# Plasma Rotation Effects on the Trigger of Reversed Shear Plasma Disruptions

Y. Ishii<sup>1)</sup>, A.I.Smolyakov<sup>2)</sup> and M.Takechi<sup>1)</sup>

1) Japan Atomic Energy Agency, Naka, Ibaraki-ken, 311-0193, Japan

2) University of Saskatchewan, Saskatoon, SK, S7N 5E2, Canada

e-mail: ishii.yasutomo@jaea.go.jp

**Abstract.** Plasma rotation effects on the trigger and evolution of MHD modes in a reversed shear profile are studied by nonlinear MHD simulations. It is found that, in rotating plasma, magnetic islands formed around inner and outer magnetic resonant surfaces, which are stable for the tearing mode, by an external perturbation, (driven magnetic island) evolve with different growth rates during an initial growth phase. After an initial growth phase, an outer magnetic island grows explosively earlier and triggers an explosive growth of an inner one. At a final phase, enlarged magnetic islands flatten a q-profile in wide radial region including the plasma center. Though this final phase resembles closely a nonlinear destabilization of a spontaneous double tearing mode (DTM) [1], this process can explain time delay of a plasma edge oscillation to trigger an internal MHD events and disruption in RS plasmas[2].

### 1 Introduction

Reversed magnetic shear configuration is considered as one of the possible way to attain high performance fusion plasma. However, reversed shear plasma encounters plasma collapses even at lower  $\beta$  than that of the ideal MHD limit. Therefore, it is an urgent issue to make the mechanism of low  $\beta$  RS disruption clear and propose an effective control method to avoid it.

Resistive MHD modes, such as the double tearing mode (DTM), resistive interchange mode and also the coupled mode between a tearing and interchange one, have been considered as an origin of RS plasma disruptions in low beta regime. However, some aspects of experimental observations have not been explained by the theoretical analysis and numerical simulation studies. One of these is the relationship between the fast time scale collapse and the resistive time scale precursor oscillation, which are sometimes observed in RS disruptions. For this question, the nonlinear destabilization process of the DTM has been proposed to explain a time-scale transition from the resistive to the fast one. However, in this process, magnetic islands grow both at the inner and outer resonant surfaces at the same time, which is inconsistent with the experimental observation. In experiments, the precursor oscillation is usually observed



FIG. 1: tearing stable and unstable reversed q-profiles and a rigid poloidal rotation profile.

only at the outer resonant surface. Another one is the trigger mechanism of the low  $\beta$  RS plasma disruption. In an experimental analysis [2], the correlation between MHD modes in the edge and in the core region has been reported. The interaction processes between MHD modes occurred at different spatial positions have been studied recently as the trigger process of the resistive wall mode and the neoclassical tearing mode. It is pointed out that the background plasma rotation is important to the interaction between MHD modes. However, these processes have not been investigated for RS plasma disruptions. In this study, we will investigate the excitation process and nonlinear evolution of

magnetic islands by an external perturbation in rotating RS plasma in order to explain some aspects observed in RS plasma disruptions.

### 2 Basic Equations

In order to investigate the time evolution of a spontaneous double tearing mode (DTM) and also a forced magnetic island driven by an external perturbation at the plasma edge in a rotating plasma, the resistive reduced magnetohydrodynamics (MHD) equations are used in the cylindrical geometry,

$$\frac{\partial}{\partial t}\Psi = \frac{1}{r}[\Psi,\phi] + \frac{B_0}{R_0}\frac{\partial}{\partial\varphi}\phi + \eta J \tag{1}$$

$$\frac{\partial}{\partial t}U = \frac{1}{r}[U,\phi] + \frac{1}{r}[\Psi,J] + \frac{B_0}{R_0}\frac{\partial J}{\partial\varphi} + \mu\nabla_{\perp}(U-U_0)$$

$$J = \nabla_{\perp}^2\Psi, \ U = \nabla_{\perp}^2\phi.$$
(2)

