

Symmetry breaking effects of toroidicity on toroidal momentum transport.

J. Weiland¹, R. Singh², H. Nordman¹, P. Kaw², A.G. Peeters³ and D. Strinzi⁴

1. Department of Radio and Space Science, Chalmers. University of Technology and Euratom-VR Association, S41296 Gothenburg, Sweden
2. Institute for Plasma Research, Bhat Gandhinagar 382 428, India
3. Center for Fusion, Space and Astrophysics, Physics department, University of Warwick, CV4 7AL, Coventry UK
4. Department of Electrical and Copmputer Engineering, National Technical University of Athens, GR-157 73 Athens, Greece, Association Euratom – Hellenic Republic

Abstract

A derivation of symmetry breaking toroidicity effects on toroidal momentum transport has been made from the stress tensor. This effect is usually stronger than the symmetry beaking caused by the flows themselves on the eigenfunction. The model obtained generalizes a recent derivation of diagonal transport elements¹ from the stress tensor to convective elements of turbulent equipartition or thermoelectric types. This gives possibility for interpretation of the same type of effects previously obtained from a phase space conserving nonlinear gyrokinetic equation^{2,3}.

1. Introduction

The present interest in momentum transport¹⁻⁸ has recently focused on symmetry breaking effects^{2, 4, 7} on toroidal momentum transport. The first effect of this type was identified, as the effect of an asymmetric eigen-function on the average of the parallel mode number is needed of the parallel momentum transport. For practical purposes we may approximate the toroidal momentum by the parallel momentum. Recently also symmetry breaking effects of toroidicity were found⁷. For an improved understanding of such effects and for a consistent derivation of fluid equations for momentum transport it is highly desirable to also make the fluid derivation of such effects. Since the magnetic drift is not a fluid drift such effects originates from the stress tensor in fluid equations. We are thus faced with the rather complicated task of including magnetic curvature effects from the stress tensor. On the other hand gyrokinetic derivations have to start from a phase space conserving formulation⁹ of the gyrokinetic equation. Thus quite ambitious approaches are needed both in gyrokinetic^{7, 8} and in fluid derivations. The fluid derivation is, however, more straight forward, showing the strength of fluid theory in deriving the advanced dynamic equations. The stress tensor is important mainly for low frequency phenomena in magnetized plasmas. It was already used for deriving the diagonal magnetic drift terms¹⁰. In the collisionless case stress tensor contributions are associated with finite gyroradius effects for the motion perpendicular to the magnetic field. Here the first results included the gyro viscous cancellations^{11, 12} between convective diamagnetic effects. At this time also kinetic calculations for drift and MHD type modes had just started and full Vlasov descriptions with simplified geometry were used¹¹. The agreement for the lowest order FLR effect between fluid and kinetic derivations^{11, 12} could thus be regarded as a significant achievement at this time. While the FLR effect, for the perpendicular motion is obtained through an expansion in the ratio of characteristic mode frequency and cyclotron frequency, and thus usually treated as small, the toroidal effects from the stress tensor enter as order $\epsilon_n (L_n / R)$ in the parallel momentum equation. Although ϵ_n can be treated small in the edge and it is typically of order 1 in the core. Thus this effect is important for the parallel motion. A simpler way to obtain the diagonal curvature effects is to derive equations of motion for guiding centers (gyro fluid equation). Such equations have magnetic drifts included as guiding center drifts and the equation of motion can be obtained by taking moments of a gyro kinetic equation¹². The diagonal curvature terms then appear as due to a convective magnetic drift (sum of $g \text{ard} \vec{B}$ and curvature terms) with a factor 2 as a convection of parallel momentum flow. In the present work we will start from the Braginskii fluid equation and derive symmetry breaking effects corresponding to Ref. 7, using entirely fluid equations for the toroidal ion temperature gradient mode (ITG).

