Nonlinear Dynamics of Transport Barrier Relaxations in Fusion Plasmas

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Abstract. Relaxation oscillations of transport barriers are studied by three dimensional turbulence simulations. Barriers generated by an imposed ExB shear flow are found to relax intermittently on confinement time scales, even when fluctuations of the ExB shear flow are suppressed. A relaxation event has a complex dynamical behavior, characterized by the intermittent growth of a mode at the barrier center. An analytical study reveals that this dynamics is governed by the ExB velocity shear. A crucial ingredient therein is a time delay for effective ExB velocity shear stabilization.

1. Introduction

The most promising operational regime of future thermonuclear reactors such as ITER is characterized by the existence of a transport barrier at the plasma edge in the high confinement modes (H-mode). During a transition from low to high confinement (L-H transition), an edge transport barrier builds up spontaneously [1]. These barriers are not stable but relaxes quasi-periodically. Such relaxation oscillations can have a strong impact on energy and particle confinement. During these relaxation events, turbulent transport through the barrier increases strongly and the pressure inside it drops. Thereafter, the barrier builds up again on a slow, collisional time scale. The basic physical mechanism underlying such relaxation oscillations is not fully understood. In particular, there is no explanation why the plasma instead of remaining in a statistically stationary state close to pressure gradients or current stability limits, oscillates quasi periodically close to these limits. These relaxation oscillations are currently modeled by phenomenologically constructed dynamical equations for the amplitudes of unstable modes [2, 3, 4]. In this paper, we investigate the nonlinear dynamics of barrier relaxation using 3D turbulence simulations based on first principles. The turbulence model used is the resistive ballooning model for edge turbulence (RBM) [5, 6]. In the RBM simulations at the plasma edge, transport barriers can be generated by an externally imposed ExB shear flow (where E is the radial electric field and B is the equilibrium magnetic field) and it is found to relax quasi-periodically in a range of ExB shear decorrelation rates. Note that this behavior is robust as it persists even if the ExB flow is frozen, i.e. turbulence flow generation is suppressed. In this case, a steady state shear flow is imposed externally to generate the barrier, but the flow does not participate in the dynamics. The mechanism responsible of the barrier oscillation is fundamentally different from previous reported theories based essentially on turbulent shear flow generation [7]. The relaxation dynamics is found to be governed by the intermittent growth of a mode localized at the plasma center, characterized by low poloidal and toroidal wavenumbers. An analytical study based on a derivation of a reduced (one dimensional) model shows that the key element here is that the dynamics is governed by a time delay for effective velocity shear stabilization. Thus, the effect of the ExB shear flow differs from a shift of the linear instability threshold.

2. Model of External Transport Barrier Formation

Resistive ballooning mode (RBM) turbulence at the edge of a tokamak plasma is modeled by reduced resistive magneto–hydrodynamical (MHD) equations for the normalized electrostatic potential ϕ and pressure p [8],

$$\partial_t \nabla_\perp^2 \phi + \left\{\phi, \nabla_\perp^2 \phi\right\} = -\nabla_{||}^2 \phi - \mathbf{G} p + \nu \nabla_\perp^4 \phi \, , \tag{1}$$

$$\partial_t p + \{\phi, p\} = \delta_c \mathbf{G} \phi + \chi_{||} \nabla_{||}^2 p + \chi_\perp \nabla_\perp^2 p + S \, . \tag{2}$$

where ν stands for the viscosity, $\chi_{||}$ and $\chi_\perp$ are effective collisional heat diffusivities parallel and perpendicular to the magnetic field B, and $S(r)$ is an energy source. The curvature operator $\mathbf{G}$ arises from the compressibility of diamagnetic current and ExB drift, and $\delta_c = (5/3)2L_p/R_0$ is essentially the ratio between the pressure gradient length L_p and the major radius R_0. The Poisson bracket is $\{\phi, \cdot\} = r^{-1}\left(\partial_r \phi \partial_\theta - \partial_\theta \phi \partial_r\right)$, the curvature operator is $\mathbf{G} = \sin\theta \, \partial_r + \cos\theta \, r^{-1} \partial_\theta$ and the gradient along field lines is $\nabla_{||} = R^{-1}\left(\partial_\varphi + q^{-1} \partial_\theta\right)$. Assuming a monotonically increasing safety factor $q(r)$, the simulations cover a domain between $q = 2$ and $q = 3$ in the vicinity of a reference surface r_0 at the edge.

