Lambda^2 \hat{\Omega}_q / 2 \right)^2 + \hat{\varepsilon} \cos k \equiv \left(\sqrt{2\hat{\varepsilon}} / \Lambda^2 \right) \sigma - 2\hat{\varepsilon} \sin^2(k/2) \quad (15)$$

with $\hat{\varepsilon} = -h/g$ evaluated at $\alpha_{\max}$ and $\hat{\gamma} = \gamma/\gamma_{\max}$. Since G is periodic in k , we seek solutions of the Mathieu equation (15) of the Floquet type; the eigenvalue equation then takes the form [9]

$$H(2\pi) = \cosh(\Lambda^4 \hat{\Omega}_q \pi) \quad (16)$$

where H is a solution of eqn. (15) with boundary conditions $H(0) = 1$, $H'(0) = 0$. Since $\Lambda^2 \gg 1$, we can seek a solution of eqn. (15) in the WKB form: $H^\pm = \exp(\pm i\Lambda^2 \int dk Q^{\frac{1}{2}})$. We can distinguish two cases of interest in this MHD problem: (i) $Q < 0$ everywhere; and (ii) $Q = 0$ at two 'transition' points in the range, i.e. $0 < k_1 < k_2 < 2\pi$. In case (i) the relevant solution yields:

$$H(2\pi) = \cosh(\Lambda^2 \oint dk (-Q)^{\frac{1}{2}}) \quad (17)$$

and, taking account of eqn. (16), this provides the eigenvalue condition

$$\oint dk (-Q)^{\frac{1}{2}} = \Lambda^2 \hat{\Omega}_q \pi + 2i\ell\pi / \Lambda^2 \quad (18)$$

The case (ii) is more complex. The WKB solutions must be connected appropriately through the transition points $k_{1,2}$. For the MHD problem the relevant situation is $\sigma \sim 0(1)$ when $k_{1,2}$ are close together. This involves continuing the WKB solutions through the transition points by asymptotic matching to solutions of the parabolic cylinder equation. It is found that [9]

$$H(2\pi) = \left(\frac{\pi}{2}\right)^{1/2} \frac{\exp(4\Lambda^2 (2\hat{\epsilon})^{1/2})}{(16\Lambda^2 (2\hat{\epsilon})^{1/2})^\sigma} \left(\Gamma\left(\frac{1}{2} - \sigma\right)\right)^{-1} \quad (19)$$

For small $\hat{\Omega}_q$ the consistent solution has σ negative so that one matches eqn. (19) to (16). Since $\Lambda^2 \gg 1$, σ must approach a pole of the gamma function; the most unstable mode has $\sigma = 1/2$, with $\hat{\gamma} = 1 + \hat{\epsilon} - (\Lambda^2 \hat{\Omega}_q / 2)^2 - \sqrt{(2\hat{\epsilon})/(2\Lambda^2)}$: the standard high- n ballooning result with finite- n corrections. As $\hat{\Omega}_q$ increases the solution for σ decreases to zero at a critical value $\hat{\Omega}_q^{\text{crit}} = (4\sqrt{2\hat{\epsilon}}/\pi\Lambda^2) \sim 0(1/n)$ when $\hat{\gamma} = 1 + 0.199\hat{\epsilon}$. With further increases of $\hat{\Omega}_q$, solution (19) matches smoothly onto (18) as σ rapidly becomes large and negative; eventually $\hat{\gamma} \rightarrow 1 + (\pi^2 \hat{\epsilon} / 64)(\hat{\Omega}_q^{\text{crit}} / \hat{\Omega}_q)^2$. Figure 3 shows this evolution from γ_{max} to $\langle \gamma(k) \rangle_k$ as $\hat{\Omega}_q$ increases.

This treatment is informative but fails to capture all the effects of rotation shear. For example we only considered small s and also avoided discussing the issue of continuum damping [10] by considering $\hat{\epsilon} \leq 1$ to ensure $\gamma(k) > 0$ for all k . To overcome these limitations we have solved the full, incompressible MHD eigenmode equations for the $s - \alpha$ model [8], a two-dimensional problem. This allows investigation of the influences of magnetic shear, rotation shear and finite- n profile effects. For the case $s = 1$, $\alpha = 2$, studied by Miller et al. [11], and with $\Lambda^2 = 2.34$, we see a reduction in the growth rate with flow shear, with stability achieved when $\hat{\Omega}_q \cong 1$. Moreover we recover the results of Ref. 11 for $\Lambda^2 \rightarrow \infty$ which, in particular, correspond to averaging $\gamma(k)$ over k at low $\hat{\Omega}_q$; i.e. there is an instantaneous reduction in γ the moment rotation shear is introduced.

However, at this high value of α , the interpretation of the results is confused by second - stability effects when we introduce the effects of finite- n and rotation shear. For example, in the case of a quadratic α - profile there are two radial positions of maximum growth rate and the mode can lock to the corresponding values of rotation frequency to give a real part to the frequency, $\propto \hat{\Omega}_q$. Furthermore at modest n , radial harmonics of the stationary problem are readily coupled by rotation shear [8]. We therefore consider here a more modest value of $\alpha = 1$, still with $s = 1$, and show the results in Fig. 4. This demonstrates that for finite- n ($\Lambda^2 = 1.82$) the growth rate reduces as $\hat{\Omega}_q$ increases, again leading to stability at $\hat{\Omega}_q \cong 1$. With increasing Λ^2 this reduction occurs more quickly and for this choice of α the mode can become stable, as exemplified by the limit $\Lambda^2 = \infty$. However it passes through another unstable region as $\hat{\Omega}_q$ increases further, eventually stabilising again when $\hat{\Omega}_q \cong 1$.

5 Conclusions

Internal transport barriers are characterised by low magnetic shear and significant rotation shear. A formalism for treating high- n ballooning modes at low magnetic shear, a situation when the standard theory fails, has been developed. Applying it to the MHD stability of an ITB near $q_{\min}$ it is found that stable, high pressure gradients can exist there. In the case of drift waves, toroidal coupling is only significantly weakened at long wavelength: For the important toroidal branch of ITG modes, different mode structures exist on either side of $q_{\min}$, so this can act as a 'barrier' to the radial mode structure. In addition, the effect of rotation shear on high- n MHD stability has been explored. It is found that calculations of MHD stability using standard theory are not robust to the introduction of even a small amount of rotation shear, $\hat{\Omega}_q \sim 0(1/n)$: there is then a rapid reduction in growth rate. Although there may be a rather complex behaviour as $\hat{\Omega}_q$ increases further, stability occurs for $\hat{\Omega}_q \sim 1$. In physical terms this means a flow speed $v \sim sv_A$, with v_A the Alfvén speed. Thus one can expect the experimentally observed flow shears to strongly influence stability at low s , i.e. near an ITB.

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