Vortex nucleation in strongly sheared poloidal rotation and effects on velocity saturation and generation of ELM modes

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Abstract

The vorticity field in tokamak evolves to self organization and is concentrated in a narrow layer at the edge. The particle density and the current density have local maxima on the same layer, leading to destabilization of the rotation, breaking up of the layer and filamentation. We propose this as a basic model for the Edge Localized Modes.

Large scale vorticity is injected in tokamak plasma via external heating (NBI, ICRH) and evolves to an equilibrium profile via the balance of torque. The drive-dissipation is not the unique factor in the dynamics since the vorticity cannot have an arbitrary equilibrium profile in a $2D$ plasma. It has natural profiles corresponding to the spatial distributions of the streamfunction ψ of the poloidal velocity which are extrema of a particular action functional [1]. The extrema of that action functional are governed by a differential equation

$$\Delta\psi + \frac{1}{2}\sinh\psi\left(\cosh\psi - 1\right) = 0 \qquad (1)$$

Solving this equation one identifies *natural* profiles of the vorticity which in the phase space represent attractors. Essentially the vorticity separates into two regions with opposite signs: in the center it is collected the vorticity of one sign and at the perifery it is expelled the vorticity of the opposite sign. This is a particular form of dipolar structure, of the same nature as, for example, the Larichev-Reznik modon, but in cylindrical geometry it has superior stability compared to the situation where the regions of opposite vorticity are side-by-side, and it is compatible with global rotation.

The states consisting of this radial separation are actually not stationary, they continue to evlove on a slow time scale toward the strict localisation of the vorticity of the appropriate sign in a region close to the edge, leading to a narrowing of the layer of poloidal rotation. An indication of the direction of evolution is given by the following density of "energy" ($v^2 = \Omega_{ci}$, $v^2/\kappa = \rho_s^{-1}$)

$$\mathscr{E} = v^2 \left(\frac{v^2}{\kappa}\right)^2 \frac{1}{4} \left[\frac{11}{8} (\sinh\psi)^2 (-2 + \cosh\psi) + \frac{3}{8} \cosh\psi\right] \tag{2}$$

According to the Ertel's theorem the particle density $n(r,t)$ will tend to create a local maximum superimposed on the maximum of the vorticity $\omega(r,t)$. We argue that a similar process leads to concentration of the current density $j(r,t)$ in the form of a layer of local maximum, (a current sheet) coinciding with the layer of the extremum of the peripheric vorticity. The position of the current density extremum evolves such as to coincide with the vorticity extremum since it removes terms of the "baroclinic" (but MHD-) type in the dynamics. The fact that the current density evolves such as its extrema coincide with the extrema of vorticity have been seen in both relaxed or transient MHD states and has been found experimentally in DIII-D [7]. The states described above, essentially based on vorticity radial distribution derived from Eq.(1) have relevance for the H mode. Several experimental studies of the H mode in tokamak have shown the presence of a narrow layer of sheared poloidal rotation which acts as a barrier to the transport of energy. With the continuous (slow time scale) concentration of vorticity into the narrow layer near the last closed magnetic surface, and the induced increase of particle density and current density in the same layer, this state becomes fragile to perturbations that have not been included in the action functional. Two perturbations are known to appear : vorticity concentration into filaments; and tearing of the current sheet with formation of local concentrations of current limited by a separatrix with two Y-type singular points. These two processes are acting together and synergetically.

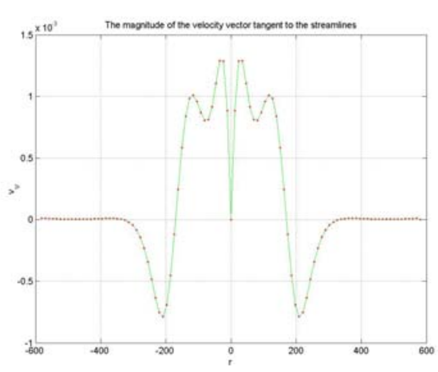

Figure 1: Profile of poloidal rotation resulting from Eq.(1).

The Edge Localised Modes (ELM) appear as a fast, very strong, perturbation of this layer eventually leading to the suppression of the sheared rotation and its replacement by a periodic chain of filaments. The evolution which starts from the strongly sheared rotation layer and leads to a set of filaments with high concentration of vorticity, particle density and current density has its origin in the vortex nucleation.

