OV-TH

Turbulence Theory and Gyrokinetic Codes

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Abstract: This overview is an assessment of the progress made in the theory of turbulence in fusion plasmas, supported by gyrokinetic simulations. After a brief description of the main micro-instabilities leading to turbulent transport, the processes underlying turbulence self-organisation are commented. The second part is dedicated to specific issues such as dimensionless scaling laws and various transport channels. The last part deals with improved confinement and transport barriers, with some emphasis on the role of shear flows and magnetic shear.

1. Introduction

Mastering turbulent transport is a key issue on the way towards commercially viable fusion reactors, as it controls the confinement properties of any magnetically confined plasma. It turns out that this field of research has made tremendous progress thanks to a combination of analytical results, numerical simulations and dedicated experiments. In particular comprehensive gyrokinetic codes have been helpful to clarify a number of issues. This overview presents an assessment of the main advances in theory and numerical simulations, and of the questions which remain to be solved. Due to lack of space, the discussion will be mostly restricted to core turbulence.

This grand tour starts with a presentation of some basics, namely the main micro-instabilities that underlie turbulent transport, and the mechanisms that lead to self-organisation. Fusion plasmas are weakly collisional so that a gyrokinetic approach is necessary to describe turbulence with accuracy. This means that 5D kinetic equations coupled to Maxwell equations must be solved - an unprecedented challenge for numerical computing. The second part is dedicated to the main physical issues and summarises the outcomes of gyrokinetic simulations. It covers dimensionless scaling laws, and gives some insight into the various transport channels. While ion transport is rather well understood, it turns out that electron heat transport is still hotly debated. Particle and impurity transport is another important issue. Theory and simulations have now reached some maturity and a rather clear picture, though incomplete, has emerged with time. Momentum transport is a much less mature subject and will certainly evolve in a near future. Due to limited space, the comparison to fluctuation measurements will not be presented. The third part addresses the question of improved confinement and transport barriers. It is now well established that shear flows and some optimisation of the magnetic configuration, in particular via magnetic shear, can reduce turbulent transport. Other mechanisms are possible, but less generic. The stabilising effect of shear flow is now well documented. Still the mechanisms that determine the mean shear flow dynamics, i.e. the radial electric field, are not fully mastered. Regarding the effect of the magnetic configuration, the situation is even less satisfactory. In particular the role of low order rational resonant surfaces at the onset of transport barriers remains puzzling. The last part of this overview is dedicated to a discussion of the open issues and foreseeable developments.

2. Basics of turbulent transport

2.1. The gyrokinetic framework

Turbulence simulations have long been performed by solving fluid equations [1, 2, 3, 4]. However, reactor-grade plasmas are weakly collisional. Also fluctuations cover a large range of spatial scales, which can be smaller than the ion gyroradius. This implies using a kinetic description, in order to account for resonant interactions between particles and waves, and to describe finite orbit width effects. Hence distribution functions must be computed for all species, and coupled to Maxwell equations via charge and current densities. This means in principle solving a 6D problem (3 directions for space and 3 for velocities). In practice, the cyclotron motion of particles is much faster than the dynamics of turbulent structures. This allows reducing the dimensionality by properly separating the fast cyclotron motion from the slow particle gyrocenter motion. A Vlasov equation is then written for the distribution function of gyrocenters, which is a function of the position, parallel velocity, and adiabatic invariant. The gyrocenter dynamics is therefore 4D, parameterized by the adiabatic motion invariant. The new Vlasov equation, which lives in a 5D space, is called a gyrokinetic equation. The charge and current densities are obtained by calculating the moments of the distribution function. However a difficulty is that the particle distribution function differs from the gyrocenter distribution. The difference lies in a polarisation term, which is the kinetic version of the polarisation drift in fluid equations. This term plays a crucial role in the generation of zonal flows. Once current and charge densities are implemented in the Maxwell equations, the electromagnetic field is updated (see [5] for details).

Solving 5D gyrokinetic equations for each species coupled to Maxwell equations is a difficult problem due to the large range of scales that must be covered. Nevertheless gyrokinetic simulations are now routinely run thanks to the progress made in supercomputers and numerical techniques. A detailed description of the various numerical recipes is beyond the scope of this paper. A brief summary is given here. Three techniques are commonly used to solve numerically gyrokinetic equations: Eulerian, Lagrangian and semi-Lagrangian. Eulerian codes solve the kinetic equation on a fix grid in the phase space. The advantage is that usual techniques for solving partial differential equations can be used. Also the distribution function is well described everywhere. These codes have proved their efficacy, but require a careful choice of numerical schemes in order to guarantee both stability and accuracy (see e.g. [6, 7, 8, 9]). Lagrangian codes (typically Particle in Cell codes) take benefit from of a long standing and widespread experience (see e.g. [10,11]). They are well suited to massively parallel calculations. However they can be affected by numerical noise due to random sampling [12, 13, 14, 15]. This difficulty is cured by techniques of "optimal loading" and filtering [16, 17, 18] or weight spreading [19, 20]. A third method is based on a semi-Lagrangian scheme [21,22,23]. This algorithm combines some of the advantages of Eulerian (fixed grid) and Lagrangian (characteristic methods) methods. In particular it is not subject to CFL constraint on the time step. However it requires a large amount of computer memory. Also accuracy requirements impose in practice time steps comparable to the other techniques. Overviews on gyrokinetic simulations can be found in [24, 25].

