Potential Control and Flow Generation in a Toroidal Internal-Coil System -- a New Approach to High-beta Equilibrium --

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Abstract. Potential control and flow generation have been studied on the Proto-RT device that is equipped with an internal ring coil producing a stationary magnetic field. Biasing the surface of the internal coil, the radial electric field is controlled. Supersonic flow has been generated when the plasma is negatively biased. The present experiment is limited in a low-density (~ 10^{14} m^{-3}) regime, and hence, only electrostatic effects dominate the structure and stability. Higher density plasma will be produced in a new device Mini-RT that has a super-conducting levitated internal ring. When the flow velocity becomes comparable to the Alfven speed, the hydrodynamic pressure can produce a new type of high-beta (diamagnetic) equilibrium, so-called “double Beltrami (DB) state”. Recent theory predicts “self-organization” of a DB state that may have Lyapunov stability. This regime is the target of the new device Mini-RT that is equipped with a super-conducting levitated internal coil system.

1. Introduction

A fast flow in a plasma brings about a variety of interesting effects on the equilibrium, stability, turbulence and self-organization phenomena. Plasmas in the universe are almost always flowing, and large hydrodynamic forces often dominate their structures. For example, the Jupiter magnetosphere, which is co-rotating with the planet at an Alfvenic speed, confines a very high-beta (>1) plasma. Interesting hydrodynamic effects stem from inhomogeneity of a flow. A shear flow stretches perturbations, resulting in transport of wave numbers, phase mixing, transient excitations, and so on. So-called “transport barriers” are accompanied by shear flows, suggesting that small-scale turbulences are quenched by the shear-flow effect. On the contrary, a shear flow adds a new source of energy to excite instabilities –cousins of the Kelvin-Helmholtz (KH) instability. In a plasma, the coupling of the KH-type modes and the conventional instability/wave modes are not so simple, and the notion of dispersion relation falls short to predict transient and long-term behavior of perturbations. The complexity of a flowing plasma may reduce in some self-organized states that are, however, far richer than those without flow, because of the nonlinear coupling of the flow and magnetic fields [1,2].

In this paper, we describe a theoretical prediction (Sec. 2) and some experimental observations (Sec. 3) of a new regime of plasma confinement that is characterized by a fast cross-field flow. The experiment was done on the Proto-RT device [3] that has an internal ring coil and can produce stationary magnetic-surface configurations. Biasing the surface of the coil, as well as other electrodes, we can control the electric field. In the present experiment, however, the plasma parameters still remain in the electrostatic regime because of the low density (~ 10^{14} m^{-3}). If the density is raised (>10^{17} m^{-3}) and the flow velocity becomes comparable to the Alfven speed, the hydrodynamic pressure can produce a new type of high-beta (diamagnetic) equilibrium. This regime is the target of a new device Mini-RT [4] that has a super-conducting levitated internal coil. Preliminary results from the Mini-RT experiment will be presented in Sec. 4.

2. Theoretical Predictions

2.1. Equilibrium with Fast Flow

The aim of this project is to explore a new regime of plasmas where flow effects dominate the equilibrium, stability and nonlinear phenomena. The standard magnetohydrodynamics (MHD) model omits some essential characteristics of flowing plasmas. Invoking the two-fluid (Hall) MHD model, we may discover far richer structures characterized by fast flows (with perpendicular velocity components with respect to the magnetic fields) and a wide scale hierarchy.

It is often convenient to view a plasma as a complex system consisting of “vortex quanta”. A general solenoidal vector field, such as a magnetic field or an incompressible flow, can be decomposed into an orthogonal sum of Beltrami fields, the eigenfunctions of the curl operator [5]. Nonlinear dynamics of a plasma induces couplings among these Beltrami fields. In the MHD plasma model, however, the energy condensates into a single Beltrami magnetic field [6] resulting in the self-organization of a force-free equilibrium, that is, the Taylor relaxed state. By relating the velocity and the magnetic fields, the Hall term in the two-fluid model leads to a singular perturbation that enables the formation of a “double Beltrami (DB) field” given by a pair of two different Beltrami fields (belonging to two eigenvalues λ_+ and λ_-). This new set of relaxed states, despite the simple mathematical structure, includes a variety of plasma states such as a high-beta equilibrium [1].

