Results From the International Collaboration on Neoclassical Transport in Stellarators (ICNTS)

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Abstract: The International Collaboration on Neoclassical Transport in Stellarators (ICNTS) is one of several activities initiated by the stellarator community to develop the tools and know-how necessary to describe and predict the behavior of non-axisymmetric plasmas during high-performance discharges. This contribution presents an overview of benchmarking results for mono-energetic transport coefficients determined from various numerical solutions of the drift kinetic equation for a wide range of devices representative of the extensive configuration space available to stellarators. Emphasis is placed on the radial-transport and bootstrap-current coefficients which both exhibit a strong dependence on the radial electric field which is lacking for axisymmetric tokamaks.

1. Introduction

This contribution describes work carried out within the *IEA Implementing Agreement for Cooperation in Development of the Stellarator Concept*, with the ultimate goal of providing a comprehensive description of neoclassical transport processes in stellarator experiments. Such a description is mandatory for analyzing experimental results and carrying out predictive simulations as neoclassical transport for stellarators with reactor-relevant plasma parameters is a much stronger function of temperature than for axisymmetric tokamaks and, indeed, even in experiments of moderate size, high-performance discharges have been found to obey neoclassical expectations for particle and energy confinement as well as the bootstrap current [1–3].

The first results of this International Collaboration on Neoclassical Transport in Stellarators (ICNTS), presented below, document a thorough benchmarking of the various numerical methods used to calculate mono-energetic neoclassical transport coefficients in realistic 3-D magnetic-field topologies with multiple classes of trapped particles. These transport coefficients are flux-surface-averaged moments of the solution to the linearized drift kinetic equation and were obtained using the field-line-integration techniques of the NEO family of codes [4, 5], Monte Carlo simulations employing either *full-f* [6–8] or δf schemes [8–11], the variational approach of the Drift Kinetic Equation Solver, DKES [12] and (where appropriate) a numerical solution of the ripple-averaged kinetic equation, GSRAKE [13]. The devices for which the benchmarking has been performed are representative of the extensive configuration space available to stellarators: the Large Helical Device (LHD) heliotron, in operation at Toki, Japan; the quasi-axisymmetric National Compact Stellarator Experiment (NCSX), under construction at Princeton, NJ, USA; the quasi-Helically-Symmetric Experiment (HSX), in operation at Madison, WI, USA; the Quasi-Poloidally-Symmetric device (QPS), a design study initiated by Oak Ridge National Laboratory, USA; the heliac TJ-II, in operation at Madrid, Spain; and two advanced stellarators of the Wendelstein line, W7-AS which ended operation in 2002 at Garching, Germany, and W7-X which is under construction at Greifswald, Germany.

2. Basics

Neoclassical theory describes transport processes which are assumed to be *radially local* and described by the linearized drift kinetic equation

$$\mathcal{V}(f_1) - \nu\mathcal{L}(f_1) = -\frac{\mathrm{d}r}{\mathrm{d}t}\left(\frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}r} - \frac{qE_r}{T} + \left(K - \frac{3}{2}\right)\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)f_M - pv\,\frac{B}{B_0}\frac{qV_L}{R_0T}\,f_M \qquad (1)$$

where f_1 is the (small) deviation of the distribution function from Maxwellian and

$$\mathcal{V}(f_1) = \left(pv\frac{\mathbf{B}}{B} + \frac{\mathbf{E}_r\times\mathbf{B}}{\langle B^2\rangle}\right)\cdot\boldsymbol{\nabla} f_1 - \frac{v\,(1-p^2)}{2B^2}\,\mathbf{B}\cdot\boldsymbol{\nabla} B\,\frac{\partial f_1}{\partial p}$$

is the Vlasov operator, with $\mathbf{B}$ the magnetic-field vector, B its magnitude, v is the particle speed, $p = v_{||}/v$ the pitch-angle variable, $\mathbf{E}_r = E_r\mathbf{e}_r$ is the *radial* electric field with $\mathbf{e}_r = \boldsymbol{\nabla} r/|\boldsymbol{\nabla} r|$ the unit vector in the direction normal to the flux surface, r is the flux-surface label, angle brackets denote the flux-surface average and $\mathcal{L}$ is the Lorentz pitch-angle-scattering collision operator