2.2. Micro-instabilities

The spectrum of instabilities in tokamaks is quite rich. Let us start first with low wave number electrostatic micro-instabilities, i.e. such that $k_\perp\rho_i<1$, where $k_\perp$ is the perpendicular wave number and ρ_i is the ion Larmor radius ($\rho_i=(m_iT_i)^{1/2}/e_iB$ where m_i is the ion mass, and T_i is the ion temperature, e_i the ion charge). The dominant instabilities are the Ion Temperature Gradient driven modes (ITG) and Trapped Electron Modes (TEM) (see overviews [26, 27]). When the plasma beta $\beta=2\mu_0 p/B^2$ (p is the total pressure, and B the magnetic field) is vanishingly small, perturbations of the magnetic field can be ignored and the modes are essentially electrostatic. An important property of these micro-modes is the existence of an instability threshold, expressed as a function of the logarithmic gradients of the density, ion and electron temperature gradients. ITG/TEM modes are predominantly driven by an interchange-like effect. There exists also a branch of "Slab ITG" modes, which are unstable in absence of field curvature. At higher wave numbers $k_\perp\rho_i>1$, instabilities called Electron Temperature Gradient modes appear [28, 29, 8, 30]. These modes can contribute to electron transport when electron heating is dominant.

When β increases, ITG/TEM modes become less unstable. However, above a critical value of β, a new branch of kinetic electromagnetic modes appear, called "kinetic ballooning modes" [31] or Alfvén Ion Temperature Gradient modes [32, 33]. The onset of these electromagnetic modes is potentially highly detrimental to the confinement since their growth rates are quite high. This state should indeed correspond to an Alfvénic (MHD) turbulence, and correspondingly a strong degraded confinement. The electrostatic assumption is also questionable in the edge of tokamaks, where electromagnetic effects are known to be important [34,35]. In some particular situations, in particular in low aspect ratio tokamaks, micro-tearing modes can become unstable. These modes may lead to an ergodisation of field lines, and enhance the electron heat diffusivity [36].

2.3. Reduced transport models

An estimate of turbulent transport is given by the quasi-linear theory [37,38]. In essence this procedure consists in plugging the linear response of the gyrokinetic equation into the expressions of the particle, momentum and energy fluxes. It appears that the various transport channels are coupled, i.e. that the transport matrix is not diagonal. This has very important implications on transport, since it leads to "pinch" effects. Unfortunately the quasi-linear theory, though useful, is incomplete since the fluxes are found to depend on the turbulence intensity, i.e. the spectrum of potential fluctuations, which remain to be computed. An estimate of the fluctuation level is provided by the mixing-length rule [39]. The simplest version yields a level of fluctuation of the form $e\phi_k/T_e=1/k_\perp L_p$ (L_p is a pressure gradient length – here ϕ_k is an r.m.s. level averaged over time for a wave vector k). This approximation is certainly the weakest part of the derivation of any transport model. Various recipes have been proposed to improve it. The Weiland [40], GLF23 [41], CDBM [42] and subsequent attempts [43, 44, 45, 46] are examples of transport models based on this procedure. In fact the development of reduced transport models face a major difficulty: turbulence self-organises via the generation of structures (usually at large scales) which back-react on the fluctuation background. These complex mechanisms cannot be easily captured by simple recipes, as shown in the following section.

2.4. Turbulence self-organisation

Two types of processes play a crucial role: turbulence regulation by zonal flows, and large scale transport events. Both are typical of interaction between large and small turbulence scales, and are quite different from the dynamics of 3D fluid turbulence.

2.4.1. Zonal flows

Zonal flows are fluctuations of the poloidal (toroidal symmetric) velocity and play an important role in turbulence simulations (for an overview see [47]). Indeed suppressing artificially these flows in computations leads to a transport that is substantially larger [48, 49, 50]. The mechanism which is often invoked for the generation of zonal flows is the force associated to the Reynolds stress. When magnetic fluctuations are large, the Maxwell stress plays a similar role, though its sign is usually opposite to the Reynolds stress. Also oscillations of the flow can be excited due to the coupling between the vorticity and the pressure due to perpendicular compressibility [51, 52]. The frequency of these quasi-coherent modes, called Geodesic Acoustic Modes (GAMs), is proportional to c_s/R, where c_s is the sound velocity (also proportional to the ion thermal velocity up to some ratio of electron and ion temperatures– for simplicity we will use the same notation for both)[53, 54]. The generation of GAMs via the turbulent stress tensor bears some similarities with zonal flow generation [55]. Another mechanism for flow generation has been investigated [56, 57, 58, 59], which relies on the Kelvin-Helmholtz (KH) instability. The point is that a sheared velocity can drive a secondary instability of the KH type, which in turn may destroy the primary vortex. This robustness of this mechanism depends sensitively on the magnetic shear, which stabilises KH modes. A very important property of zonal flows is that they remain essentially undamped in collisionless regime. More precisely, an initial poloidal flow relaxes towards a finite residual value, called Rosenbluth-Hinton residual flow [60], via the emission of GAMs. This relaxation process has become a crucial test when verifying a gyrokinetic code (see example on *FIG.1*).

