Theory and simulation of non-local thermal smoothing for arbitrary scale length modulation

M. Chen 1), Y. Kishimoto 1), J. Li 1), D. Saitou 1)

1) Graduate School of Energy Science, Kyoto University, Uji, Kyoto 611-0011, Japan

e-mail contact of main author: mo@center.iae.kyoto-u.ac.jp

Abstract. We present a theory that describes the non-local heat transport and associated thermal smoothing modeling the non-uniform laser illumination by solving the steady state Fokker-Planck (FP) equation. A method that connects higher order Chapman-Enskog expansion for low energy electrons and convolution formula for high energy ones through the self-consistent determination of the electric field is developed. The theory explicitly expresses the heat flux for arbitrary periodic temperature modulation given by $\delta T/T_a = \varepsilon_T \sin(kx)$ with moderate wave number $k\lambda_e \le 0.1$, where λ_e is the electron mean free path. The theory is compared with one-dimensional FP simulations by which we investigate the relaxation of sinusoidal temperature perturbations. Reduction of the heat flux from the Spitzer-Härm (SH) theory and the hysteresis nature are found to be reproduced. As the wavelength of the modulation becomes shorter, the contribution from high energy electrons to the heat flux is found to increase whereas the total amount of the heat flux is reduced in proportion to $(k\lambda_e)^2$.

1. Introduction

It is well known that the heat transport described by the classical SH theory [1] does not fit the experimental results well in laser fusion when the temperature gradient scale length L_T is less than $200\lambda_e$ (λ_e the electron-ion collision mean free path) nearby the critical surface. Here, non-local electron heat transport plays an important role, which determines the coupling efficiency between laser energy and hydrodynamic implosion [2-4]. Among them, transverse temperature modulations to the direction of laser propagation due to uneven energy deposition caused by non-uniform laser illumination become a seed of Rayleigh-Taylor instability, which deteriorates the implosion efficiency [5]. The associated thermal smoothing can be beneficial in reducing this non-uniformity during the ablation and implosion processes [6]. It is known that the heat flux is reduced comparing to the SH theory according to $1/[1+\alpha(k\lambda_e)^\beta]$ from the FP simulations, where the factor $\beta \sim 2$ for $k\lambda_e \le 0.1$ and $\beta \sim 1$ for $k\lambda_e \ge 0.3$, α is an adjusted numerical constant and *k* is the wave number of temperature modulation [7-9]. However, explicit heat flux formulas describing non-local thermal smoothing based on local plasma parameters have been less developed. Here, we present a theoretical model which explicitly represents the reduction of the heat flux from the SH theory and compare with one-dimensional FP simulations.

This paper is organized as follows. In Sec.2 we develop a 1D fully kinetic FP code in the diffusive approximation by applying physical model and numerical methods implemented by Epperlein [10] and simulate the thermal smoothing problem originated from laser non-uniform illumination comparing with the SH theory. Sec.3 describes our theory on non-local thermal smoothing based on a mixing method with a higher order Chapman-Enskog expansion [3] and a convolution approach [4]. Different contributions to heat flux from each term of our formula are investigated and compared with FP simulations which show a scaling law of $k\lambda_e$. The conclusion is given in Sec.4.

2. Fokker-Planck Simulations

We develop a one-dimensional fully kinetic FP code with the diffusive approximation and the inverse Bremsstrahlung (IB) heating term [11] based on Epperlein's [10]. We refer readers to Epperlein's paper for details and neglect the physical modeling and numerical methods here due to the page limitation. In our FP simulations, we set the space coordinate as $0 \le x \le L$ to the direction normal to that of laser propagation and assume the periodic boundary condition. As initial conditions, a fully ionized plasma with charge state $Z = 4$, electron number density $n_e = 10^{21} cm^{-3}$ and electron temperature $T_0 = 100eV$ is assumed. The time and space are normalized by the electron-electron collision time and mean free path with the temperature $4T_0$.

