Non-local Models of Perturbative Transport: Numerical Results and Application to JET Experiments

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Abstract. Perturbative experiments in fusion plasmas have shown that edge cold pulses travel to the center of the device on a time scale much faster than expected on the basis of diffusive transport. An open issue is whether the observed fast pulse propagation is due to non-local transport mechanisms or if it could be explained on the basis of local transport models. To elucidate this distinction, perturbative experiments involving ICRH power modulation in addition to cold pulses have been conducted in JET for the same plasma. Local transport models have found problematic to reconcile the fast propagation of the cold pulses with the comparatively slower propagation of the heat waves generated by power modulation. In this paper, a non-local model based on the use of fractional diffusion operators is used to describe these experiments. The proposed non-local model is able to reproduce the profiles of the amplitude and the phase of the electron temperature heat waves excited by the ICRH modulation in JET. Most importantly, for the same model parameter values, the model can successfully accommodate the propagation of pulses with time delays comparable to those in the experiment, ~4 ms. We also present a numerical study of the parameter dependence of the transport properties of the fractional model, and discuss the role of non-locality in the flux-gradient dependence.

1. Introduction

Cold pulse experiments in JET and other machines have shown that perturbations applied at the edge travel to the center much faster than expected on the basis of diffusive time scales compatible with plasma confinement, see e.g. [1]. An open issue in describing these experiments is whether the observed fast propagation is due to non-local transport mechanisms, or if it could be explained on the basis of non linear but local transport models. Determining the role of non-locality using only cold pulse experiments is difficult because both non-linear local and non-local models can exhibit fast propagation for properly chosen parameters and conditions. Thus, to discriminate between the two transport mechanisms conclusively, it is necessary to consider, in addition to cold pulses, other type of perturbations with different propagation properties. The use of RF power modulation perturbations is a natural choice as it has been experimentally observed that the resulting heat waves exhibit clear propagation asymmetries compared to pulses. In particular, cold pulses can propagate fast even in regions where heat waves slowdown. A promising modeling approach is then to focus on experiments in which both type of perturbations, cold pulses and power modulation, are present for the *same* plasma. This type of experiments has been carried out in JET [1,2], and their modeling is the main object of study in the present paper. Previous attempts to account for these experiments using local diffusive models have found problematic to reconcile the fast propagation of the cold pulses with the comparatively slower propagation of the heat waves generated by power modulation. In this paper, a non-local model based on the use of fractional diffusion operators [3] is used to describe these experiments [4].

[#] See the Appendix of F.Romanelli et al., Fusion Energy 2008 (Proc. 22nd Int. Conf. Geneve, 2008) IAEA, (2008)

2. Non-local transport model

We restrict attention to energy transport in a one-dimensional domain

$$\partial_t\left[3/2\, nT\right] = -\partial_x q + S \tag{1}$$

where x denotes the radial coordinate in the slab approximation. In the standard diffusive transport model, the flux is determined according to the local Fourier-Fick's prescription, $q_d = -\chi_{eff}\, n\partial_x T$, where χ_{eff} denotes the effective diffusivity, and for simplicity we have not included a drift, pinch term. In the fractional-diffusion model [3] this prescription is replaced by

$$q_{nl} = -\chi_{nl}\, n\left[l\ {}_aD_x^{\alpha-1} - r\ {}_xD_b^{\alpha-1}\right]T \tag{2}$$

where ${}_aD_x^{\alpha-1}$ and ${}_xD_b^{\alpha-1}$ are non-local integro-differential operators defined as

$${}_aD_x^{\alpha-1}T = \frac{1}{\Gamma(2-\alpha)}\frac{\partial}{\partial x}\int_a^x \frac{T(y,t)}{(x-y)^{\alpha-1}}dy\,, \qquad {}_xD_b^{\alpha-1}T = -\frac{1}{\Gamma(2-\alpha)}\frac{\partial}{\partial x}\int_x^b \frac{T(y,t)}{(y-x)^{\alpha-1}}dy \tag{3}$$

where l and r are constants, Γ is the gamma function, (a,b) is the domain of the system, and the parameter $0<\alpha<2$ determines the degree of non-locality. In Fourier space, the non-local flux is

$$F\left[q_{nl}/(\chi_{nl}\, n)\right] = -\left[l(-ik)^{\alpha-1} - r(ik)^{\alpha-1}\right]F\left[T\right] \tag{4}$$

