Electron Cyclotron Current Drive in Spherical Tokamaks with Application to ITER

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Abstract. The high-β spherical tokamaks (ST), such as NSTX and MAST, are attractive fusion devices for studying the physics of current drive by electron cyclotron (EC) waves. While ST plasmas are overdense to conventional EC waves, electron Bernstein waves (EBW) can be used to generate plasma currents. Besides providing better confinement, EBW driven current can also help suppress neoclassical tearing modes. This paper examines the characteristic features of EBW current drive. It is shown that the propagation and damping of EBWs and their interaction with electrons in STs provides useful insight into the propagation and damping of O waves and their interaction with electrons in ITER. The physics of current drive has also many similar features. In theoretical and computational studies of current drive the interaction of electron cyclotron waves with electrons is modeled by a quasilinear diffusion coefficient. The usual approach has been to use a diffusion coefficient that is valid for a homogeneous plasma in a slab geometry. Thus, it lacks the toroidal effects necessary for current drive in STs and ITER. A new relativistic wave-particle diffusion operator in toroidal plasmas that includes spatial and momentum transport due to RF waves has been derived. It is suitable for numerical implementation and could explain the observed broadening of the current profile due to ECRF waves. The diffusion operator is relevant for studies on heating and current drive by EC waves in present day fusion devices and in ITER. The derivation and the final form of this diffusion operator is discussed in this paper.

1. Introduction

The ordinary O wave and the extraordinary X wave in the electron cyclotron range of frequencies (ECRF) have been successfully used for generating plasma current and for modifying the current profile in many conventional tokamaks. In high-β spherical tokamaks (ST), e.g., in NSTX and MAST. electron Bernstein waves (EBW) can be used for the same purpose [1, 2, 3]. We carry out a fully relativistic analytical and computational analysis of the characteristics of EBWs and their interaction with electrons for driving plasma currents. We show that a study of EBWs in STs will provide detailed insights into the physics of O wave propagation and its interaction with electrons in ITER.

A Fokker-Planck description of RF driven current depends on modeling the the interaction of RF waves with electrons. Toward this end, we derive the relativistic quasilinear operator for momentum and spatial diffusion of electrons due to their interaction with RF waves in non-axisymmetric toroidal plasmas. The plasma equilibrium is expressed in terms of the magnetic flux coordinates of an axisymmetric plasma. The electron motion is treated fully relativistically and is expressed in guiding center coordinates using the action-angle variables of motion in an axisymmetric toroidal equilibrium. The non-axisymmetry in the equilibrium due to magnetic field perturbations and the effect of RF waves on electron motion are treated perturbatively. The magnetic field perturbations can be due to magnetic islands as in neoclassical tearing modes. It is well-known that relativity needs to be included in a proper description of the damping of EC waves [1, 4]. The relativistic generalization of the quasilinear operator is necessary for the interaction of electrons with EC waves. Our study is a generalization of an earlier work by Kaufman [5] and is in contrast to the Kennel-Engelmann [6] description of quasilinear diffusion in a uniform plasma. We carry out the Lie perturbation technique to first order in the perturbation parameters. The generalized quasilinear evolution equation is accurate to second order in the perturbation parameters. The diffusion operator is time dependent and describes resonant and non-resonant momentum space diffusion, and non-resonant radial transport of electrons. The former leads to current generation in a plasma and the latter to the broadening of the current profile. The final results are expressed in a form that is suitable for numerical implementation.

2. Relativistic Effects in Wave Damping

We have developed a fully relativistic code R2D2 which solves for wave propagation and damping of all waves in the electron cyclotron range of frequencies (ECRF) [1]. This code has been used in a number of studies on electron cyclotron waves and electron Bernstein waves (EBW) [1, 7]. Results from R2D2 on the damping of the ordinary O waves in plasmas which are ITER-like and the damping of EBWs in plasmas which represent present day STs are discussed below.