In the present model, a transport barrier is generated by externally imposing a locally sheared poloidal ExB flow. A corresponding drive is added to the equation for the poloidal flow, i.e. the magnetic flux surface average $\langle \ldots \rangle_{\theta\varphi}$ of Eq. (1),

$$\partial_t \bar{u}_\theta = -\frac{1}{r^2} \partial_r r^2 \left\langle \tilde{u}_\theta \tilde{u}_r \right\rangle_{\theta\varphi} + \nu \partial_r \frac{1}{r} \partial_r r \bar{u}_\theta - \mu \left(\bar{u}_\theta - U\right) \, , \tag{3}$$

where $\bar{u}_\theta = \langle u_\theta \rangle_{\theta\varphi}$ is the flow profile and $\tilde{u}_{r,\theta} = u_{r,\theta} - \bar{u}_{r,\theta}$ are the fluctuations of radial and poloidal velocity. The first two terms on the right hand side of (3) correspond to the divergences of the Reynolds stress and the viscosity stress, respectively, and the last term has been added artificially to account for the friction with an external sheared flow U. In the absence of external drive (i.e. $\mu = 0$), a poloidal flow is generated by turbulent fluctuations via Reynolds stress. This mechanism generates both, a mean component (finite time average) and a time fluctuating component (zonal flow) [9, 10]. In the limit $\mu \to \infty$, the poloidal flow $\bar{u}_\theta$ becomes identical to the external flow U (frozen flow case). This limit is simulated in the numerical code by using a finite value μ much larger then $\nu \left(\xi_{bal}/d\right)^2$ (here $\mu = 2$) and suppressing the Reynolds stress term in Eq. (3).

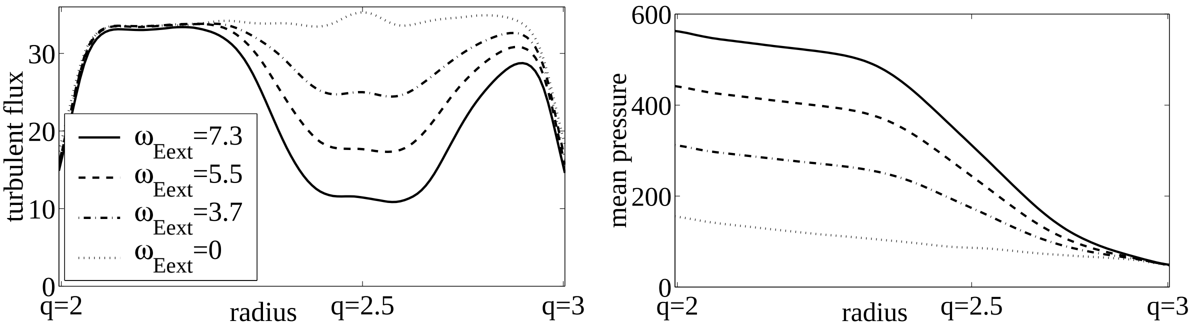

Figure 1: *Time averaged profiles of turbulent flux and pressure in normalized units for different values of the maximal shear and* $\Gamma_{tot} = 36$, $\mu \to \infty$.

Fig. 1 shows time averaged profiles of turbulent flux and pressure for different values of shearing rates $\omega_{E\,ext}$. We observe as $\omega_{E\,ext}$ increases a local reduction of turbulent transport [11] leading to a strong steepening of the pressure profile leading to the formation of a transport barrier.