The scaling of the Fourier transform of the flux as a fractional power of k, motivates the mathematical interpretation of the non-local operators as fractional derivatives. In the limit $\alpha=2$, the model reduces to the Fourier-Ficks prescription, and in the limit $\alpha=1$ the model reduces to a free-streaming non-local model.

As it is well known, the standard diffusion model is closely related to the Brownian random walk. In a similar way, in the case of particle transport, the fractional diffusion model is closely related to the theory of generalized random walks that allow the incorporation of no-Gaussian (Levy) and non-Markovian stochastic processes. This connection provides a foundation of the fractional diffusion model in the context of non-equilibrium statistical mechanics, see for example Ref.[5], and Ref.[6] for a discussion in the context of plasma physics. Some previous applications of the fractional diffusion model include the description of tracer transport in pressure-gradient-driven turbulence [7], and the study of basic non-diffusive transport phenomenology including anomalous scaling of confinement time, up-hill transport, pinch-effects and fast propagation phenomena [3]. In this paper we discuss the application of the model to perturbative experiments in JET [4].

The parameters a and b in the integration limits in Eq.(3) define the lower and upper boundaries of the domain $x \in (a,b)$. In the case of a finite size domain, which is the case of interest here, the definition of the non-local flux requires the regularization of the fractional derivatives. Here, we adopt the regularization described in [3], and solve Eq.(1) with the total flux including a diffusive component, $q_d = -\chi_{eff}\, n\partial_x T$, and the nonlocal fractional component q_{nl} in Eq.(2). The integration domain is the interval, $x \in (0,1)$, and the boundary conditions are

$$q(x=0,t) = [q_d + q_{nl}](x=0,t) = 0, \qquad T(x=1,t) = 0 \tag{5}$$

where $x=0$ denotes the magnetic axis and $x=1$ denotes the plasma edge. Based on the assumption that in magnetically confined plasmas there is a qualitative difference between

core transport and edge transport, we will assume a non-local diffusivity of the form

$$\chi_{nl}(x) = \frac{\chi_{nl0}}{2}\left[\tanh\left(\frac{x - x_c}{L}\right) + \tanh\left(\frac{x_c}{L}\right)\right] \tag{6}$$

That is, in the core region, $x \sim 0$, $\chi_d >> \chi_{nl}$, and transport is dominated by diffusive processes. The transition to non-diffusive transport occurs in a boundary layer at $x = x_c$ of width ~L in which χ_{nl} changes from zero to the edge value χ_{nl0}.

3. Fast pulse propagation in the non-local transport model

In this section we present numerical results on fast cold pulse propagation in the non-local fractional transport model. Given an equilibrium temperature profile $T_0(x)$, we consider the evolution of a localized edge temperature perturbation. Figure 1 shows the spatio-temporal evolution of the perturbed temperature, $\delta T(x,t) = T(x,t) - T_0(x)$, and the perturbed flux, $\delta q = q(x,t) - q_0(x)$, where $q_0(x)$ is the equilibrium steady state flux. For the fractional model we assumed a symmetric, $l=r$, non-local operator with α=1.25. For further details on the numerical simulations and results corresponding to other parameter values see Ref.[4].