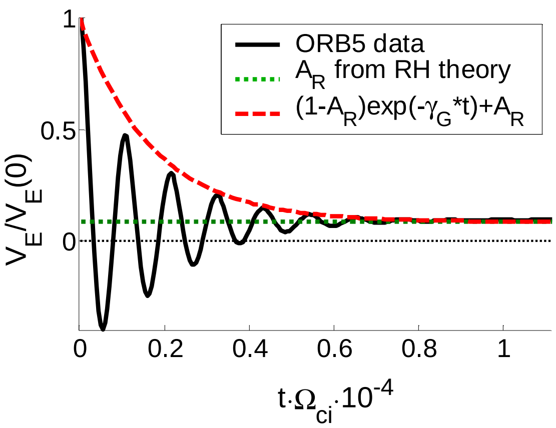

FIG.1: Relaxation of a poloidal flow in a gyrokinetic simulation (ORB5 code [18]). The GAM damping rate agrees with the theoretical prediction [53], while the residual flow agrees with the Rosenbluth-Hinton prediction [60].

Switching on zonal flows in simulations, their amplitude is observed to increase at the expense of small scale fluctuations. The process is made easier by their weak damping. Zonal flows back react on fluctuations by tearing apart small scale vortices. This process leads to a reduction of turbulent transport. It takes place provided that zonal flows evolve on a time scale that is longer than a vortex typical life time. Regarding this criterion, GAMs are expected to be less efficient than zonal flows to reduce turbulent transport [61, 62], except when their frequency is low enough, i.e. at low temperature. Moreover, GAMs are usually found to drive energy from zonal flows [63]. Finally, in contrast with zonal flows, GAMs are damped by sideband Landau resonances. The damping of long wavelength GAMs is proportional to $exp[-(qR\omega_{GAM}/c_s)^2]$ and is therefore small when the safety factor is large. Hence weak damping is reached near the separatrix, reason why GAMs are prominent in edge plasmas [64]. The generation of flows, and feedback on fluctuation via shearing, is a very important mechanism of turbulence self-regulation.

2.4.2. Avalanches and streamers

Turbulence simulations frequently exhibit large scale transport events, which are detrimental to confinement. Two mechanisms have been identified: avalanches and streamers. Avalanches appear via a domino effect: if a gradient of temperature (or density) locally exceeds an instability threshold, it generates a burst of transport that expels some heat or matter, thus increasing the gradient on a neighbouring radial position, where the same process is reproduced [65]. Avalanches are commonly observed in numerical simulations, in particular during transients [66, 67, 68]. The question of avalanches is closely connected to the concept of turbulence spreading [69, 70, 71]. One expects indeed turbulence to spread from unstable to stable regions. The balance between growth and diffusion yields an estimate of the spreading extent $d_{spr}\approx(\chi/\gamma_{lin})^{1/2}$. Another manifestation of spreading is the propagation of fronts whose velocity is $c_{front}\approx(\gamma_{lin}\chi)^{1/2}$ [72, 73]. Using a growth rate of the order of c_s/a, and a gyroBohm diffusion coefficient $\chi\approx\rho_*T/eB$ (see section 3.1), it is found that c_{front} scales as ρ_*c_s, where ρ_* is the normalized gyroradius $\rho_*=\rho_i/a$ (a is the minor radius). Hence c_{front} is a fraction of the sound speed. Strong shear flows tend to prevent spreading, and more generally any type of large scale structures [74].

Streamers are E×B eddies elongated in the radial direction. They have also been observed in various turbulence simulations [8, 30, 75, 76]. They are obviously competing with shear flows [77]. Streamers have been proposed as a possible cause for boosting the flux associated to small scale turbulence (see section 3.2.2).

The existence of mesoscale structures, i.e. structures whose size is intermediate between a correlation length and the plasma size, raises the question of the assumption of locality, which underlies the traditional diffusion/convection form of fluxes in reduced transport models. It is stressed however that the existence of ballistic fronts do not necessarily contradict the diffusion/convection paradigm [72, 73]. Also the Fokker-Planck form of transport equations appears to be very robust [78]. Nevertheless, this question is legitimate and several non-local models have been proposed in the literature [79, 80, 81], often based on fractional kinetic approach.

3. Main physical issues

3.1. Dimensionless scaling laws

An important feature of turbulent transport is the existence of a similarity principle, which states that properly normalised physical quantities depend mainly on 3 dimensionless parameters once the geometry is fixed and some other dimensionless parameters frozen. These main parameters are the normalized gyroradius $\rho_*=\rho_i/a$, collisionality ν^* (typically a collision frequency normalized to a transit time c_s/R – the details do not matter here) and β. Global [82] and local [83, 84] versions have been formulated (see overviews in [85, 86]). Let us first consider ITG turbulence. Some elementary theoretical considerations indicate that the correlation lengths and times should scale respectively as ρ_i and a/c_s. Let us assume that particles experience a random walk. In that case, the diffusion coefficient, which scales as the square of a correlation length divided by a correlation time, should be proportional to the GyroBohm diffusion coefficient $\chi_{gb}=\rho_*T/eB$. Since early 3D fluid simulations of ion turbulence [87, 88], and more recent gyrokinetic simulations [89, 90, 91, 92], it is now widely admitted that the correlation lengths and times, and fluxes follow the gyroBohm prediction in the limit of small values of ρ_*. A consequence of this behaviour is a fair agreement between global and local codes in the limit of small values of ρ_* (see *FIG.2,* right panel). A departure from gyroBohm scaling is usually observed for large values of ρ_* (see *FIG.2*, left panel). Three explanations at least have been proposed for this behaviour. Linear modes exhibit a radial width that scales as $\sqrt{a\rho_s}$, which leads to Bohm scaling, i.e. diffusion coefficients that scale as T/eB. Although observed in some simulations [93, 94], this effect does not appear to be the main one. Indeed the shearing effect of zonal flows leads to small scale vortices whose size is proportional to the ion Larmor radius (see *FIG.3*). The two other explanations are based on shear flow stabilisation [87] and turbulence spreading [89]. Quite interestingly both theories lead to a scaling of the form $\chi \approx \chi_{gb}(1-C\rho_*)$, where C is some constant. Hence for large values of ρ_*, the scaling is not monomial. Unfortunately the tendency of gradients to stay close to the instability threshold (stiffness) makes the practical definition of the scaling very difficult to settle both in experiment and theory [95]. Moreover it is quite difficult to explain why the scaling of electron transport is always gyroBohm in experiments, while the ions are not.