II. Toroidal momentum flow

The general momentum equation is obtained from ion and electron momentum equations and the continuity equation:

$$m_i N_i \left(\frac{\partial}{\partial t} + \vec{U}_i \cdot \vec{\nabla} \right) \vec{U}_i = -\vec{\nabla} P_i - \vec{\nabla} \cdot \vec{p}_i + e N_i (\vec{E} + \vec{U}_i \times \vec{B}) \quad (1a)$$

$$0 = -\vec{\nabla} P_i - e N_e (\vec{E} + \vec{U}_e \times \vec{B}) \quad (1b)$$

$$\frac{\partial N_i}{\partial t} + \vec{\nabla} \cdot N_i \vec{U}_i = S_n \quad (1c)$$

Here \vec{p}_i is the ion stress tensor given by Braginskii⁷ (see appendix) and S_n is the density source term. We use the generalized co-ordinate system $(\hat{p}, \hat{b}, \hat{n})$ which is tied to the magnetic field. Here $\hat{n} = \vec{B}/B$ is the unit vector along the magnetic field lines, \hat{p} is orthogonal to the magnetic surface, and $\hat{b} = \hat{n} \times \hat{p}$. The co-ordinates basic $(\hat{p}, \hat{b}, \hat{n})$ are related to the flux co-ordinates $(\mathbf{y}, \mathbf{c}, \mathbf{f})$, where \mathbf{y} is the poloidal magnetic flux, \mathbf{c} the generalized poloidal angle and \mathbf{f} is the toroidal angle. Both co-ordinates system are related as

$$\begin{aligned} \hat{p} &= \hat{e}_y, \quad \hat{b} = \left(\frac{B_f}{B} \right) \hat{e}_c - \left(\frac{B_c}{B} \right) \hat{e}_f, \\ \hat{n} &= \left(\frac{B_c}{B} \right) \hat{e}_c + \left(\frac{B_f}{B} \right) \hat{e}_f \end{aligned} \quad (2)$$

For large aspect ratio ($\mathbf{e} = r/R \ll 1$) and toroidal symmetry (i.e., $\partial/\partial \mathbf{f} \equiv 0$), the differential operators can be written as:

$$\begin{aligned} \hat{p} \cdot \vec{\nabla} &\equiv h_y^{-1} \frac{\partial}{\partial \mathbf{y}} \sim \frac{\partial}{\partial r} \sim L_y^{-1}, \quad \hat{b} \cdot \vec{\nabla} \equiv \left(\frac{B_f}{B} \right) h_c^{-1} \frac{\partial}{\partial \mathbf{c}} \sim \frac{1}{r} \frac{\partial}{\partial \mathbf{q}} \sim r^{-1} \\ \hat{n} \cdot \vec{\nabla} &\equiv \left(\frac{B_c}{B} \right) h_c^{-1} \frac{\partial}{\partial \mathbf{c}} \sim \frac{1}{qR} \frac{\partial}{\partial \mathbf{q}} \sim (qR)^{-1} \end{aligned} \quad (3)$$

Here $h_y = 1/h_f B_c$, $h_c = J B_c$, $J = h_y h_c h_f$, the Jacobian of the transformation $(\vec{r} \rightarrow \mathbf{y}, \mathbf{c}, \mathbf{f})$, $\mathbf{n} = h_c B_f / h_f B_c \sim r B_f / R B_c$, the pitch of the field lines, $q = \oint \mathbf{n} d\mathbf{c}$, the safety factor and for large aspect ratio ($\mathbf{e} = r/R_0 < 1$), $h_f = R_0(1 + \mathbf{e} \cos \mathbf{c})$ and $B_f = B_{f0}(1 - \mathbf{e} \cos \mathbf{c})$, where r and R_0 the minor and major radii, respectively.

The sum of toroidal components of the Eqs (1a) and (1b) can be written as:

$$m_i N_i \left[\hat{e}_f \cdot \left(\frac{\partial}{\partial t} + \vec{U}_i \cdot \vec{\nabla} \right) \vec{U}_i \right] = -\hat{e}_f \cdot (\vec{\nabla} \cdot \vec{p}_i) + \frac{B_c}{c} J_y \quad (4)$$

Where the last term corresponds to the radial angular momentum flux driven by background turbulence and $\hat{e}_f \cdot \vec{\nabla} P_i = 0$.

