Analysis of the Relaxed States in the Rotating Plasmas with no Momentum Input

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Abstract Plasma rotation has many beneficial effects on tokamak operation including stabilization of MHD modes and suppression of microturbulence to sustain high beta equilibria and improve energy confinement. Plasma rotation has been observed in NBI, ICRF heated plasmas as well as in pure Ohmic plasma, in which case there is no momentum input**.** In this paper, the relaxed state with a mass flow but no momentum input is investigated. The analysis for the dissipation, injection and transform of generalized helicity shows that the pure ohmic plasma with an AC magnetic helicity injection may relax to a state with flow duo to the generalized helicity transform, which gives a possible explanation for plasmas rotation without momentum input. The relaxed state of plasma with rotation in Ohmically driven tokamaks with an arbitrary aspect ratio is explored using the principle of minimum total energy dissipation rate subject to the generalized helicity balance and the energy balance. The resulting Euler–Lagrange equations for plasmas with flow are solved analytically. The solutions describe the structure, transition, and sensitive parameters of the relaxed state with plasma rotation. It is found that there exist different types of relaxed states in the different regions of the parameter space for a specific device. The different plasma current profiles include the typical experimental profiles and the profiles with hole or reversed current in the central region. The results show that there exists a key parameter $E_0\nu/B_0\eta$ (where η is plasma resistivity, ν is plasma viscosity, B_0 and E_0 is related to boundary toroidal magnetic field and boundary electric field respectively) in determining the final relaxed states and there exists the critical value of this key parameter to induce the abrupt state transition. The results indicate the rotation characteristics for the minimum dissipation state. It is shown that plasma fluid vorticity and the plasma current density is in parallel with co-current or counter-current directions. The flow can even be reversed from co- to counter- (or counter- to co-) current direction during transitions.

1. Introduction

Plasma rotation in tokamaks has been observed in NBI (Neutral Beam Injection) [1-4], ICRF (Ion Cyclotron Resonance Frequency) [5,6] ,ECH（Electron Cyclotron Heating）[7] heated plasmas as well as in pure Ohmic plasma [8], in which case there is no momentum input. Profuse experimental phenomenon about plasma rotation not only presents various driven modes， also various rotation direction, plasma velocity magnitude and profile, the dependence of experimental parameters, the relativity to L-H transition and so on. Much effort has been devoted to investigate the properties of tokamak plasmas in the presence of macroscopic flow. There exist many experimental observations need theoretical explanation, including plasmas rotation in pure Ohmic discharge without momentum input.

Experiments have shown that in some cases tokamak plasmas tend to evolve to a 'self-consistent' natural profile and under some conditions may evolve to other forms of states [9-11]. It implies that a relaxation mechanism may exist in tokamak plasmas. Basically, three different variation principles have been used in plasma physics. The first one is the minimum magnetic energy principle developed by Taylor [12, 13]. The principle leads to the force-free plasma equilibrium, which has successfully represented reversed-field-pinch (RFP) equilibria. The second is the minimum entropy production principle proposed by Hameiri and Bhattacharjee [14]. The third is the principle of minimum rate of energy dissipation presented for the first time by Montgomery and Phillips [15]. Many authors have applied the minimum energy dissipation principle to predict features of driven steady state for RFPs [15-17], direct current helicity injected torus [18-20] as well as inductively driven tokamaks [21-23]. In all the above papers no account was taken of mass flow. The effects of mass flow on the minimum dissipation states have been discussed for the first time by Montgomery et al.[24]. In recent years, experimental observations of tokamaks indicate that plasma rotation can be routinely obtained in experiments and has many beneficial effects on tokamak operation including stabilization of MHD instabilities and turbulence to improve the beta limit and confinement. So it is necessary to introduce a mass flow in the plasma relaxation theory.

In this paper, the relaxed state of plasma with a mass flow but no momentum input in Ohmically driven tokamaks is investigated using the principle of minimum total energy dissipation rate subject to the generalized helicity balance and the energy balance. The resulting Euler–Lagrange equations are solved analytically. The solutions describe the structure, transition, and sensitive parameters of the relaxed state with plasma rotation, and may give a possible explanation for the experimental plasmas rotation without momentum input.