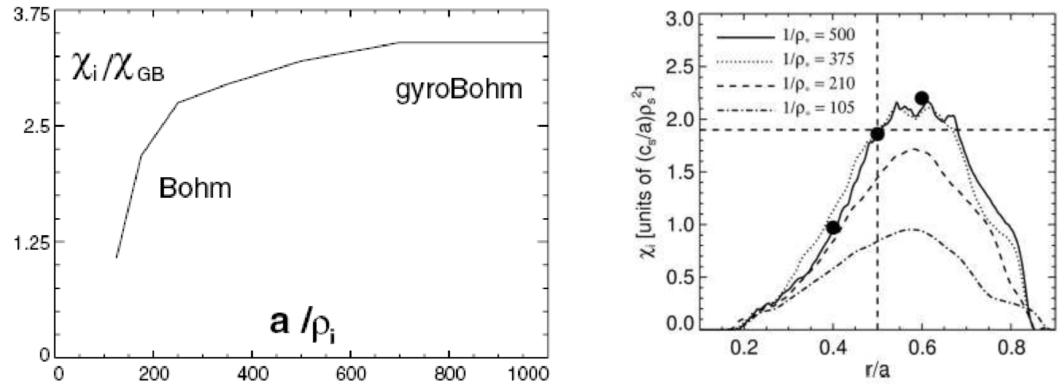

FIG.2: Left panel: Ion heat diffusivity vs 1/ρ_* showing the transition from Bohm to gyroBohm scaling computed with the GTC ([89]). Right panel: radial profiles at decreasing ρ_* of ion heat diffusivity for a sequence of global GYRO simulations and local GS2 simulations [91] (dots).

The situation is less clear the dependence on collisionality, because of competing effects. Collisionality has a stabilising effect on electron (TEM) modes due to electron collisional detrapping [96]. On the other hand, collisional friction damps zonal flows [97, 98, 99]. Moreover, collisions play an important role in the dissipation of small structures in the velocity space. Indeed, the entropy increase can only be due to collisional dissipation at small scales in the velocity space. This property can be verified by writing a balance equation for the entropy production rate [100]. The process of generation of small scales by Landau damping, and dissipation by collisions have been verified in gyrokinetic simulations [101, 102].

The dependence on β is mainly a signature of electromagnetic effects. The parameter β is also involved in the compression of magnetic surfaces (the Shafranov shift, which is stabilising, see section 4). As mentioned in section 2.2, increasing β stabilises ITG/TEM modes, but trigger (kinetic) ballooning modes above a critical value of β. Fluid [103,104] and gyrokinetic [105, 106] simulations have confirmed this behaviour, i.e. a decrease of transport with increasing β, followed by a sharp increase of the diffusion coefficient at high β. In fact it appears that the ion heat transport is weakly sensitive to β, while the electron heat and particle diffusion coefficients are much more sensitive to electromagnetic effects [106]. This is an interesting finding, as it might explain why in usual cases where ion transport is dominant, no dependence on β is found experimentally. Nevertheless this question is in fact quite controversial and debated both from the experimental and theoretical points of view.

Assuming a collisionless electrostatic turbulence, it appears that a gyroBohm scaling can be translated into a scaling law for the confinement time, namely $\omega_c\tau_E \equiv \rho_*^{-3}$ ($\omega_c=eB/m$ is the cyclotron frequency). This result can be compared to the ITER scaling (H-mode) $\omega_c\tau_E \equiv \rho_*^{-3.0}\beta^{-0.9}\nu_*^{0.0}$. It appears that the exponent of the normalized gyroradius is the same. However the experimental scaling law exhibits a strong β dependence, which suggests some effect of electromagnetic instabilities. However, this strong dependence was not recovered in some dedicated experiments [85].

Other dimensionless parameters play an important role, such as the safety factor or the charge number. The improvement of the confinement with plasma current, which translates into a diffusivity that increases with the safety factor, has received several explanations based on the downshift of wave number spectra due to Landau damping [107,108,109], damping of GAMs [62], and finite orbit effects. The effect of charge number can be understood essentially from linear theory, and can be either stabilising or destabilising. Above the threshold, the stabilising effect is dominant, and is a consequence of the dilution effect on the interchange drive [110, 111, 112, 113]. Issues such as the isotope effect and scaling with the Mach number are still unresolved (see section 3.2.4 for momentum transport).

