**Equilibrium Reconstruction of KSTAR Plasmas with Large Uncertainty on Magnetics**

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**Abstract.** For magnetically confined plasmas such as in tokamaks and stellarators, understanding its magnetics is a basic and essential element for better physical understanding of experiments. However in the last two year campaigns of KSTAR, it was not as usual as in other devices, although all of campaign missions had been successfully conducted by achieving 350kA circular plasmas of 3.5 sec discharge length with 1.5 sec flattop. It's conjectured due to (1) large uncertainties on magnetic measurements such as non-linear drifts and (2) the existence of a ferromagnetic material (called as Incoloy 908) in PF and TF coils that adds significant non-linear uncertainties on magnetics. And the most critical one in view of equilibrium analysis is (3) the existence of unidentified up-down asymmetric field sources. In order to get a reliable equilibrium reconstruction under these obstacles, a new code has been developed based on a simplified plasma model, and applied to equilibrium analysis of KSTAR plasmas for 2009 campaign. As results, reconstructed position and shape of plasmas are described in comparison with CCD image analysis by a newly developed image analysis technique, and some results of basic stability analysis are described in view of plasma operation control. Finally the effects of ferromagnetic material on magnetics are discussed based on field reconstructions.

**1. Introduction**

Korea Superconducting Tokamak Advanced Research (KSTAR) [1] has been successfully operated during last two year campaigns since the first plasma in 2008. Plasma operation was extended from the first plasma of 100kA to 350kA circular plasma with 3.5 sec discharge length and 1.5 sec flattop, and many of magnetic diagnostics were newly installed and upgraded. However, the magnetics of KSTAR plasmas was not close to our predictions and understandings. First of all, magnetic perturbations from ferromagnetic materials were significant, so that it was difficult to make stable plasma start-up and current ramp-up. And there were unidentified magnetic field sources, so that all plasmas from the first one were produced downward shifted by about 10 cm from the mid-plane with up-down symmetric PF coil currents applied. Also some of magnetic measurements had large non-linear drifts and noises. Particularly most of magnetic flux loop measurements are not available for use in analysis due to the signal’s saturation.

Meanwhile, many efforts to understand the equilibrium of KSTAR plasmas, particularly using an EFIT code [2], were devoted. However, this first attempt was not satisfactory. Both the convergence and accuracy of EFIT runs were not good although a simple polynomial representation of plasma was used. It was conjectured that the plasma modeling used in the EFIT code has more freedoms than one that can be solved under constraints in 2009 campaign of KSTAR. Therefore, a new equilibrium analysis code has been developed using a simplified plasma model.

**2. Developed Formulations for Equilibrium Reconstruction**

Equilibrium reconstructions of axi-symmetric tokamak plasmas are usually formulated as a least-square minimization problem as follows.
Here \( J_{q,\text{plasma}} = J_{q,\text{plasma}}(R, Z) \) is the plasma current distribution to be found. Therefore it means that we find the optimal location and distribution of plasma currents that minimize the total fitting error to diagnostic measurements. Since the main reason of difficulty on equilibrium reconstruction for 2009 campaign plasmas in KSTAR is conjectured due to too many freedoms on plasma model, we adopted simplified plasma models in this study.

At first, as a simplest model, plasma is assumed as a single current-carrying toroidal filamentary ring and implemented to a code named as IDK-RZIp. In this case, the above minimization problem can be described as follows.

\[
\min_{I_{\text{plasma}}} E_{\text{total}}(J_{q,\text{plasma}}) = \min_{I_{\text{plasma}}} \left\{ \sum_l [m_{\text{measured}} - m_{\text{estimated}}(I_{q,\text{plasma}})]^2 \right\}
\]

Although it’s a nonlinear minimization problem, it can be solved easily. Note that using this model we can get only plasma position \((R_p, Z_p)\) information. To get the shape information, plasma can be assumed as a set of current-carrying toroidal filamentary rings, so that the plasma location is fixed and we focus on finding a best distribution of plasma currents that minimize the total fitting error to diagnostic measurements. Hence the above equation can be described with another simplified plasma model as follows and implemented to a code named as IDK-ECFIT.