Here,  $\Psi$  is the poloidal flux function,  $\phi$  is the flow potential, J is the plasma current, U is the vorticity,  $\varphi$  is the toroidal angle,  $B_0$  is the toroidal magnetic field at the magnetic axis,  $R_0$  is the aspect ratio and set to 10 in this study. These equations include the resistivity  $\eta$  and the viscosity  $\mu$ .  $U_0$  is the initial background component of the vorticity U. The last term in eq.(2) is added to keep the background flow to the initial profile. In these equations, the parameters are normalized by the plasma minor radius a, the poloidal Alfven time  $\tau_{pa} = \frac{\sqrt{\rho a}}{B_{\theta}(a)}$ , where the plasma density  $\rho$  is set to 1. The resistivity  $\eta$  is normalized such that  $\eta = \frac{\tau_{pa}}{\tau_{\eta}}$ , where  $\tau_{\eta}$  is the plasma skin time. In this study, we treat the reversed shear profile with the q=3 resonant surface as the minimum rational surface. We also investigate an externally driven two tearing mode, which is excited by an external perturbation at the tearing stable q=3 resonant surfaces. In order to form a tearing stable reversed q-profile, the q-profile is locally modulated at the inner and outer q=3 rational surfaces by using the following functional form of q,

$$q = q_0 \cdot \{1 + (\frac{r}{r_0})^2\} [1 + f_q \cdot exp\{-(\frac{r-r_1}{\delta})^2\}] \cdot [1 + h_{q1} \cdot (r - h_{q1})exp\{-(\frac{r-h_{q1}}{\delta_{q1}})^2\}] \cdot [1 + h_{q2} \cdot (r - h_{q2})exp\{-(\frac{r-h_{q2}}{\delta_{q2}})^2\}].$$
(3)

Here, parameters are set to  $q_0 = 0.95$ ,  $r_0 = 0.412$ ,  $f_q = 3$ ,  $r_1 = 0$ ,  $\delta = 0.273$ ,  $h_{q1} = 0.1$ ,  $h_{q2} = -0.5$ ,  $h_{q1} = 0.266$ ,  $h_{q2} = 0.576$ ,  $\delta_{q1,q2} = 0.05$ .

In this study, the single helicity of m/n = 3/1 is assumed for simplicity. Two types of the background flow are treated in this study. One is a rigid background flow, which is introduced by the m/n = 0/0flow potential  $\phi_{0/0} = -\frac{2\pi}{\lambda\tau}(1-r^{\lambda})$ . Here,  $\lambda$  is set to 2 to form the rigid rotation. Another one is a sheared background flow, which will be described in Sec.5. Figure 1 shows the tearing stable and unstable reversed qprofiles and the rigid rotation profile used in this study. The external perturbation is added at the plasma surface as the linearly increasing poloidal flux function,  $\psi_{2/1}(r = a) = \psi_{2/1}(a) \cdot (t - t_0)$ . Here,  $\psi_{2/1}(a)$  is the increasing rate of the poloidal flux function at the plasma edge.



FIG. 2: Time evolution of the 3/1-harmonics of the magnetic and kinetic energy.

# 3 Externally Driven Magnetic Island Evolution in a Reversed Shear Plasma

Linear and nonlinear features of spontaneous DTM change largely depending on the distance between two resonant surfaces [6]. However, it has not been investigated what happens when an external perturbation is applied to a tearing stable reversed magnetic shear profile. In this section, we will investigate linear and nonlinear responses of a tearing stable reversed magnetic shear profile to an external perturbation.