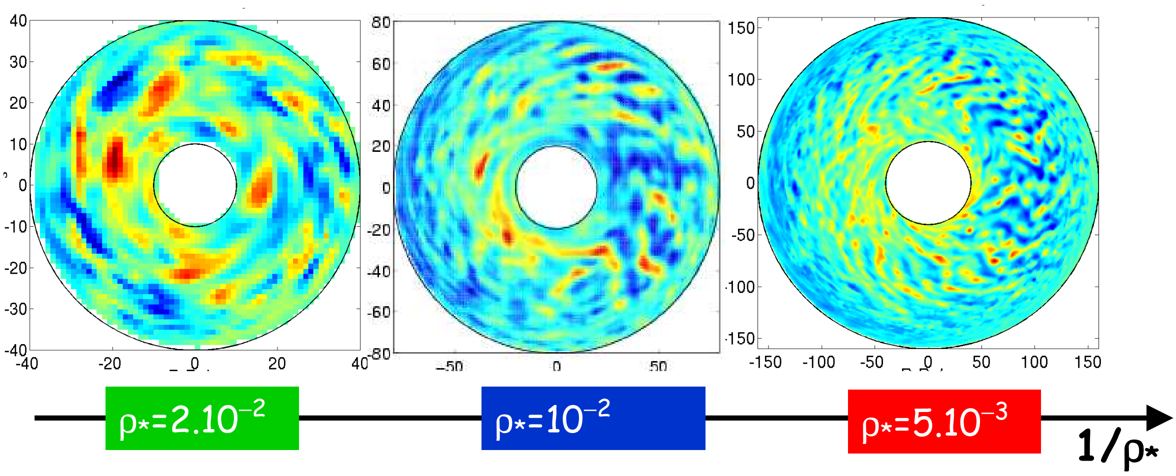

FIG.3: Contour lines of the electric for 3 different values of the normalised gyroradius. The size of the vortex follows the gyroradius, indicating a gyroBohm scaling (from [91]).

3.2. Transport channels

3.2.1. Ion heat transport

Ion heat transport was historically addressed in the first place by numerical simulations and can be considered as mature. In fact, the computation of the ion heat diffusivity has become a way of comparing codes. This was done within the frame of the Cyclone project [114] and TF-ITM IMP4 project [115]. The later work also made a comparison of fluctuation and flux spectra. Several results came out of these exercises. First the actual threshold was found to be larger than the linear stability threshold (Dimits shift). Second the heat diffusivity was found to match the approximate relation $\chi_i=7.9\chi_{i,gb}(1\text{-}6L_{Ti}/R)$ ("LLNL fit"), where L_{Ti} is the temperature gradient length. This expression illustrates the concept of profile stiffness: since transport becomes quite large above the threshold, the temperature gradient length tends to stay close to the threshold, leading to a resilience of the temperature profile. Also heat diffusivities calculated by gyrokinetic simulations were found to be lower than the fluid values in the Cyclone exercise. In the TF-ITM project, some of the diffusivities are below the LLNL fit. Nevertheless the same trend remains qualitatively true. Comparison to experimental data appears to be challenging. This is believed to be due to stiffness: a slight change of the gradients lead to large variations of the heat diffusivity. It is expected that the next generation of gyrokinetic codes, which will be flux driven instead of running at fixed gradient, will allow direct comparison between codes and experiments.

3.2.2. Electron heat transport

Electron heat transport is still subject to discussion. First it is stressed that dominant ITG turbulence does lead to some amount of electron transport, which agrees with the quasi-linear prediction. Nevertheless it is found that the corresponding contribution is too small to explain the measured diffusivity when electron heating is dominant. Hence it is expected that trapped electron modes (TEM) and/or electron temperature gradient (ETG) driven modes play some role.

Gyrokinetic simulations show indeed that TEM modes contribute to electron transport above the stability threshold [109]. The diffusivity can be fit by a formula of the LLNL type. Also a Dimits shift is observed, but in density gradient (TEM modes are driven by density and electron temperature gradients) [116]. The saturation mechanism seems different from ITG modes. Whereas zonal flows are found to play a prominent role for ITG turbulence, it appears that mode coupling to small scale fluctuations, which act as a diffusion, is a key ingredient for TEM turbulence. This behaviour provides some ground for applying the quasi-linear theory [117]. However the question of saturation is not fully settled, since zonal flows, and also profile relaxation play a role in some cases [118].

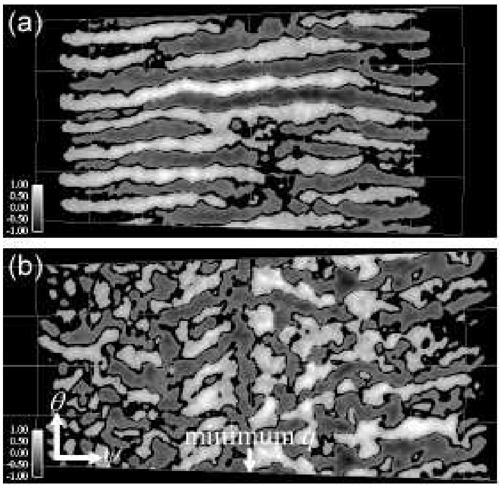

FIG.4: Contour plots of the electric potential in ETG turbulence for simulations with (a) positive magnetic shear, dominated by streamers, and (b) reversed magnetic shear dominated by zonal flows (from [123])