The non-local response to the edge perturbation gives rise to a fast drop of the temperature at the core, even larger than the drop experienced at intermediate places as evidenced by the detached green blob near the core at t~0.05 in Fig.1 (a). The evolution of the temperature pulse can be understood by looking at the corresponding perturbed flux in Fig.1 (b). In the local diffusive case, δq depends on the local gradient of δT and therefore no δT is detected near the core since δq is highly localized at the edge where $\partial_x \delta T$ is large However, in the non-local case, δq exhibit long-range ‘tongues’ that extend all the way to the core. The positive flux contribution of these tongues is responsible for the transport of heat leading to the rapid cooling of the core.

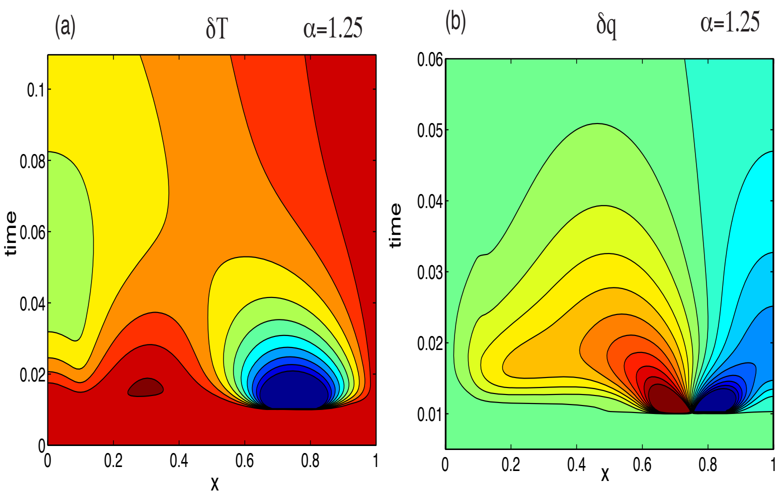

FIG. 1. Space-time evolution of the temperature perturbation (a), and the flux perturbation (b,) during a cold pulse simulation according to the non-local, symmetric, l=r, fractional transport model for α=1.25 . On (a) blue contours denote large negative values of δT, and red denotes δT=0. On (b) red (blue) denotes large positive (negative) values of δq.

Figure 2 shows the time traces of the normalized temperature perturbations at different locations, for different values of the non-locality parameter α in the symmetric case, $l=r$. At the point where the pulse is introduced, x=0.75, the temperature relaxation is dominated by diffusive local transport and very similar behavior is observed independent of the value of α. However, at the core, x=0, a significant delay of the signal is observed in the diffusive case, and the pulse speed is observed to increase with decreasing α.

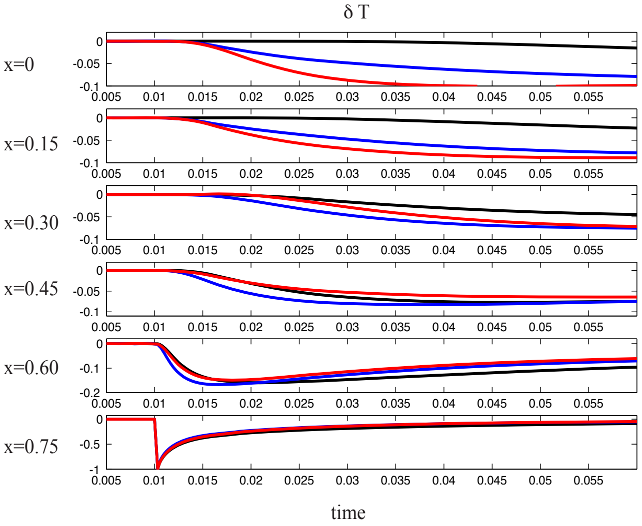

FIG. 2. Dependence of the non-local temperature response on the fractional diffusion parameter α. The plot shows time traces of the normalized temperature perturbation at various spatial locations, with x=0.75 denoting the initial location of the cold pulse. The fastest response, shown in red, corresponds to α=1.25. The blue curve corresponds to α=1.75. For reference, the slow local diffusive response is shown in black.