At first we give an analysis for the dissipation, injection and transform of generalized helicity in sec.2. The Euler–Lagrange equations are given in sec.3. The main results for plasma current density and plasma rotation are given in Sec.4 and Sec.5. Finally, we give the summary in Sec.6.

2. Analysis for the dissipation, injection and transform of generalized helicity

The generalized helicity consists of magnetic helicity, cross helicity and kinetic helicity. We have generalized helicity for ion system as following

$$H = \int [(m/e)^2 \nabla \times \vec{u} \cdot \vec{u} + (2m/e)\vec{u} \cdot \vec{B} + \vec{A} \cdot \vec{B}] d\tau \qquad (1)$$

Where $\vec{B} = \nabla \times \vec{A}$.

From plasma MHD equations we have $\frac{dH}{dt} + \oiint d\vec{\sigma} \cdot \vec{S} = \int Q d\tau$ (2)

where $\vec{S}$ is the generalized helicity flow，Q is the dissipation density of the generalized helicity. We have $Q = Q_m + Q_c + Q_k$ (3)

Where Q_m, Q_c, Q_k is the dissipation density of magnetic helicity, cross helicity and kinetic helicity respectively. For idea plasmas, we have

$$Q_c = 2\vec{E}\cdot\vec{B} - \frac{2m}{e}\vec{E}\cdot\nabla\times\vec{u} - \frac{2m}{e}(\vec{u}\times\vec{B})\cdot\nabla\times\vec{u} \quad (3.1)$$

$$Q_m = -2\vec{E}\cdot\vec{B} \quad (3.2)$$

$$Q_k = \frac{2m}{e}\vec{E}\cdot\nabla\times\vec{u} + \frac{2m}{e}(\vec{u}\times\vec{B})\cdot\nabla\times\vec{u} \quad (3.3)$$

We can see that there exist the transform between magnetic helicity, cross helicity and kinetic helicity. The dissipation of magnetic helicity is absorbed by cross helicity through electric field force. Meanwhile, the transform between cross helicity and kinetic helicity is driven by electric field force as well as Lorontz force. It is found that the same transform also exists in non-ideal plasmas. Considering the non-ideal effects, we got the generalized helicity balance equation as following

$$\frac{dH}{dt} = -\int 2(\frac{m}{e}\vec{\omega}+\vec{B})\cdot(\frac{1}{ne}\nabla\cdot\vec{P}+\eta\vec{j})d\tau - \oiint d\vec{\sigma}\cdot\left[(\phi+\frac{mu^2}{2e})\vec{G}+\vec{g}\times(\vec{u}\times\vec{G})+(\frac{1}{ne}\nabla\cdot\vec{P}+\eta\vec{j})\times\vec{g}\right] \quad (4)$$

where $\vec{g} = \frac{m}{e}\vec{u}+\vec{A}$, $\vec{G} = \nabla\times\vec{g}$, $\vec{\omega}$ is the fluid vorticity $\vec{\omega} = \nabla\times\vec{u}$

The total dissipation density of the generalized helicity for non-ideal plasmas is as following

$$Q = 2(\frac{m}{e}\vec{\omega}+\vec{B})\cdot(\frac{1}{ne}\nabla\cdot\vec{P}+\eta\vec{j}) \quad (5)$$

The helicity flow is $\vec{S} = (\phi+\frac{mu^2}{2e})\vec{G}+\vec{g}\times(\vec{u}\times\vec{G})+(\frac{1}{ne}\nabla\cdot\vec{P}+\eta\vec{j})\times\vec{g}$ (6)

The total generalized helicity will be conservation for steady states, so the helicity injection will be balance with dissipation. The analysis for the dissipation, injection and transform of generalized helicity shows that the pure ohmic plasma with an AC magnetic helicity injection may relax to a state with flow duo to the generalized helicity transform. It gives a possible explanation for plasmas rotation without momentum input.

