# PLASMA SHAPING EFFECTS ON TEMPERATURE GRADIENT-DRIVEN INSTABILITIES AND GEODESIC ACOUSTIC MODES

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**Abstract**. Plasma shaping effects on temperature gradient-driven instabilities and geodesic acoustic oscillations are investigated with a gyrokinetic theory and a local magnetohydrodynamics equilibrium model. In specific, we focus on the effect of the elongation $\kappa$, including its radial derivative $s_\kappa = (r/\kappa)(\partial\kappa/\partial r)$, in the large aspect ratio limit. An analytical formula of the dependence of the GAM frequency on the elongation is given. It is found that the GAM frequency sharply decreases with an increasing elongation by dependence of $\left[(2-s_\kappa)/(\kappa^2+1)\right]^{1/2}$, which comes from the modification of classical ion polarization balanced by that of curvature drift polarization. The dependence of the critical threshold of the ETG/ITG instability on the elongation is numerically studied and a semi-analytical formula is given as $(R/L_{Tc})/(R/L_{Tc})_{s_\kappa=0,\kappa=1} = (1+0.36 s_\kappa)[1+0.11(\kappa-1)]$.

## 1. Introduction

Progress in understanding the anomalous transport in plasmas has been continuing for decades. It is now widely accepted that the anomalous transport is induced by turbulent plasma fluctuations with small scales, the so-called microinstabilities. Simultaneously, micro-turbulence can drive zonal flows, including the low frequency residual flow and the higher frequency geodesic acoustic mode (GAM), [1] and these coherent structures sequently moderate turbulent transport.[2-8] However, effects of plasma shape on the transport have not been considered as adequately as on the magnetohydrodynamic (MHD) stability property, where the dependence of the MHD stability limit on plasma shape is well known both theoretically and experimentally. The influence of plasma shape on confinement has been studied experimentally, and also, increasing attention [9-12] has been paid with numerical simulation of microinstabilities in noncircular plasmas. However, some results are confused and, moreover, there is considerable interest in understanding the physics inside. In addition, it becomes urgent to investigate the effect of plasma shaping on ZFs. For example, based on experimental data of ASDEX-Upgrade, [13] the GAM frequency is found to depend on the elongation $\kappa$ roughly by the scaling of $1/(1+\kappa)$. Simulations [14-16] also show the effect of $\kappa$ on the frequency, but usually suggest as a $1/\kappa$ dependence due to the geodesic curvature drift term. Therefore, it is important to investigate the effect of plasma shape on these coherent modes, as well as those on microinstabilities. Especially, an analytical treatment is still desirable since the physics is not clearly displayed yet.

In this paper, we make a generalization of previous gyrokinetic theory [17, 18] in circular geometry to the noncircular flux surface to study the drift instability and the GAM, respectively. Among plasma shape parameters, the elongation is a leading deformation comparing to the Shafranov shift gradient and the triangularity deformation. Finite aspect ratio is known to have a strong effect on microinstabilities, but the velocity modulation by the poloidal variation due to finite aspect ratio will cause big difficulty in analytical treatment. Moreover, it is known that the leading effect of the elongation is decoupled from the effect of finite aspect ratio. Therefore, we may leave the effect of finite aspect ratio and other noncircular parameters in a future work. Here we will focus on the effect of the elongation, including its radial derivative, in the large aspect ratio limit. An analytical formula of the dependence of the GAM frequency on the elongation is presented. The dependence of the critical threshold of the ETG/ITG instability on the elongation is numerically studied, but the physics is analyzed and a semi-analytical formula is given.

