Long Pulse FRC Sustainment with Enhanced Edge Driven Rotating Magnetic Field Current Drive

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Abstract: FRCs have been formed and sustained for up to 50 normal flux decay times by Rotating Magnetic Fields (RMF) in the TCS experiment. For these longer pulse times a new phenomenon has been observed: switching to a higher performance mode delineated by shallower RMF penetration, higher ratios of generated poloidal to RMF drive field, and lower overall plasma resistivity. This global data is not explainable by previous RMF theory based on uniform electron rotational velocities or by numerical calculations based on uniform plasma resistivity, but agrees in many respects with new calculations made using strongly varying resistivity profiles. In order to more realistically model RMF driven FRCs with such non-uniform resistivity profiles, a double rigid rotor model has been developed with separate inner and outer electron rotational velocities and resistivities. The results of this modeling suggest that the RMF drive results in very high resistivity in a narrow edge layer, and that the higher performance mode is characterized by a sharp reduction in resistivity over the bulk of the FRC.

1. Introduction

The use of Rotating Magnetic Fields (RMF) to drive toroidal currents and both create and sustain Field Reversed Configurations (FRC) has a long history, primarily in rotamak devices [1]. Most analysis has been based on assuming a uniform plasma resistivity and a uniform electron rotational speed $\omega_e$ at some fraction $\zeta = \omega_e / \omega$ of the RMF frequency $\omega$ [2,3,4]. For a given RMF amplitude, $B_\omega$, this analysis produces a relationship between applied RMF torque, the plasma resistivity, and the achievable plasma density [5,6]. However, experimentally, when the resistivity was estimated based on the ratio of absorbed RMF power to the square of the driven current, serious discrepancies were noted. Much higher plasma densities and driven currents were produced than calculated, especially when taking into account FRC lengths $l_s$ greater than the RMF antenna length $l_a$ [7]. Under some low $\zeta$ drive conditions experimental features were also observed that could not be accounted for by simple uniform resistivity modeling; most notably the RMF not penetrating to the FRC magnetic field null. Although we have always modeled RMF drive in terms of total RMF and resistive torques, assuming that radial plasma flow could redistribute the RMF drive forces, it was difficult to understand how the FRC flux could be maintained with so little RMF drive at the null, where the RMF flow driven EMF, $v_\parallel \times B_z$, was unavailable.

The significance of the RMF penetration profile has become more apparent in recent experiments on the TCS (Translation, Confinement, & Sustainment) device [8] where the discharge pulse lengths were extended from 2.5 to 10 msec by increasing the size of the RMF capacitor bank from 30 to 60 kJ. It was then observed that the performance, as indicated by spontaneous switching between different ratios of external confinement field $B_e$ to $B_\omega$, would undergo abrupt changes. This was accompanied by abrupt switches in RMF profile $B_\theta(r)$ between shallower penetration (higher performance) and deeper penetration (lower performance). Recent calculations with non-uniform resistivities, which illuminate the ‘edge driven’ process [9], provide a mechanism for understanding this phenomenon. The plasma must have high resistivity in the driven, low density edge region, but much lower resistivity throughout the bulk of the FRC. Many features, such as a non-steady behavior with cyclical
RMF tearing and reconnection caused by vastly different edge and central electron rotational speeds, which are predicted by the non-uniform resistivity calculations, are seen in the experimental measurements [9]. A double rigid rotor model with different central and edge resistivities has been developed to analyze experiments, and is used to examine the above performance mode switching.

2. Basic Experimental Operation

TCS has a 2.6-m long, 80-cm internal diameter vacuum chamber, consisting of two 1.25-m long quartz tubes joined by a metal foil lined plastic ring containing vacuum access ports. Various measurements, such as internal magnetic field probing, CO₂ interferometry, and bolometry, are performed through these access ports. An axial bias field is provided by a parallel arrangement of 56-turn coils at an effective flux confining radius of 47 cm. The coils are powered by a small capacitor bank with a rise time of about 4 msec, and then crowbared. There are also copper straps placed directly on the vacuum vessel, with effective L/R field penetration times (in vacuum) of ~4 msec. When the RMF is activated in the presence of a slightly pre-ionized D₂ gas fill and initial axial bias field, an FRC is rapidly formed, compressing the external flux against the flux confining boundaries, and raising the external field. The basic experimental operation is indicated by the time traces of external and internal magnetic fields, and the traces of \( B_\omega \) (always scaled from the measured RMF antenna currents), shown in Fig. 1. The FRC internal field has approximately the same magnitude as the external field, which is typical of RMF driven FRCs. The operation is very insensitive to the initial puff gas fill pressure (within limits of about a factor of 3). Spectroscopic studies of C and O impurity lines indicate that the density is sustained by recycling after the first half msec, with a magnitude determined by the applied RMF torque and plasma resistivity.