In order to simulate transverse temperature modulation and associated thermal smoothing in laser fusion, a sinusoidal laser intensity modulation $I = I_0\left[1+\varepsilon_L \sin(kx)\right]$ is input by IB heating term, where I_0, k and ε_L represent the average laser intensity, modulation wave number and amplitude, respectively detailed as reference [12]. A parameter scan is given with $I_0 = 1\times10^{14} \sim 2\times10^{16} Wcm^{-2}$, $\varepsilon_L = 0.1 \sim 1$ and $k = 0.25 \sim 16$ until the average temperature T_a becomes $3.4T_0$. Then we cease the heating and investigate the subsequent relaxation process. We also calculate the temperature evolution with the SH heat equation predicted by the local SH theory, in which the initial temperature profile is given as that recorded when the heating time is ended in FP simulations.

FIG.1 illustrates the time evolution of the temperature profile from the FP (a) and SH (b) simulations for $k\lambda_e \cong 0.1$. The initial spiky profile at *t*=100 results from the non-linear IB heating. It is found that the relaxation of the FP becomes slower than that of the SH. The corresponding heat flux Q/Q_{FS} (Q_{FS} is free streaming value) versus L_T/λ_e is shown in *FIG.2*. A hysteresis nature [2] is seen at *t*=110, but becomes weak as the relaxation is advanced. Even after L_T/λ_e becomes small enough, the reduction of the heat flux from the SH theory can be seen at *t*=1500 in *FIG.2*, suggesting that the reduction of the heat flux persists at small perturbation limit. The wave number dependence of Q/Q_{SH} is illustrated in *FIG.3* and shows a better agreement with the simulation for $k\lambda_e \le 0.1$ [7, 8]

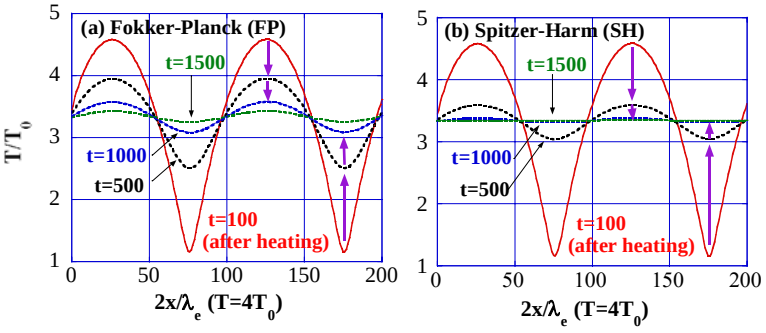

FIG. 1. Temporal evolution of the temperature profile from (a) FP and (b) SH simulation for $k\lambda_e$=0.1.

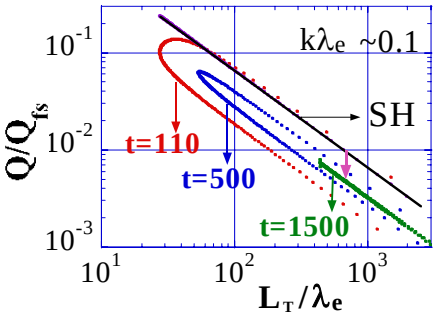

FIG. 2. Heat flux Q/Q_{FS} versus L_T/λ_e at different time for $k\lambda_e$=0.1.

3. Theory of Non-local Thermal Smoothing

Since the classical SH theory breaks down in laser fusion due to the steep temperature gradient, many methods for non-local electron heat transport have been investigated instead for more than 30 years such as SH flux-limiter, non-local convolution [4], linear non-local FP analysis [8, 9] and beam deposition [13] besides direct FP simulations [6, 12]. Kishimoto et al. give an extension SH theory based on Legendre expansion for the second order of electron distribution functions and Chapman-Enskog perturbation expansion in powers of the smallness parameter $\varepsilon \equiv \lambda_e / L_T$ [3] (details given there).

Assuming the temperature perturbation $T = T_a\left[1+\varepsilon_T \sin(kx)\right]$ where ε_T is the temperature modulation amplitude, formula (29)-(31) in [3] can give the heat flux expression in small perturbation situation. However it does not quantitatively fit the FP simulation results but qualitatively because the free streaming part is calculated by the unsuitable zero current condition with the second order electric field E_2. Based on an idea that the non-local heat flux is a direct consequence of super-thermal electrons with long mean free path, we employ a higher order Chapman-Enskog expansion for low energy electrons with $v \le v_c$ [3], whereas directly solve the FP equation with the Krook type collision operator for those with $v \ge v_c$ by a convolution method [4]. Two distribution functions are self-consistently connected through the electric field determined by the zero current condition, which yields