**2. The Gyrokinetic Model**

We consider a toroidal axisymmetric plasma with a flux surface ( $R_s$ , $Z_s$ ), written as $R_s = R_0 + r\cos\left(\theta + \sin^{-1}\delta\sin\theta\right)$, $Z_s = \kappa r\sin\theta$, where $R_0$ and r are the major and minor radius, $\theta$ is the generalized poloidal angle, and $\kappa$ and $\delta$ are elongation and triangularity deformation. Employing the Miller's local equilibrium model, [19] the toroidal and poloidal fields in the flux surface can be described as $B_t = B_0 R_0/R_s$ and

$$B_p = B_0 \frac{r}{qR_0}\frac{dl/d\theta}{J_s R_s/R_0}\frac{1}{2\pi}\oint d\theta \frac{J_s/r}{R_s/R_0},$$

where $J_s = \left(\partial R_s/\partial r\right)\left(\partial Z_s/\partial\theta\right) - \left(\partial Z_s/\partial r\right)\left(\partial R_s/\partial\theta\right)$ and $dl/d\theta = \sqrt{\left(dR_s/d\theta\right)^2 + \left(dZ_s/d\theta\right)^2}$ are the Jacobian and the differential of poloidal arc length with respect to poloidal angle, and $B_0$ is the field at the magnetic axis. We consider an electrostatic potential, $\phi = \sum_{\omega,k}\hat{\phi}\exp\left[-inS + ik_{x0}x\left|\nabla r\right| - i\omega t\right]$ with $\left|\nabla r\right| = \left(dl/d\theta\right)/J_s$, where $x$ is a normal distance from the flux surface, the eikonal $S$ can be expanded as $S = \phi + S_0(\theta) + xS_1(\theta)$ and is determined by the relation $\mathbf{b}\cdot\nabla S = 0$. In the locally orthogonal coordinate system $\left(\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_l, \hat{\mathbf{e}}_\xi\right)$, the derivative of the perturbation, in other words, the wave vector, is

$$k_l = -n\hat{\mathbf{e}}_l\cdot\nabla S = k_\theta\left(\frac{rB}{qR_s B_p}\right), \tag{1}$$

$$k_x = k_{x0}x\left|\nabla r\right| - n\hat{\mathbf{e}}_x\cdot\nabla S = k_{x0}\frac{\mathrm{d}l/\mathrm{d}\theta}{J_s} - k_l g\left(S_1\right), \tag{2}$$

where, $k_\theta = nq/r$ and $g = R_s B_p S_1/B$. Then, the plasma response can be solved as $\hat{f} = -qF_0\hat{\phi}/T + \hat{h}J_0(\delta)$ and the nonadiabatic part satisfies the linear gyrokinetic equation,

$$\left(\omega - \omega_d + i\omega_t\frac{\partial}{\partial\theta}\right)\hat{h} = \frac{e\hat{\phi}}{T}\left(\omega - \omega_*^T\right)J_0(\delta)F_0\,. \tag{3}$$

Here, $F_0$ is Maxwellian at temperature $T = mv_t^2/2$, $J_0$ is a zero order Bessel function, $\delta = \sqrt{k_x^2 + k_l^2}\,\rho_0\left(v_\perp B_0/v_t B\right)$ is the finite gyroradius parameter, where $\rho_0 = v_t/\Omega_0$ is the gyroradius with the thermal velocity and the field of $B_0$, and $\omega_t = v_{//}B_p/\left[\left(dl/d\theta\right)B\right]$, $\omega_D = \nabla S\cdot\mathbf{b}\times\left(\mu\nabla B + mv_{//}^2\mathbf{b}\cdot\nabla\mathbf{b}\right)/m\Omega$, and $\omega_*^T = \left(ncT\left|\nabla r\right|/eR_s B_p\right)\partial\ln F_0/\partial x$ are the transit frequency, magnetic drift frequency and pressure-driven-diamagnetic drift frequency, respectively. In the above, all the subscripts to represent particle species are neglected for simplicity of description. After solving the responses for ions and electrons, the quasi-neutrality condition provide us a governing equation for both microinstabilities and GAM oscillations.