$$\nu\mathcal{L}(f_1) = \frac{\nu}{2}\frac{\partial}{\partial p}\left(\left(1-p^2\right)\frac{\partial f_1}{\partial p}\right)\ ,$$

with ν the collision frequency. The radial drift velocity is given by

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{mv^2\,(1+p^2)}{2qB^3}\,(\mathbf{B}\times\boldsymbol{\nabla} B)\cdot\boldsymbol{\nabla} r$$

where m is the particle mass and q its charge, n and T are the density and temperature, respectively, of the local Maxwellian $f_M = n(m/2\pi T)^{3/2}\exp(-K)$, $K = mv^2/2T$ is the normalized kinetic energy, B_0 is the reference value of B on the flux surface, R_0 is the major radius of the torus and V_L is the loop voltage.

It will be noted that derivatives of f_1 with respect to r and v are lacking in eq. (1) making it possible to treat these two variables as mere parameters and express the first-order distribution function

$$f_1 = \frac{qV_L}{T}f_M\widehat{f}_{I} + \frac{v_dR_0}{v}\left(\frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}r} - \frac{qE_r}{T} + \left(K - \frac{3}{2}\right)\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)f_M\widehat{f}_{II}$$

where $v_d = mv^2/(2qR_0B_0)$ is characteristic of the radial drift velocity. Written in dimensionless form, the resulting differential equations for $\widehat{f}_{I}$ and $\widehat{f}_{II}$ (which are themselves dimensionless) are expressed

$$\frac{R_0}{v}\,\mathcal{V}(\widehat{f}_I) - \frac{R_0\nu}{v}\,\mathcal{L}(\widehat{f}_I) = -p\,\frac{B}{B_0} \tag{2}$$

$$\frac{R_0}{v}\,\mathcal{V}(\widehat{f}_{II}) - \frac{R_0\nu}{v}\,\mathcal{L}(\widehat{f}_{II}) = -\frac{1}{v_d}\,\frac{\mathrm{d}r}{\mathrm{d}t}\ . \tag{3}$$

Examination of the terms appearing in these equations shows that the solutions can depend only on the normalized $\mathbf{E}\times\mathbf{B}$ drift velocity, $E_r/(vB_0)$, the 'collisionality' $R_0\nu/v$ and the structure of the confining magnetic field (but not its magnitude).

Within this neoclassical formalism, the relationships between the flux-surface-averaged flows, I_i, and the thermodynamic forces which drive them, A_j, may be expressed

$$I_i = -n\sum_{j=1}^{3} L_{ij}A_j\ .$$

Conforming to the standard convention, I_1 is related to the radial component of the particle flux density, $\mathbf{\Gamma}$, through

$$I_1 = \langle \mathbf{\Gamma}\cdot\boldsymbol{\nabla} r\rangle = \left\langle \int \mathrm{d}^3v\ \frac{\mathrm{d}r}{\mathrm{d}t}\, f_1 \right\rangle\ ,$$

I_2 to the radial component of the energy flux density, $\mathbf{Q}$,

$$I_2 = \left\langle \frac{\mathbf{Q}}{T}\cdot\boldsymbol{\nabla} r \right\rangle = \left\langle \int \mathrm{d}^3v\ K\,\frac{\mathrm{d}r}{\mathrm{d}t}\, f_1 \right\rangle$$

and I_3 to the parallel component of the current density, $\mathbf{J}$,

$$I_3 = \frac{\langle \mathbf{J}\cdot\mathbf{B}\rangle}{qB_0} = \left\langle \int \mathrm{d}^3v\ pv\frac{B}{B_0}\, f_1 \right\rangle\ .$$