In Fig. 1(a) we plot the imaginary part of $n_\perp$, calculated from R2D2, as a function of ω/ω_{ce} for the O wave in ITER-type of plasma conditions. Here $n_\perp$ is the wave index perpendicular to the magnetic field, ($n_\parallel$ being the parallel wave index), ω is the wave angular frequency, and ω_{ce} is the electron cyclotron angular frequency. The imaginary part of $n_\perp$ is a measure of the damping of the wave. In this figure we compare the relativistic (solid red) and the non-relativistic (dashed blue) results. It is evident that the relativistic results are significantly different from the non-relativistic results. In approaching the cyclotron resonance from the high-field side ($\omega/\omega_{ce} \leq 1$), the relativistic damping starts to occur before the non-relativistic damping. In the approach to the resonance from the low field side, relativity tends to narrow the deposition profile so that deposition starts closer the cyclotron resonance. Overall, relativity tends to broaden the deposition profile and reduce the maximum value.

The propagation and damping physics of O waves is being studied in present tokamaks. However, an ST plasma is not a suitable candidate for O waves since the plasma is overdense. At low harmonics the O wave is cutoff near the edge of the plasma while at high harmonics the plasma is essentially transparent to the wave. Since EBWs do not have density cutoffs and are well absorbed by electrons in the Doppler-shifted vicinity of low order cyclotron resonances, they are well suited for ST plasmas [2] such as those encountered in NSTX and MAST. In Fig. 1(b) we plot the imaginary part of $k_\perp\rho_e$ as a function of ω/ω_{ce} for parameters relevant to a NSTX plasma. Here $k_\perp$ is the perpendicular component of the wave vector and ρ_e is the electron Larmor radius. The figure shows the differences between the relativistic (solid red) and the non-relativistic (dashed blue) results when approaching the resonance from the high field side. We find that, for EBWs in a ST, relativity narrows the absorption profile when approaching the resonance from the low field side, and broadens it for the high field side approach. In a high-β NSTX-

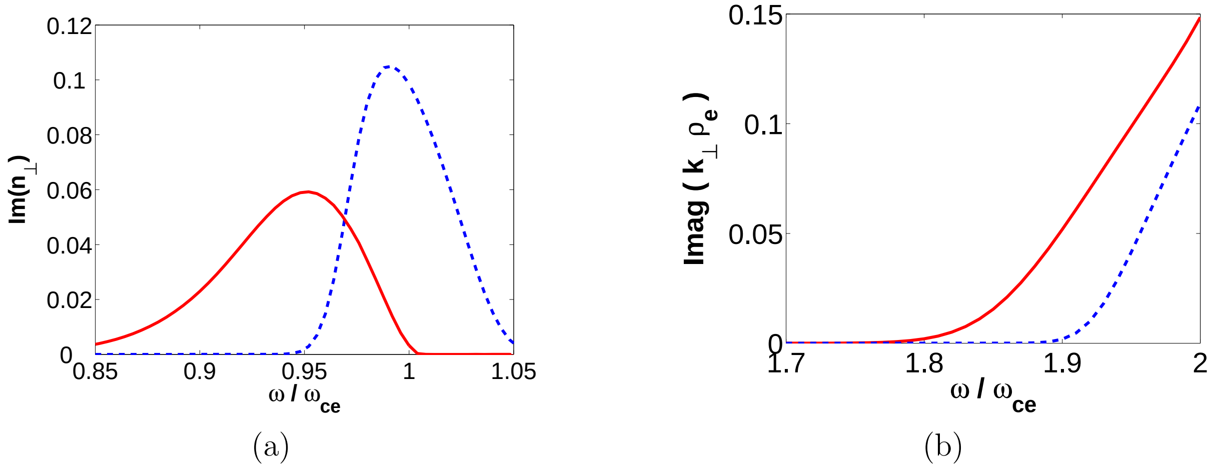


Figure 1: (a) Imaginary-$n_\perp$ for an O wave versus ω/ω_{ce} for ITER-type parameters with $\omega/2\pi = 170$ GHz, electron temperature $T_e = 10$ keV, $\omega_{pe}/\omega = 0.75$ (ω_{pe} is the electron plasma frequency), and $n_\parallel = 0.1$; (b) imaginary part of $(k_\perp \rho_e)$ for an EBW versus ω/ω_{ce} for NSTX-type parameters with $\omega_{pe}/\omega_{ce} = 6$, $T_e = 3$ keV, and $n_\parallel = 0.2$.

type plasma, there is a dip in the magnetic field along the equatorial plane [2]. By an appropriate choice of EBW frequency, the wave can approach the cyclotron resonance either from the low field or from the high field for the same launching position. Thus, an ST offers an extended test of waves in the EC range of frequencies.