Generally, the $\omega_t$, $\omega_D$, $\omega_*^T$, and $\delta$ are complicated functions of $\theta$ and shape parameters. However, in an elliptic nest surface with infinite aspect ratio, they are simplified as follows,

$$\omega_t = \frac{1+s_\kappa/2}{1+s_\kappa \sin^2\theta}\frac{v_t}{qR_0}, \tag{4}$$

$$\omega_d = \left[\omega_* \frac{L_n}{R_0}\frac{(1+s_\kappa \sin^2\theta)(\kappa\cos\theta + g\sin\theta)}{(1+s_\kappa/2)(\sin^2\theta+\kappa^2\cos^2\theta)} + \omega_{dx0}\frac{\sin\theta}{\kappa(1+s_\kappa\sin^2\theta)}\right]\left(\frac{v_\perp^2}{v_t^2}+\frac{2v_{//}^2}{v_t^2}\right), \tag{5}$$

$$\omega_*^T = \frac{\omega_*}{\kappa}\frac{1}{(1+s_\kappa/2)}\left[1+\eta\left(\frac{v^2}{v_t^2}-\frac{3}{2}\right)\right], \tag{6}$$

$$\delta = k_{x0}\rho_0\left(\frac{v_\perp}{v_t}\right)\frac{\sqrt{\sin^2\theta+\kappa^2\cos^2\theta}}{\kappa(1+s_\kappa\sin^2\theta)} + k_\theta\rho_0\left(\frac{v_\perp}{v_t}\right)\frac{(1+s_\kappa\sin^2\theta)\sqrt{1+g^2}}{(1+s_\kappa/2)\sqrt{\sin^2\theta+\kappa^2\cos^2\theta}}, \tag{7}$$

where $\omega_* = k_\theta cT/(eB_0L_n)$, $\omega_{dx0} = k_{x0}cT/(eB_0R_0)$, $\eta = L_n/L_T$, $L_T^{-1} = d\ln T/dx$ and $L_n^{-1} = d\ln n/dx$. The relative simple expression makes it possible to make an analytical treatment in studying the effect of elongation on microinstabilities and GAMs, which is displayed in next sections.

## 3. Applied to Geodesic Acoustic Modes

For GAMs, the toroidal number of the potential perturbation is zero, $n=0$, then $\omega_* = k_\theta = 0$. The expressions in Eq. (4)-(7) can be simplified further. We can analytically solve the gyrokinetic equation as follows,[17]

$$h = \exp\left[i\int\left(\frac{\omega}{\omega_t}-\frac{\omega_d}{\omega_t}\right)d\theta\right]\int d\theta \frac{-iqF_0\hat{\phi}}{T}\frac{\omega}{\omega_t}J_0(\delta)\exp\left[-i\int\left(\frac{\omega}{\omega_t}-\frac{\omega_d}{\omega_t}\right)d\theta\right], \tag{8}$$

where the periodic boundary condition is automatically satisfied. Moreover, the GAM is mostly governed by the ion dynamics and the electron response to the $m \neq 0$ components of potential induces a $T_e/T_i$ correction in the GAM frequency. For simplicity, we assume the potential $\hat{\phi}$ has a strict $m=0$ structure, then only the ion dynamics is left and the quasi-neutrality condition reduces to that the flux average of the ion density perturbation is zero,

$$\int d\theta dEd\mu \frac{BR_s}{|v_{//}|}\left[-q_jF_{0j}\hat{\phi}/T_i + \hat{h}_iJ_0(\delta_i)\right] = 0. \tag{9}$$