\[
\min_{R_p, Z_p} E_{\text{total}}(R_p, Z_p) = \min_{I_{\text{plasma}}} \left\{ \sum_l [m_{\text{measured}} - m_{\text{estimated}}(R_p, Z_p)]^2 \right\}
\]

Here \( I_k (k = 1, 2, \cdots, N) \) is an equivalent current element on each specific location and \( K(l, k) \) is a kernel that maps the k-th equivalent current to the l-th diagnostic data. With an assumption that equivalent current elements are properly positioned to represent the real plasmas, we can easily find out a solution that minimizes the total fitting error. In practical applications, it’s important to locate equivalent current elements to the best position. In this study, this issue can be easily solved by using results from IDK-RZIp.

It is worth noticing that if the plasma model is extended more from one in IDK-ECFIT, then it becomes almost same with EFIT-type full equilibrium reconstructions. Therefore this study is only focused on plasma position and shape reconstruction using above two models.

On the other hand, for equilibrium reconstruction of KSTAR plasmas, an additional method to handle up-down asymmetry is required. A simplest approach is adding an eddy current model into the equilibrium reconstruction. In this study, total 32 additional current elements are assumed to be uniformly distributed on the outer vacuum vessel wall in poloidal direction, with an assumption that these current elements can equivalently represent any contributions from outside including the effects of ferromagnetic material.

3. Validations of Developed Equilibrium Reconstruction Code (IDK)

Developed equilibrium reconstruction codes, IDK-RZIp/ECFIT, were validated with various reference equilibriums of KSTAR plasmas (generated by TES code [3]). Some of them are shown in figure 1. Three types of plasma equilibriums with different shapes (upper row) are considered with high beta (\( \beta_p = 1.5 \)) and 500kA plasma currents. First one is circular plasma that is downward shifted by 10 cm, second one is double null D-shape plasma, and the final
one is lower single null plasma. As shown in the second row, reconstructed plasma boundaries (blue dotted lines) are almost not distinguishable compared with referenced ones (red lines). Additionally blue circle markers indicate the equivalent current elements used in these reconstructions. As mentioned, these circular equivalent currents are defined on the locations given from IDK-RZIp reconstructions. Note that even though the region of equivalent current elements is much different compared with reference distributions, the reconstruction results show very accurate plasma position and shape estimations.

4. Results of Equilibrium Analysis for KSTAR 2009 Plasmas

Many of (more than 30) plasma shots in 2009 campaign of KSTAR have analyzed with IDK successfully. Several important results of them are described and discussed here.

4.1. Validations of Plasma Position and Shape Reconstruction

Since the most reliable and intuitive estimation for the position and shape of KSTAR plasmas in 2009 campaign is using 2-D images measured by a fast framing camera (CCD), the reconstructed positions by IDK codes are assessed with that. To accurately define the plasma position parameters from images, an image analysis tool built using IDL [4] was used.

One representative comparison results are shown in figure 2. In this shot 2074, plasma radial position was actively controlled inward and outward (black dotted line), while the vertical position remained uncontrolled. Since the plasma position controller was not fully optimized, the resultant radial position was roughly traced to its reference.
waveform. As easily captured from the upper time evolutions, the reconstructed positions from both IDK-RZIP and IDK-ECFIT are well matched with them from CCD image analysis except one discrepancy from 2.2 sec (point a) to 2.6 sec (point b) on Z position evolution. In the second row, the reconstructed plasma shapes from IDK-ECFIT are overlapped onto the CCD images and it confirms that the shape reconstruction by IDK-ECFIT is also reliable and accurate even for the initial and terminating phase of discharge where the plasma currents are so low. Interestingly, the sudden drop on Z evolution of CCD image analysis can be explained by IDK results as shown in fig.3.