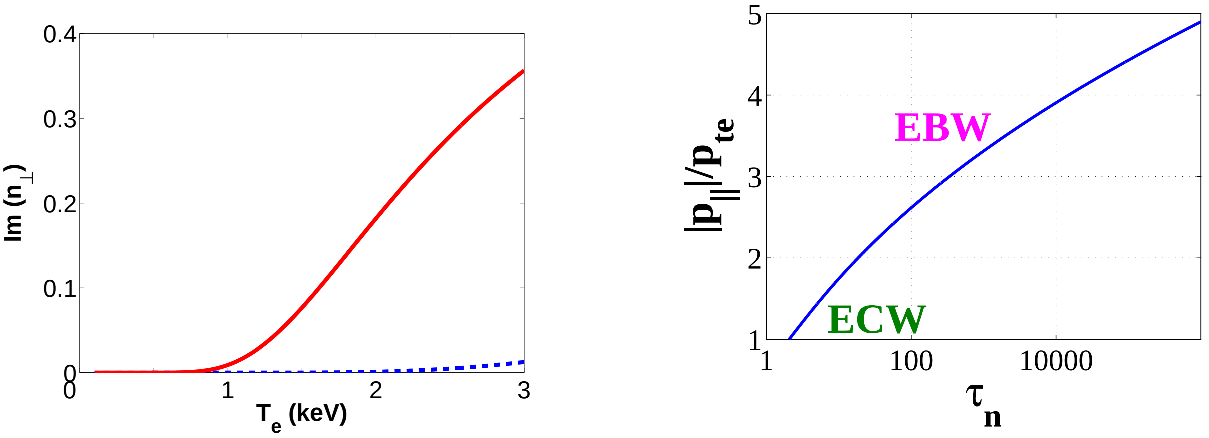
In Fig. 2(a) we compare, as a function of the plasma temperature, the relativistic (solid red) and the non-relativistic (dashed blue) values of the imaginary part of $n_\perp$ for EBWs in typical NSTX-type plasmas. From this figure it is evident that relativistic effects become important for EBWs for temperatures above 1 keV. Since temperatures in this range can be easily achieved in as ST, the dispersion characteristics of EBWs will provide useful insight into the relativistic modifications to the damping of O waves in ITER.

3. Current Drive by Electron Cyclotron Waves

The primary role of EC waves in ITER will be to drive localized plasma currents. The current drive physics depends on the momentum of the electrons in the distribution function that interact with the EC waves. A measure of this interaction is the optical depth of the EC waves. The optical depth can be determined from the linear theory of EC wave propagation and damping [8]. In Fig. 2(b) we plot the momentum of the electrons, normalized to the thermal momentum, as a function of the optical depth of EC waves. The optical depth of X and O waves is around 10 (represented by ECW in the figure) so that these waves interact with electrons near the thermal momentum. For EBWs the optical depth is in the range $200 \lesssim \tau_n \lesssim 1000$, so that EBWs interact with electrons in the range $3 \lesssim p_\parallel/p_{te} \lesssim 4$. Thus, EBWs interact with electrons which are approximately an order of magnitude more energetic than the electrons with which the O wave interacts in present day tokamaks. If the plasma temperature in the region where EBWs damp in an ST is 3 keV, the effective energy of the electrons with which the EBWs interact is around 30 keV. In ITER the O mode will be interacting with electrons in this approximate energy range. Thus, EBW experiments in a ST can provide insight into the physics of the interaction of EC waves with highly energetic electrons.

The EBWs can drive plasma currents in a ST either through the Fisch-Boozer scheme [9] or the Ohkawa scheme [10, 11]. The latter means of current drive is possible since, on the outboard side, a large fraction of the electrons in a ST are magnetically trapped. Even though the Ohkawa current drive is not envisioned for ITER, experiments on a ST can be useful in extending our understanding of the wave-particle interactions.