In contrast to the conventional MHD model, the two-fluid plasma equilibrium is characterized by two different potentials ψ (magnetic flux function) and ϕ (stream function) that are governed by a coupled generalized Grad-Shafranov equations [7]:

$$-\left[r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)+\frac{\partial^2}{\partial z^2}\right]\psi=\frac{\partial}{\partial\psi}F(\psi,\phi)+r^2\frac{\partial}{\partial\psi}P(\psi,\phi),$$
$$-\left[r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)+\frac{\partial^2}{\partial z^2}\right]\phi=-\frac{\partial}{\partial\phi}F(\psi,\phi)-r^2\frac{\partial}{\partial\phi}P(\psi,\phi).$$

The DB fields are solutions for the case of

$$F(\psi,\phi)=\frac{1}{2}\left[\left(\phi-a^{-1}\psi\right)^2-\left(b\phi-\psi\right)^2\right],\quad P(\psi,\phi)=\text{constant}.$$

2.2. Stability of Flowing Plasmas

In a flowing plasma, inhomogeneity of vorticity distribution provides a free energy to excite Kelvin-Helmholtz (KH)-type modes. On the contrary, flow shear brings about the stretching and mixing effects that yield stabilizing effects on some different modes [8,9]. While the couplings between the conventional MHD modes and the KH modes are rather complicated, we may draw a simple stability diagram for a Beltrami equilibrium such that

$$\nabla \times \boldsymbol{B} = \lambda \boldsymbol{B}, \qquad \mathbf{V} = \alpha \mathbf{B}, \tag{1}$$

where λ (scaling the shear) and α (scaling the flow velocity) are real numbers. Solving the ideal MHD dispersion relation for slab geometry, we obtain the necessary and sufficient conditions for stability as summarized in Fig. 1 (a) [10]. We find that the KH mode, which is unstable when $|\lambda| > |\lambda_0|$ (the smallest eigenvalue of curl operator), is stabilized by the magnetic field if $\alpha^2 < 1$, i.e., the flow is sub-Alfvenic. We note that this slab geometry inhibits kink modes.

For a Beltrami equilibrium (1), we may extend the framework of analysis for general three-dimensional geometry, as well as for general non-exponential transient/long-term behavior. The latter issue is essential in a flowing plasma, because the generator of the linear dynamics is no longer a Hermitian (self-adjoint) operator, and hence, the dispersion relation (normal mode analysis) is not sufficient to predict stability of fluctuations. Here we may invoke a Lyapunov function constructed from some constants of motion [11,12]. The Beltrami equilibrium (1) is characterized by a variational principle for the functional combining three different constants of motion, i.e., the energy $E(\boldsymbol{B},\boldsymbol{V})$, magnetic helicity $H_1(\boldsymbol{B})$ and cross helicity $H_2(\boldsymbol{B},\boldsymbol{V})$. Due to this variational principle, a certain combination of $E(\tilde{\boldsymbol{B}},\tilde{\boldsymbol{V}})$, $H_1(\tilde{\boldsymbol{B}})$, $H_2(\tilde{\boldsymbol{B}},\tilde{\boldsymbol{V}})$ turns out to be a constant of motion for the perturbations $\tilde{\boldsymbol{B}}$ and $\tilde{\boldsymbol{V}}$. This constant, determined by the initial condition, can yield a bound for the energy norm of the perturbations (we say "coercive"), when $|\lambda| < |\lambda_0|$ and $\alpha^2 < 1$ (see Fig. 1 (b)). The bound for the energy of the perturbations implies a sufficient condition of absolute stability (including any non-exponential behavior).

The analysis of the Lyapunov stability of the DB fields is more complicated, because the Hall MHD demands a bound for the enstrophy of the perturbation. While the enstrophy is not bounded in general DB fields, we find bounds in two different classes of DB fields; (1) a longitudinal flow with a spiral magnetic field, (2) a longitudinal magnetic field with a spiral flow. In both cases, two Beltrami parameters (eigenvalues) satisfy $\lambda_+ = -\lambda_-$ $(=\lambda)$. The sufficient condition for the stability is, for (1), $\lambda^2 < \lambda_0^2 - 1$, and for (2), $\lambda^2 < \lambda_0^2$ [13].