3. The Euler-Lagrange equations of Rotating Plasmas in Tokamak

The relaxed state is explored using the principle of minimum total energy dissipation rate subject to the generalized helicity balance and the energy balance. Applying the force balance condition for the non-idea rotating plasmas and assuming that plasma viscidity is contributed mostly from ion, we got generalized helicity dissipation density from (5) as following

$$Q = \frac{m\nu}{ne^2}\vec{\omega}\cdot(\nabla\times\vec{\omega}) + (\frac{\nu}{ne}+\frac{2m\eta}{e})\vec{\omega}\cdot\vec{j} + 2\eta\vec{j}\cdot\vec{B} \quad (7)$$

Assuming plasma velocity and vorticity vanish on the plasma boundary, also B_n=0, we obtain the normal helicity flow on the surface from equation (6) as

$$S_n = (\eta\vec{j} \times \vec{A})_n \tag{8}$$

Which is the AC magnetic helicity injection rate related to a pure Ohmic driven plasma. Finally, we obtained the functional for Ohmic plasmas with a magnetic helicity injection as

$$\begin{aligned} W &= \int_V (\eta j^2 + \nu\omega^2) d\tau \\ &+ \tilde{\lambda} \int_V \left[\frac{m\nu}{ne^2} \vec{\omega} \cdot (\nabla \times \vec{\omega}) + \left(\frac{\nu}{ne} + \frac{2m\eta}{e}\right) \vec{\omega} \cdot \vec{j} + 2\eta\vec{j} \cdot \vec{B} - 2\vec{E} \cdot \vec{B}\right] d\tau \\ &+ \tilde{\beta} \int_V (\eta j^2 + \nu\omega^2 - \vec{E} \cdot \vec{j}) d\tau \end{aligned} \tag{9}$$

Where $\tilde{\lambda}$ and $\tilde{\beta}$ are Lagtangian multipliers, η and ν is the resistivity and viscosity respectively. Comparing the first and the second teams of the second line, we got the scale for the ratio of both terms as

$$K = \frac{r_c}{L} \cdot \frac{R_m}{R_m + R_v}$$

Where r_c is Larmor radius, L is the device scale, R_m and R_v is the magnetic-Reynolds number and Reynolds number respectively. We can neglect the first team of the second line for the states with Larmor radius much smaller than the device scale. Taking the first variation, we got Euler-Lagrange equation and the natural boundary condition as following

$$2(1+\tilde{\beta})\nabla \times \eta\vec{j} + \frac{\tilde{\lambda}}{2}\left(\frac{\nu}{ne} + \frac{2\eta m}{e}\right)\nabla \times \vec{\omega} + 2\tilde{\lambda}\,\eta\vec{j} - \tilde{\lambda}\vec{E} - \tilde{\beta}\nabla \times \vec{E} = 0 \tag{10}$$

$$2(1+\tilde{\beta})\nu\vec{\omega} + \frac{\tilde{\lambda}}{2}\left(\frac{\nu}{ne} + \frac{2m\eta}{e}\right)\vec{j} = 0 \tag{11}$$

$$\left[\frac{\tilde{\lambda}}{2}\left(\frac{\nu}{ne} + \frac{2\eta m}{e}\right)\vec{\omega} + \tilde{\lambda}\,\eta\vec{B} + 2(1+\tilde{\beta})\eta\vec{j} - \tilde{\beta}\vec{E}\right]_b = 0 \tag{12}$$

4. The Analytical solution of plasma current density

The equation（10）is predigested to give an analytical analysis for plasma current prifiles. Selecting a square minor cross, η and ν is assumed uniform through whole plasma. The applied electric field and vacuum toroidal magnetic field is assumed to be inversely proportional to r. We got Euler-Lagrange equation for toroidal current density and magnetic field in the cylindrical coordinate as following.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial j_\varphi}{\partial r}\right) + \frac{\partial^2 j_\varphi}{\partial z^2} - \frac{j_\varphi}{r^2} + \lambda^2 j_\varphi - \frac{\lambda^2}{2\eta}E_\varphi = 0 \tag{13}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial B_\varphi}{\partial r}\right)+\frac{\partial^2 B_\varphi}{\partial z^2}-\frac{B_\varphi}{r^2}-\lambda j_\varphi+\frac{\lambda}{2\eta}E_\varphi=0 \tag{14}$$