At the origin of the vortex nucleation is the fact that, for a fluid in rotation, it is energetically more favorable to generate localised vortical structures immersed in the flow than to maintain the uniform structure of the flow field. This has been shown for rotating Navier-Stokes fluids, protoplanetary disks, planetary atmosphere; the standard examples are the rotating Bose-Einstein condensates and the superfluids. In a $3D$ study of the *atmospheric mesocyclone* [2] it has been identified a very rapid concentration of the vorticity, in which the narrow circular band of vorticity is replaced with a set of strongly concentrated filaments disposed periodically on the circle, a process in which the two-dimensional aspect is dominant (the concentration takes place due to negative convergence of vorticity toward the final very localized structures). The example of superfluids deserves some discussion [4]. The velocity of a superfluid is potential $\mathbf{v}_s = \nabla \chi$ (*i.e.* irrotational) as long as there are no vortices present in the fluid. When vortices nucleate, they appear as *line deffects*, stable string-like structures with a central hard *core* and with the order parameter (complex function whose phase is χ) vanishing in the center of the core. A vortex is a *line singularity* in the coherent order-parameter field. For superfluids the phase of the order parameter varies with $2\pi n$ when turning around the core. The *circulation* around the core is quantized $\int \mathbf{dl} \cdot \mathbf{v}_s = \kappa n$. This means that the azimuthal velocity is $v_{s\theta} = \kappa n / 2\pi r$. The energy per unit length (the *line tension* of the vortex string) is

$$\varepsilon_V = \frac{\rho_0 \kappa^2}{4\pi} n^2 \ln \left(\frac{r_V}{r_c} \right)$$

$\frac{r_V}{r_c}$ is the ratio is of the distance between two vortices and the radius of the core. The condition of creation of a vortex *ring* with radius R and core radius a has been formulated by Feynman : the velocity of the superfluid component must be greater than the velocity self-induced by the ring $V_{sf} > V^{ring} = \frac{C}{R} \ln \left(\frac{8R}{a} - 1 \right)$ and the self-energy of the flow in a vortex ring is

$$E^{ring} = \frac{C'}{R} \left[\ln \left(\frac{8R}{a} \right) - 3 \right]$$

Since it is a threshold for nucleation, this energy is taken by **Langer Fischer** at the exponent of a barrier-type expression for the probability of nucleation

$$\Gamma = \Gamma_0 \exp \left(- \frac{E^{ring}}{kT} \right)$$

A rotating superfluid should be seen as a metastable state of a supersaturated vapor. There is a finite probability of nucleation of the stable phase. The generation of a vortex is similar to the condensation of a droplet of a liquid phase from a gas at the critical state and the energy is lowered. It is proved by numerical simulations and by mathematical analysis of the bounds for the *minimisers* that a superconducting fluid rotated in a two dimensional geometry evolves through a series of transitions consisting of nucleation of vortices. The uniform state, rotating but without vortices is stable up to a certain angular velocity. Then a vortex nucleates in the fluid. For even higher velocities another vortex is nucleated and so on. This process manifests *hysteresis* due to the interplay between the branch with vortices and the branch with uniform fluid. Summarizing, for superfluids the nucleated vortices are topological and the equation is the Nonlinear Schrodinger Equation. It is actually the same as what results from the field-theoretical model of point-like vortices for the Euler equation [3]. We will use this below.

The physical process of generation of filaments inside the sheared velocity layer, followed by their growth until the suppression of the poloidal flow cannot be attributed to a spontaneous nucleation as in superfluids, *i.e.* the superfluid paradigma of vortex nucleation cannot be directly applied to the ELM problem. However there is a physical process of generation of localised structures of vorticity due to a transient Kelvin-Helmholtz (KH) event. In such an event a piece of fluid from the region of high vorticity is transported inside the flow and deformed into a double spiral. We can now invoke the field-theoretical model of the ion-hydrodynamics, build on the discrete model of point-like vortices interacting in plane by the short range potential, $K_0(r/\rho_s)$ [5]. This leads to Eq.(1) but this model is non-Abelian and has trivial topology. It has been noted, however, that this model descends to an *Abelian dominated* dynamics [6], where the equation of the stationary states (instead of Eq.(1)) is

$$\Delta\psi - \exp(\psi)\left[\exp(\psi) - 1\right] = 0 \tag{3}$$

Notably, this model restores the topological constraint (the energy is bounded from below by a topological flux Φ),

$$\mathscr{E} \geq v^2 \Phi \tag{4}$$

and has *ring*-type vortical solutions. We can consider that such a state saturates and stabilizes the double-spiral distribution of vorticity emerging from the KH event.