The data shown in this paper were obtained using 1-m long strip antennas, which generally produced deeper penetration profiles than obtained with the original 1.6-m long low inductance tube antennas [7]. The RMF was run at a frequency of 105 kHz. In the discharge shown in Fig. 1 there is an initial startup phase, lasting about one-half msec, where a relatively higher value of \( B_x \) is obtained. This peak can be up to 50% higher than during the following period, which is attributed to an initially higher plasma temperature. The temperature is subsequently reduced by recycling impurity ingestion and strong radiation. After startup, the FRC settles into a phase we now call ‘lower performance’. However, after about 4 msec \( B_{\text{ext}} \) and \( B_{\text{int}} \) begin to vary in an oscillatory manner, and then settle into a ‘higher performance’ mode after 5-6 msec. This higher performance mode is always accompanied by the generation of a steady toroidal field of about 1-2 mT near the FRC field null (which is usually a precursor to the first jump in \( B_x \)). The behavior is highly repeatable, although the exact timing and number of transitions can vary somewhat from shot to shot.
The resistivity inferred from achieved densities and external fields is up to a factor of two lower during the higher performance mode.

$B_\omega$, and the resultant external field and FRC separatrix radius, decrease during the discharge, primarily due to multi-MW losses in the drive tubes of the RMF power supply. There is also some decay due to the $1.5 - 0.5$ MW absorbed by the FRC during the pulse, as seen by the differences between the plasma and vacuum traces of $B_\omega$ in Fig. 1. After the first half msec, bolometric measurements show that a large fraction of the absorbed power is radiated away, which limits the plasma total temperature to 20-40 eV. The peak plasma density decays from about $2 \times 10^{19}$ m$^{-3}$ to $1 \times 10^{19}$ m$^{-3}$ during the discharge, roughly proportional to the decay in $B_\omega$. It is seen however, that the ratio of $B_e/B_\omega$ increases, which is an unambiguous indication of higher performance.

3. Analysis

The azimuthal RMF force on the electrons, $\langle j e B_z \rangle$, is due to the RMF induced in-phase axial oscillation of the electrons interacting with the radial component of the RMF. This translates into an applied torque equal to

$$ T_{\text{RMF}} = \frac{2 \pi r_s \delta^* B_\omega^2}{\mu_0} \ell_a , $$

where $r_s$ is the FRC separatrix radius and $\delta^*$ is the RMF penetration distance [3]. Examination of the RMF internal structure in Fig. 2 gives an intuitive idea of why this is so. The RMF magnitude is approximately doubled over its vacuum value $B_\omega$ near the FRC separatrix due to the presence of the axial shielding currents $j_z$, but the forces act in a sine squared manner with respect to azimuthal angle. We analyze current drive in terms of torques since the RMF forces can be transferred to the FRC interior by various mechanisms, and must balance the total resistive retarding torque on the electrons. In the past, analytical expressions have been used for the RMF penetration distance $\delta^*$ ($\delta^* \sim (2 \eta/\mu_0 \omega)_{1/2}$, with $\omega = \omega_0 - \omega_e$), but these are inappropriate for a plasma with non-uniform resistivity and rotation velocity. We will use actual measured values of $\delta^*$, along with Eq. (1), in our data reduction analysis. Numerical calculations show this gives reasonable values for $T_{\text{RMF}}$ even with non-uniform resistivity.