Spontaneous DTM is thought to be one of the origins of reversed shear plasma disruptions [1, 5]. In case that the distance between two resonant surfaces is short, a spontaneous DTM, which is called a strongly coupled one, causes the q-profile flat within a short radial region between an inner and outer resonant surface with the same q-value. Therefore, the nonlinear evolution of a strongly coupled DTM is considered as an origin of a minor RS plasma disruption. On the other hand, in case that the distance between two resonant surfaces is long, a spontaneous DTM, which is called a weakly coupled one, causes the q-profile flat in a long radial region including the plasma center. Therefore, the nonlinear evolution of a weakly coupled DTM is considered as one of the origins of a major RS plasma disruption. In order to investigate the trigger process of the disruptive instability by an external perturbation, in this section, we will focus on



FIG. 3: Temporal growth rate of the magnetic island width for the different resistivity  $\eta = 10^{-5}$ ,  $5 \times 10^{-6}$  and  $3 \times 10^{-6}$ .

the linear and nonlinear response of a tearing stable reversed shear profile, whose distance between two resonant surfaces is long.

Figure 2 shows the time evolution of magnetic and kinetic energy, which include also those of an externally applied perturbation, for an externally driven two magnetic islands formed around the inner and outer q=3resonant surfaces. The energy evolution phase can be divided into three ones; an initial growth phase, a slow growth and rapid growth one. These aspects of two magnetic island evolution are similar to nonlinear destabilization process of the double tearing mode [1]. One of important features of this process is a weak dependence of the temporal growth rate on the resistivity that can explain the fast time scale collapse after a resistive time scale precursor by a single MHD mode evolution. Figure 3 shows the temporal growth rate of outer magnetic island width, W, for different resistivity ;  $\eta = 10^{-5}, 5 \times 10^{-6}$  and  $3 \times 10^{-6}$ . During an ini-



FIG. 4: Time evolution of the 3/1-harmonics of the magnetic and kinetic energy.

tial phase,  $t \leq 250$ ,  $\dot{W}$  does not depend on the resistivity. However, during a slow growth phase,  $\dot{W}$  shows the resistivity dependence of  $\alpha \sim 0.55$ , which is estimated from  $\dot{W} = A \cdot \eta^{\alpha}$ . This finite dependency of  $\dot{W}$  on the resistivity means that the time evolution of an externally driven magnetic island is not dominated only by an external driving force, but also by plasma response like as the magnetic reconnection. During the rapid growth phase,  $\dot{W}$  changes almost one order from that at the slow growth phase. In this phase, the magnetic island growth is accelerated. Therefore, it is difficult to estimate the dependency of  $\dot{W}$  on the resistivity for the whole rapid growth phase by averaging  $\dot{W}$ . During the final period of  $\Delta t \simeq 10$ ,  $\dot{W}$  for three resistivity overlap each other. This means that the magnetic island growth depends weakly on the resistivity that is similar to the nonlinear destabilization of the spontaneous DTM.

#### Rigid Rotation Effects on an Externally Driven Magnetic Island in RS 4 Plasma

In this section, rigid plasma rotation effects on the evolution of an externally driven two magnetic islands in reversed shear plasma will be investigated. The initial rotation profile is plotted in Fig.1. Figure 4 shows the time evolution of magnetic and kinetic energy of an externally driven two tearing mode in a rotating plasma. In case of a finite plasma rotation, the energy evolution is also divided to three phases. This feature is the same as that in the case without plasma rotation shown in fig.3, except for the modulation of kinetic energy at  $t \simeq 600$ . Figure 5 shows the time evolution of an externally driven magnetic island width in non-rotating and rotating plasma. During an initial growth phase, magnetic island evolution is suppressed in rotating plasma. After this flow suppressed growth phase, an outer magnetic island begins to growth at  $t \simeq 600$  preceding to an inner magnetic island, which begins to grow at  $t \simeq 800$ .