The question of ETG driven turbulence is much more controversial. If one assumes an homothetic behaviour to ITG turbulence, a gyroBohm estimate predicts an electron diffusion coefficient that is $(m_e/m_i)^{1/2}$ smaller that the ion heat diffusivity. Since the measured electron diffusion coefficient is of the same order as the ion value, it appears that the expected value for ETG turbulence is too small by an order of magnitude. However it was argued in [8,30] that zonal flows are less efficiently generated in ETG than in ITG turbulence, because the adiabatic responses of ions (in ETG) and electrons (in ITG) are different. A smaller intensity of zonal flows in ETG turbulence favours the emergence of streamers, which may boost the heat transport. It was found indeed that zonal flows play a little role in the saturation of ETG modes, rather due to wave-particle decorrelation [119]. Nevertheless the role of streamers is controversial. Although there is an agreement on their existence, the enhancement factor associated to streamers varies in the literature [120, 121, 122]. Certainly the magnetic shear plays an important role in that matter: streamers tend to be dominant for high magnetic shear, while zonal flows are dominant at low shear [8, 123, 124] (see *FIG.4*). Also it turns out that numerical issues make the exercise difficult. One may quote the question of noise due to random sampling [13, 14, 15]. It was also mentioned that ETG turbulence with adiabatic ions is sometimes an ill-posed problem due to the interaction with TEMs at low wave numbers [125, 126]. Simulations of the whole spectrum of instabilities (ITG, TEM, ETG) with the GYRO code showed that in fact the contribution to large wave numbers $k_{\perp}\rho_i>1$ to the electron diffusivity is less than 15% in a typical case where electron and ion temperature gradient lengths are equal [124] (see *FIG.5*, upper panel). This was confirmed by a recent simulation with the GENE code where a comparable number was found in similar conditions [127]. However the latter work also shows that when the electron temperature gradient length is smaller than the ion temperature gradient length (and also vanishing density gradient), small scales contribute significantly to electron turbulent transport, while the ion heat diffusivity gets closer to experimental values (see *FIG.5*, lower panel).

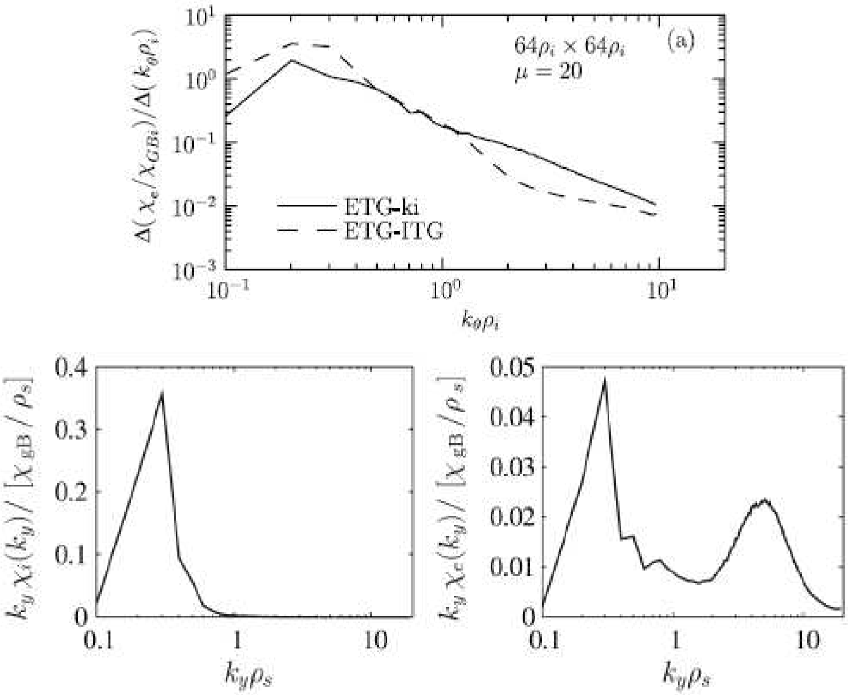

FIG.5: upper panel: simulations of ITG/TEM/ETG turbulence with the GYRO code for equal electron and ion temperature gradient length $R/L_{Ti}=R/L_{Te}=6.89$ and $R/L_{ne}=2.22$ [124]. Lower panel: simulations with the GENE code of ITG/ETG/TEM turbulence for electron temperature gradient length smaller than the ion gradient length, i.e. $R/L_{Ti}=5.5$, $R/L_{Te}=6.9$ and $R/L_{ne}=0$ – ion heat diffusivity on the left, electron heat diffusivity on the right [127].

3.2.3. Particle transport

Particle transport is obviously an important question for a fusion reactor. On the one hand, one would like the density to be the highest as possible to enhance the fusion power. Since the edge density must remain low enough to avoid a disruption (Greenwald limit), peaked density profiles are desirable. On the other hand, peaked density profiles might lead to impurity neoclassical accumulation in the core. One key characteristic of the particle flux is that it is not correctly described by a pure diffusion. Indeed it is well known that density profiles are peaked, even when the ionization source is localized in the edge. To account for this behaviour, the particle flux is traditionally written as $\Gamma = -D\nabla n + Vn$ [128, 129], where V is the pinch velocity and D is the particle diffusion coefficient. Fluid and gyrokinetic simulations (quasi-linear and fully non linear) show indeed that a finite pinch velocity is driven by turbulence. This velocity contains "curvature" (or "compressional") and thermodiffusion contributions, i.e. schematically $VR/D = C_{curv} + C_{\nabla T} R\nabla T/T$ [27, 130,131]. The coefficient C_{curv} is related to compressional effects and is well described by turbulence equipartition theory [132, 133]. It can be shown that when trapped electrons behave as trace particles in a dominant ITG turbulence, one gets $C_{curv}=1/2+4s/3$ ($s=dlnq/dlnr$ is the magnetic shear). Hence the curvature pinch introduces a link between the density and safety factor profile. However it was also shown that passing electrons also participate in the pinch process [134]. The thermodiffusion coefficient $C_{\nabla T}$ can be shown to change sign with the phase velocity of fluctuations, i.e. when moving for instance from ITG to TEM turbulence [130, 131]. Typically, the thermal pinch velocity is directed outward ($C_{\nabla T}<0$) for ITG turbulence. Collisionality plays an important role in that matter. Indeed the ratio *VR/D*, which is representative of the peaking factor of the density, decreases as $1/\nu^*$ [135].