4. Heat wave propagation in the non-local transport model

To study the propagation of "heat waves" due to power modulation, we integrated the non-local fractional model with a source consisting of an on-axis component, and an off-axis component localized at *x=0.5* including a time-periodic amplitude modulation. To study the response of the system to the modulation we considered the Fourier decomposition of the perturbed temperature $\delta T = T(x,t) - \langle T \rangle (x)$

$$\delta T(x,t) = \sum_{n=1}^{\infty} A_n(x)\cos\left[2\pi n \nu t + \Phi_n(x)\right] \tag{7}$$

where $\langle T \rangle = (1/\tau)\int_0^\tau T(t')dt'$ is the time averaged equilibrium profile. Figure 3 shows that, compared to the cold pulse, non-locality does not seem to have a significant impact on the relatively slow propagation of temperature perturbations due to power modulation. Although, as shown in Fig.3 (a), in the fractional diffusion case the temperature perturbation is stronger near the core, the difference is not very significant. As discussed in Ref.[4], the difference between the fractional and diffusive dynamics becomes smaller as the power modulation frequency increases.

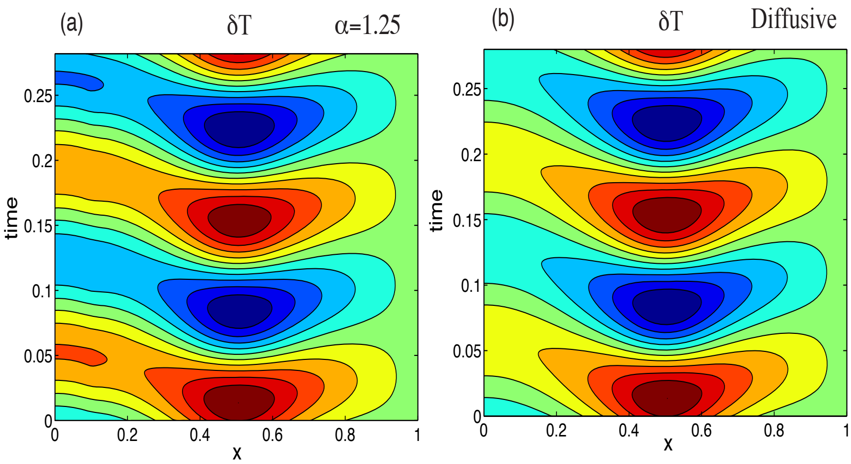

FIG. 3. Spatio-temporal dynamics of heat waves generated by power modulation. Panel (a) corresponds to the non-local, symmetric (l=r) fractional diffusion model with a=1.25. Panel (b) shows the standard, local, diffusive case.

5. Application of model to JET experiments

Several previous experiments in JET have shown that cold pulse perturbations applied at the edge, either via laser ablation of metallic impurities or shallow deuterium pellet injections, travel to the center much faster than expected on the basis of diffusive time scales compatible with plasma confinement. However, in those experiments it was never clarified whether such high propagation speed needed a non-local transport component or if they could be described in the context of local, non-linear models. Indeed, the fast propagation of pulses can be described in the context of the critical gradient model (CGM) with large stiffness. However, a potential problem with this approach is that high levels of stiffness imply the fast propagation of *all* type of perturbations, and this seems to be in contradiction with the relatively slow propagation of heat waves observed in JET and other machines.

A convincing test to discriminate between the non-local and the non-linear, critical gradient length driven, transport can be made only if the two different types of perturbations, power modulation and cold pulses, are applied to the same plasma. These experiments have been carried out on JET [2,8] and are the main object of study here. Figure 4 shows the measured amplitude and phase profiles in Eq.(7) of the 1st and 3rd harmonics of the electron temperature. At the end of the power modulation phase, a cold pulse is applied to cool the plasma edge. Figure 5 shows the time evolution of Te at different radii following the edge cooling. It is observed that the signal at the core exhibits a temperature drop of 30 eV in about 4 ms.