At the final phase,  $t \simeq 1100$ , both inner and outer magnetic islands grow rapidly in almost the same growth rate. The initial flow suppressed growth phase is originated from the modulation of a current sheet profile around the q=3 resonant surfaces by a background flow,  $V_{\theta 0/0}$ . However, the background flow is damped by three types of torque in a poloidal direction; electromagnetic torque  $T_{\theta}^{EM}(r,\theta) = r\vec{e}_{\theta} \cdot \vec{J} \times \vec{B}$ , nonlinear inertial torque  $T^{I}_{\theta}(r,\theta) = r\vec{e}_{\theta} \cdot (\vec{V} \cdot \nabla)\vec{V}$  and viscous torque  $T^{V}_{\theta}(r,\theta) =$  $r\nu \vec{e_{\theta}} \cdot \nabla^2 \vec{V}$ . The total torque is defined as a sum of these components;  $T_{\theta}^{T} = T_{\theta}^{EM}(r, \theta) + T_{\theta}^{V}(r, \theta) - T_{\theta}^{I}(r, \theta)$ . Figure 6 shows the time evolution of the background flow  $V_{\theta 0/0}$ . The Background flow begins to decrease around the outer q=3 resonant surface. Though at t = 600,  $V_{\theta 0/0}$  at the outer q=3 resonant surface is damped to almost 1/3 of that at t = 0,  $V_{\theta 0/0}$  at the inner one changes



FIG. 5: Time evolution of an inner and outer magnetic island width in a static and rotating plamsas.



FIG. 6: Time evolution of the background flow  $V_{0/0}^{\theta}$  radial profile.

little. Around t = 600, the outer magnetic island begins to grow because of the background flow damming around the outer q=3 surface. However, the inner magnetic island growth is suppressed almost until to t = 800.

In rotating plasma, a magnetic island caused by a spontaneous MHD mode moves in the poloidal direction.

That is not for an externally driven one. For an externally driven magnetic island, it moves in the poloidal direction, when the background flow becomes less some value. Figure 7 shows the time evolution of the poloidal angle  $\theta$  for the outer and inner magnetic islands. Until to  $t \simeq 600$ , the O-points of the inner and outer magnetic islands remain around at  $\theta = 85$  for the outer one and at  $\theta = 105$  for the inner one. Around t = 600, both the outer and the inner O-points begin to move in the poloidal direction. However, at this time, background flow around the inner q=3 resonant surface changes little. Therefore, there is a possibility that the trigger mechanism of the inner magnetic island moving in the poloidal direction is different from that of the outer one.

Figures 8 show the contour plots of the poloidal flux function, i.e. magnetic island, and m/n = 3/1 component of the flow potential ,  $\phi_{3/1}$ . In an initial phase,  $t \leq 400$ ,  $\phi_{3/1}$  is localized around the outer magnetic island. As the outer magnetic island grows and the background flow is damped,  $\phi_{3/1}$  extends inner and outer radial directions. The amplitude of  $\phi_{3/1}$  between the inner and outer magnetic islands becomes large rapidly from t = 600 to 700. In case of a spontaneous DTM, the coupling strength between two resonant surfaces varies depending on the distance between these. As this distance becomes short, the coupling strength becomes strong and the amplitude of the eigen mode  $\phi_{m/n}$  becomes large. Therefore, trigger process of an inner magnetic island evolution resembles to form an eigen mode structure without a background rotation. Before the trigger of



FIG. 7: Time evolution of the poloidal angle  $\theta$  of the X- and O-points.

a poloidal movement, the phase difference between an inner and outer magnetic island is about 20 degree. This phase relation is different from that of the most unstable DTM without a background flow. As magnetic islands move in poloidal direction, the magnetic island grows and the phase difference between an inner and outer island approaches to 60 degree, which is the same one in case without a background flow. From  $t \simeq 1060$ , the growth of an inner an outer magnetic island accelerates. After this rapid growth phase, the inner magnetic island is pushed out and squeezed and the outer magnetic island spreads out from  $r/a \simeq 0.1 \sim 0.7$ , as shown in fig.8(d). Figure 9 shows the time evolution of the safety factor, q-profile. At the final phase, t = 1200, the q-profile becomes flat in the wide radial region of  $r/a \simeq 0 \sim 0.7$ . Therefore, this situation will lead to the plasma disruption.