A related question is the issue of impurity transport. In that case again, a pinch velocity is driven by turbulence. Here it is found that the thermodiffusion coefficient is directed outward for ITG turbulence, which is favourable (thermal screening). Unfortunately the coefficient of the thermodiffusion term decreases as *1/Z*, where *Z* is the charge number [136,137]. Hence it is negligible for heavy impurities. The compressional term contains two contributions due to perpendicular and parallel compressibility [136]. The pinch velocity associated to perpendicular compressibility is constant and directed inward, while the associated to parallel compressibility scales as *Z/A*, and its sign depends on the phase velocity of the fluctuations. It is outward for dominant TEM turbulence. Gyrokinetic simulations are roughly in agreement with the quasi-linear picture [138], although a more quantitative assessment of the dependence on charge and mass numbers is needed. This global picture of particle transport predicts non flat density profiles in Iter, without impurity accumulation [139].

Let us also mention that the transport of alpha particles belongs also to this category. Fast ions are expected to be well confined because of finite orbit width effects. Theoretical considerations suggest that fast ion fluxes might be larger than expected [140]. Gyrokinetic simulations indicate that though significant, losses remain reasonable [141, 142, 143, 144], and probably acceptable in Iter plasmas.

3.2.4. Toroidal momentum transport

Toroidal momentum transport has been investigated in detail only quite recently. These studies were motivated by the discovery of plasma "spontaneous" spin-up, i.e. situations where a significant toroidal rotation was measured, in absence of any external torque (see [145] for an overview). Moreover, in many case co-rotation was observed (toroidal velocity in the same direction as the current), which cannot be explained by simple effects as ripple losses or direct losses of ions in the edge. The theoretical question of momentum transport is quite difficult to address. One first obstacle is actually to define properly "momentum transport". One would be tempted to look into the conservation of angular momentum density. However the later depends on particle density, which was seen to be subject to various pinch effects. In order to decouple particle and "true" momentum pinches, it is common to investigate the transport of parallel velocity $U_{//}$. The quasi-linear theory predicts a radial flux of parallel velocity $\Gamma_U=-\chi_U \nabla U_{//}+V_U U_{//}+S_U$ [146,147, 148, 149, 150, 151, 152]. When compared to the expression for particle transport, an extra term S_Ω appears, called residual stress, proportional to the ExB shear rate [146, 147, 148, 156]. The later can be expressed as a function of the parallel velocity, density and temperature derivatives when using the force balance equation. There exists two ways of deriving the various contributions. One solution consists in writing the equations in the frame of reference of the plasma, which requires accounting for the Coriolis and centrifugal forces (the centrifugal force is in practice negligible) [150]. The second way consists in writing the gyrokinetic equations in the laboratory frame, accounting for the fact that the distribution function is shifted in the parallel direction by an inhomogeneous mean velocity [153]. The normalised pinch velocity RV_U/χ_U contains two contributions: a constant term (this scaling is similar as the curvature pinch for particle transport) and a term proportional to the density (possibly temperature) gradient [150, 151]. As for particle transport, part of the pinch velocity is consistent with turbulence equipartition theory [154, 155]. Gyrokinetic simulations did find evidence of a momentum pinch velocity and residual stress [156, 155]. Another important parameter is the Prandtl number (ratio of viscosity χ_U to heat diffusivity χ_{Ti}). This ratio is usually found to be of the order 0.7 [149]. Recent simulations indicate that it can be as low as 0.2 in some cases [157]. Work is actively being done to determine the relative weights of the various mechanisms at play.

4. Improved confinement

The physics of transport barriers is a broad subject that is already covered by several overview papers for external [158, 159, 160, 161] and internal transport barriers [162, 163, 164]. Two generic key parameters are known to play a stabilising role: flow shear and magnetic shear. Other ingredients may be involved (density gradient, ratio of electron to ion temperature, impurity content, …), but are less generic.

4.1. Shear flow stabilisation

The physics of turbulent transport reduction due to $E\times B$ shear flow is well documented. The interested reader might consult overviews on theory [165] and experiments related to shear flow stabilisation [166]. Stabilisation is obtained above a critical value of the shear flow rate. Several criterions have been proposed and tested [167, 49, 168, 169], which confirm the stabilising effect of shear flows except in a few well identified cases [170]. The radial electric field is constrained by the ion force balance equation $ne(\mathbf{E}+\mathbf{V}\times\mathbf{B})-\nabla p=0$. Once a barrier is formed, a positive loop takes place where density and ion temperature gradients increase, thus boosting the velocity shear rate. The transition to improved confinement bears some similarity with 1st order phase transition [171, 172, 173, 174]. Shear flow stabilisation has been tested with fluid and gyrokinetic simulations. One difficulty however is that most gyrokinetic codes do not calculate self-consistently the mean radial electric field. A new generation of codes is emerging, which aim at this objective [175, 176, 177, 178, 179]. These codes are global, and calculate the full distribution function. This is a demanding task as it requires the calculation of the full neoclassical equilibrium as well as the turbulent generation of flow. A related question is the generation of a strong poloidal shear flow when an internal transport barrier is produced.