$$Q = Q_{SH}\left\{R(\bar{v}_c)+\left[\delta_1(\bar{v}_c)\left(\frac{\lambda_e}{T}\frac{dT}{dx}\right)^2+\delta_2(\bar{v}_c)\left(\frac{\lambda_e^2}{T}\frac{d^2T}{dx^2}\right)+\delta_3(\bar{v}_c)\lambda_e^2\left(\frac{dT}{dx}\right)^{-1}\frac{d^3T}{dx^3}\right]\right\}+Q_{FS}(\bar{v}_c) \quad (1)$$

$$\cong Q_{SH}\left\{R(\bar{v}_c)+(\lambda_e k)^2\left[\delta_1(\bar{v}_c)\varepsilon_T^2\cos^2 kx-\delta_2(\bar{v}_c)\varepsilon_T \sin kx-\delta_3(\bar{v}_c)\right]+F(\bar{v}_c)\right\} \quad (2)$$

where $Q_{FS} = Q_{SH}F$ represents the free streaming heat flux and Q_{SH} is the local SH heat flux. Here, the normalized velocity is determined by $\bar{v}_c = (\alpha_c/\lambda_e k)^{1/4}$ with an adjustable value $\alpha_c = 1.0$. R and $(\delta_1, \delta_2, \delta_3)$ are all the functions of $\bar{v}_c$, whose expressions are given in [3], and $F(\bar{v}_c) \approx Y_9(\bar{v}_c)/12$, where

$$Y_n(x) = \int_x^\infty \bar{v}^n(\bar{v}^2-3/2)\exp(-\bar{v}^2)/[1+(Z+1)(\lambda_{e0}k)^2\bar{v}^8/6]d\bar{v} \quad (3)$$

instead of formula (31) in [3], satisfying $R \to 1$ and $F \to 0$ for $\bar{v}_c \to \infty$.

The wave number dependence of Eq. (2) is investigated and found to explain the simulation results well for $\lambda_e k \le 0.1$ as shown in *FIG.3(a)*. The hysteresis nature in *FIG.2* originates from the term proportional to δ_2 in Eq. (2), which becomes smaller as the relaxation is advanced as seen in *FIG.2*. In the limit $\varepsilon_T \to 0$, $Q/Q_{SH} \cong R-\delta_3(\lambda_e k)^2+F$ is obtained, suggesting that the hysteresis nature vanishes, whereas the reduction of the heat flux from the SH theory persists as shown in *FIG.2*. When the modulation wave number *k* becomes larger, our theory does not fit the FP simulation results well for $\lambda_e k \ge 0.1$ as shown in *FIG.3(b)*, which is due to our theoretical modeling based on steady state and relatively simple collisional term of Krook type. As $k \to \infty$ limit, the free streaming heat flux is the main part of heat flux, so from Eq. (3) and (4), $F \propto 1/(k\lambda_e)^2$, whereas the scaling law given by FP simulations is $Q \propto 1/k\lambda_e$.