The code R2D2 has been coupled to a code LUKE [12] which solves or RF driven current using a quasilinear diffusion operator. Some results on EBW current drive obtained from this combination of codes have been discussed in [1].



(a) (b)

Figure 2: (a) $Im(n_\perp)$ versus electron temperature for NSTX-type plasmas with $\omega/\omega_{ce} = 1.9$, $\omega_{pe}/\omega_{ce} = 6$, and $n_\parallel = 0.2$; (b) The parallel electron momentum normalized to the thermal momentum versus the optical depth for the electron cyclotron waves. ECW represents the X and O waves.

4. Quasilinear Theory for Momentum and Spatial Diffusion due to Radio Frequency Waves

A theoretical description of the interaction of radio frequency (RF) waves with electrons in tokamaks requires an accounting of the toroidal magnetic field geometry. For EC waves, the description has to be relativistic so that the damping of the waves and their interaction with electrons are described correctly. In this section we derive the quasilinear diffusion operator for the interaction of RF waves with electrons using the Lie transform perturbation technique. We use the magnetic flux coordinates of an axisymmetric toroidal plasma, and the electron motion is expressed in terms of the canonical guiding center variables. The electron motion is perturbed by RF waves and by non-axisymmetric perturbations to the confining magnetic field. The magnetic perturbations could be due to magnetic islands in a plasma.

The quasilinear action diffusion equation describes transport in momentum space and in the radial spatial direction induced by magnetic perturbations and RF waves. The diffusion tensor is non-singular and time-dependent. As a result of applying the perturbation theory for finite time intervals we avoid the presence of the usual Dirac delta function. The Dirac delta function results in a non-vanishing diffusion tensor only on a discrete set of action surfaces which satisfy exactly a resonance condition [5]. The appearance of Dirac's delta function is a consequence of two assumptions. The first assumption is that the RF waves are plane waves and hence occupy all space. For a finite beam size, as in EC heating and current drive, this assumption is not valid. When the finite beam size for the RF waves is properly taken into account the dependence on the resonance condition becomes non-singular. This is due to the resonance broadening effect [13]. The second assumption is related to dynamical features of the electron motion. In particular, singularities appear when the Markovian assumption for decorrelation of the particle orbits due to perturbations, is invoked [5]. However, in many cases of interest, such a statistical assumption do not necessarily hold. The underlying phase space of the system contains not only chaotic areas but also islands of "regular", quasiperiodic motion. Consequently, the Markovian assumption is violated and the dynamics of the particles have to be accounted for properly.

In a general magnetic field configuration, consisting of nested toroidal magnetic surfaces, the covariant representation of the magnetic field is [14]

$$\mathbf{B} = g(\psi_p)\nabla\zeta + I(\psi_p)\nabla\theta + \delta(\psi_p,\theta)\nabla\psi_p \tag{1}$$

where ψ_p, ζ, and θ are, respectively, the poloidal flux, the toroidal angle, and the poloidal angle. The functions g and I are related to the poloidal and toroidal currents, respectively, and δ is related to the degree of non-orthogonality of the coordinate system. The magnetic field lines are straight lines in the (ζ,θ) plane. The guiding center Hamiltonian [14] is

$$H_{gc} = \left(m^2c^4 + m^2c^2\rho_\parallel^2B^2 + 2mc^2\mu B\right)^{1/2} + \Phi \tag{2}$$

where $\rho_\parallel = v_\parallel/B$, $v_\parallel$ is the component of $\mathbf{v}$ along $\mathbf{B}$, m is the mass of the electron, μ is the magnetic moment, and Φ is the electrostatic potential. The two sets of canonically conjugate variables are (P_θ,θ) and (P_ζ,ζ) [14], where

$$P_\theta = \psi + \rho_\parallel I, \qquad\qquad P_\zeta = \rho_\parallel g - \psi_p \tag{3}$$