Under the large aspect ratio assumption, the velocity-space integral in Eq. (9) can be written as expressions of the plasma dispersion function. Then, Eq. (9) is simplified as follows,

$$\frac{\delta_0^2}{2}(1-2\lambda) = \frac{\omega_{dt0}^2}{2}\left[Q(\hat{\omega}) - \frac{\lambda}{4}Q\left(\frac{\hat{\omega}}{2}\right)\right], \tag{10}$$

where $\delta_0 = k_{x0}\sqrt{(\kappa^2+1)/2\kappa^2}$, $\omega_{dt0} \equiv \omega_{dx0}/\omega_{t0} = k_{x0}q/[2\kappa(1+s_\kappa/2)]$, $\lambda = s_\kappa/(2+s_\kappa)$, $Q(\hat{\omega}) = -3-2\hat{\omega}^2 - (\hat{\omega}^{-1}+2\hat{\omega}+2\hat{\omega}^3)Z(\omega)$ and $\hat{\omega} = qR\omega/v_{ti}$. Here, the expansion based on the assumption $\lambda << 1$ is performed. The first order term $O(\lambda)$ is retained except for the terms with the order of $\lambda(\kappa^2-1)/(\kappa^2+1)$. Also, in Eq. (10), only the leading order $O(k^2)$ terms are retained for small $k$, that is, we only consider the lowest order relation of the GAM frequency to the plasma shape. The enhanced damping due to higher order $k$ terms and corresponding higher order harmonic resonances was discussed in circular plasmas [20, 21]and will not considered here. Now we return to the physics in Eq. (10). The LHS is the so-called the classical polarization due to the gyro-motion; and the RHS term is the polarization due to the toroidal curvature and gradient drift. The toroidal drift polarization is proportional to $1/\kappa^2$, while the classical polarization is characterized by $\delta_0^2$, which is proportional to $(1+\kappa^2)/2\kappa^2$. Therefore, the dependence of the GAM frequency on $\kappa$ is not simply the factor of $1/\kappa$ in the geodesic curvature drift term, but the result of the balance between the classical and drift polarization. As for $s_\kappa$, its modification to the classical polarization, $1-2\lambda$, is just cancelled by the $1/(1+s_\kappa/2)^2$ in the $\omega_{dt0}^2$ term for $\lambda = s_\kappa/(2+s_\kappa) << 1$ and only the $\lambda Q(\omega/2)/4$ take into effects. At high $q$, it is expected $|\hat{\omega}| \sim |qR\omega/v_{ti}| >> 1$. Asymptotically expanding the plasma dispersion function and neglecting higher order small terms, Eq. (10) reduces to

$$-\frac{\kappa^2+1}{2q^2}+\frac{7(1-s_k/2)}{4\omega^2}+\frac{23(1-2s_k)}{8\omega^4}-i\omega^3\left[\exp\left(-\omega^2\right)-\frac{s_k}{64}\exp\left(-\frac{\omega^2}{4}\right)\right]=0. \tag{11}$$

Then, the eigen-frequency of GAM is obtained

$$\omega=\frac{\sqrt{7}v_{ti}}{2R}\sqrt{\frac{2-s_k}{1+\kappa^2}}\left[1+\frac{23(\kappa^2+1)(1-s_k)}{98q^2}\right], \tag{12}$$

$$\gamma=-i\frac{2\sqrt{\pi}\left(R\omega_{GAM}/v_{ti}\right)^6}{7(1-s_\kappa/2)}\left[\exp\left(-\omega^2\right)-\frac{s_\kappa}{64}\exp\left(-\frac{\omega^2}{4}\right)\right]. \tag{13}$$