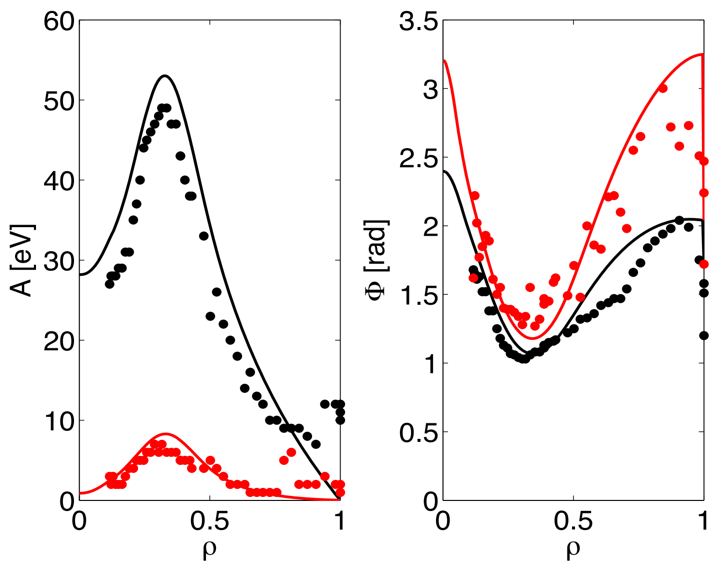

FIG. 4. Experimental (dots) and non-local model (lines) profiles of A and F corresponding to the 1st (black) and the3rd (red) harmonics.

Both perturbative experiments, the cold pulse and the heat wave, have been simulated using the critical gradient transport model (CGM) in Ref.[7]. Results are described in detail in Ref.[2]. The stiffness level and other parameters of the CGM are first determined by fitting the modulation data, and then the model is used to predict the propagation speed of the cold pulse. In this case, the CGM predicts a pulse delay of 22 ms that is much longer than the experimentally observed delay of 4 ms. In addition to the semi-empirical CGM, other attempts to describe these experiments using first principle based transport models or turbulence codes have found problematic to reconcile the fast propagation of the cold pulses with the relatively slower propagation of the heat waves. For example, the Weiland model [10] accounts reasonably well for the heat wave propagation but predicts a delay of the order of 50 ms for the cold pulse. On the other hand, the 3D fluid turbulence code TRB [11] predicts a very fast propagation speed for *both* pulses and heat waves. In the 3D global electromagnetic code CUTIE [12] cold pulses damp fast in the outer region without reaching the center, and power modulation simulations are not feasible due to the long time scales involved in JET. Turbulence spreading models tend to do a better job accounting to faster responses while at the same time succeeding in reproducing the modulation data [13]. This apparent lack of successes of local transport models is our main motivation to study the application of the non-local fractional diffusion model discussed in Sec.2. to describe the perturbative experiments at JET.