Here ψ, the toroidal flux, is given by $d\psi/d\psi_p = q(\psi_p)$ with $q(\psi_p)$ being the safety factor. Note that ψ_p and $\rho_\parallel$ are functions of P_θ and P_ζ only. The third set of canonically conjugate variables is (μ,ξ), with ξ being the gyration angle. For an axisymmetric magnetic field the three-degree of freedom system (2) has three independent conserved quantities (μ, P_ζ, W), and the particle motion is completely integrable. The Hamiltonian describes magnetically trapped particles moving in banana orbits, and passing particles circulating in the toroidal direction. An action-angle transformation can be used to eliminate θ from the Hamiltonian. A new action $\hat{P}_\theta$ where $\hat{P}_\theta = \oint P_\theta(\theta;\mu,P_\zeta,W)d\theta$ along with the canonical transformation is obtained from the generating function $S(\xi,\zeta,\theta;\hat{\mu},\hat{P}_\zeta,\hat{P}_\theta) = \xi\hat{\mu} + \zeta\hat{P}_\zeta + \int_0^\theta P_\theta(\theta';\hat{\mu},\hat{P}_\zeta,\hat{P}_\theta)d\theta'$. The hatted variables are the new action-angle variables with $\hat{\mu} = \mu$ and $\hat{P}_\zeta = P_\zeta$. We will use the new action-angle variables and drop, without leading to any confusion, the hat over this variable set.

The non-axisymmetric magnetic perturbations have the form $\tilde{\mathbf{A}} = a\mathbf{B}$ with $a(\psi_p,\theta,\zeta) = \sum_{m_1,m_2} a_{m_1,m_2}(\psi_p)e^{i(m_1\theta+m_2\zeta)}$. Such perturbations modify the parallel canonical momentum $\rho_c = \rho_\parallel + a$ [14]. The scalar and vector potentials corresponding to RF wave fields are represented in an eikonal form $\Phi_{rf}(\mathbf{x},t) = \tilde{\Phi}_{rf}(\mathbf{x})e^{i\Psi(\mathbf{x},t)}$, $\mathbf{A}_{rf}(\mathbf{x},t) = \tilde{A}_{rf}(\mathbf{x})e^{i\Psi(\mathbf{x},t)}\mathbf{P}_{rf}$ where $\tilde{\Phi}$ and $\tilde{A}$ are amplitudes of the scalar and vector potentials, respectively, Ψ is the phase, and $\mathbf{P}_{rf}$ is the wave polarization vector. The local wave vector $\mathbf{k}$ and the angular frequency ω of the wave fields are given by $\mathbf{k}(\mathbf{x},t) = \nabla\Psi(\mathbf{x},t)$, $\omega(\mathbf{x},t) = -\frac{\partial\Psi(\mathbf{x},t)}{\partial t}$.

To second order in the ordering parameters ϵ (RF wave perturbations) and $\lambda(\sim\epsilon)$ (non-axisymmetric magnetic perturbations) $H = H_0 + \epsilon H_1 + \epsilon^2 H_2$, where $H_0 = mc^2\Gamma_0 + \Phi$ and

$$H_1 = -\frac{1}{\Gamma_0}\left(\rho_c B\hat{b} + (2\mu B)^{1/2}\hat{c}\right)\cdot\mathbf{A}_{rf} + \Phi_{rf} - \frac{\lambda}{\epsilon}\frac{m}{\Gamma_0}\rho_c B^2 a \tag{4}$$

$$
\begin{aligned}
H_2 &= \frac{1}{2mc^2\Gamma_0}\left[c^2A_{rf}^2 - \frac{1}{\Gamma_0^2}\left\{\left(\rho_c B\hat{b} + (2\mu B)^{1/2}\hat{c}\right)\cdot \mathbf{A}_{rf}\right\}^2\right] \\
&\quad + \frac{\lambda^2}{\epsilon^2}\frac{B^2}{2c^2\Gamma_0^2}\left(1 - \frac{\rho_c^2 B^2}{c^2\Gamma_0^2}\right)a^2 + \frac{\lambda}{\epsilon}\frac{aB}{\Gamma_0}\hat{b}\cdot\mathbf{A}_{rf}
\end{aligned} \tag{5}
$$

The unit vector $\hat{b}$ is along the axisymmetric magnetic field, $\hat{a}$ and $\hat{c}$ are perpendicular to $\hat{b}$ ($\hat{a} = \hat{b}\times\hat{c}$) and gyrating with the particle, and $\Gamma_0 = (1 + \rho_c^2 B^2/c^2 + 2\mu B/mc^2)^{1/2}$.