For toroidal symmetry (i.e., $\partial/\partial\mathbf{f}\equiv 0$), the convective term in Eq. (8) can be expressed as:

$$\begin{aligned} N_i \hat{e}_f \cdot [(\vec{U}_i \cdot \vec{\nabla}) \vec{U}_i] &= -U_{fi} [h_y^{-1} \frac{\partial}{\partial \mathbf{y}} (N_i U_{yi}) + h_c^{-1} \frac{\partial}{\partial \mathbf{c}} (N_i U_{ci})] + N_i U_{fi} \vec{U}_i \cdot \vec{\nabla} \ln h_f \\ &+ h_y^{-1} \frac{\partial}{\partial \mathbf{y}} (N_i U_{yi} U_{fi}) + h_c^{-1} \frac{\partial}{\partial \mathbf{c}} (N_i U_{ci} U_{fi}) \end{aligned} \quad (5)$$

By using Eq. (5) and the continuity equation, the total derivative of momentum density – the left hand side of Eq. (4) can be expressed as:

$$\begin{aligned} m_i N_i \hat{e}_f \cdot \left[\frac{\partial \vec{U}_i}{\partial t} + [(\vec{U}_i \cdot \vec{\nabla}) \vec{U}_i] \right] &= -m_i U_{fi} \left[\frac{\partial N_i}{\partial t} + h_y^{-1} \frac{\partial}{\partial \mathbf{y}} (N_i U_{yi}) + h_c^{-1} \frac{\partial}{\partial \mathbf{c}} (N_i U_{ci}) \right] \\ &+ m_i \left[\frac{\partial (N_i U_{fi})}{\partial t} + h_y^{-1} \frac{\partial}{\partial \mathbf{y}} (N_i U_{yi} U_{fi}) + h_c^{-1} \frac{\partial}{\partial \mathbf{c}} (N_i U_{ci} U_{fi}) \right] + m_i N_i U_{fi} \vec{U}_i \cdot \vec{\nabla} \ln h_f \\ &\approx -m_i U_{fi} S_n + m_i \left[\frac{\partial (N_i U_{fi})}{\partial t} + h_y^{-1} \frac{\partial}{\partial \mathbf{y}} (N_i U_{yi} U_{fi}) + h_c^{-1} \frac{\partial}{\partial \mathbf{c}} (N_i U_{ci} U_{fi}) \right] + m_i N_i U_{fi} \vec{U}_i \cdot \vec{\nabla} \ln h_f \end{aligned} \quad (6)$$

Here we have made use of a tensorial relation $\vec{\nabla} \hat{e}_f = -\hat{e}_f \vec{\nabla} \ln h_f$ or $\hat{e}_f \cdot \vec{\nabla} \hat{e}_f = -\vec{\nabla} \ln h_f$, $\hat{e}_y \cdot \vec{\nabla} \hat{e}_y = 0$, $\hat{e}_c \cdot \vec{\nabla} \hat{e}_c = 0$ and S_n is the particle source term.

Note from Appendix A, the definition of \vec{p}_i is of the form $\vec{p}_i = \vec{a}\vec{b} + \vec{b}\vec{a}$, the stress force along toroidal direction can easily be expressed as:

$$\begin{aligned} \hat{e}_f \cdot (\vec{\nabla} \cdot \vec{p}_i) &= \frac{1}{Jh_f} \left[\frac{\partial}{\partial \mathbf{y}} h_c h_f^2 (\mathbf{a}_y \mathbf{b}_f + \mathbf{a}_f \mathbf{b}_y) + \frac{\partial}{\partial \mathbf{c}} h_y h_f^2 (\mathbf{a}_c \mathbf{b}_f + \mathbf{a}_f \mathbf{b}_c) \right] \\ &\approx h_y^{-1} \frac{\partial \vec{p}_{yf}}{\partial \mathbf{y}} + h_c^{-1} \frac{\partial \vec{p}_{cf}}{\partial \mathbf{c}} \end{aligned} \quad (7)$$