3. Dynamics of Relaxations of Transport Barriers

In typical simulations of RBM turbulence with a transport barrier, the latter is not steady state but relaxes intermittently. This can be seen from Fig. 2 where time evolutions of the edge energy confinement time, the pressure gradient, the turbulent flux at the barrier center and the poloidal flow shear at the barrier position are presented for two different values of μ corresponding respectively to a case with zonal flows and a case where the poloidal flow profile is frozen.

The evolution of the pressure gradient is characterized by phases of a slow increase quasi periodically interrupted by rapid crashes. The latter correspond to relaxations of the barrier and are associated with large peaks of the turbulent flux at the barrier position. Typically, fluctuations of the velocity shear are also observed during the relaxations. The edge confinement time is defined as the ratio of the energy confined in the volume considered and the total energy flux across a magnetic surface. It represents a measure for the "strength" of the barrier. As can be seen from Fig. 2, the evolution of the confinement time follows the pressure gradient oscillations. Let us note that relaxation oscillations are found to persist even if the poloidal flow

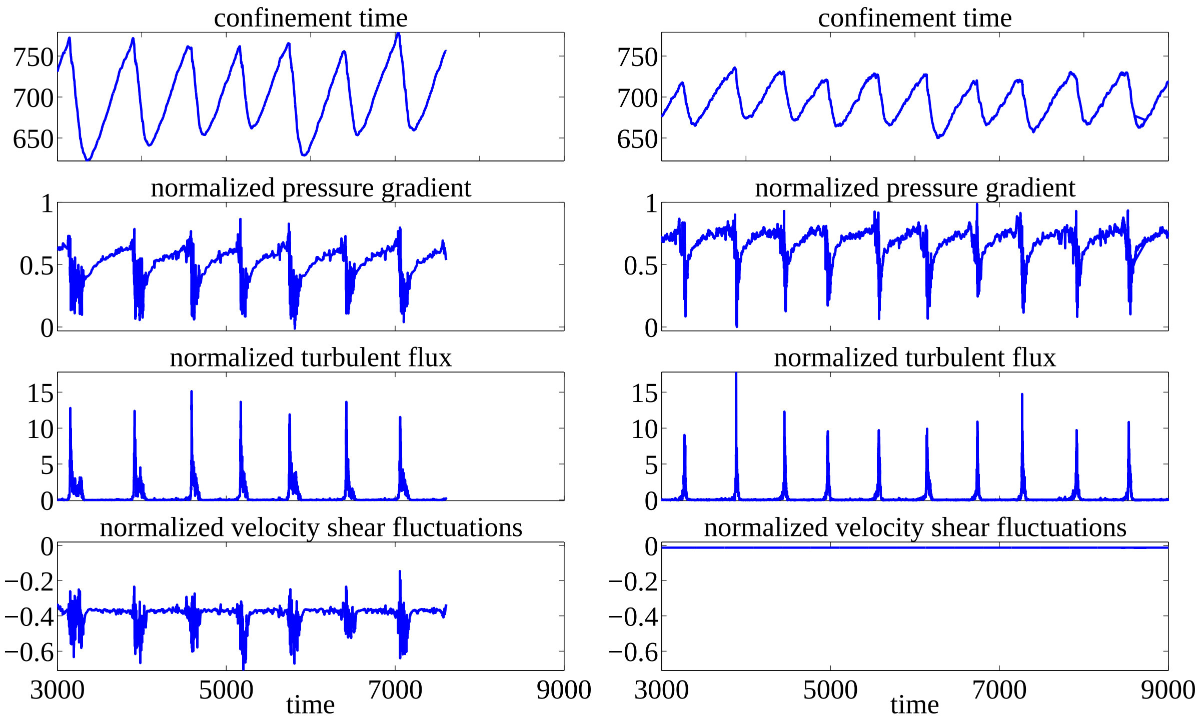

Figure 2: *Time evolution of edge energy confinement time, pressure gradient normalized to the diffusive value,* $\partial_r\bar{p}/(-\Gamma_{tot}/\chi_\perp)$*, turbulent flux normalized to the total incoming flux* $\langle\tilde{p}\,\tilde{u}_r\rangle_{\theta\varphi}/\Gamma_{tot}$*, and relative deviations of the poloidal flow shear from the imposed value* $(\partial_r\bar{u}_\theta - \omega_{E\,ext})/\omega_{E\,ext}$*, at the center of the barrier. Here,* $\omega_{E\,ext} = 8$*,* $\Gamma_{tot} = 36$*, and* $\mu = 0.02$ *(left) respectively* $\mu \to \infty$ *(right).*