The natural boundary condition is

$$j_{\varphi b}=j_\varphi(r,z)\big|_b=\frac{1}{r|_b}\left(\frac{\beta E_0 r_0}{2\eta}-\frac{\lambda B_0 r_0}{2}\right) \tag{15}$$

Where

$$\lambda=\frac{\tilde{\lambda}/(1+\tilde{\beta})}{[1-C\tilde{\lambda}^2/(1+\tilde{\beta})^2]}, \tag{16}$$

$$\beta=\frac{\tilde{\beta}/(1+\tilde{\beta})}{[1-C\tilde{\lambda}^2/(1+\tilde{\beta})^2]} \tag{17}$$

$$C=\frac{(\nu/ne+2m\eta/e)^2}{16\eta\nu} \tag{18}$$

B_0 and E_0 is the vacuum toroidal magnetic field and applied electric field at $r = r_0$ respectively (r_0 is the major radius of tokamak). We can see that (13)-(15) has same forms as the equations without mass flow [23] but the λ and β is different, which includes plasma parameter C, assumed uniform through whole plasma here. Actually, the solutions of the plasma current profiles represent the configuration in the plasma center region duo to above uniform assumation for parameters η, ν and C.

The resulting Euler–Lagrange equations for plasmas with flow are solved analytically. We get the analytical solution for toroidal current density [22, 23]

$$j_\varphi(r, z) = Y(r, z) + E_0 r_0/2\eta r \tag{19}$$

Where Y(r, z) is the solution of the homogeneous equation related to (13), with the boundary condition

$$Y_b= (\beta E_0/2\eta- E_0/2\eta-\lambda B_0/2) r_0/r_b = \alpha (\beta, \lambda) r_0/r_b \tag{20}$$

We have the solution of Y as

$$Y(r, z)=Y_1(r, z)+Y_2(r, z). \tag{21}$$

For $k_n^1 \le \lambda < k_{n+1}^1$, we have

$$Y_1=\sum_{m=1}^{n}\frac{a_m J_1(k_m^1 r)}{\sin(u_m h)}\left[\sin(u_m z)+\sin(u_m(h-z))\right]+\sum_{m=n+1}^{\infty}\frac{a_m J_1(k_m^1 r)}{\sin(u_m h)}\left[\sinh(u_m h)+\sinh(u_m(h-z))\right] \tag{22}$$

With $u_m^2=\lambda^2-\left(k_m^1\right)^2$ for $1 \le m \le n$ and $u_m^2=\left(k_m^1\right)^2-\lambda^2$ for m>n. $k_m^1=x_m^1/r_{00}$, x_m^1 is the m[th] zero point of the Bessel function of order 1. For $\nu_m \le \lambda < \nu_{m+1}$ with $\nu_n = n\pi/h$

$$Y_2 = \sum_{n=1}^{m} \left[c_n J_1(k_n r) + d_n N_1(k_n r)\right]\sin(v_n z) + \sum_{n=m+1}^{\infty} \left[c_n I_1(k_n r) + d_n K_1(k_n r)\right]\sin(v_n z) \tag{23}$$

Where J_1, N_1, I_1 and K_1 are respectively Bessel and modified Bessel functions.

For given ν, E_0/η and B_0, the values of α and λ should be obtained consistently by the energy and helicity balance conditions. However, the process cannot be accomplished using an analytical method.

The analytical analyses indicate that for a device of given dimensions, Y is only related to λ and α, where λ determines the form of Y. Different forms of Y are obtained in different λ ranges as analyzed in Ref.[22-23]. The final current profile is mainly determined by λ, also related to the relative magnitude of two terms of Y and $E_0 r_0/2\eta r$. Similar to the case without plasma flow, there exist different types of relaxed states in the different regions of the parameter space for a specific device. The different plasma current profiles include the typical experimental profiles and the profiles with reversed shear, while the current may have a hole or even reverse in the central part as shown in FIG. 1.