From the Lie transform perturbation theory [15], the first order Lie generator w_1, obtained from the solution of the equation $\partial w_1/\partial t + [w_1, H_0] = K_1 - H_1$ by setting $K_1 = 0$, is $w_1 = -\int_{t_0}^{t} H_1(\mathbf{J}, \boldsymbol{\theta}, s)ds$ where $\mathbf{J} = (P_\theta, P_\zeta, \mu)$ and $\boldsymbol{\theta} = (\theta, \zeta, \xi)$. The integration is along the orbits of the unperturbed, integrable, Hamiltonian H_0, and are given by $\mathbf{J}(s) = \text{const.}$ and $\boldsymbol{\theta}(s) = \boldsymbol{\theta}(t) + \boldsymbol{\omega_\theta}(s-t)$ with $\boldsymbol{\omega_\theta} = \partial H_0/\partial \mathbf{J}$. The resulting w_1 is

$$
\begin{aligned}
w_1 &= \sum_{n_1,n_2,l} G_{n_1,n_2,l}(\mathbf{J}) e^{i\mathbf{N}_{n_1,n_2,l}\cdot(\boldsymbol{\theta}-\boldsymbol{\omega_\theta} t)} \frac{e^{i\left(\mathbf{N}_{n_1,n_2,l}\cdot\boldsymbol{\omega_\theta}-\omega\right)t} - e^{i\left(\mathbf{N}_{n_1,n_2,l}\cdot\boldsymbol{\omega_\theta}-\omega\right)t_0}}{i(\mathbf{N}_{n_1,n_2,l}\cdot\boldsymbol{\omega_\theta}-\omega)} \\
&+ \sum_{n_1,m_1,m_2} F_{n_1}(\mathbf{J}) a_{m_1,m_2}(\mathbf{J}) e^{i\mathbf{M}_{n_1,m_1,m_2}\cdot(\boldsymbol{\theta}-\boldsymbol{\omega_\theta} t)} \frac{e^{i\mathbf{M}_{n_1,m_1,m_2}\cdot\boldsymbol{\omega_\theta} t} - e^{i\mathbf{M}_{n_1,m_1,m_2}\cdot\boldsymbol{\omega_\theta} t_0}}{i(\mathbf{M}_{n_1,m_1,m_2}\cdot\boldsymbol{\omega_\theta})}
\end{aligned} \tag{6}
$$

where $\mathbf{N}_{n_1,n_2,l} = (n_1 + k_\theta, n_2 + k_\zeta, l)$ and $\mathbf{M}_{n_1,m_1,m_2} = (n_1 + m_1, m_2, 0)$. The first sum includes resonance between the RF waves and the particles and depends on the three angles. The second sum includes resonance between the magnetic perturbations and the particles and depends on the two angles θ and ζ. This form is derived from the Fourier series representation of the first order Hamiltonian H_1 in Eq. (4)

$$
\begin{aligned}
&\sum_{n_1,n_2} G_{n_1,n_2}(\mathbf{J}) e^{i(n_1\theta + n_2\zeta)} = \\
&\quad \left[(1/\Gamma_0)\tilde{A}_{rf}(\mathbf{X})\left(\rho_c B P_{rf\|} J_l + (2\mu B)^{1/2}\left(P_{rf}^{+} J_{l-1} + P_{rf}^{-} J_{l+1}\right)\right) - \tilde{\Phi}_{rf}(\mathbf{X})\right] e^{ik_{\psi_p}\psi_p}
\end{aligned} \tag{7}
$$

$$
\sum_{n_1} F_{n_1}(\mathbf{J}) e^{in_1\theta} = \frac{m}{\Gamma_0}\rho_c B^2 \tag{8}
$$

where the polarization vector has been decomposed into one parallel ($P_{rf\|}$) and two counter-rotating circular polarizations (P_{rf}^{+}, P_{rf}^{-}), and $J_l = J_l(k_\perp\rho)$ is the $l-$th order Bessel function. Both sums in Eq. (6) include a functional dependence on the actions of the form

$$
\mathcal{R}(\Omega; t, t_0) = \frac{e^{i\Omega t} - e^{i\Omega t_0}}{i\Omega} = \int_{t_0}^{t} e^{i\Omega s} ds \tag{9}
$$

This function is smooth and localized around $\Omega = 0$. It indicates a resonance between the particle motion and the perturbations. For long times $\lim_{t\to\infty}, \mathcal{R}(\Omega; t, -t) = 2\pi\delta(\Omega)$, where $\delta(\Omega)$ is the Dirac delta function commonly appearing in quasilinear theory [5].