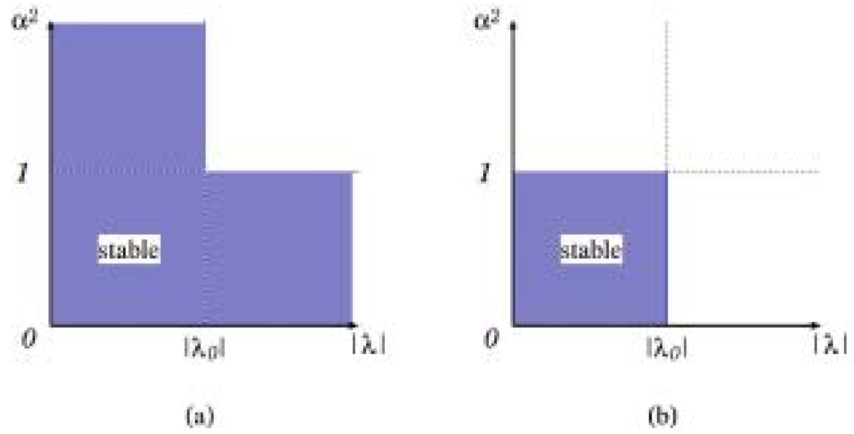


FIG. 1. (a) Necessary and sufficient condition for exponential stability of Beltrami flows in slab geometry. (b) Sufficient condition for absolute stability of Beltrami flows in an arbitrary geometry.

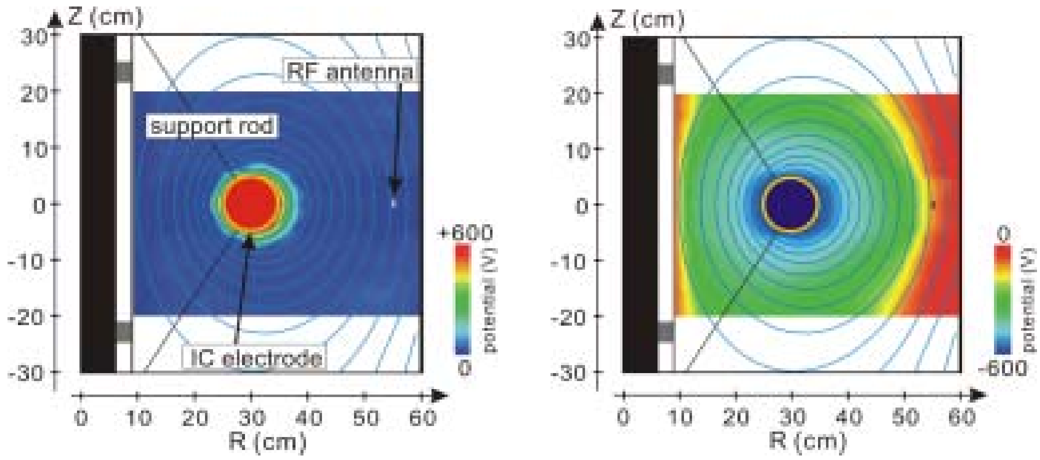


FIG.2. Two-dimensional potential profiles when the IC electrode is positively (left) and negatively (right) biased.

3. Experiments on Proto-RT

An internal coil device has various advantages in studying interesting effects of flows in magnetized plasmas. Proto-RT [3] has an internal ring coil producing dipole magnetic field (~ 10^{-2} T) and external coils providing vertical and toroidal fields. We started basic studies on various plasma flows with producing pure electron plasmas. By its self-electric field, a non-neutral plasma produces an $\boldsymbol{E}\times\boldsymbol{B}$ -drift flow. With appropriate control of the electrostatic potential, the equi-potential surfaces approximate the magnetic surfaces, and then, we observe a long confinement time (> 0.1 sec) of the pure electron plasma [14].

By inductively coupled 13.56MHz RF discharge, we can also produce low-density electron-ion plasmas, and, applying the potential control, we may produce a strong flow [15]. We have generated a supersonic flow by biasing the surface of the internal coil. Figure 2 shows the two-dimensional structure of the electrostatic potential (approximates the stream function ϕ of the flow). We observe that the potential contours self-regulate to coincide with the magnetic surfaces (calculated). The potential distribution is sensitive to the magnetic surface configuration. In Fig. 3, we compare a simple dipole and separatrix configurations. In the latter configuration, we observe a boundary layer of localized potential gradient (flow shear) near the separatrix of the magnetic surfaces.