By choosing to combine the thermodynamic forces in the following manner

$$A_1 = \frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}r} - \frac{qE_r}{T} - \frac{3}{2}\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}r} \qquad A_2 = \frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}r} \qquad A_3 = \frac{qV_L}{R_0T}\ ,$$

the *mono-energetic* solutions of the kinetic equations (2) and (3) may be used to determine the transport coefficients by energy convolution with the local Maxwellian

$$L_{ij} = \frac{2}{\sqrt{\pi}}\int_0^\infty \mathrm{d}K\ \sqrt{K}\,e^{-K}\,D_{ij}(K)\,h_ih_j$$

where $h_1 = h_3 = 1$, $h_2 = K$ and the D_{ij} are *mono-energetic transport coefficients* defined by

$$D_{11} = D_{12} = D_{21} = D_{22} = -\frac{v_d^2R_0}{2v}\left\langle \int_{-1}^{1}\mathrm{d}p\ \frac{1}{v_d}\,\frac{\mathrm{d}r}{\mathrm{d}t}\,\widehat{f}_{II}\right\rangle$$

$$D_{13} = D_{23} = -\frac{v_dR_0}{2}\left\langle \int_{-1}^{1}\mathrm{d}p\ \frac{1}{v_d}\,\frac{\mathrm{d}r}{\mathrm{d}t}\,\widehat{f}_{I}\right\rangle$$

$$D_{31} = D_{32} = -\frac{v_dR_0}{2}\left\langle \int_{-1}^{1}\mathrm{d}p\ p\,\frac{B}{B_0}\,\widehat{f}_{II}\right\rangle$$

$$D_{33} = -\frac{vR_0}{2}\left\langle \int_{-1}^{1} \mathrm{d}p\, p\, \frac{B}{B_0}\, \widehat{f}_I \right\rangle \, .$$

Of these mono-energetic coefficients, D_{11} is said to describe the radial transport, D_{33} the parallel transport, D_{13} is characteristic of the Ware pinch and D_{31} of the bootstrap current. Only three of these coefficients are independent, however, as $D_{13} = -D_{31}$ due to Onsager symmetry. It is also worth noting that the neoclassical fluxes may be corrected so as restore the conservation of parallel momentum (which is violated by use of the Lorentz collision operator) by solving a linear system of moment equations in which the coefficients are differently weighted energy moments of the mono-energetic transport coefficients [14, 15]. Thus, for benchmarking purposes, the determination and comparison of mono-energetic coefficients is the first order of business.

3. Benchmarking Results

Due to the space limitations placed on this paper, benchmarking results presented here will concentrate on just three of the many configurations considered by the ICNTS, which illustrate quite different philosophies regarding neoclassical transport: the classical heliotron LHD, with a large fraction of localized particles and consequently high rate of radial transport while simultaneously exhibiting significant bootstrap current; the helias W7-X, also with a large fraction of localized particles but in regions of the torus with little curvature of B in an attempt to strongly reduce both radial transport and bootstrap current; and the quasi-axisymmetric NCSX, which has only a tiny fraction of localized particles (as B is nearly axisymmetric in magnetic flux coordinates) and thus neoclassical transport similar to that of rippled tokamaks.

A number of numerical techniques has been developed to determine the radial transport coefficient D_{11}. The most general methods treat eq. (3) in its entirety and include both *full-f* [6–8] and δf [8–11] Monte Carlo approaches as well as the Drift Kinetic Equation Solver (DKES) [12] which employs a variational principle in which the solution is expressed using a series of Fourier-Legendre test functions. Such methods demand a considerable price in computational resources but can be employed for arbitrarily complex magnetic fields and (usually) at all relevant values of collision frequency.

To reduce the cost in computational resources, efficient methods for solving ‘simplified’ kinetic equations have also been developed and two such approaches are included here in the benchmarking. In the first approach, eq. (3) is solved ignoring the $\mathbf{E} \times \mathbf{B}$ drift in the Vlasov operator, making it possible to determine D_{11} in the long-mean-free-path (*lmfp*) limit by performing a weighted integral of the geodesic curvature along a field line of ‘infinite’ length (i.e. sufficiently long to cover the magnetic flux surface). This field-line-integration technique, carried out numerically by the NEO code [4], provides a rapid means for determining the radial transport in the stellarator $1/\nu$ regime (where D_{11} is inversely proportional to the collision frequency) for arbitrarily complex magnetic fields. The influence of E_r on the radial transport coefficient cannot be determined with NEO, however, due to the initial assumptions. The second approach, a numerical solution of the *ripple-averaged* kinetic equation, GSRAKE [13], does not suffer this shortcoming. The ripple average is a generalization of the common bounce average (time average over the periodic motion of reflected particles) so as to encompass all of phase space (including passing particles) and is performed as a separation of time scales to