It is clearly seen from Eq. (12) that, when $\kappa$ increases, the real frequency dramatically decreases and, correspondingly, the damping becomes strong due to a decrease in the frequency in strongly elongated plasmas. We can compare the analytical formula to the experimental scaling and the simulation result. Experimental data on the ASDEX-Upgrade [13] shows that the elongation decrease the GAM frequency by the scaling of $\omega_{GAM} \propto 2/(1+\kappa)$. The analytical formula we obtained gives $\omega_{GAM} \propto \sqrt{2/(\kappa^2+1)}$, which is a slightly stronger dependence than the experimental scaling, but is closer than the $1/\kappa$ dependence. The analytical dependence is also consistent with that from theoretical [22] and simulation results [14-16] in trend. As for $s_\kappa$, it slightly decrease the GAM frequency. However, it should be noted, since finite $s_\kappa$ introduces high harmonics, the analytical process based on the expansion technique in $s_\kappa$, equivalently $\lambda$, may not give a right estimation on the effect of $s_\kappa$ on the damping rate. (It is because, even $f_0(\omega) >> \lambda f_1(\omega)$, it cannot get $\mathrm{Im}[f_0(\omega)] >> \lambda\,\mathrm{Im}[f_1(\omega)]$). The explicit expression of the damping rate can be given by numerical calculation rather than the analytical derivation by direct expansion technique used here, which is scheduled in future work. However, the dependence of real frequency is still reasonable since the damping rate is much less than the real frequency.

## 4. Applied to Temperature Gradient Instabilities

In this section, we turn to temperature gradient instabilities. Here, the potential perturbation has a high $n$, then the periodic boundary condition cannot be easily satisfied due to the existence of magnetic shear. However, the ballooning representation can be used, and then Eq. (4) can be easily integrated with the boundary condition $h(\theta)=0$ as $|\theta|\to\infty$ as follows

$$h^{\pm}=\frac{\mp iqF_0\hat{\phi}}{T}\int_{\mp\infty}^{\theta}d\theta'\left(\frac{\omega}{\omega_t}-\frac{\omega_*^T}{\omega_t}\right)J_0(\delta')\exp\left[\mp i\int_{\theta}^{\theta'}d\theta''\left(\frac{\omega}{\omega_t}-\frac{\omega_d}{\omega_t}\right)\right], \tag{14}$$

where the signs "$\pm$" denote different parallel velocity direction. Substituting the integral form of $h(\theta)$ into the quasi-neutrality condition, a integral eigenmode equation is obtained,

$$\sum_{j=i,e} q_j \int \left[ -q_j F_{0j}\hat{\phi}/T_j + \left(\hat{h}_j^+ + \hat{h}_j^-\right) J_0\left(\delta_j\right)\right] d^3v = 0 \quad , \tag{15}$$

Without the plasma shape parameters, the integral equation direct reduces to the equation in the $\hat{s}-\alpha$ configuration,[18] which has been extensively studied and the code can be developed to study the effect of plasma shape without any intrinsic difficulty.

However, before solving Eqs. (14) and (15) numerically, an analysis on the local model may be instructional. We can see the drift instability and the GAM are almost two limits of one problem. Firstly, the variation of zonal potential in perpendicular plane is across the flux surface, while that the potential of the drift instability mainly inside the flux surface. Although the radial wave number $k_{x0}$ also exists in the study of the two-dimension structure of the drift instability, the faster growing mode is usually characterized by $k_{x0} = 0$. Secondly, the GAM depend on the coupling of the curvature drift to the transit motion, therefore it is close related to the poloidal dependence of the curvature drift and the transit motion. While, the drift instabilities is due to the coupling of the diamagnetic drift to the curvature drift (in the toroidal limit) and/or the parallel transit (in the slab limit), therefore, the perturbation has a clear streamer structure and is poloidally asymmetric, strongest in the bad-curvature region and weakest in the good- curvature region. Then, in the local limit with $\theta \approx 0$, we have

$$\omega_t = \left(1 + s_\kappa/2\right)\left(v_t/qR_0\right), \tag{16}$$

$$\omega_d = \frac{\omega_* L_n/R_0}{\left(1+s_\kappa/2\right)\kappa}\left(\frac{v_\perp^2}{v_t^2} + \frac{2v_{//}^2}{v_t^2}\right), \tag{17}$$

$$\omega_*^T = \frac{\omega_*}{\kappa}\frac{1}{\left(1+s_\kappa/2\right)}\left[1 + \eta\left(\frac{v^2}{v_t^2} - \frac{3}{2}\right)\right], \tag{18}$$

$$\delta = \frac{k_\theta \rho_0}{\kappa\left(1+s_\kappa/2\right)}\left(\frac{v_\perp}{v_t}\right). \tag{19}$$