Figure 4 shows the radial profiles of the potential with different biasing of the coil surface. With negative biasing, we can produce a smoothly distributed electric field across the plasma. Positive biasing, however, yields a gap (low-density layer) between the electrode and the plasma (see also Fig. 2 (a)). This is because the radial current (primarily ion current) sweeps the ions away from the coil surface. The relative permittivity of the plasma is of order 10^2, and hence, the electric field concentrates into the gap.

In the present experiments where only the poloidal magnetic fields were applied, the radial electric field yields toroidal rotation of the plasma. Neglecting the pressure term in the equation of motion for ions and electrons, the velocity perpendicular to the magnetic field is

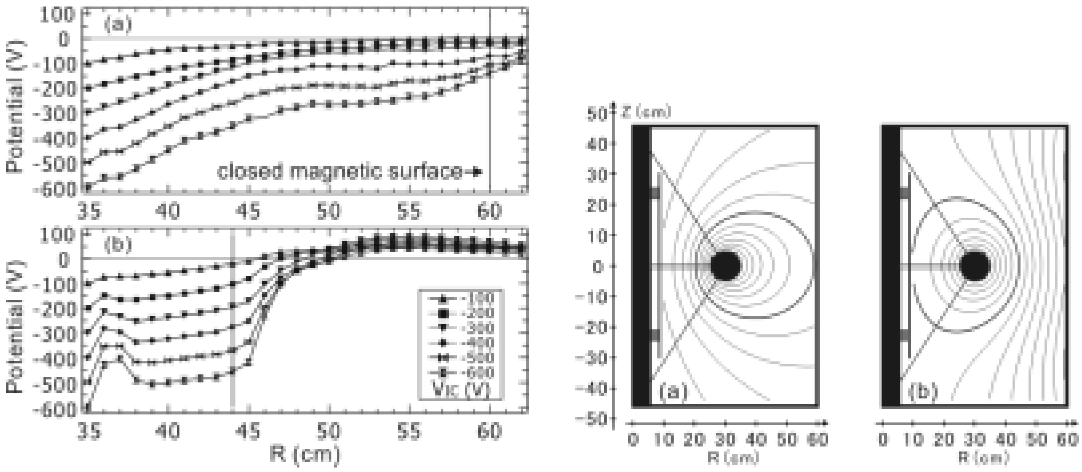


FIG. 3. (Left) Radial potential profiles at Z=0 in the variation of electrode bias V_{IC},, in (a) pure dipole field (I_{IC}=7kAT) and (b) dipole and stronger vertical field (I_{IC}=7kAT, I_{VF}=4.2kAT) configurations. (Right) Magnetic surfaces when (a) I_{IC}=7kAT, I_{VF}=0kAT, and (b) I_{IC}=7kAT, I_{VF}=4.2kAT.

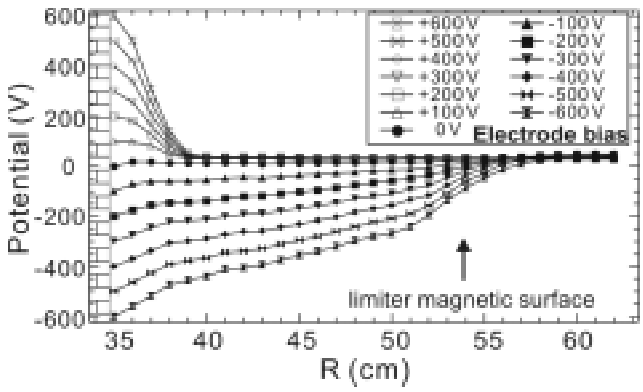


FIG.4.. Radial distribution of the electrostatic potential with different biasing.

given by

$$\boldsymbol{V}_{\perp} = \frac{q\nu_{nk}\boldsymbol{E}}{m_k\omega_{ck}^2} + \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2} \quad ,$$

where k indicates either ion or electro, q is the charge, $\nu_{\rm nk}$ is the collision frequency between neutral particles and charged particles, m_k is the mass of charged particles, and ω_{ck} is the cyclotron frequency. The first term on the right-hand side estimates to the radial motion of charged particles across the magnetic surfaces, and the second term, the toroidal drift speed. We found that the $\boldsymbol{j}\times\boldsymbol{B}$ torque due to the radial ion current balances the collisional friction force between the flow and the neutrals (neutral density is of order 10^{19} m^{-3}). The internal electric field (~ 10^3 V/m) yields the $\boldsymbol{E}\times\boldsymbol{B}$ drift speed of order 10^5 m/sec that is much higher than the ion sound speed.