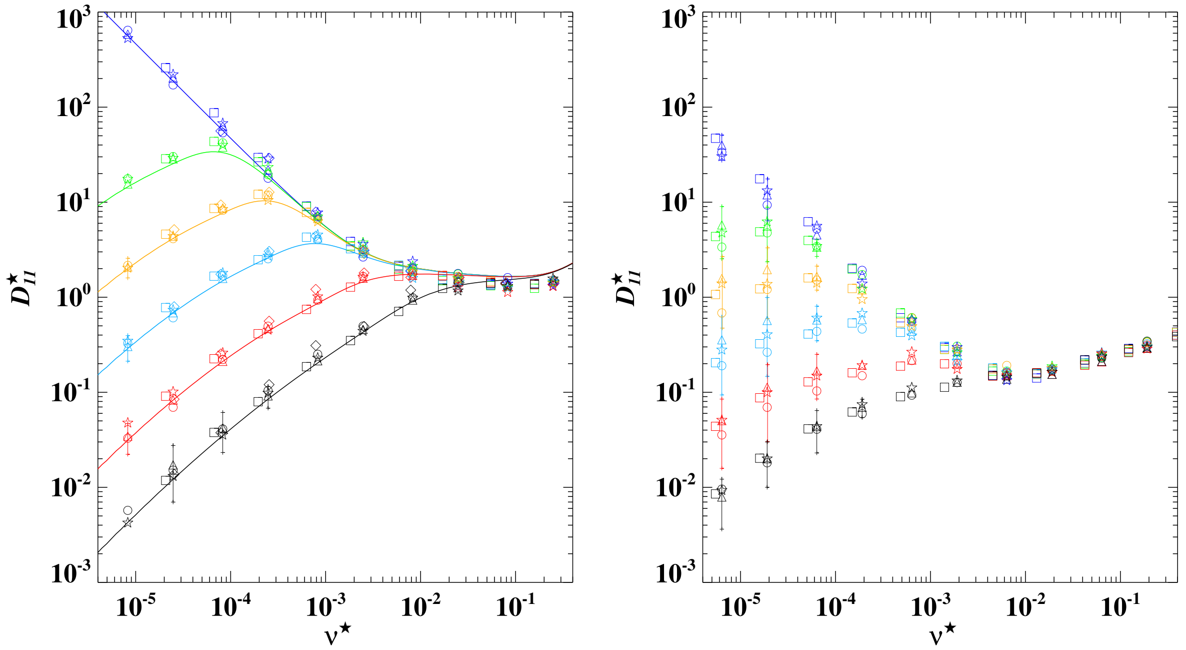
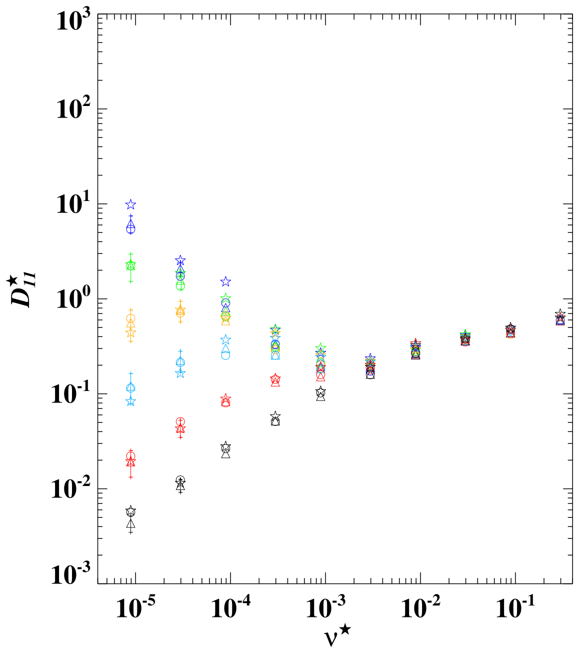