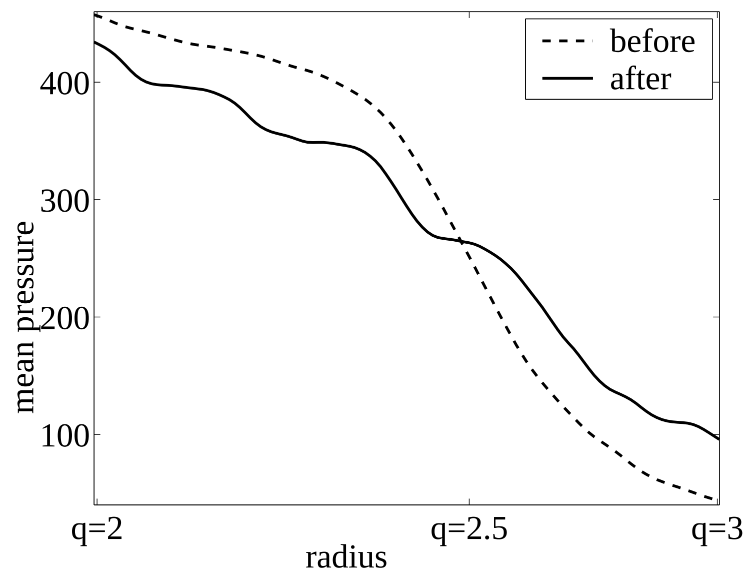

Figure 3: *Instantaneous pressure profiles before respectively after a relaxation.*

profile is frozen.

The ExB flow shear in tokamaks increases with heating power. It is found here that if this increase is faster than linear, the relaxation frequency decreases with power. These properties, onset of a transport barrier, relaxation oscillations associated to resistive ballooning modes, and the oscillation frequency that decreases with power, are reminiscent of so-called type III