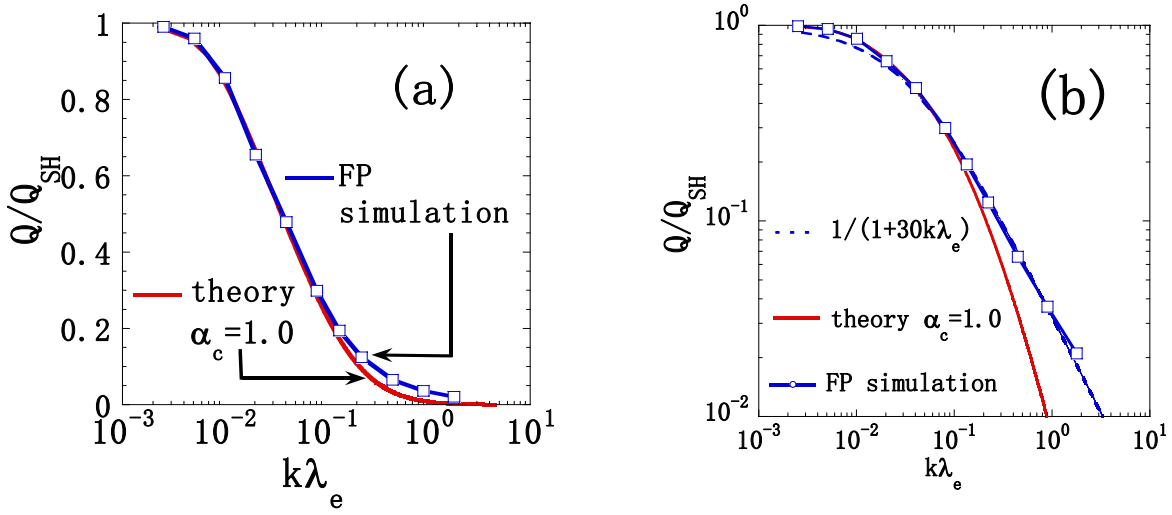

FIG. 3. Heat flux Q/Q_{SH} versus $k\lambda_e$ from the FP simulation and present theory. (a) is linear scale and (b) is logarithm scale.

The contributions from each term in Eq. (2) are investigated in *FIG.4*, where (Q_1, Q_2, Q_3) corresponds to the term proportional to $(\delta_1, \delta_2, \delta_3)$. The SH term Q_R provides a major contribution for $k\lambda_e \sim 0.005$ which is the moderate temperature modulation case as shown in *FIG.4(a)*, whereas Q_3 which is always negative plays a role in reducing the heat flux. Q_2 shows a characteristic feature of hysteresis which is consistent with *FIG.2*. For $k\lambda_e \sim 0.1$ which is in the semi-collisional regime in *FIG.4(b)* where the reduction rate is large, the free streaming part maintains the major part of the heat flux. At the same time, Q_R is giving less contribution and (Q_1, Q_2, Q_3) can be neglected.

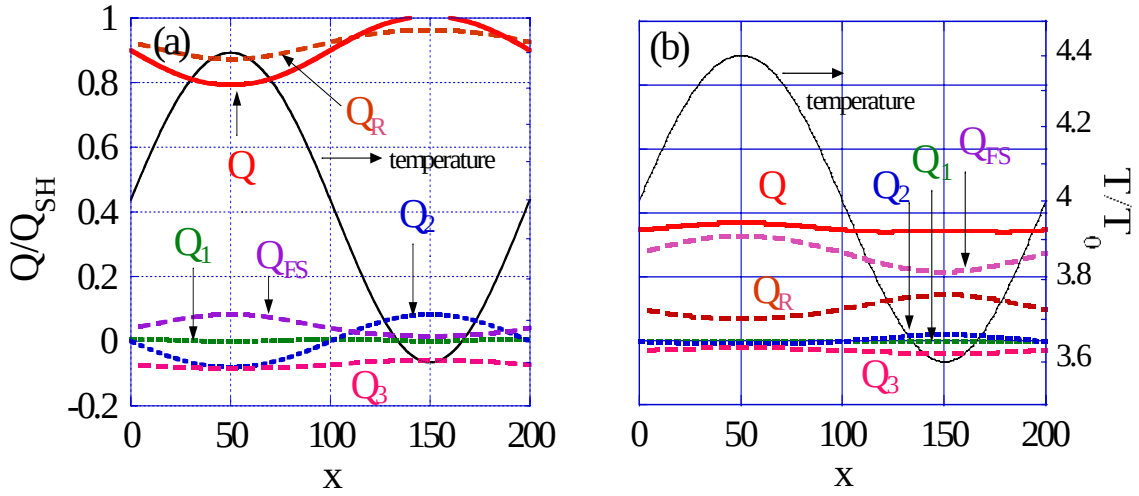

FIG. 4. Spatial distribution of heat flux Q/Q_{SH} and contributions from each term of Eq. (2) for (a) $k\lambda_e$~0.005 and (b) $k\lambda_e$~0.1.

4. Conclusions

We have investigated the thermal smoothing in transverse direction in laser fusion by simulations and theory. A 1D FP code with IB heating term has been used in numerical modeling of sinusoidal temperature perturbations. For large wave number modulations, the magnitude of the heat flux is strongly inhibited and the thermal smoothing is reduced as compared to the SH results. A modified non-local heat flux theory based on both expansion methods for low energy electrons and a consistent convolution approach for super-thermal electrons is investigated for different wave numbers. In strong and semi-collision regimes, we can get a perfect result compared with FP simulations, but in collisionless regime, it is not consistent with the scaling law given by FP calculations, which is maybe due to the simplified collision operator and steady state solution of FP equation as the assumptions in [4].

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