If we define

$$k_{//eff} = k_{//}\left(1 + s_\kappa/2\right) \tag{20}$$

and

$$k_{\theta eff} = \frac{k_\theta}{\kappa\left(1+s_\kappa/2\right)}, \tag{21}$$

the dispersion equation reduces to the same form as in circular plasmas. Therefore, if $s_\kappa = 0$, the effect of $\kappa$ only cause an expansion in $k_\theta$-spectrum of the frequency and growth rate but do not influence the maximum value of the growth rate. Since the critical threshold of temperature gradient, $R_0/L_{Tc}$, is obtained by scanning the poloidal wavenumber $k_\theta$, it is expected that $R_0/L_{Tc}$ is hardly influenced by $\kappa$ lonely. Of course, if the nonlocal effect is included, the $\kappa$ can influence the stability property through changing the ratio between the poloidal and radial component of curvature drift driving term, i.e. $\omega_d$ in Eq. (5), and introducing the term $g(S_1)$ in Eq. (2), which involves the effect of magnetic shear $\hat{s}$ and plasma pressure $\alpha$ in circular case. The expression is so complex that only a qualitative estimation can be made. If $\kappa$ is enough large, the driving force become the same as that in the local limit, but the stabilizing force due to the increase of finite Larmor radius effect increases. It means that the $\kappa$ has a stabilizing effect in the nonlocal consideration.

Now, we stare at the effect of $s_\kappa$. It significantly changes the parallel wave vector. This change can be understood as the decrease in the effective safety factor due to finite $s_\kappa$, $q_{eff} = q/(1+s_\kappa/2)$. Then we can reach the dependence of the critical gradient on $s_\kappa$ by generalizing existing formulae. For example, if we use the $q$ dependence obtained in toroidal geometry as $R/L_{Tc} \propto 1.33+1.91\hat{s}/q$ ,[10] it can reach $(R/L_{Tc})/(R/L_{Tc})_{s_k=0} \sim 1+ (s_k/2)/(1+1.33q/1.91\hat{s}) \sim 1+0.23s_k$ for nominal parameters $s=0.8$ and $q=1.4$. However, it underestimates the effect of $s_\kappa$ since the effect of the variation of $q_{eff}$ in $k_{\theta eff}$ is eliminated by scanning $k_\theta = nq/r$. If we use the dependence obtained in sheared slab, where $R/L_{Tc} \propto R/L_s \sim \varepsilon_n \hat{s}/qR$, [23] a simple dependence is gotten as $(R/L_{Tc})/(R/L_{Tc})_{s_k=0} = 1+0.5s_\kappa$. In fact, in the study [24] of the transition from toroidal to slab temperature gradient driven modes, the increase of parallel transit frequency has been shown to increase the threshold value of $\eta$, where a fitting formula $\eta_c = (1+x^4)/(1+x^4/3)$ is given with $x=\omega_t/\omega_d \approx (1+s_\kappa/2)/k_{\theta eff}\rho q$. It is easily understood that it gives a medium dependence.