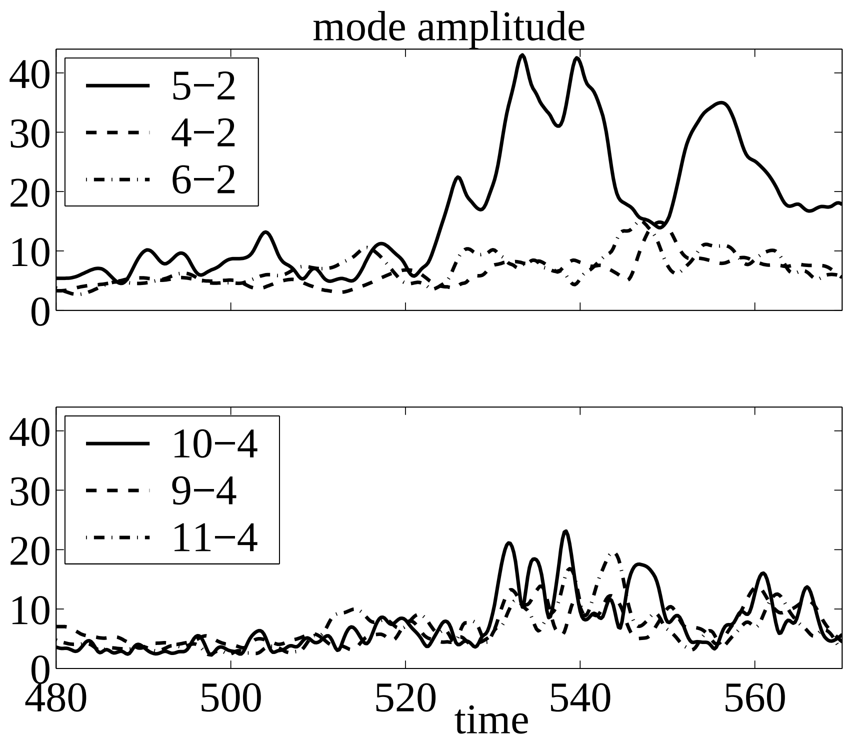

Figure 4: *Time evolution of amplitudes (of the potential) of different* (m,n) *modes during a flux peak.*

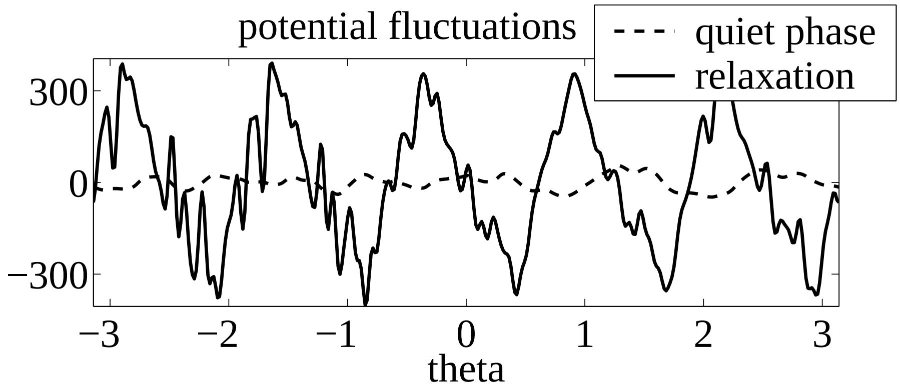

Figure 5: *Instantaneous electric potential fluctuations at the barrier center as a function of the poloidal angle in a quiescent phase and during a relaxation.*

edge localized mode (ELM) dynamics in tokamak edge transport barriers [12].

As shown Fig. 3, a relaxation event leads to a flattening of the pressure gradient at the barrier center and an isolation of two regions of steep gradient on both sides. A further analysis reveals that these steep gradients are then propagating radially away from the barrier center.

As relaxation oscillations are observed even when fluctuations of the ExB shear flow are suppressed, a “predator-prey” like mechanism involving energy exchange between turbulent fluctuations and ExB shear flow does not apply. Other possible mechanisms for relaxation oscillations can be excluded as well, i.e. a Kelvin–Helmholtz instability due to strong velocity shear at the barrier center or toroidal mode coupling, pumping energy from unstable modes outside the barrier to a mode localized at the barrier center. Indeed, as can be seen from Fig. 4, a relaxation event is characterized by the intermittent growth of a mode localized at the barrier center [here $(m,n) = (5,2)$]. However, no precursor on the directly coupled neighbors at the barrier shoulder, $(m,n) = (4,2)$ and $(6,2)$, is observed. This is also true for the neighbors $(m,n) = (8,4)$ and $(12,4)$ of the next order central mode $(m,n) = (10,4)$.

Fig. 5, shows that a relaxation event is dominated by a $(m,n) = (5,2)$ mode which is the lowest order (m,n) mode localized at the barrier position. This is a counter intuitive result as one expects fluctuations localized at this position are strongly stabilized by the velocity shear. However, as will be shown in the following by means of a reduced model, a transitory growth of a perturbation is possible due to the existence of a time delay for velocity shear stabilization which is an intrinsically nonlinear effect.