The evolution of any function $f(\mathbf{z})$ of the phase space variables over an infinitesimal time interval $[t_0, t_0 + \Delta t]$ is

$$
f\left(\mathbf{z}(t_0 + \Delta t; t_0), t_0 + \Delta t\right) = T(\mathbf{z}_0, t_0) S_K(t_0 + \Delta t; t_0) T^{-1}(\mathbf{z}_0, t_0) f\left(\mathbf{z}_0, t_0\right) \tag{10}
$$

where $T = e^{-L}$, $Lf = [w, f]$. As a result of applying the canonical perturbation theory for finite time intervals $[t_0, t]$, one can easily show that $w_n(\mathbf{z}_0, t_0) = 0$. Thus, $T(\mathbf{z}_0, t_0) = I$. Furthermore, we have chosen $K_n = 0$ for $n = 1, 2$. Then the time evolution of S_K is given by the H_0, i.e., by integrating along unperturbed orbits $S_K = S_{K_0} = S_{H_0}$. Upon taking the limit $\Delta t \to 0$ we obtain $\partial f(\mathbf{z},t)/\partial t = \partial\left[T^{-1} - I\right](\mathbf{z},t)/\partial t f(\mathbf{z},t)$. For the case where $f(\mathbf{z})$ is the distribution function, this equation is an approximation, up to the same order as T^{-1}, of the original Vlasov (Liouville) equation. For a function $F(\mathbf{J})$ which is an average of f over the angles, $F(\mathbf{J}) = \langle f(\boldsymbol{\theta},\mathbf{J})\rangle_{\boldsymbol{\theta}}$ we have

$$\frac{\partial F(\mathbf{J},t)}{\partial t} = \frac{\partial \left\langle [T^{-1} - I](\mathbf{z},t)\right\rangle_{\boldsymbol{\theta}}}{\partial t} F(\mathbf{J},t). \tag{11}$$

To second order in ϵ we have $T^{-1} - I = L_1 + (1/2)L_2 + (1/2)L_1^2$ with $L_n F = [w_n, F]$ [15]. Upon integration by parts, and using the fact that the dependence on all the angles is periodic, we find that $< L_n F(\mathbf{J}) >_{\boldsymbol{\theta}} = 0$ for $n = 1,2$ and $< L_1^2 F(\mathbf{J}) >_{\boldsymbol{\theta}} = \nabla_{\mathbf{J}} \cdot [< (\nabla_{\boldsymbol{\theta}} w_1)^2 >_{\boldsymbol{\theta}} \cdot \nabla_{\mathbf{J}} F(\mathbf{J})]$. An important point emerges from these equations. The angle-averaged operators that are needed in the evolution equation (11) can be calculated up to second order in the perturbation parameter using results from first order perturbation theory, namely w_1 [16]. Then the evolution equation (11) becomes

$$\frac{\partial F(\mathbf{J},t)}{\partial t} = \nabla_{\mathbf{J}} \cdot \left[\mathbf{D}(\mathbf{J},t) \cdot \nabla_{\mathbf{J}} F(\mathbf{J},t)\right], \qquad \text{where} \qquad \mathbf{D}(\mathbf{J},t) = \frac{1}{2}\frac{\partial \left\langle (\nabla_{\boldsymbol{\theta}} w_1)^2 \right\rangle_{\boldsymbol{\theta}}}{\partial t} \tag{12}$$

is the generalized quasilinear tensor. It can be shown that the first order momentum variation can be written as $< (\Delta\mathbf{J})^2 >_{\boldsymbol{\theta}} = < (\nabla_{\boldsymbol{\theta}} w_1)^2 >_{\boldsymbol{\theta}}$, from which we find that $D(\mathbf{J},t) = \lim_{\Delta t \to 0} < (\Delta\mathbf{J})^2 >_{\boldsymbol{\theta}} / 2\Delta t$ corresponding to the common definition of the quasilinear diffusion tensor. The evolution equation (12), can be transformed to the physical variables $\mathbf{P} = (\psi_p, v_\parallel, v_\perp)$ describing particle transport, heating, and current drive through the variation of the distribution function with respect to (ψ_p, θ, ζ), $v_\perp$, and $v_\parallel$, respectively.