When the plasma was controlled to move outward from the time point A, the plasma hit the outer (poloidal) limiter so that it generated very intense visible lights. When it moved back to the time point B, the intense visible emission was reduced reversely. Since the CCD image analysis is based on fitting the measured intensity of visible lights, it probably overestimated the Z position in this range to have more negative values than real ones, due to too much intensity from the lower part of outer limiter.

4.2. Basic Stability Analysis of KSTAR Plasmas Using IDK-ECFIT

An axi-symmetric mode (n=0) stability such as vertical or radial positional instabilities can be addressed as an equilibrium problem. The axi-symmetric positional stability of plasma can be described in terms of a field decay index, \( n_{\text{decay}} = -\frac{dB_r}{dZ} \frac{1}{B_Z R} \), which has a negative value for vertically unstable plasma and a positive value larger than 1.5 for radially unstable plasma. Therefore positional stable plasma should have a positive value ranged from zero to 1.5.

Figure 4 shows results of basic stability analysis by IDK for several representative shots in KSTAR 2009 campaign. The field decay index was calculated at the effective center of plasma column with reconstructed magnetic fields excluding plasma contributions. As shown in the figure, the field decay index of the shot 2275 exceeded 1.5 around the time point of B, so that the plasma was terminated by RDE (Radial...
Displacement Event) after that. It can also be confirmed from the reconstructed radial position evolution in the second raw that shows an exponential decay from that time point. Similarly other shots such as 1924, 2048, 2074, and 2112 show that the field decay index became negative around the time points of A and C, so that those shots were terminated by VDE (Vertical Displacement Event). In the similar way, it can be confirmed from the reconstructed vertical position evolution in the third raw that show exponential growths from those time points.

4.3. Density Limit Disruption

One special shot (#2277) in 2009 campaign was devoted to investigate the disruption phenomena in KSTAR. To make the plasma disrupted by density limit, an excessive gas was injected at t=1.2 sec as shown in fig. 4. After certain amounts of time delay from the gas valve, the line averaged density was increased linearly. Then roughly 70msec later, the electron temperature from ECE measurements shows a sudden radiative collapse, while the plasma current was sustained in the same level of amounts. At this moment, it’s expected that the line averaged density meets the Greenwald density. For low density and circular plasmas, the Greenwald density scaling law can predict the achievable density limit quite well. The Greenwald density, \( n_G = \frac{I_p [\text{MA}]}{\pi a^2} \times 10^{20} \text{ m}^{-3} \), is a function of plasma currents and minor radius. The estimated density limit based on this formula using minor radius from IDK reconstructions is shown with a blue dotted and marked line in the second raw on the figure. It’s clearly shown that the timing, that both line-averaged and calculated Greenwald densities are coincident, is well compatible with that the radioactive collapse occurred on the electron temperature evolution. Therefore it confirms that the

![Figure 5. Controlled disruption experiment driven by excessive gas injection at t=1.2 sec. On the time around t=1.27 sec when the line-averaged density meets the Greenwald density calculated using IDK-ECFIT results, the plasma thermal energy was radiatively collapsed.](image-url)
reconstructed plasma shape as well as position by IDK is so reliable and accurate.

4.4. Vacuum Field under Existence of a Ferromagnetic Material

A careful analysis for effects of ferromagnetic materials on magnetics has been conducted in 2009 campaign both experimentally [5] and theoretically [6]. It turns out that the vacuum magnetic field distribution would be affected by ferromagnetic effects and the disturbance is not small. Particularly an initial magnetization would be much different than the original one without this effect. Figure 5 shows comparison of reconstructed initial magnetic profiles with predicted ones. The most important difference is the $B_Z$ profile versus $R$ on the mid-plane. The initial magnetization was designed to have a negative derivative of $B_Z$ profile, but the calculated one including ferromagnetic effects shows a positive derivative as shown in the figure. The IDK reconstruction shows a quite similar profile with the calculated one rather than originally designed one. The IDK code doesn’t include the ferromagnetic effect directly on its reconstruction. Therefore it’s a quite promising result for the reliability of IDK formulation. On the other hand, the $B_R$ profile versus $Z$ on $R=1.8$ position shows a discrepancy compared with the prediction including ferromagnetic effects, although the direction of change is same. At this moment it’s not clear what the source of this overestimation is. It will be investigated more as a future work.