3. Analytical Study and Reduced Model for Barrier Relaxations

A one dimensional (1D) dynamical model for the radial dynamics of one dominant (m,n) mode coupled to the dynamics of the profiles can be obtained using the representation

$$\begin{pmatrix} \phi \\ p \end{pmatrix} = \begin{pmatrix} \bar{\phi} \\ \bar{p} \end{pmatrix}(r,t) + \begin{pmatrix} \tilde{\phi} \\ \tilde{p} \end{pmatrix}(r,t)\exp\left(\mathrm{i}m\theta - \mathrm{i}n\varphi\right) \tag{4}$$

and a subsequent Galerkin projection of the evolution equations (1, 2). The resulting system for the fields $\bar{\phi}$, $\tilde{\phi}$, $\bar{p}$, $\tilde{p}$ is simplified further by assuming a fixed relation between $\tilde{\phi}$ and $\tilde{p}$ given by the linear mode structure: $\tilde{\phi} = \mathrm{i}\,(\gamma_0/k_\theta)\,\tilde{p}$. Imposing a poloidal shear flow of the form

$\partial_r\bar{\phi} = \bar{u}_\theta = \omega_E x$, a 1D model is obtained that consists of an equation for the amplitude of the perturbation $\tilde{p}$ coupled to the dynamics of the pressure profile $\bar{p}$,

$$\begin{aligned} \partial_t \tilde{p} &= -ik_\theta \omega_E x \tilde{p} + \gamma_0 \left(-\partial_x \bar{p} - \kappa_0\right) \tilde{p} - \omega_t x^2 \tilde{p} + \chi_\perp \partial_x^2 \tilde{p}\ , && (5) \\ \partial_t \bar{p} &= -2\gamma_0 \partial_x \left|\tilde{p}\right|^2 + \chi_\perp \partial_x^2 \bar{p} + S\ . && (6) \end{aligned}$$

Here, $x = r - r_0$, $k_\theta = m/r_0$, and $\omega_t = \chi_{||}/\left(r_0 L_s\right)^2$, where L_s is the shear length. The parallel gradient has been evaluated for a Fourier mode localized at the barrier center $\nabla_{||} p_{mn} = \mathrm{i}x/(r_0 L_s) p_{mn}$. The system (5, 6) reproduces barrier relaxation oscillations for finite values of ω_E.

The short-term dynamics in the model (5, 6) is described by the evolution of an initial pulse $\tilde{p}(x,t=0) = \hat{p}\delta(x)$, infinitely localized at $x = 0$, that can be calculated analytically from Eq. (5) for a given pressure gradient $-\partial_x \bar{p} = \kappa$ and when neglecting the $\chi_{||}$ term. A solution of the equation

$$\partial_t p_{mn} + \mathrm{i}k_\theta \omega_E x p_{mn} = \gamma_0 p_{mn} + \chi_\perp \partial_x^2 p_{mn} + \hat{S}\delta(x)\delta(t)\ , \qquad (7)$$

is given by

$$p_{mn} = \frac{\hat{S}}{\sqrt{4\pi\chi_\perp t}} \exp\left(-\frac{x^2}{4\chi_\perp t} - \frac{\mathrm{i}k_\theta \omega_E x}{2} t\right) \exp\left(\gamma_0 t - \frac{t^3}{3\tau_D^3}\right)\ , \qquad (8)$$

where $\tau_D = \left[\frac{1}{4}\chi_\perp \left(k_\theta \omega_E\right)^2\right]^{-1/3}$. Note that for $\omega_E = \gamma_0 = 0$, the usual solution of the diffusion equation is recovered. The solution (8) describes an initial transient growth of the perturbation for $t < \tau_D$ before the cubic term in the last exponential takes over the linear term, leading to a stabilization. The characteristic time τ_D for the transient growth is large for small values of the perpendicular diffusivity $\chi_\perp$ (close to the collisional value at the barrier center) and low poloidal wave numbers k_θ.

4. Conclusions

3D turbulence simulations based on first principles with imposed ExB shear flow reveal the nonlinear dynamics of transport barriers characterized by relaxation oscillations. The analysis of these simulations shows that this dynamics is governed by an effective time delay in the stabilization by the shear flow. This is confirmed by a reduced 1D model. As this dynamics bears similarities to ELMs, this suggests that such effect of ExB shear flow might be included into the ELM modeling.

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