Figure 1. Benchmarking results for the standard configurations of LHD (upper left), W7-X (upper right) and NCSX (right). The normalized radial transport coefficient is plotted versus the collisionality assuming normalized values of the radial electric field to be $E_r/vB_0 = 3\times10^{-3}$, 1×10^{-3}, 3×10^{-4}, 1×10^{-4}, 3×10^{-5} *and zero. Numerical results for the* $\rho = 0.5$ *flux surface are depicted by: triangles for DKES (with upper and lower bounds indicated by the 'error bars'), squares [6] diamonds [7] circles [8] and stars [9] for four different Monte Carlo simulations and by the continuous curves for GSRAKE.*



eliminate the most rapidly varying spatial coordinate in the kinetic equation. This separation can only be carried out efficiently if the structure of B is described accurately within the so-called *multiple-helicity* model [16] which is appropriate here only in the case of LHD.

In Figure 1, the mono-energetic radial transport coefficient normalized to the plateau value of the equivalent axisymmetric tokamak, $D^\star_{11} = D_{11}/D_p$ with $D_p = \pi v_d^2 R_0/(4v\iota)$ and ι the rotational transform, is plotted as a function of the collisionality $\nu^\star = \nu R_0/(v\iota)$ for the flux surface at half the plasma radius. Six different values of the radial electric field have been considered, illustrating the strong dependence of $D^\star_{11}$ on this quantity in the *lmfp* regime. Good agreement of the results from DKES, four Monte Carlo codes and GSRAKE is documented in these plots. It will be noted that each configuration obeys $D^\star_{11} \propto 1/\nu$ in the *lmfp* regime with the range of collisionalities over which this holds depending on the magnitude of the radial electric field (the $1/\nu$ regime being entirely suppressed for values of E_r/vB_0 sufficiently

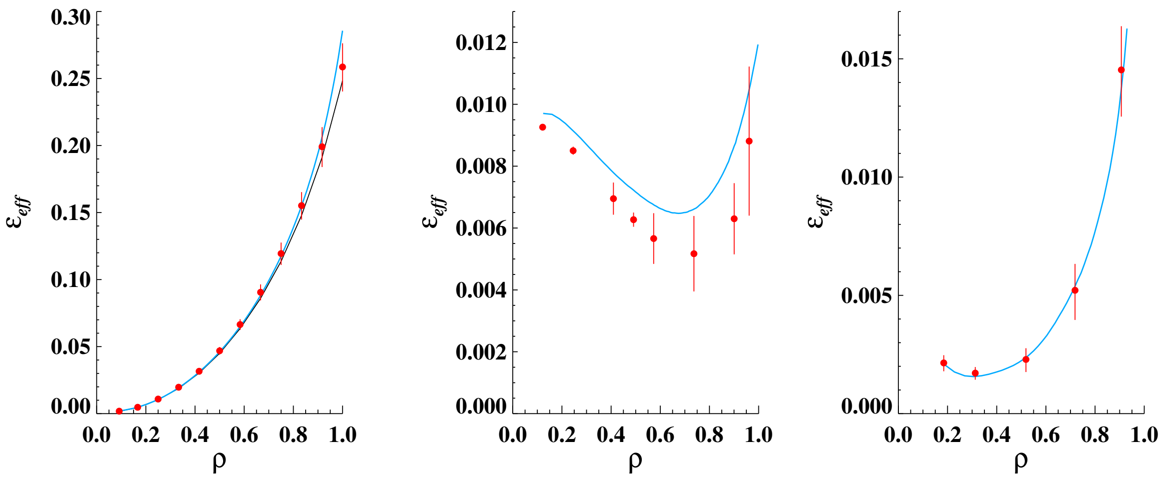
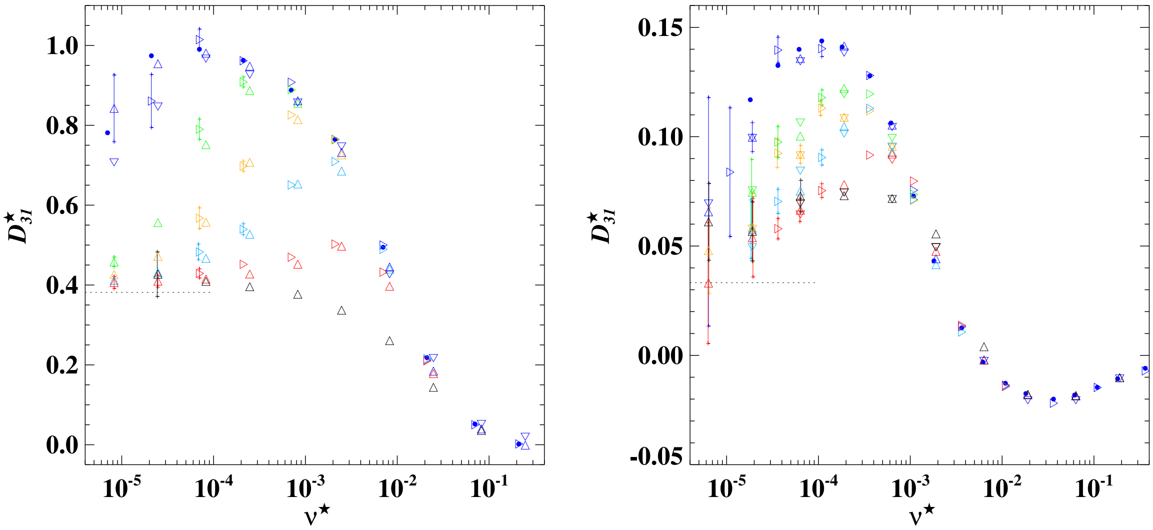