5. Role of Flux Loop Measurements and Its Impact on Equilibrium Analysis

Basic equilibrium reconstructions of tokamak plasmas were primarily based on magnetic measurements. For this purpose, usually magnetic probe and flux loop measurements are used. However in KSTAR 2009 campaign, flux loop measurements are not available for use in this analysis due to the signal’s saturation from integrators. It’s thought as one major reason why the attempts of equilibrium reconstruction using EFIT-type full equilibrium reconstruction codes was not satisfactory.

In principle, magnetic probe measurement provides a local magnetic field so that it relates to relative current distributions of existing current sources, while magnetic flux-loop measurement provides a total poloidal flux inside that loop so that it relates to absolute amounts of current on the region of interest. Therefore, it can be thought that with magnetic probe measurements only, we cannot fully resolve both plasma region and plasma current.
density profile simultaneously. This is why the EFIT-type analysis had bad convergences and why the simplified plasma model was adopted in IDK code. To validate this hypothesis, the effect of magnetic flux loop measurements on equilibrium reconstruction was investigated using TEFIT full equilibrium reconstruction code [7]. To have a similar condition with KSTAR plasmas, two circular plasmas with 500kA plasma currents are selected. One is for low beta plasma (\( \beta_p = 0.1, \beta_N = 0.2 \)) and the other is for high beta plasma (\( \beta_p = 1.5, \beta_N = 3.5 \)). Figure 7 shows the reconstructed plasma boundaries and current profiles at each iteration loops of equilibrium reconstruction without magnetic flux loop measurements.

![Figure 7. Plasma shape and profile changes during reconstruction iteration without magnetic flux-loop measurements. Plasma shapes are shown on the left and profiles on the right at each iterations with reference ones (black line).](image)

Black line shows the reference plasma boundary and current profile. For the low beta plasma (upper row), the effective center of current density is located inboard side, so that inboard magnetic probes have relatively larger values. It makes the reconstructed plasma center shifted outward with current profile peaking, and finally the plasma shrinks to a point nearby the outer limiter. On the contrary, for the high beta plasma (lower row), the reconstructed plasma is shifted inward with current profile peaking in the similar way, and finally shrinks to a point nearby the inboard limiter. Therefore it turns out that the magnetic flux loop data is essentially required to resolve both plasma location and current profile simultaneously and without flux loop data it usually shows a numerical instability in radial direction. This is why the EFIT-type codes were not successful for KSTAR 2009 plasmas.

6. Conclusion

In this paper, we introduced an equilibrium reconstruction code (IDK) using simplified plasma models to solve KSTAR equilibrium under large uncertainties on magnetics. Many of
shots in 2009 campaign were analyzed and some important results are described in view of validation of this method, including plasma position and shape reconstructions, vacuum magnetic field reconstructions, and density limit disruption analysis. As described, all results are well consistent with experimental measurements even though there are large uncertainties on magnetics, and they are not treated and resolved individually. It’s worth noticing that the effect of ferromagnetic material makes the vacuum magnetic field so much different than originally designed one. Although the IDK doesn’t treat this effect directly, the reconstructed vacuum fields show that the changes of vacuum field by ferromagnetic materials are indeed significant as that much as predicted.

In addition, a subtle issue, why the EFIT reconstructions for KSTAR 2009 plasmas were not as usual as in other tokamaks, is discussed. A dedicated investigation on reconstruction process of EFIT-type full equilibrium reconstruction code showed that both plasma boundary and current profile cannot be resolved using magnetic probe data only (without flux loop data). Hence it’s understood that for successful EFIT reconstructions for 2009 campaign, additional constraints to plasma model parameters are essential due to the lack of flux loop measurements. Naturally it’s expected that since the magnetic flux loop measurements would be available in coming campaign, the difficulty on equilibrium reconstruction that we had in 2009 will be resolved easily.

7. Acknowledgement

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8. References