FIG. 8: Contour plots of the poloidal flux function  $\psi^*$ and the flow potential  $\phi$  with the helicity of m/n=3/1.

two phenomena can be explained by introducing a background flow effect into nonlinear evolution of an externally driven two tearing modes. Even though the lowest rational surfaces are stable for the tearing mode, an MHD mode occurred at the edge region can drive a MHD mode at the lowest rational surface, which is different from that at the edge region, by the toroidal coupling effect. At first, a magnetic island, the width of which is larger than the experimental resolution, appears only at the outer lowest resonant surface and grows with the resis-

As shown in this section, these

tive time scale. Then, there appears a magnetic island at the inner lowest rational surface. At the final phase, both of the inner and outer magnetic islands grow rapidly, that is corresponds to the plasma disruption with the fast time scale.

# 5 Sheared Rotation Effects on a spontaneous DTM

In this section, sheared plasma rotation effects on a spontaneous DTM will be investigated. For a spontaneous DTM, a rigid plasma rotation hardly nonlinear evolution affect on its features. However, in tokamak experiments, a plasma rotation has a sheared profile, because of a non-uniform momentum input, the radial electric field formation around the internal transport barrier, etc. In order to introduce a sheared background flow, the z-component of m/n = 0/0vortex  $U_{0/0}$  is set in the following form,

$$U_{0/0} = \Omega_a + \frac{1}{2}r^2 [1 - exp\{-(\frac{1-r}{\epsilon})^2\}]\Omega_b atan(\frac{r-r_x}{\delta}).$$
(4)

By solving the poisson equation of  $U_{0/0} = \nabla_{\perp}^2 \phi_{0/0}$  and taking a radial derivative of  $\phi_{0/0}$ , a sheared poloidal  $V_{0/0}^{\theta}$ 



FIG. 9: Time evolution of the q-profile.

is obtained as shown in fig.10, where  $\Omega_a = -0.1$ .  $\Omega_b = 0.1$  and 1.4,  $\delta = 10^{-2}$ ,  $\epsilon = 0.1$ . In these parameters,  $V_{0/0}^{\theta} = -1.63 \times 10^{-2}$  at the inner q=3 surface and  $V_{0/0}^{\theta} = -1.47 \times 10^{-3}$ at the outer q=3 one. Figure 11 shows the time evolution of an inner and outer magnetic island width of a spontaneous DTM with and without a sheared background rotation. During an initial growth phase of  $t \leq 250$ , the growth of an inner magnetic island with the background sheared rotation is suppressed compared with that without the background rotation, because of the finite rotation shear around the inner resonant surface. On the other hand, the growth of an outer magnetic island with the background sheared rotation is almost the same as that without the background rotation, because the sheared rotation is small around at the outer resonant surface. Then, an inner magnetic island with the sheared background rotation begins to grow. From  $t \simeq 200$ , the background flow around the inner q=3 resonant surface begins to decrease, while that around the outer one changes a little. In case of an externally driven magnetic island, magnetic islands do not rotate by the background flow, but are locked to an external perturbation. However, in case of a spontaneous DTM, it rotates by the background rotation. During an initial growth phase of  $t \leq 300$ , when an inner magnetic island is a flow-suppressed growth phase, an inner and outer magnetic islands rotate with a different speed. However, an inner magnetic island rotates little. From  $t \simeq 300$ , both inner and outer magnetic islands rotate with the same speed and grow with almost the same growth rate. Finally, in the same way as a spontaneous DTM without the background flow, tow magnetic islands growth rapidly. Therefore, by introducing sheared rotation effects, an experimentally observed phenomenon that a precursor oscillation with the resistive time scale appears around the outer resonant surface prior to the fast time plasma collapse, is also explained also by a spontaneous DTM. A sheared background rotation has a stabilizing effect on both of a spontaneous and an externally driven DTM at least during an initial growth phase. Therefore, there is a possibility that both of a spontaneous and an externally driven DTM can be stabilize by fixing a background rotation. This is important to avoid the low beta reversed shear disruption by controlling the plasma rotation. Figure 12 shows the time evolution of the magnetic island separatrix obtained from a spontaneous DTM under a sheared plasma rotation, which is set in constant in time. The radial profile of a poloidal flow  $V_{0/0}^{\theta}$  changes as shown in fig.10 by changing a parameter  $\Omega_b$  from  $\Omega_b = 1.4$ to 0.1. In the latter case, there is almost no difference of rotation frequency between the inner and outer resonant surfaces. This profile is very close to a rigid rotation profile. Therefore, in this case, the stabilization effect on a spontaneous DTM is hardly seen. Therefore, an inner and outer magnetic island interact each other and grow rapidly to flatten the q-profile in the wide radial region. In case of a large flow shear,  $\Omega_b = 1.4$ , though the inner magnetic island growth is suppressed until  $t \simeq 250$ , it begins to grow

