Electron Fishbones: theory and experimental evidence


1) Associazione Euratom-ENEA sulla Fusione, C.P. 65 - I-00044 - Frascati, Rome, Italy
2) Dept. of Physics and Astronomy, Univ. of California, Irvine CA 92697-4575, U.S.A.
3) Southwestern Institute of Physics, P.O. Box 432, Chengdu 610041, P.R.C.
4) Institute of Physics, Chinese Academy of Sciences, Beijing 100080, P.R.C.
5) Department of Physics and Technology, University of Tromsø, N-9037 Tromsø, Norway
6) Space Research Institute, Russian Academy of Sciences, Moscow, Russia

Acknowledgments: J.P. Graves, R.J. Hastie and X. Garbet
Outline

- Experimental observations of electron fishbones: some historic background
- Experimental measurements on FTU and HL-1M: possibility of exciting electron fishbones in extremely different conditions
- Analytic theory of electron fishbones
  - Classic fishbone theory: drive mechanism
  - Kinetic layer equations: importance of ion compressibility
  - Optimal conditions for high-frequency e-fishbones
- Relevance for burning plasmas: nonlinear evolution equations for the fishbone cycle
- Discussions and Conclusions
Experimental Observations I: historic background

- Fishbone-like internal kink instabilities have been observed on DIII-D in conjunction with ECRH on the high field side. (Wong et al, PRL 85, 996, 2000). Excitation by barely trapped suprathermal electrons, characterized by drift-reversal and destabilizing a mode propagating in the ion diamagnetic direction for inverted tail spatial gradient.

- Similar but higher frequency modes were observed in Compass-D (Valovic et al, NF 40, 1569, 2000). There $\omega \lesssim \omega_{\mathrm{TAE}}$ and the mode characterized by chirping frequency was observed with ECRH and LH.

- Observations of electron fishbones with ECRH only HL-1M; (J. Li et al., IAEA 2002; Ding, et al., NF 42, 1, 2002) and LH only FTU; (F. Romanelli et al., IAEA 2002; P.Smeulders, et al., ECA 26B, D-5.016, 2002)

- Recent observations of electron fishbone activity on Tore Supra with inverted q profiles $q_{\mathrm{min}} \gtrsim 2$ (P. Maget, et al., NF 46, 797, 2006).
Experimental Observations II: HL-1M with ECRH only

Ding, et al., NF 42, 1, 2002

• Excitation of the (1,1) mode was observed only when the ECR location is on the high field side near the q = 1 surface.

• This feature is similar to the previous result from the DIII-D tokamak.

Courtesy of SWIP
Experimental Observations III: FTU with LH only

P. Smeulders, et al., ECA 26B, D-5.016, 2002

- Fishbones are visible with only when LH power is on.
- Clear transition in nonlinear behavior as LH power is increased.

IAEA FEC 2006 F. Zonca et al.
Analytic theory of electron fishbones I


\[ i \Delta |s| = \delta \hat{W} = \delta \hat{W}_f + \delta \hat{W}_k \]

\[ \delta \hat{W}_f = 3\pi \Delta q_0 \left( \frac{13}{144} - \beta_{ps}^2 \right) \left( \frac{r_s^2}{R_0^2} \right) \]


\[ \delta \hat{W}_k = \frac{4 \pi^2}{B_0^2} m \omega_c^2 R_0 \int_{0}^{r_s} \frac{r^3}{q} \int \mathcal{E} d\mathcal{E} d\lambda \sum_{\nu \parallel /|\nu| = \pm 1} \frac{\tau_b \bar{\omega}_d^2 QF_0}{\bar{\omega}_d - \omega} \]

Analytic theory of electron fishbones II


Banana regime
\[ |\omega| \ll \omega_{bi} \ll \omega_{ti} \]

\[ \Lambda^2 = \left( \frac{\omega^2}{\omega_A^2} \right) \left( 1 - \frac{\omega_{p_{ei}}}{\omega} \right) \left[ 1 + \left( 1.6 \frac{R_0}{r} \right)^{1/2} + 0.5 \right] q^2 \]

High frequency regime
\[ |\omega| \gg \omega_{ti} \]

Inertial layer response (J.P. Graves, et al., PPCF 42, 1049, 2000) is asymmetric

Inertial layer response (F. Zonca, et al., PPCF 38, 2011, 1996) is symmetric

\[ \omega_{BAE} = q \omega_{ti} \left( \frac{7}{4} + \frac{T_e}{T_i} \right)^{1/2} \]
Analytic theory of electron fishbones III: ECRH

- Asymmetry of Alfvén continuum favors the excitation of modes propagating in the ion diamagnetic direction.