As a first step we analyze experimental results for the retarding torque on the electrons based on a simple Rigid Rotor (RR) profile, with $B_s(r) = B_e \tanh K_{\text{RR}} u$, $n_e(r) = n_e \sech^2 K_{\text{RR}} u$ and $u = (r/R)^2 - 1$ where $R = r_s/\sqrt{2}$ is the location of the field null for an elongated FRC [12]. $K_{\text{RR}}$ is a form factor near unity. Assuming a uniform resistivity $\eta_{\text{RR}} = m_e e^2 / n_e$, the torque on the electrons is then

$$ T_{\eta_{\text{RR}}} = 0.5 \pi e^2 n_e^2 \tanh K_{\text{RR}} \left( 1 - \frac{1}{3} \tanh^2 K_{\text{RR}} \right) (\omega_e - \omega_i) r_s^2 \eta_{\text{RR}} \ell_s / K_{\text{RR}} . $$

We have included the ion rotation speed $\omega_i = 2 \pi f_i$ since RMF driven FRCs are observed to spin up to about $f_i \sim 9$ kHz in the driven electron direction. Calling $\omega_{\text{RR}} = \omega_e - \omega_i$, the peak value of external field is $B_e = \mu_0 e n_m \omega_{\text{RR}} r_s^2 / 4 K_{\text{RR}}$, and the torque can be expressed in terms of the external magnetic field, which is the most accurately measured quantity.

$$ T_{\eta_{\text{RR}}} = 2 \pi e n_m \tanh K_{\text{RR}} \left( 1 - \frac{1}{3} \tanh^2 K_{\text{RR}} \right) r_s^2 \eta_{\text{RR}} \ell_s (B_e / \mu_0) $$

FIG 2. Calculated internal RMF.
Equating Eqs. (1) and (3) yields a rigid rotor value for the average plasma resistivity.

\[
\eta_{RR} = \frac{B^2_m(\delta^*/r_s)\ell_a/\ell_s}{en_m\tanh K_{RR}(1-\frac{1}{3}\tanh^2 K_{RR})B_s} = \frac{1020B^2_m(\text{mT})(\delta^*/r_s)\ell_a/\ell_s}{n_m(10^{19}\text{ m}^{-3})B_s(\text{mT})} \mu\Omega \cdot \text{m}
\]  

(4)

where we have taken \(K_{RR} = 1\), which is an appropriate fit for the measured profiles. The RR profile has an edge value \(B_z(r_s) = B_etanh K_{RR}\) and the above analysis does not take into account the resistive torque exerted by the edge plasma, where \(B_z\) increases to \(B_e\) and \(n_e(r)\) drops from \(n_{me}sech^2 K_{RR}\) to zero.

If we calculate the resistivity based on the measured absorbed power, as has been done in the past [5,6], the ratio of resistivity measured in this way, to that of Eq. (4), is given by \(P_{\text{abs}/(\omega e T)}\) (we use \(\omega e\) rather than \(\omega RR\) for power calculation since the RMF force acts on the electrons). This ratio always considerably exceeds unity, which implies that the resistivity cannot be uniform. The recent numerical calculations of Ref. 9 show that the resistivity must be much higher in the edge region than in the center in order to account for many of the observed data. We have thus developed a double rigid rotor (DRR) model, with an interior rotation speed difference between ions and electrons of \(\omega_{ri}\), and an edge rotation \(\omega_{re} = \omega - \omega_{ri}\). The ion rotation is assumed uniform, consistent with Doppler broadening measurements [5]. We assume the inner region extends out to a location \(u^* < 1\). This value, and the value of \(\omega_{ri}\), are determined from requiring the DRR profile to have the same \(\langle \beta \rangle\) as measured from internal probes, and making it produce the measured \(B_e\). For a given value of \(K_i\) for the inner region, the edge region \(K_e\) can be determined by matching \(n_e\) and \(B_z\) at \(u^*\). The solutions can be obtained from graphical plots of \(u^*\) for \(K_i\) chosen to best match the actual profile. We use \(K_i = 1\), but the final results have very little dependence on this choice. \(u^*\) always turns out to be between 0.6 and 0.85, corresponding to \(r^*\) between 1.27 and 1.36. Equilibrium constraints thus result in a very narrow edge region where the density decreases to near zero, in agreement with observations for our RMF driven FRCs. This contrasts with the simple \(K_{RR} = 1\) RR value of \(n_e(r_s)/n_m = sech^2 1 = 0.42\).