5. Conclusions

The implication of the results presented in this paper is as follows. We can study the importance of relativistic effects on EC wave propagation in present day STs. The relativistic modifications to the propagation and damping of EBWs in STs will provide an insight into the effect of relativity on O wave propagation and damping in ITER. The ECRF waves in ITER will be used for stabilizing the neo-classical tearing mode. For this to be accomplished successfully we need to account for any changes in the spatial location of wave damping. The EBWs in STs will damp on electrons whose energies are similar to those that will interact with O waves in ITER. The O waves and EBWs interact with electrons via the cyclotron resonance interaction. Consequently, we can study the interaction physics of EC waves at high ITER-like temperatures in present day ST plasmas.

We have also derived a relativistic operator for momentum and spatial diffusion of electrons due to RF waves and non-axisymmetric magnetic field perturbations. For EC current drive the relativistic treatment is necessary. An important role of EC current drive is to control the growth of the neoclassical tearing mode (NTM). The non-axisymmetric magnetic field perturbations included in the diffusion operator can be due to magnetic islands as for NTMs. Furthermore, the quasilinear operator is time dependent and does not have the singular delta function dependence on the particle and wave phase velocities as encountered in previous papers. Thus, the diffusion operator can be more easily implemented in a numerical code. Even though we have evaluated a flux-surface averaged quasilinear operator it is not necessary to do so. The steps preceding the flux surface averaging include the dependence of the quasilinear operator on the poloidal angle. So our formalism leads to a more general diffusion operator than in previous publications. The spatial dependence of the diffusion operator gives the effect of RF waves on the spatial diffusion of electrons in the radial direction. Consequently, RF induced broadening of the current profile is included in the operator.

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References

[1] Ram, A.K., Decker, J., Peysson, Y., "On electron Bernstein waves in spherical tori," J. Plasma Physics **71** (2005) 675.

[2] Ram, A.K., Schultz, "Excitation, propagation, and damping of electron Bernstein waves in tokamaks," Phys. Plasmas **7** (2000) 4084.

[3] Ram, A.K., Bers, A., "Excitation and emission of electron cyclotron waves in spherical tori," Nucl. Fusion **43** (2003) 1305.

[4] Bornatici, M., Cano, R.,De Barbieri, O., Engelmann, F., Nucl. Fusion **23** (1983) 1153.

[5] Kaufmann, A.N., Phys. Fluids **15** (1972) 1063.

[6] Kennel, C.F., Engelmann, F., Phys. Fluids **9** (1966) 2377.

[7] Decker, J., Ram, A.K., "Relativistic description of electron Bernstein waves," Phys. Plasmas **13** (2006) 112503.

[8] Decker, J., Ph.D. Thesis, Department of Electrical Engineering and Computer Science, M.I.T., Cambridge, MA 02139. USA (2005).

[9] Fisch, N.J., Boozer, A., Phys. Rev. Lett. **45** (1980) 720.

[10] Ohkawa, T., General Atomics Report No. GA-A13847 (1976).

[11] More recently Ohkawa current drive using waves in the electron cyclotron range of frequencies has been discussed by: Decker, J., in *Proc. 15th Topical Conf. on RF Power in Plasmas*, AIP Conf. Proc. No. 694 (ed. C.B. Forest), Melville, New York (2003), p. 447.

[12] Decker, J., Peysson, Y., Report EUR-CEA-FC-1736, Cadarache, France (2004).

[13] Demeio, L., Engelmann, F., Plasma Phys. Contr. Fusion **28** (1986) 1851.

[14] White, R.B., Phys. Fluids **2** (1990) 845.

[15] Cary, J.R., Phys. Rep. **79** (1981) 129.

[16] Kominis, Y., Phys. Rev. E **77** (2008) 016404.