4.2. Effect of the magnetic topology.

Negative magnetic shear and high values of β are known to decrease the interchange drive [180]. The effect of β is related to the Shafranov shift of magnetic surfaces (also called α effect, $\alpha=-q^2Rd\beta/dr$ is a measure of the Shafranov shift) [181, 182, 183]. In fact this effect has been known for long in the context of MHD stability [184, 185, 186]). For electron modes, stabilisation occurs when $s<-3/8$, while for ions the exact value depends on the poloidal structure of modes. This stabilisation scheme has been tested both in fluid and kinetic simulations. An electron transport barrier appears when the magnetic shear is negative [187]. This effect is amplified for values of α of the order of unity. For electron modes, theory predicts stability when $s<3\alpha/5-3/8$. A similar effect exists for ions, which comes from the shear dependence of the ion curvature averaged over the mode structure. However it is important to note that slab ITG modes are not sensitive to these effects, and remain unstable at negative magnetic shear. One intriguing mystery is the reason why the onset of internal transport barriers appears to be easier when the magnetic shear is zero and the minimum value of the safety factor is a low order rational number. To some extent, this is a surprising result since no special role of *s=0* is found in simulations [188]. So rational numbers seem to be the key. Three types of explanations have been proposed to explain this behaviour:

i) the onset of a large scale coherent structure which acts on turbulence. This can be an MHD mode located at a rational value of q that generates a localized velocity shear [189]. An alternative explanation is based on a loss of fast ions due to MHD that leads to a shear flow [190]. More recently, it has been proposed that a coherent electrostatic convective cell is generated and reduces transport by tapping energy on turbulence [191].

ii) generation of zonal flows [192, 193] or GAMs [194] close to rational q values.

iii) the existence of gaps in the density of magnetic resonant surfaces at low magnetic shear, which are wider when q_{min} is close to a low order rational number [195, 196, 197, 193].

Shaping effects also play a role via a direct effect on the growth rate, or by affecting the dynamics of zonal flows and GAMs. Indeed the GAM frequency decreases with the elongation. Hence the damping rate of GAMs, which should lead to less active GAMs when elongation increases [198]. Also it is found that the residual Rosenbluth-Hinton flow increases with elongation [199, 198] . One might infer that in non linear regime, this will lead to a higher level of zonal flows and a better confinement. Recent gyrokinetic simulations confirm this behaviour [200].

5. Conclusion

Hopefully this overview has convinced the reader that theory of turbulent transport has made enormous progress. These findings owe a lot to state of the art gyrokinetic simulations which incorporate a large number of ingredients, in particular effects such as Landau resonances and finite orbit effects. This effort, which involves many theoreticians and code developers, has allowed the clarification of a number of issues, such as dimensionless scaling laws, parametric dependences of the various transport channels, and improved confinement. The predicted turbulence intensities and fluxes are now getting close to the observed values. Also the processes leading to self-organisation are better understood. Still several issues remain open. In particular no first principle simulation has been able to reproduce a transition to a transport barrier, external or internal. Also the comparison of simulations to experiments suffers from the sensitivity of transport to the distance of gradients to stability thresholds (stiffness). A slight mismatch in the gradients leads to a large change of fluxes, which makes this comparison very difficult. Finally, transport transients (pulses, heat modulation) have not been simulated yet, whereas these experiments are known to find puzzling results. This issue is also linked to the simulations of true steady-state plasmas, which has not been done yet with gyrokinetic codes.

These various limitations lead naturally to a discussion of the future developments. To simulate transitions to transport barriers, the next generation of gyrokinetic codes, which is emerging now, will have to calculate the full distribution function over the whole torus. This is a very difficult task, as it requires calculating both the equilibrium quantities and fluctuations, which are tiny. Also the implementation of boundary conditions is a formidable task, in particular in the edge plasmas if one wants to describe correctly the scrape-off layer and the pre-sheath and sheath close to the divertor target plates. Concerning the comparison to experiments, it also appears that this new generation will have to be flux-driven, instead of freezing gradients. This should in principle solve the problem of stiffness. The question of simulating both steady-state and transients is even trickier. Indeed it is not foreseen at the moment to run routinely gyrokinetic codes over a confinement time and for plasmas of the ITER size – the requested computing time is indeed enormous. Presently the strategy relies on the use of reduced models. This methodology will probably remain unchanged for some time. Reduced transport models have done a tremendous progress, but are still far from being accurate enough. Whether the improvement of these models is certainly an option, one might wonder whether the use of more elaborated models of turbulence, which would predict the fluctuation spectra and fluxes, would not be a more efficient approach. This supposes however making a quantitative jump in the predictive capability of statistical theory of turbulence. It might actually happen that the progress in massively parallelized computation allows a complete gyrokinetic calculation before reduced or statistical models reach the requested level of accuracy. The coming years will certainly prove to be very interesting regarding this competition between the various approaches.

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