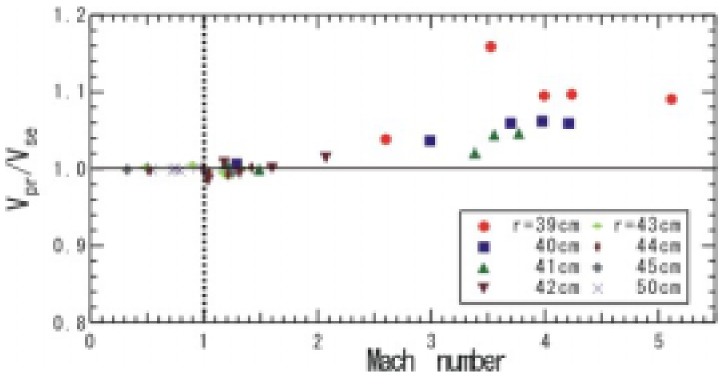


FIG. 5. Comparison of the potentials estimated by emissive probes (V_{pr}) and a photo-electric sensor (V_{se}). Here, V_{pr} and V_{se} are the potential difference between the biased internal coil and the measuring points.

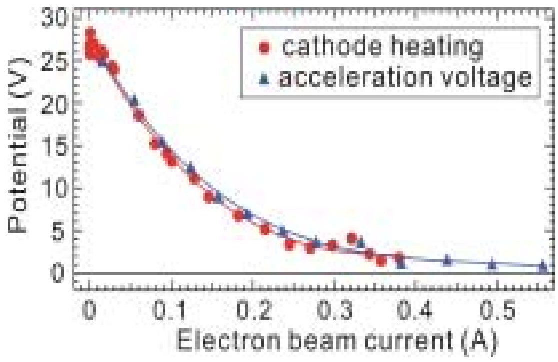


FIG.6. Potential control by electron injection. The electron beam current was controlled both by changing the cathode temperature and the acceleration voltage. Potential was measure at r=40 cm (10cm inside from the gun).

When the flow becomes super sonic, we observe anomalous response of the emissive probe (Fig. 5). Using a photo-electric (Pockels) sensor, we measured the local electric field, and compared the estimated potential with the floating potential obtained from the emissive probe. When the ion-sound Mach number exceeds unity, we observe that the probe under-estimates the space potential significantly, suggesting formation of a shock-wave potential near the surface of the probe.

Electron injection has been also tested to control the potential of the plasma. In Fig. 6, we plot the internal potential as a function of the electron beam current. The plasma potential can be reduced, while a negative potential was not achieved, suggesting anomalous loss of electrons.

4. Experiments on Mini-RT

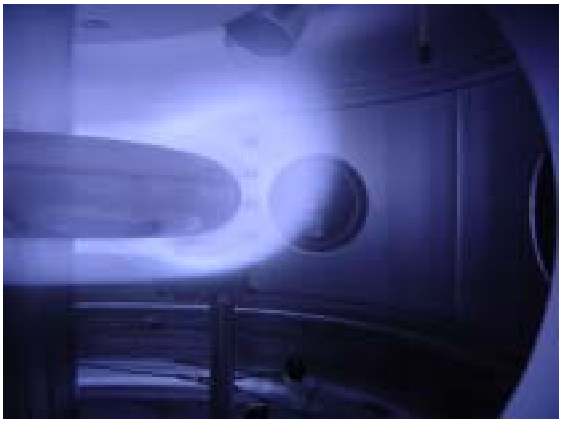
The Mini-RT device has a high-temperature super-conductor (HTS) levitated coil (a silver-sheathed Bi-2223 tape with a critical current of 108 A at 77 K). The major radius of the coil is 0.15m, and the current is 50kAturns, yielding a typical magnetic field of order 0.1T

[4,16]. The coil position, which was monitored with laser sensors, was feedback-controlled with the accuracy of less than 20 micrometers. Stable levitation has been demonstrated during one hour [17].

Figure 7 (a) shows a photograph of the plasma produced by 2.45 GHz Electron Cyclotron Heating (ECH) at the levitated coil condition. As shown in Fig. 7 (b), the magnetic surface combined with the levitation coil has a separatrix at the upper region of the vacuum vessel, and the plasma shape similar with this magnetic surface was produced.