We consider that plasma is in the collisionless regime ($qRn_i/c_i < 1$, n_i is the ion collision frequency). We now collect only \vec{p}_{yf} and \vec{p}_{cf} from the collisionless stress tensor in Appendix A.

$$\begin{aligned} (\mathbf{p}_{3-4})_{yf} - \mathbf{h}_3 \left[\frac{B_c}{B} (\hat{p} \cdot \vec{\nabla} \vec{U} \cdot \hat{p} - \hat{b} \cdot \vec{\nabla} \vec{U} \cdot \hat{b}) + 2 \frac{B_f}{B} (\hat{b} \cdot \vec{\nabla} \vec{U} \cdot \hat{n} + \hat{n} \cdot \vec{\nabla} \vec{U} \cdot \hat{b}) \right] \\ \approx -4\mathbf{h}_3 \frac{B_f}{B} (\hat{b} \cdot \vec{\nabla} \vec{U} \cdot \hat{n} + \hat{n} \cdot \vec{\nabla} \vec{U} \cdot \hat{b}) = -4\mathbf{h}_3 \frac{B_f}{B} \left[\hat{b} \cdot \vec{\nabla} (\hat{n} \cdot \vec{U}) + \mathbf{k} \cdot \hat{b} U_{\parallel} \right] \end{aligned}$$

$$\begin{aligned}
(\mathbf{p}_{3-4})_{cf} &= -\mathbf{h}_3 \left[-2 \left(\frac{B_f}{B} \right)^2 (\hat{n} \cdot \vec{\nabla} \vec{U} \cdot \hat{p} + \hat{p} \cdot \vec{\nabla} \vec{U} \cdot \hat{n}) + \left(\frac{B_c}{B} \right)^2 (\hat{n} \cdot \vec{\nabla} \vec{U} \cdot \hat{p} + \hat{p} \cdot \vec{\nabla} \vec{U} \cdot \hat{n}) \right] \\
&\approx 4\mathbf{h}_3 \left(\frac{B_f}{B} \right)^2 (\hat{n} \cdot \vec{\nabla} \vec{U} \cdot \hat{p} + \hat{p} \cdot \vec{\nabla} \vec{U} \cdot \hat{n}) = 4\mathbf{h}_3 \left(\frac{B_f}{B} \right)^2 \left[\hat{p} \cdot \vec{\nabla} (\hat{n} \cdot \vec{U}) + \hat{\mathbf{k}} \cdot \hat{p} U_{\parallel} \right]
\end{aligned}$$

where $\hat{\mathbf{k}} = (\hat{n} \cdot \nabla) \hat{n}$. Here $B_c / B < 1$ is assumed and $\mathbf{h}_{3,i} = P_i / 2\Omega_i$, $\mathbf{h}_{2i} = 4\mathbf{h}_i$. Again in the low \mathbf{b} and large aspect ratio limit, the term $\hat{e}_f \cdot (\vec{\nabla} \cdot \vec{p}_i)$ now can be expressed as :

$$\begin{aligned}
\hat{e}_f \cdot (\vec{\nabla} \cdot \vec{p}_i) &\approx \frac{\partial}{\partial r} (\mathbf{p}_{3-4})_{yf} + \frac{1}{r} \frac{\partial}{\partial \mathbf{q}} (\mathbf{p}_{3-4})_{cf} \quad (8) \\
&= 2 \frac{m_i c}{e} \left[-\frac{\partial}{\partial r} \left(\frac{P_i}{B^2} \right) \frac{1}{r} \frac{\partial}{\partial \mathbf{q}} (U_{\parallel} B) + \frac{1}{r} \frac{\partial}{\partial \mathbf{q}} \left(\frac{P_i}{B^2} \right) \frac{\partial}{\partial r} (U_{\parallel} B) \right] \approx 2 \frac{m_i c}{e} \left[\hat{n} \times \vec{\nabla} (U_{\parallel} B) \right] \cdot \nabla \left(\frac{P_i}{B^2} \right)
\end{aligned}$$