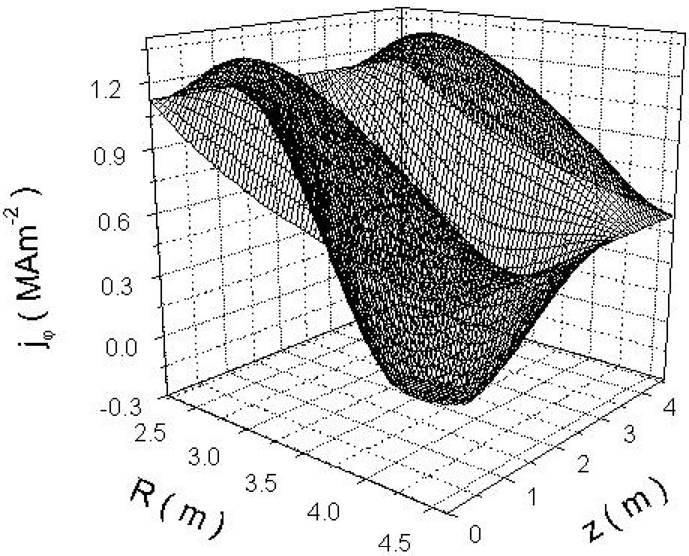

FIG 1, Plasma current profile reversed in the central region for the dimensions of JT-60U (R_0 = 3.4 m, a = 1.2m, h = 4.6m), with the parameters of λ = 2.3 m^{-1}, α = 1.71 MA/m^2, E_0/η=11.61 MA/m2

The analysis also shows that there exists a key parameter $E_0\nu/B_0\eta$ in determining the final relaxed states and there exists the critical value of this key parameter to induce the abrupt state transition. We can see that the key parameter for rotating plasma is different from the case without mass flow ($E_0/B_0\eta$).

5. The plasma fluid vorticity of the relaxed state in Ohmically driven tokamaks

The plasma fluid vorticity is obtained from equation (11)

$$\vec{\varpi} = -\frac{\tilde{\lambda}'}{4}\left(\frac{1}{ne} + \frac{2m\eta}{\nu}\right)\vec{j} \ , \qquad (24)$$

where $\tilde{\lambda}' = \dfrac{\tilde{\lambda}}{1+\tilde{\beta}}$. For a selected λ, we can get two $\tilde{\lambda}'$ solutions as following

$$\tilde{\lambda}_1' = \frac{-1+\sqrt{1+4C\lambda^2}}{2C\lambda} \ , \quad \tilde{\lambda}_2' = \frac{-1-\sqrt{1+4C\lambda^2}}{2C\lambda} \qquad (25)$$

When $\lambda>0$, we have $\tilde{\lambda}_1' > 0$ ($\vec{\varpi}$ in counter-current directions) and $\tilde{\lambda}_2' < 0$ ($\vec{\varpi}$ in co-current directions). When $\lambda<0$, we have $\tilde{\lambda}_1' < 0$ ($\vec{\varpi}$ in co-current directions) and $\tilde{\lambda}_2' > 0$ ($\vec{\varpi}$ in counter-current directions). This means that for one value of λ (or one current density form), we have two possible fluid vorticity diridtions, in co-current or counter-current directions. For the solution $\tilde{\lambda}_1'$, the fluid vorticity diriction changes from co-current to counter-current direction smoothly when λ is changing from less than zero to more than zero. But for $\tilde{\lambda}_2'$ solution, the fluid vorticity diriction changes from counter-current to co-current direction abruptly.

6. Summary

The relaxed state of plasma with rotation in Ohmically driven tokamaks with an arbitrary aspect ratio is explored using the principle of minimum total energy dissipation rate subject to the generalized helicity balance and the energy balance. The resulting Euler–Lagrange equations for plasmas with flow are solved analytically. The solutions describe the structure, transition, and sensitive parameters of the relaxed state with plasma rotation. It is found that there exist different types of relaxed states in the different regions of the parameter space for a specific device. The different plasma current profiles include the typical experimental profiles and the profiles with hole or reversed current in the central region. The results show that there exists a key parameter $E_0\nu/B_0\eta$ in determining the final relaxed states and there exists the critical value of this key parameter to induce the abrupt state transition. The results also indicate the rotation characteristics for the minimum dissipation state. It is shown that plasma fluid vorticity is in parallel the plasma current density in co-current or counter-current directions. The flow can even be reversed from co- to counter- (or counter- to co-) current direction during transitions. The results show that the pure ohmic plasma with an AC magnetic helicity injection may relax to a state with flow.

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