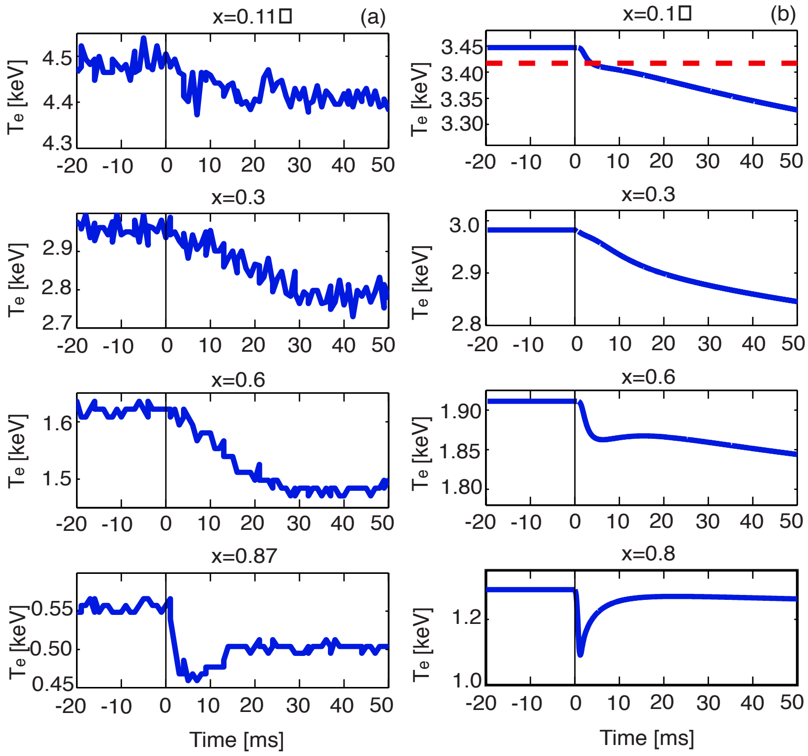

FIG. 5 Comparison between the temperature traces in JET (a) and the non-local fractional model (b). Consistent with the experiment, the model exhibits a drop of 30 eV (corresponding to the dashed red line) in about 4ms.

As a first step we calibrated the fractional diffusion model by fitting the amplitude and the phase profiles of the dominant harmonics of the electron temperature corresponding to the propagation of heat waves excited by the ICRH modulation in JET. Figure 4 shows that a good fit is achieved with $n = 2.6\times 10^{19}$ part/m^3, α=1.25, l=r, a standard diffusivity profile of the form $\chi_d = (0.75 + 6x)$ m^2/sec, and a fractional diffusivity of the form in Eq.(6) with $\chi_{nl0} = 2$ m$^\alpha$/sec, x_c=0.1, and L=0.025. Once the model parameters are determined by fitting the modulation data, the second critical step is to predict the propagation speed of the pulse. As mentioned before, it is here that the local models apparently fail by significantly underestimating the pulse speed. However, as shown in Fig.5 this is not the case with the fractional model. In particular, consistent with the experiment, the non-locality of the fractional model can successfully accommodate the propagation of pulses with a time delay of the order of ~4 ms.

To further explore the role of non-locality Figs. 6 and 7 show the flux-gradient relation at different locations for power modulation and cold pulse propagation. The data corresponds to the fractional diffusion model used to describe the JET data in Figs. 4 and 5. Figure 6 exhibits a close to linear flux-gradient relation indicative that in the case of ICRH power modulation transport is mostly local. This is consistent with Fig.3 according to which non-locality does not seem to have a significant impact in the propagation of heat waves. However, the 'loops' in Fig.7 provide evidence of strong non-local transport in the propagation of the cold pulse. In particular, the observed multivalued flux-gradient relation in Fig.7 is inconsistent with a local diffusive Fourier-Ficks prescription unless a very peculiar, ad-hoc spatio-temporal dependence is built in the effective diffusivity.

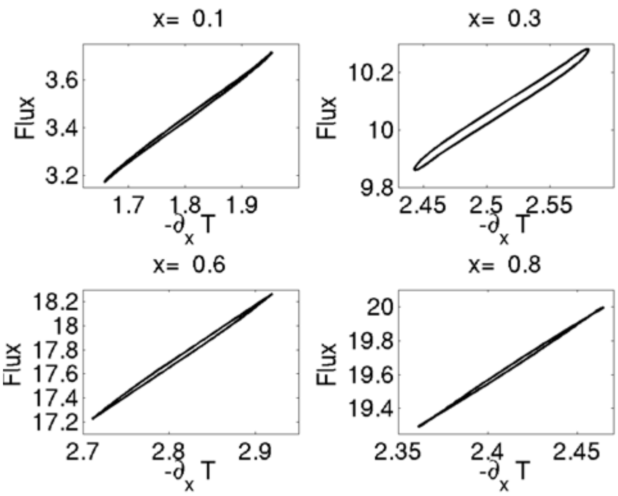

FIG. 6. Flux-gradient relation over a cycle of the ICRH power modulation at fixed spatial locations. The point x=0.3 is near the location of the peak of the modulation (see FIG. 4). Contrary to the cold pulse, in the case of modulation the flux is to a good approximation linearly proportional to the gradient, indicative of local transport.