We have carried out plasma experiments for two cases; i.e., mechanically supported and levitated cases for the HTS coil. Figure 8 shows the profiles of the electron density and the electron temperature measured by the double probe for both cases. The HTS coil surface is located at R = 212mm, and the vacuum vessel surface is at R=500mm. In the case of the mechanically supported coil, the plasma extends widely in a simple dipole magnetic field. The electron density was of order 10^{16} m^{-3}, and the density profile had a peak near the internal coil. The electron temperature was of order 10 eV with a broad profile. When the coil is levitated, remarkable improvement of the plasma parameters was observed in R= 240~250mm (inside the separatrix); The electron density (10^{17} m^{-3}) was 3-4 times as high as that of the previous case, and the electron temperature increased up to 20 eV. The energy confinement (of order $10^{-(5\sim6)}$ sec) is comparable to the electron-neutral collision time, because the operation gas pressure was still high (~ 10^{-2} Pa).

As shown in Fig. 8 (a), the electron density exceeds the cut-off density ($7.6\text{x}10^{16}$ m^{-3}) for the 2.45 GHz microwave, suggesting that the mode conversion to the Electron Bernstein Wave (EBW) occurred [18]. The efficiency of the mode conversion increases in a weak magnetic field and a sharp density gradient. In the present experiment, the magnetic field strength is order of 0.1T, and the scale length of the density gradient is of order 1 cm at the separatrix. Using these parameters, we estimate the mode conversion efficiency to be of order 0.1.



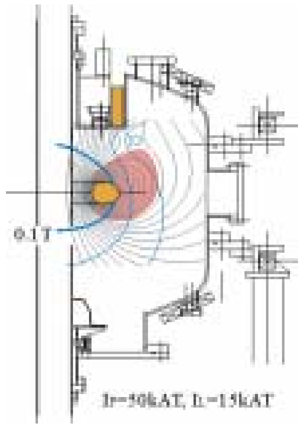


FIG. 7. (a) A photograph of a plasma produced by 2.45 GHz microwave, and (b) a corresponding magnetic field configuration. The 2.45 GHz microwave was injected through the inclined upper port.

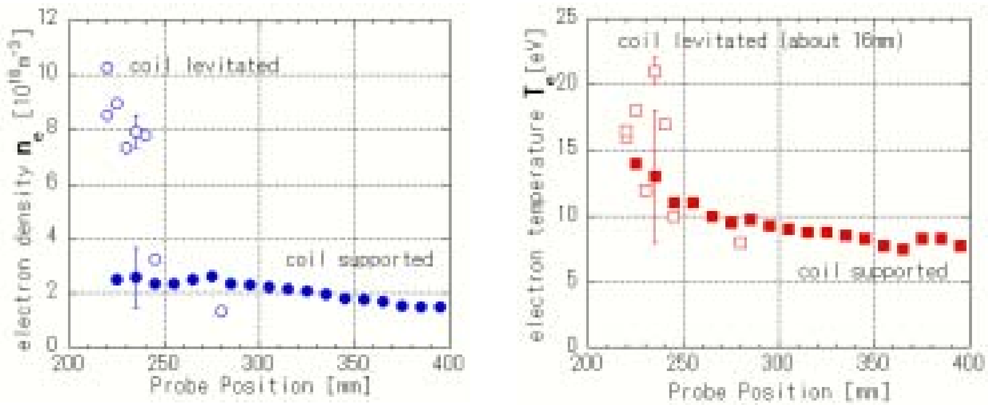


FIG. 8. Spatial profiles of (a) an electron density and (b) an electron temperature for mechanically-supported and levitated coil cases. The HTS coil surface is located at R = 212mm, and the vacuum vessel surface is at R=500mm.

5. Summary

Two-fluid (Hall) MHD theory predicts a new type of high-beta equilibrium that dominated by a combination of electromagnetic and hydrodynamic forces. Internal-conductor systems have been constructed to explore this new regime of plasma confinement; the dipole magnetic-field configuration mimics Jupiter's magnetosphere that confines a very high-beta plasma. Applying a radial electric field, we have generated supersonic flow in the Proto-RT device. While the flow effects are still in the electrostatic regime, we observe interesting phenomena such as shock formation near probes. A super-conductor device Mini-RT is aiming at producing higher-density plasmas where the flow velocity becomes comparable to the Alfven speed and electromagnetic effects dominate the equilibrium and stability.

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