Calling \(F(x) = tanh(x(1-\text{tanh}^2 x)/3)\) and \(\eta_{i}\) and \(\eta_{e}\) the resistivities in the inner and edge regions respectively, the DRR torque is calculated as

\[
T_{\eta_{\text{DRR}}} = 0.5\pi e^2 n^2_m \omega_{ri}^4 \left(F(K_iu^*)/K_i\right) \left(\eta_0 + (\omega_{ri}K_{i} / \omega_{re}K_{e})\left(F(y) \left(F(K_iu^*)\right) - 1\right)\right) \ell_s
\]  

(5)

\(K_e\) is always fairly large (to bring the separatrix density down to near zero, and the parameter \((\omega_{ri}K_{e}/\omega_{re}K_{i})\) is generally about 1.5. The parameter \(y (B_s(r_s) = B_stanh y)\) is also determined from the profile and matching conditions, and is always between 1.5 and 2.5. If we divide the torque into inner and outer contributions, \(T_{\eta_i}\) and \(T_{\eta_e}\), the absorbed power is given by

\[
P_{\text{abs}0} = (\omega_{ri} + \omega_{re})T_{\eta_i} + \omega_{re}T_{\eta_e}
\]  

(6)

Equating Eq. (5) to Eq. (1), and Eq. (6) to the measured absorbed power, gives us simultaneous equations for the inner and outer resistivities. We have used \(P_{\text{abs}0}\) to represent the portion of absorbed power attributable to the azimuthal currents, and the largest source of error in the analysis lies in choosing this fraction. We use \(P_{\text{abs}0} = P_{\text{abs}}/(1 + (B_{yi}/B_y)^2)\), which is fairly arbitrary, but choosing the full measured \(P_{\text{abs}}\) in Eq. (6) sometimes yields negative values of \(\eta_i\), and reducing \(P_{\text{abs}0}\) further (by say \(1 + (r^*/\delta^*) (B_{yi}/B_y)^2\)) sometimes gives unrealistic results with \(\eta_e\) less than the average value. In the data reduction we will use these two limits to place error bars on the calculations of \(\eta_i\).
4. Resistivity Measurements

The internal magnetic field profiles $B_\theta(r)$ and $B_\phi(r)$ (assumed to be represented by the measured $B_x$), are shown in Fig. 3 for four times in a particular discharge. The profiles were obtained with an array of 29 single axis internal probes inserted from the top of TCS and oriented at 45° to the plasma tube centerline so that they were sensitive to both $B_z$ and $B_x$. The component at the 105 kHz RMF frequency was assumed to be $B_0$ and the remainder $B_f$. The frequency content of $B_z$ alone, obtained with the probe oriented in the $z$ direction for a subsequent discharge, is also shown in the figure.

On careful examination there is a subtle but distinct and highly repeatable difference in the $B_0$ profiles during the lower and higher performance phases. During the lower performance phase $B_0$ extrapolates smoothly to zero somewhere inside the field null. This is as would be expected from simple, uniform resistivity calculations. Based on 2-dimensional $(r;\theta)$ calculations it is not understood why some 105 kHz signal reappears inside the field null, but it is probably due to 3-dimensional effects since axial oscillatory currents on outer field lines should be reflected by similar currents on their inner field line connection. During the higher performance phase, the $B_0$ profile extrapolates to zero well outside the FRC field null. This is seen to produce a relatively higher value of externally generated field $B_\phi$, and the drive performance, as characterized by the ratio of $B_\phi/B_0$, continuously improves once the higher performance phase becomes steady, as the RMF magnitude decreases.