Depending on the dimensions, the element of vorticity that is absorbed into the shear layer can have different evolutions. If the double spiral blob is small it takes a long time before being dissipated by the parallel electron dynamics and it is easily advected via the Magnus force modified by the effect of pressure variation, inside the layer. The vortex nucleation converts a part of the angular momentum in the volume of the layer into a discrete set of blobs of vorticity which later are disipated. This provides implicitely an additional mechanism of transport of angular momentum leading to saturation of the velocity in the sheared layer. When the amplitude of the double spiral from the transient KH event is high enough, the threshold given by the topological bound Eq.(4) is overcomed and the double spiral transforms into a ring vortex whose stability is protected by the topological bound.

In the local coordinate system attached to the double spiral, the evolution consists of a contractive motion of the tip of the spiral, toward the center of the spiral

$$\frac{\rho}{B^2}\left(\frac{\partial}{\partial t}+v_r\frac{\partial}{\partial r}\right)\nabla^2\phi=\nabla_{\parallel}j_{\parallel}$$

This is actually a pinch of the full structure and consists of the stretching of the initial plasma element along the path of the spiral with simultaneous advancement of the rotating spiral body toward the center (diffusion takes place simultaneously). The radial compression v_r is determined from the spiral field

$$\psi=A(r)\exp\{i[\varphi(r)-\omega t-m\theta]\}$$

The wavenumber of the spiral is $k=\frac{d\varphi}{dr}$ and the winding number of the spiral is given by the angle between the tangent to the spiral and the circle centered at the center of the spiral $\tan(\theta)=\frac{m}{r\varphi'(r)}=\frac{m}{kr}$. This is a *trailing* spiral, that come from exterior and arrive in the center $\frac{d\theta}{dr}<0$, $k<0$, with $v_r<0$. The parallel current is pushed to the parallel direction by the local pinch of the vorticity associated with the evolution of the double spiral in a KH event. Being a transient event and at low collisionality, the only damping of the current in the parallel direction comes from the poloidal magnetic pumping, but this is reduced because the squeezing factor is high, due to the strong radial electric field in the sheared velocity layer. The current in the parallel direction being enhanced from the vorticity pinch (in the double spiral) it will provoke a local magnetic structure that will produce a swirl, and this swirl enhances the stabilization of the vorticity filamentation process.

This dynamics can explain the formation and stabilization of the vortical structures inside the layer of poloidal rotation but we still need to explain the break up of the layer. Two processes can be invoked, both leading to dynamics which is typical for the Chaplygin gas with strange polytropic or negative temperature. They have been discussed by Trubnikov and by Bulanov and Sasorov.

The concentration of the vorticity in the rotation layer induce a concentration of current density in the same layer, *i.e.* a current sheet (see Fig.9 of Burrell *et. al [7]*). The current sheet is unstable to the tearing instability and it can be torn apart into strips of current. The geometry adopted by Trubnikov [8] is adequate for studying the tearing of the particle density distribution in the layer. The width is initially L_0 and it evolves to a profile L which is variable along the direction y of the layer (poloidal). The coordinate x is perpendicular on the layer in the equilibrium position (radial). The magnetic field has a shear $B = B_y(x) = -B_0 \tanh(x/L)$ (this should be the Harris profile) and the z component of the magnetic potential $A \equiv A_z$ gives $B_y = \partial A/\partial x$ with $A = -LB_0 \ln\cosh(x/L)$ in the unperturbed state. The magnetic field has the magnitude B_0 at the upper and lower limits of the layer (with opposite directions).

The process consists of the deformation of the profile of the leyer $L_0 \rightarrow L(t,y)$. In the layer there is the current and on every unit length of the layer along x the total current is i_z^0.

The first assumption is that in the *long wave* limit the magnetic field at the surfaces of the deformed layer does not differ too much of B_0. Then the total current per unit of y-length is always the same $i_z = L(t,y)\, j_z = i_z^0 = cB_0/(2\pi)$. The current density is $j_z = en(v_{iz} - v_{ez})$ ($e = |e|$, $n_e = n_i = n$). Then

$$v_{iz} - v_{ez} = \frac{cB_0}{2\pi enL(t,y)} \tag{5}$$

We note that the product nL is actually the density of the plasma and the equation of continuity is $\frac{\partial}{\partial t}(nL) + \frac{\partial}{\partial y}(vnL) = 0$ where v is the velocity of plasma along the direction of the layer, y. One can introduce a normalized density of plasma $\rho(t,y) = nL(t,y)/(n_0L_0)$ and the previous equation becomes the usual density conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial y}(\rho v) = 0 \tag{6}$$

The equation of motion is

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} = \frac{1}{nm_i c}(-j_z B_x) = \frac{e}{m_i c}(v_{iz} - v_{ez})\frac{\partial A}{\partial y} \tag{7}$$