Figure 2. The effective helical ripple for $1/\nu$ transport is shown as a function of the normalized minor radius for LHD (left), W7-X (center) and NCSX (right). Analytical results are shown by the black curve (LHD only), those from NEO in light blue and DKES results with their bounds are given in red.

large), and one concludes that even a small departure from axisymmetry is sufficient to evoke this 'stellarator-like' behavior of the mono-energetic radial transport coefficient. It will also be noted that the upper and lower bounds on the DKES results (indicated in the plots by 'error bars') differ more strongly at low collisionality for W7-X than for LHD and NCSX; as a general rule, this divergence of the bounds is observed to increase with the complexity of the magnetic field and as the degree of drift optimization increases. In contrast, the statistical accuracy of the Monte Carlo algorithms is not affected by the magnetic configuration but instead by the number of simulation particles, which is generally chosen large enough here to produce error bars smaller than the data symbols.

A common figure of merit for the neoclassical transport in stellarators is the so-called *effective* helical ripple, ε_{eff}, determined in the $1/\nu$ regime from $D^{\star}_{11} = (4/3\pi)^2(2\varepsilon_{eff})^{3/2}/\nu^{\star}$. The radial dependence of this quantity determined using DKES, NEO and an analytic expression [17] (appropriate only for LHD) is plotted in Figure 2; again the agreement is excellent. It is also worth noting that *drift-optimized* configurations can be realized in LHD by displacing the magnetic flux surfaces towards the high-field side of the device. For example, the configuration with a major radius of $R_0 = 3.6$ m (compared to $R_0 = 3.75$ m in the standard case) exhibits significantly reduced $1/\nu$ transport with $\varepsilon_{eff} < 0.06$ at the plasma edge. This *inward-shifted* configuration has also been fully benchmarked within the ICNTS with success equal to that found for the standard LHD.