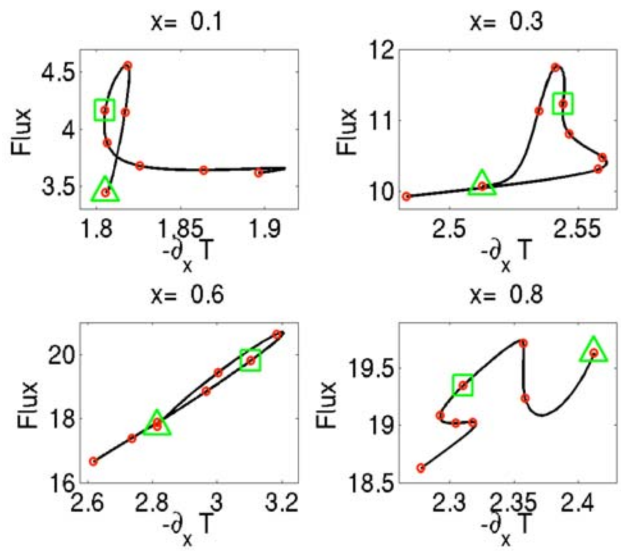

FIG.7. Flux-gradient relation during the cold pulse propagation at fixed locations. The spatial locations correspond to those shown in the temperature traces in Fig.5. The circles along the paths correspond to t=0, 1.3, 2, 4, 6, 10, 15 and 50 ms. *The triangles mark the initial time, t=0, when the pulse was introduced, and the squares mark the time of arrival of the pulse to the core, t=4* ms.

6. Conclusions

Non-local fractional transport models are a natural generalization of diffusive models that provide a unifying framework to describe non-diffusive transport including anomalous scaling, spatial non-locality and non-Markovian (memory) effects. These models have been successfully applied in the past to describe basic non-diffusive transport phenomenology in fusion plasmas, and quantitative aspects of test particle transport in plasma turbulence.

Here we have shown that fractional diffusion is able to reproduce cold pulse and power modulation perturbative experiments conducted in JET that have not been satisfactorily described using local transport models. The JET experiments discussed here show an asymmetry between the propagation of perturbations due to heat modulation and cold pulses. For $x > x_s$, where x_s denotes the location of the ICRH power deposition, waves and pulses propagate fast. However, for $x < x_s$ the heat wave slows down and is damped, but the cold pulses still travel fast. Local transport models have found problematic to simultaneously describe both types of perturbations. In particular, when these models are calibrated to reproduce the slow modulation data, they significantly underestimate the fast propagation of pulses. Here we have shown that a transport model that incorporates a fractional diffusion non-local transport channel as well as a local diffusive channel is able to reproduce satisfactorily both the modulation data and the fast propagation of the pulses.

We have also presented a numerical study of the parameter dependence of the transport properties of the fractional model. It was observed that decreasing α leads to an increase of the propagation speed of pulses. The parameter dependence of the transport properties in the case of power modulation is weaker. In particular, for high frequency perturbations, the amplitude and the phase of the first harmonic of the temperature perturbation are not very sensitive to changes in α. For low frequencies, consistent with the cold pulse results, the speed of the heat wave increases with decreasing α.

Non-locality and critical gradient non-linearities play a complimentary role, and a complete model of perturbative transport most likely should include both. The main motivation to limit attention in this paper to linear non-local models without critical gradients is conceptual and mathematical simplicity. The relatively simple linear model discussed here, has allowed us to make evident the previously overlooked role played by non-locality, independent of further potential complications due to nonlinearity.

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