The system is invariant along the z direction which means that the generalized momenta of the electrons and of ions are conserved $m_i v_{iz} + \frac{e}{c}A =$const , $m_e v_{ez} - \frac{e}{c}A =$const$'$, leading to

$$\frac{\partial A}{\partial y} = -\frac{cm_i m_e}{e(m_e + m_i)}\frac{\partial}{\partial y}(v_{iz} - v_{ez}) \tag{8}$$

The difference of the two velocities is obtained from the continuity equation, expressed in terms of the quantity nL, $v_{iz} - v_{ez} = \frac{cB_0}{2\pi}\frac{1}{nL} = \frac{cB_0}{2\pi n_0 L_0}\frac{1}{\rho}$ and the equation of motion becomes

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} = c_0^2 \frac{1}{\rho^3}\frac{\partial \rho}{\partial y} \tag{9}$$

The constant is $c_0^2 = \frac{cm_e}{(m_e+m_i)}\left(\frac{cB_0}{2\pi n_0 L_0}\right)^2 = \frac{2v_A\delta_0}{L_0}$. This is a Chaplygin gas and is subject to the so-called "drop-on-ceil" instability, known that transforms a uniform layer of density into a discrete set of patches with high concentration of the density separated by regions with almost vanishing magnitude of the density.

The equations (6) and (9) are solved by Trubnikov using a *hodograph* transformation. The formulas are

$$\frac{nL}{n_0L_0} = \rho\,(t,y) = \frac{\sinh(|\tau|)}{\cosh(\tau) - \cos\chi} \quad , \quad \frac{v}{c_0} = -\frac{\sin\chi}{\sinh(|\tau|)}$$

where $\tau = t/t_* < 0$, $\chi = y/c_0t_*$. The *time*-like variable τ is introduced such that the unperturbed state is located at $t \to -\infty$ and the complete *tearing* of the layer is done when $\delta\chi = \pi$, at $\tau = 0$. Trubnikov obtains a solution that exhibits modulation of the particle density of the layer in the form of periodic, very narrow, quasi-singular maxima, between which the density is extremely small. The quickest growing solution is the periodic one

$$\begin{aligned} \pm\frac{y}{c_0t_*} &= \chi\,(\tau,\rho) = z + \arctan\left(\frac{z}{r-1}\right) \\ z &= \sqrt{\exp(-\tau/2) - \left(\frac{1}{\rho} - 1\right)^2} \end{aligned}$$

The density varies between the limits

$$\frac{1}{1+\exp(-|\tau|)} = \rho_{\min} < \rho < \frac{1}{1-\exp(-|\tau|)}$$

The solution describes periodic hills whose maxima become infinite at $\tau \to -0$.

The break up of a strong current shear ($v_{ez}/v_{eTh} \gg 1$) and the concentration of the current density in periodic filamentary structures has been studied by Bulanov and Sasorov [9]. When a tearing takes place and a strip of width $2a$ on the y (poloidal) direction is formed, the motion of the edge of the tear $a\,(t)$ has a uniform acceleration in time,

$$a\,(t) = \frac{1}{12}\frac{B_0^2}{\mu_0 n_s m_i L}t^2$$

This velocity is higher than the sound speed and shows that the tearing progresses very fastly, leading to the vanishing of the current density $j\,(r,t)$ on large poloidal intervals and concentration to quasi-singular value of $j\,(r,t)$ at certain filaments which are disposed periodically.

This dynamics is particularly interesting since it starts from a perturbation of $j\,(r,t)$ which can be produced by the stabilized vorticity filamentation discussed above. The two process mentioned above are strictly nonlinear, *i.e.* they cannot be identified perturbatively.

In conclusion, we summarize the connections that have been identified. The natural distribution of the vorticity in the meridional plasma section has a dipolar character but with circular symmetry, which means a ring of vorticity at the plasma edge. This state is not the absolute extremum of the action functional, therefore it still evolves and the concentration of vorticity is enhanced. Ertel's theorem and variational constraints impose that the particle density and the current density follow the vorticity such that they accumulate and create local maxima superposed on the layer where the vorticity is concentrated. Isolated KH events generate double spiral vortex structures which can be stabilized in the form of a ring-type (tubular) vortex if a threshold is exceeded such as the topological constraint applies. The spiral stretching of the vorticity induces a transient current density increase which is also favorable to the swirl stabilization of the vorticity filament. The current perturbation initiates a nonlinear tearing of the current sheet, leading to filamentation of the current density. These connections presents the possibility to generate filaments that are local maxima of the three parameters: vorticity, particle density and current density. The filamentation process is fast and is a possible explanation of the large ELMs.

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