Numerical tools for determining the mono-energetic bootstrap current coefficient in stellarators include δf Monte Carlo methods with 'advanced' weighting schemes to overcome the statistical difficulties due to localized particles (which are most strongly weighted in the 'standard' scheme although they make a negligible contribution to D_{31}) [10, 11], DKES [12] and the field-line integration technique [5]. A sample of the results from these codes is given in Figure 3 where the normalized bootstrap current coefficient $D^{\star}_{31} = D_{31}/D_{BG}$ is plotted as a function of collisionality and radial electric field for LHD, W7-X and NCSX. The normalization factor $D_{BG} = 0.48665(R_0/r)^{1/2}mv^2/(qB_0\iota)$ is the collisionless asymptote for the equivalent axisymmetric tokamak. Also shown by the dotted line is the appropriate non-axisymmetric result derived analytically for $\nu^{\star} = 0$ [18] (which is independent of E_r).



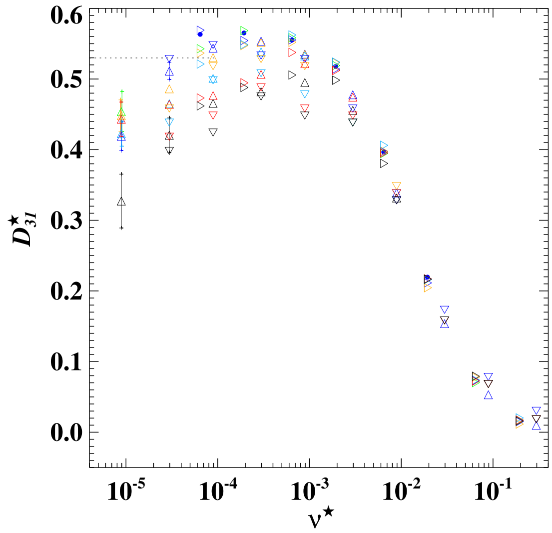


Figure 3. Normalized bootstrap current coefficients as a function of collisionality and radial electric field for LHD (upper left), W7-X (upper right) and NCSX (right). Numerical results for the $\rho = 0.5$ flux surface are given by: triangles for DKES [12], upside-down triangles [10] and rotated triangles [11] for δf Monte Carlo simulations and solid circles for the field-line-integration technique [5]. Color coding for the radial electric field value is the same as used in Figure 1. The value of $D_{31}^{\star}$ predicted analytically for $\nu^{\star} = 0$ is indicated by the dotted line.

A number of observations can be made concerning these results. Both LHD and W7-X possess mono-energetic bootstrap current coefficients which exhibit clear dependence on the magnitude of the radial electric field in the *lmfp* regime. This dependence, which is typical for stellarators, becomes much weaker in the NCSX results which are nearly ‘tokamak-like’ in this regard. Within the ICNTS investigations, the only other configuration found to have $D_{31}^{\star}$ results independent of E_r was the quasi-helically symmetric HSX, although in this case the bootstrap current coefficients are negative (and would thus lead to a reduction in the rotational transform) as expected theoretically for a ‘straight’ helix. A simplistic strategy for minimizing the bootstrap current in stellarators is thus a combination of appropriate toroidal and helical components in B such that the sum of their individual contributions to D_{31} vanishes. This strategy is realized approximately in W7-X, yielding normalized bootstrap current coefficients much less than one but nevertheless of sufficient magnitude to alter the rotational transform profile, requiring counter measures to insure the proper functioning of the island-divertor concept envisaged for this device. It is also possible, however, to further decrease $D_{31}^{\star}$ in W7-X by using the flexibility of the coil system to increase the fraction of localized particles, as confirmed within the ICNTS for the W7-X ‘high-mirror’ configuration. NCSX, on the other hand, relies on the bootstrap current to produce a significant portion of its rotational transform but is not as successful in this respect as the equivalent rippled tokamak which has $D_{31}^{\star} \approx 0.7$ in the collisionless limit.

4. Concluding Remarks

Benchmarking of the various numerical methods for determining mono-energetic neoclassical transport coefficients in stellarators has been performed for a broad range of realistic magnetic field configurations as the first task of the International Collaboration on Neoclassical Transport in Stellarators. The small sample of results presented here exemplifies the high quality of agreement obtained throughout this activity. With the benchmarking of mono-energetic transport coefficients largely completed, emphasis in the ICNTS is shifting to development of the theoretical and numerical tools required for practical application of these results, including methods to redress the neglect of momentum conservation in the original solutions of the kinetic equation.

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