where we used

$$\hat{b} \cdot \vec{\nabla} (\hat{n} \cdot \vec{U}) + \hat{\mathbf{k}} \cdot \hat{b} \hat{n} \cdot \vec{U} = \frac{1}{B} (\hat{b} \cdot \nabla) (U_{\parallel} B), \quad \hat{\mathbf{k}} \approx \frac{\nabla B}{B}$$

and analogously for \hat{p} .

By using equations (6) and (8) in equation (4), the toroidal momentum equation can now be expressed as

$$\begin{aligned}
\frac{\partial}{\partial t} (m_i N_i U_{fi}) + m_i \left[h_y^{-1} \frac{\partial}{\partial \mathbf{y}} (N_i U_{yi} U_{fi}) + h_c^{-1} \frac{\partial}{\partial \mathbf{c}} (N_i U_{ci} U_{fi}) + N_i U_{fi} \vec{U}_i \cdot \vec{\nabla} \ln h_f \right] + m_i^{(9)} U_{fi} S_n \\
= -2 \frac{m_i c}{e} \left[\hat{n} \times \vec{\nabla} (U_{\parallel} B) \right] \cdot \nabla \left(\frac{P_i}{B^2} \right)
\end{aligned}$$

We now first derive the linear parallel ion velocity perturbation from Eq.(9). In the limit $k_{\perp} L_n \gg 1$ (k_{\perp} is the perpendicular wave vector of background instability), and by taking perturbations $F = F_0 + \mathbf{d}f$ [$F_0(r)$ and $\mathbf{d}f$ are the background equilibrium and perturbation, respectively], the linear form of Eq. (9) can be expressed as:

$$m_i N_i \left(\frac{\partial}{\partial t} + (U_{\parallel 0} + \vec{U}_{0E} + 2\vec{U}_{Di}) \cdot \vec{\nabla} \right) \mathbf{d}u_{\parallel} = - \left[\hat{e}_{\parallel} \cdot \nabla + U_{\parallel 0} \frac{m_i \vec{U}_{Di} \cdot \nabla}{T_i} \right] (\mathbf{d}p_i + e N_i \mathbf{f}) + \frac{B_q}{c} J_r \quad (10)$$

Here $\vec{U}_{Di} = 2 \frac{cT_i}{eB^2} \hat{n} \times \nabla B$, $\vec{U}_{De} = -\frac{T_e}{T_i} \vec{U}_{Di}$, $\hat{n} = \vec{B}/B$ and $U_f \approx U_{\parallel}$.

In Eq (10) the curvature term involving $\nabla \mathbf{f}$ corresponds to *turbulent equipartition* (TEP) The curvature term involving $\nabla \mathbf{dp}$ has two parts where the temperature perturbation part corresponds to the *thermoelectric* convective term. If we assume Boltzmann electrons and quasineutrality we can rewrite (10) as:

$$m_i N_i \left(\frac{\partial}{\partial t} + (U_{\parallel 0} + \vec{U}_{0E} + 2\vec{U}_{Di}) \cdot \vec{\nabla} \right) \mathbf{du}_{\parallel} = - \left[\hat{e}_{\parallel} \cdot \nabla + U_{\parallel 0} \frac{m_i \vec{U}_{Di}}{T_i} \cdot \nabla \right] \left[\mathbf{dT}_i + eN_i \left(1 + \frac{1}{t} \right) \mathbf{f} \right] + \frac{B_q}{c} J_r$$