around at  $t \simeq 250$ . In this simulation, the background flow is fixed in time. However, as shown in the previous section, a magnetic island can be formed by the coupling with another magnetic island even though a background flow is damped little. Around  $t \simeq 600$ , the inner separatrix of an outer magnetic island and the outer separatrix of an inner one reach to almost the same radial position. However, both magnetic islands do not grow further more and saturate. Therefore, by keeping the background flow shear strong, it is possible to make the saturated magnetic island width smaller than that in case without the flow shear and to avoid the plasma disruption.



FIG. 10: The radial profile of the sheared flow  $V_{0/0}^{\theta}$  for  $\Omega_b =$ 1.4 and 0.1.

### 6 Summary

In this study, the nonlinear evolution process of two magnetic islands, which is driven for a tearing stable equilibrium by an externally applied perturbation, was investigated numerically. Similar to a spontaneous weakly coupled DTM, which is one of the possible MHD instabilities leading to a low  $\beta$  RS disruption, an externally driven two magnetic islands evolve through three phases; the initial growth phase, the slow growth phase, during which the growth rate depends on the resistivity, and the explosive growth phase, during which the growth rate depends weakly on the resistivity. Therefore, not only a spontaneous DTM, but also the interaction process between externally driven two magnetic islands can reproduce a fast time scale disruption accompanied with the resistive time scale precursor, which is sometimes observed in the low  $\beta$  RS disruption. The background rotation effects on both of a spontaneous DTM and externally driven two magnetic islands were also investi-



FIG. 11: Time evolution of the magnetic island width of spontaneous DTM in a static and a sheared rotation plasma.

gated. For a spontaneous DTM, a rigid rotation does not affect on its evolution process. On the other hand, an externally driven two tearing mode is affected by the shielding of an external perturbation penetration, because of the finite different rotation speed between the static edge perturbation and the resonant surface around the minimum q value. In this case, while a magnetic island formed at the outer resonant surface grows gradually, a magnetic island formed at the inner one stays in a flow-suppressed phase. Then, the inner magnetic island also beings to grow by the coupling with the outer one. At the final phase, the inner and outer magnetic islands grow rapidly and the q-profile becomes flat including the plasma center. Almost all of the same process can be reproduced for a spontaneous DTM by including the sheared background flow. These processes obtained by a numerical simulation coincide with some aspects observed in low  $\beta$  RS plasma disruption. We also investigated the effect of a fixed sheared rotation, which is set to constant in time. If there is a large flow shear between two resonant surfaces, the inner and outer magnetic islands becomes smaller than that under a non-fixed sheared flow and do not interact with each other. Therefore, there is a possibility to avoid the low  $\beta$  collapse of a RS plasma by keeping a large flow shear between two resonant surfaces.

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Supervised letter  $\Omega_b = 0.4$   $\Omega_b = 1.4$   $\Omega_b = 1.4$  $\Omega_b = 1.4$ 

FIG. 12: Time evolution of the magnetic island separatrix in a fixed sheared rotation.



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