The average difference between electron and ion rotation frequencies needed to produce the measured $B_\phi$ based on the measured density profile is about 35 kHz for all the long pulse data. The calculated DRR central value of $f_{\text{ei}} = \omega_{\text{ei}}/2\pi$ is about 25 kHz. The contour plots of frequency content show a distinct peak at ~30 kHz, which is seen in the non-uniform resistivity numerical calculations (generally showing up when the outer resistivity is specified to be about ten times the inner resistivity) due to an inner electron rotation at $f_{\text{ei}} + f_i$ [9]. In the calculations this frequency is due to some inner RMF rotating at $f_{\text{ei}} + f_i$ and

![FIG 3. Profile data for discharge 12939 with $B_z$ frequency data for discharge 12957.](image)
continually tearing and reconnecting with the outer RMF rotating at the full \( f_{\omega} \). The non-steady behavior produces a non-steady torque, which also results in an oscillation in all \( B_0(r) \) and \( B_r \). The behavior also produces sidebands about \( f_{\omega} \) which are clearly seen in both \( B_r \) and \( B_0 \). The \( f_n + f_i \) and sideband oscillations are much more prominent during the low performance phase.

Resistivities inferred from the simple RR calculations and from the DRR model for the four times shown in Fig. 3 are listed in Table 1, along with data from lower (red entries) and higher performance modes for two other discharges. The values of \( \delta^*/r_s \) are taken from the \( B_0(r) \) traces based on the position where \( B_0 \) decreases to one-half its peak value.

Since the inferred resistivity is linearly dependent on this value, there is some room for error, but the maximum possible RMF torque given by analytic calculations \([3]\) corresponds to an effective value of \( \delta^*/r_s = 0.38 \), and the procedure is reasonable. The listed value or \( \eta_{RR} \) is given by Eq. (4). \( \eta_{inner} \) and \( \eta_{outer} \) are measured accurately, but our cross-tube CO2 interferometer is only valid for the first 2 msec due to vibrations, and the line density values for later times are scaled from cross-tube continuum radiation measurements. The peak density is calculated from the pressure profile, including contributions from both the ion centrifugal force and the RMF \( j_z B_0 \) radial force, assuming the total temperature is uniform. This procedure should be reasonably accurate, even after the formation phase, since the density doesn’t vary much. It is notable that \( \eta_{abs} \), along with \( B_c \), increases during the transition from the lower to higher performance mode.

The values of \( \eta_{pabs} \) are given by the RR formula \( (P_{abs}/\omega_e T_{RMF})\eta_{RR} \) where \( \omega_e \) is taken as \( \zeta \omega + \omega_i \). \( \zeta \) is determined from internal probe measurements and the measured line density, and is between 0.29 and 0.35 for all conditions except the 9.3 msec entry, where it is 0.24. It can be seen that \( \eta_{pabs} \) always exceeds \( \eta_{RR} \) by at least a factor of two, which we infer is attributable to a high edge resistivity where most of the RMF power (always over 50%) is absorbed. The greater the difference, the greater will be the calculated difference between \( \eta_i \) and \( \eta_e \) in our DRR modeling. It can be seen that during the higher performance mode \( \eta_i \) is generally under 30 \( \mu \Omega \)-m. This value is reasonable given the relatively high ratios \( v_{Dr}/v_{ti} \) of electron drift velocity to ion thermal speed \( (v_{Dr} = \omega_e r \sim 5 \times 10^4 \text{ m/s near the field null and } 2 \times 10^5 \text{ m/s near the edge, with } v_{ti} \sim 3 \times 10^4 \text{ m/s everywhere}) \), which tend to lead to drift wave micro-turbulence. The much higher ratios of \( v_{Dr}/v_{ti} \) in the outer edge, along with any RMF induced turbulence, would be expected to result in much higher resistivities.

The inner and outer resistivities calculated using the DRR model are plotted in Fig. 4 versus the simple RR resistivity \( \eta_{RR} \). Error bounds are shown on \( \eta_i \) based on the various assumptions mentioned earlier of how much of \( P_{abs} \) is assigned to \( P_{abs0} \). The leftmost

<table>
<thead>
<tr>
<th>Shot</th>
<th>( \delta^*/r_s )</th>
<th>( n_m ) (10^19m^-3)</th>
<th>( \eta_{m} ) (( \mu \Omega )-m)</th>
<th>( \eta_{pabs} ) (( \mu \Omega )-m)</th>
<th>( \eta_{inner} ) (( \mu \Omega )-m)</th>
<th>( \eta_{outer} ) (( \mu \Omega )-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12939 0.4 ms</td>
<td>0.16</td>
<td>1.70</td>
<td>109</td>
<td>260</td>
<td>30</td>
<td>235</td>
</tr>
<tr>
<td>12889 6.3 ms</td>
<td>0.23</td>
<td>0.95</td>
<td>269</td>
<td>425</td>
<td>204</td>
<td>291</td>
</tr>
<tr>
<td>12951 4.2 ms</td>
<td>0.21</td>
<td>1.12</td>
<td>212</td>
<td>404</td>
<td>127</td>
<td>333</td>
</tr>
<tr>
<td>5.0 ms</td>
<td>0.18</td>
<td>1.38</td>
<td>114</td>
<td>349</td>
<td>3</td>
<td>351</td>
</tr>
</tbody>
</table>