Thus now the density perturbation part adds up to the TEP part and we recover the Coriolis pinch of Ref 8 after substituting \mathbf{du}_{\parallel} into the flux $\Gamma_f = \langle \mathbf{v}_{Er} \mathbf{du}_{\parallel} \rangle$. We here have only a factor 2 since our U_D includes a factor 2 making it equivalent to the total magnetic drift for low β . This part can also be recovered from Ref 9 where, however, more effort was made to separate curvature and grad B parts as well as including temperature anisotropy. The original TEP part comes from the convective inertial term, i.e. the term in brackets in Eq 9. We note from the way Eq. (10) was written that the toroidal curvature effect adds up as a new symmetry breaking effect to the parallel gradient. This increases substantially the effect of flowshear. We can now formally generalize the eigenvalue solution in Ref. 18 by adding the new symmetry breaking term to the parallel gradient. However, also the symmetry point in the radial direction will be shifted due to the toroidicity. From Eq. (9), the equation for mean toroidal velocity can be obtained by averaging over magnetic flux surfaces. In the large aspect ratio limit, the equation for mean U_f can be written:

$$m_i N_i \frac{\partial}{\partial t} \langle U_{fi} \rangle + m_i N_i \frac{\partial}{\partial r} \langle \mathbf{du}_r \mathbf{du}_{\parallel} \rangle = - \frac{m_i N_i}{P_{i0}} \vec{U}_{Di} \cdot \vec{\nabla} \langle \mathbf{du}_{\parallel} \mathbf{dp}_i \rangle - m_i N_i \frac{\mathbf{dp}_i}{P_{i0}} \vec{U}_{Di} \cdot \vec{\nabla} \mathbf{du}_{\parallel} + \frac{B_q}{c} \langle J_r \rangle \quad (11)$$

In deriving Eq (11) we kept only curvature terms from the stress tensor. We note that we have contributions both from the Reynolds stress (inertial part) and from the stress tensor.

APPENDIX A: THE STRESS TENSORS ($\bar{\mathbf{p}}_i$)

In the limit $\mathbf{n}_i / \Omega_i \ll 1$, the stress tensor⁷ can be split into three parts

$$\bar{\boldsymbol{\pi}}_i = \bar{\boldsymbol{\pi}}_{0,i} + \bar{\boldsymbol{\pi}}_{3-4,i} + \bar{\boldsymbol{\pi}}_{1-2,i} \quad (\text{A1})$$

The parallel stress tensor or diagonal matrix ($\bar{\boldsymbol{\pi}}_{0,i}$), the gyro-stress tensor ($\bar{\boldsymbol{\pi}}_{3-4,i}$) and the perpendicular stress tensor ($\bar{\boldsymbol{\pi}}_{1-2,i}$) are given as⁸

The parallel stress tensor:

$$\bar{\mathbf{p}}_{0,i} = -3\mathbf{h}_{0,i} \left(\hat{\mathbf{n}}\hat{\mathbf{n}} - \frac{\vec{\mathbf{I}}}{3} \right) \left[\hat{\mathbf{n}} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{\mathbf{n}} - \frac{\vec{\nabla} \cdot \vec{U}_i}{3} \right] \quad (\text{A2})$$

The gyro stress tensor:

$$\begin{aligned} \bar{\mathbf{p}}_{3-4,i} = & -\mathbf{h}_{3,i} \left\{ (\hat{p}\hat{p} - \hat{b}\hat{b}) [\hat{b} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{p} + \hat{p} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{b}] - (\hat{p}\hat{b} + \hat{b}\hat{p}) [\hat{p} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{p} - \hat{b} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{b}] \right\} \\ & - 2\mathbf{h}_{3,i} \left\{ (\hat{p}\hat{n} + \hat{n}\hat{p}) [\hat{b} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{n} + \hat{n} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{b}] - (\hat{b}\hat{n} + \hat{n}\hat{b}) [\hat{p} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{n} + \hat{n} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{p}] \right\} \end{aligned} \quad (\text{A3})$$