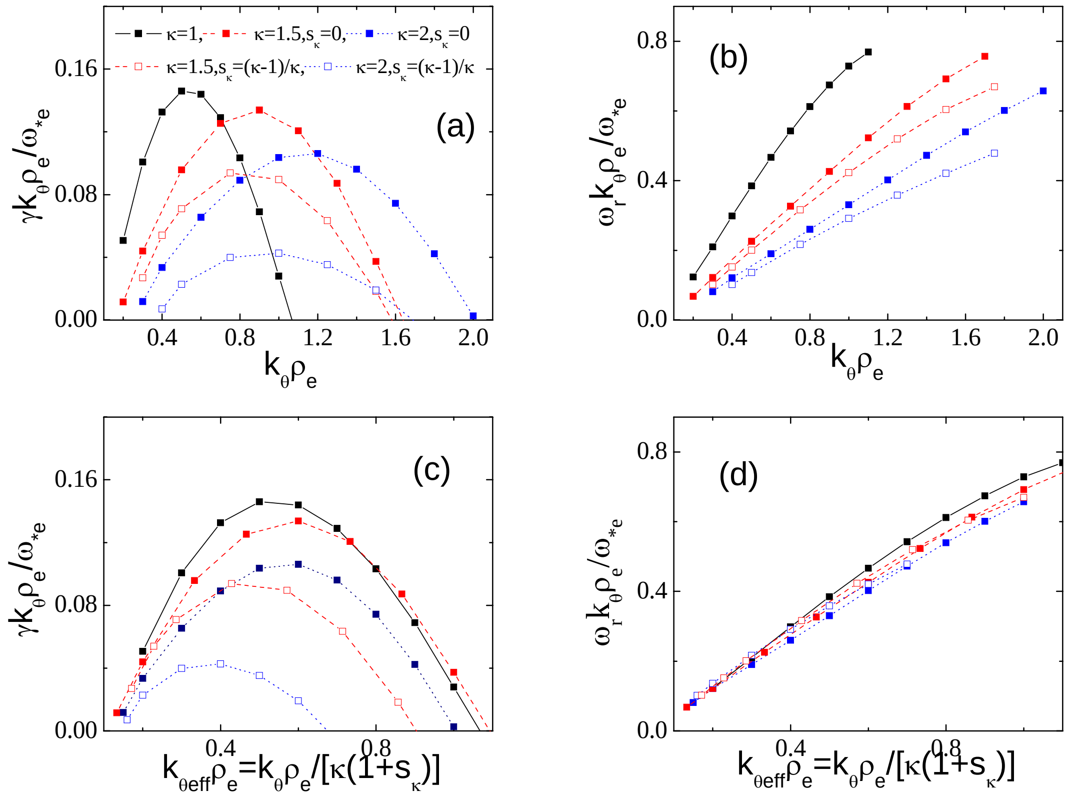


*FIG. 1. Spectrum of growth rates (a,c) and real frequencies (b,d) of ETG modes.*

Numerical results are consistent with our analysis. Calculations are performed for electron temperature gradient (ETG) modes with adiabatic ions. In fact, if we do not consider the trapped particle effect and the coupling to Alfven waves due to finite $\beta$, the ITG and ETG have a similar character and only the roles of electrons and ions are interchanged. Figure 1 shows the normalized growth rate and real frequency as functions of $\rho_e k_\theta$ for $\kappa=1.0$, 1.5 and 2.0 and $s_\kappa=(\kappa-1)/\kappa$ and 0, respectively. Other parameters used are $L_n/L_{Te}=2.5$, $ZT_e/T_i=1$, $\hat{s}=0.8$, $q=1.4$ and $R_0/L_n=0$. As $\kappa$ increases, the $k_\theta$-spectrum of growth rate is greatly shifted towards larger values of $k_\theta\rho_e$, while the maximum of the growth rate only slightly decreases at $s_\kappa = 0$. Only when $s_\kappa = (\kappa - 1)/\kappa$ is employed, the growth rate decreases significantly as $\kappa$, actually $s_\kappa$, increases. It is also easily to understand the decrease in the real frequency due to finite $\kappa$ and $s_\kappa$ since the major driving force on the ballooning-type instability induces the relation of $\omega^2 \sim |\omega_* \omega_D| \propto k_{eff}^2$. Figs. 1(c)-(d) display the same results as Figs.1(a)-(b) but $k_{\theta eff}$ is used as a variable, where the spectra of growth rate and frequency at different $\kappa$ converge. It is also trivial that, as $s_\kappa$ increases, the growth rate decreases and the spectrum shrinks towards the large wavelength region.