TABLE 1: RESISTIVITIES INFERRED FROM DRR MODEL

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grouping corresponds to the higher performance mode, or to the startup condition for discharge 12939 (orange dashed lines). The blue diamonds are based on assuming $P_{\text{abs}\theta} = P_{\text{abs}}$, which clearly leads to non-physical negative values of $\eta_i$. The upper limit orange triangles are also somewhat non-physical since, in the lower performance cases they sometimes lead to values of $\eta_e$ being less than $\eta_i$. Only the most likely value of edge resistivity is shown.

Lower central resistivities during the higher performance phase could be the reason why the RMF penetration is shallower. The lower resistivity would also correlate with the smaller $f_i + f_t$ frequency content seen in the $B_z$ frequency content contour plots in Fig. 3, since less inner torque would be required to maintain the plasma current there. In the latter stages of the discharge a strong component appears at 18 kHz. This is exactly twice the ion rotation frequency, and is due to an $n=2$ distortion which normally develops for such rapidly rotating FRCs. The radial pressure imposed by $B_\omega$ has been measured to have a strong stabilizing effect on the rotating $n=2$ instability [11], but the ion rotation frequency is almost independent of the magnitude of $B_\omega$. It is thus not surprising that the $n=2$ distortion develops as the RMF magnitude runs down.

5. Summary and Conclusions

It is possible to both form FRCs and sustain them in quasi steady-state using rotating magnetic fields. A much better understanding of the current drive process has been obtained through the use of detailed profile and frequency spectrum measurements coupled with numerical calculations. The RMF drive process appears to be characterized by strong drive in the edge region near the FRC separatrix, accompanied by fairly high plasma resistivity. The measured interior and edge values are fairly typical of Chodura type resistivity ($\eta \sim (1000/n_e^{1/2}(10^{19} \text{ m}^{1/2}))[1-\exp(v_{De}/f_{ni})] \mu\Omega\cdot\text{m}$ with $f \sim 3$) which was used to model previous FRC flux decay [13].

A higher performance mode appears to develop spontaneously for longer RMF pulse lengths, at least under the circumstances under which the TCS experiments were run. A trigger for this mode has not been identified, but it may be related to the aforementioned appearance of toroidal field. The net result appears to be fairly low interior plasma resistivities. Plasma turbulence is minimized when the RMF magnitude is small compared to the axial confinement field. These conditions were only achievable in TCS at low values of $B_\omega$ since the plasma temperature, which along with plasma density sets the diamagnetic current in high
beta FRCs, was held low by impurity radiation. Better performance at higher values of $B_\omega$ should be achievable if impurity content can be lowered and temperatures raised.

The edge drive properties of RMF, along with the resultant high edge resistivity and high edge thermal deposition, may rule out RMF as a stand-alone FRC current drive mechanism. However, RMF can be extremely useful for startup, and has been seen to greatly enhance particle lifetimes and interchange stability. It should be ideally suited to be used in conjunction with tangential neutral beam injection (TNBI) current drive near the field null, since the RMF torque, which leads to plasma rotation, could be balanced by TNBI momentum injection in the opposite direction. It may even be possible to influence the azimuthal current and velocity profiles, which could lead to controllable enhancements of performance rather than the spontaneous transitioning between performance modes seen in the pure RMF drive experiments. Monte-Carlo TNBI calculations show that to effectively utilize TNBI the TCS flux levels will have to be raised from present 2 mWb levels (when using the low inductance RMF antennas) to over 5 mWb [14]. This can be done if the plasma temperature can be increased to the 100 eV level. Modifications to minimize impurities are currently underway.

REFERENCES