The perpendicular stress tensor:

$$\begin{aligned} \bar{\mathbf{p}}_{1-2,i} = & -\mathbf{h}_{1,i} \left\{ \hat{p}\hat{p} - \hat{b}\hat{b} \right\} [\hat{p} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{p} - \hat{b} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{b}] + (\hat{p}\hat{b} + \hat{b}\hat{p}) [\hat{p} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{b} + \hat{b} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{p}] \\ & - 4\mathbf{h}_{1,i} \left\{ (\hat{p}\hat{n} + \hat{n}\hat{p}) [\hat{p} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{n} + \hat{n} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{p}] + (\hat{b}\hat{n} + \hat{n}\hat{b}) [\hat{b} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{n} + \hat{n} \cdot \vec{\nabla} \vec{U}_i \cdot \hat{b}] \right\} \end{aligned} \quad (\text{A4})$$

where $\mathbf{h}_{0,i} = 0.96 P_i \mathbf{n}_i^{-1}$, the index 3-4 and 1-2 refer to Braginskii's coefficients⁷
 $\mathbf{h}_{3,i} = P_i / 2\Omega_i$, $\mathbf{h}_{1,i} = 3P_i \mathbf{n}_i / 10\Omega_i^2$, $\mathbf{h}_{2,i} = 4\mathbf{h}_{1,i}$ and $\eta_{4i} = 2\eta_{3i}$.

Reference

1. J.E. Rice, E.S. Marmor, F. Bombarda and L. Qu, Nuclear Fusion **37**, 421 (1997).
2. X. Garbet, Y. Sarazin, P. Ghendrich, S. Benkadda, P. Beyer, C. Figarella and I. Voitsekovitch, Phys. Plasmas **9**, 3893 (2002).
3. J.S. deGrassie, K.H. Burrell, L.R. Baylor, W. Houlberg and J. Lohr, Phys. Plasmas **11**, 4323 (2004).
4. A.G. Peeters and C. Angioni, Phys. Plasmas **12**, 072515 (2005).
5. P.C. deVries, K.M. Rantamäki, C. Giroud, E. Asp, G. Corrigan, A. Eriksson, M. deGreef, I. Jenkins, H.C.M. Knoops, P. Mantica, H. Nordman, P. Strand, T.

- TALA, J. Weiland, K-D Zastrow and the JET-EFDA contributors, Plasma Phys. Control. Fusion **48**, 1693 (2006).
6. J. Weiland, A. Eriksson, H. Nordman and A. Zagorodny, Phys. Plasmas **14**, 072302 (2007).
 7. T.S. Hahm, P.H. Diamond, O.D. Gurcan and G. Rewoldt, Phys. Plasmas **14**, 072302-1 (2007).
 8. A. G. Peeters, C. Angioni and D. Strinzi, Phys. Rev. Lett. **98**, 265003 (2007).; A.G. Peeters, D. Strintzi, Y. Camenen et al, Physics of Plasmas, to be published.
 9. T. S. Hahm, Phys. Fluids **31**, 2670 (1988).
 10. D. Strinzi, A.G. Peeters and J. Weiland, Phys. Plasmas **15**, 044502 (2008).
 11. R.E. Waltz, R.R. Dominguez and G.W. Hammett, Phys. Fluids **B4**, 3138 (1992).
 12. K.V. Roberts and J.B. Taylor, Phys. Rev. Lett. **8**, 197 (1962).
 13. M.N. Rosenbluth, N.A. Krall and N. Rostoker, Nucl. Fusion (Suppl.) **1**, 143 (1962).
 14. A. Rogister, Phys. Plasmas **7**, 5070 (2000).
 15. J.J. Ramos, Phys Plasmas **12**, 0512102-1 (2005)
 16. J.J. Ramos, Phys. Plasmas **12**, 112301-1 (2005)
 17. H.A. Classen, H. Gerhauser, A. Rogister and C. Yarim, Phys. Plasmas **7**, 3699 (2000).
 18. J. Weiland and H. Nordman, Proceedings of the 33rd EPS conference, Rome 2006, ECA Vol. 301, P-2, 186.