For obtaining the critical electron temperature gradient, generally, it should choose a set of $k_\theta$'s in the region of fast-growing linear modes to calculate and find the minimum. At typical parameters, this fastest growing region is believed at $k_\theta \rho_e$~0.2-0.4. We expect that, in elongated plasmas this region should shift to the larger $k_\theta \rho_e$ region, i.e. $k_{\theta eff} \rho_e$~0.2-0.4. Numerical results verified our supposition. The scaling of $R/L_{Tec}$, with respect to $\kappa$ for different $s_\kappa$ is shown in Fig.2. The results can be fitted by this formula well

$$(R/L_{Tc})/(R/L_{Tc})_{\kappa=1} = (1 + 0.36 s_\kappa)[1 + 0.11(\kappa - 1)]. \quad (22)$$

We can compare this fitting formula to our semi-analytical scaling and other fitting formula based on simulation results. Our analysis gives the $1 + (0.23 \sim 0.5) s_\kappa$ and suggests a weakly positive dependence on $\kappa$, which agrees well with Eq. (22). Ref. [10] performs a similar numerical result as here and also find the increase of $R/L_{Tec}$ is mainly due to the radial derivative of $\kappa$. However, they used the $r\partial\kappa/\partial r = \kappa s_\kappa$ as a variable and reaches the scaling of $(R/L_{Tc})/(R/L_{Tc})_{\kappa=1} = (1 + 0.3 r\partial\kappa/\partial r)$, which is consistent with Eq. (22) in trend, but neglect the effect of $\kappa$ and overestimate the effect of $s_\kappa$ a little.

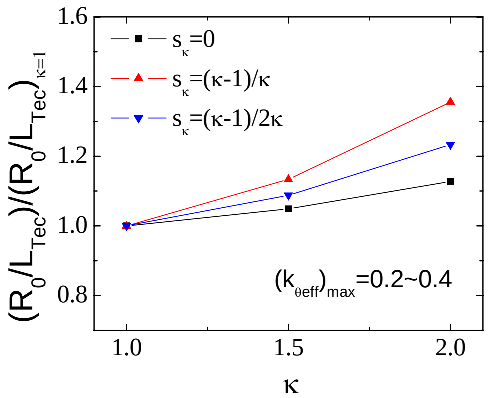


*FIG. 2. Critical temperature gradient vesus elongation.*

## 5. Summary

In summary, a gyrokinetic theory is established in noncircular toroidal plasmas by employing the local MHD equilibrium model. The temperature gradient driven instability and the GAM are investigated as two limits of this problem. The GAM is close related to the poloidal dependence, or its poloidal average, of the curvature drift and the transit motion, while the temperature gradient driven instability is mainly decided by the local behavior around $\theta \approx 0$. Dependence of the GAM frequency on elongation and its radial derivative are analytically in the large aspect limit. It is found that the GAM frequency sharply decreases with an increasing elongation by dependence of $[(2 - s_\kappa)/(\kappa^2 + 1)]^{1/2}$, which comes from the modification of classical ion polarization divided by that of curvature drift polarization, where the elongation is a major factor and $s_\kappa$ is a minor one. However, for temperature gradient driven instability, as $\kappa$ increases, the $k_\theta$-spectrum of growth rate is greatly shifted towards larger values of $k_\theta$, while the maximum of the growth rate only slightly decreases at $s_\kappa = 0$. However, the radial deviation of elongation $s_\kappa$ can significantly influence the stability property of temperature gradient instability by modifying the parallel wave number. Dependence of the critical temperature gradient on the elongation deformation can be well described by this formula as $(R/L_{Tc})/(R/L_{Tc})_{\kappa=1} = (1+0.36 s_\kappa)[1+0.11(\kappa-1)]$.

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