- High field side ECRH fulfills this requirement and guarantees both drift-reversal of the barely trapped supra-thermal electrons as well as the inverted spatial gradient of the supra-thermal tail (K.L. Wong, et al., PRL 85, 996, 2000).

- Consistent with experimental observations (DIII-D, HL-1M).
Analytic theory of electron fishbones IV: LH

- Asymmetry of Alfvén continuum favors the excitation of modes propagating in the ion diamagnetic direction.
- Trapped and barely circulating supra-thermal electrons produced by LH give less selective mode drive than ECRH.
- Well circulating supra-thermal electrons modify the current profile, eventually reversing the magnetic shear and broadening the fraction of trapped particles characterized by drift reversal.

\[
\bar{\omega}_d = \frac{\mathcal{E} q}{\omega_c R_0} \frac{1}{\kappa} \left[ 2 \frac{\mathcal{E} (1/\kappa)}{\kappa K (1/\kappa)} - 1 + 4s \left( \frac{\mathcal{E} (1/\kappa)}{\kappa K (1/\kappa)} + \frac{1}{\kappa^2} - 1 \right) - \frac{\alpha}{2q^2} - \frac{4\alpha}{3} \left( 1 - \frac{1}{\kappa^2} + \frac{2}{\kappa^2} - 1 \right) \right] \frac{\mathcal{E} (1/\kappa)}{\kappa K (1/\kappa)}
\]

- With \( s=0 \) but \( S=(r/q)\sqrt{q}>0 \), fishbone dispersion relation is modified (R.J. Hastie, et al., PF 30, 1756, 1987). \( \Delta q=q-1 \)

\[
-S (\Delta q^2 - \Lambda^2)^{3/4} \left[ 1 + \Delta q/\sqrt{\Delta q^2 - \Lambda^2} \right]^{1/2} = \delta \hat{W}_f + \delta \hat{W}_k
\]
Critical threshold for electron fishbones on FTU with LH

Real frequency

\[ \delta \hat{W}_f + \Re \delta \hat{W}_k = (S/\sqrt{2}) \Lambda^{3/2} \simeq 0 \]

Growth rate

\[ \gamma = \Gamma \left[ \int_0^{r_s} \left( \frac{r}{r_s} \right) \left( \frac{\partial \beta_{h,res}}{\partial r} \right) dr - \beta_{h,c} \right] \]

\[ \Gamma = -\left( \frac{R_0}{r_s} \right) \left( \frac{\partial \Re \delta \hat{W}_k}{\partial \omega} \right)^{-1} \]

\[ \Im \delta \hat{W}_k \equiv \left( \frac{R_0}{r_s} \right) \int_0^{r_s} r dr \partial_r \beta_{h,res} \]

- Critical threshold for electron fishbones on FTU with LH only

\[ \beta_{h,c} = \left( \frac{r_s}{R_0} \right) \left( \frac{S}{2^{1/2}} \right) \Lambda^{3/2} \simeq 1.43 \left( \frac{r_s}{R_0} \right)^{1/4} S \left( \frac{\omega}{\omega_A} \right)^{3/2} (1 - \omega_{\pi} \omega)^{3/4} \]

- Typical values: \[ \beta_{h,c}/\tilde{S} \simeq 3 \times 10^{-4} \]

- Consistent with FTU observations: \[ \beta_{h,res} \gtrsim 0.7 \times 10^{-4} \] for 1MW and \[ \beta_{h,res} \gtrsim 1.2 \times 10^{-4} \] for 1.7MW, with S=0.1÷0.2.
Optimal condition for high frequency (electron) fishbones

- Symmetry of Alfvén continuum favors the excitation of modes propagating in both ion and electron diamagnetic direction.

\[
\Lambda^2 = \frac{\omega^2}{\omega_A^2} - \frac{\omega_{BAE}^2}{\omega_A^2} \left[ 1 + \frac{\omega_{BAE}^2}{q^2\omega^2} \frac{(46/49) + (32/49)(T_e/T_i) + (8/49)(T_e/T_i)^2}{(1 + (4/7)(T_e/T_i))^2} \right] \quad |\omega| \gg \omega_{ti}
\]


- Effective mode excitation would require high power ECRH (ICRH) on axis and \( T_h \gtrsim 200\text{keV} \) on FTU (typical \( T_h \approx 30\text{keV} \))

- For consistency requirements \( T_e \gg T_i \) and/or \( q \gtrsim 2 \)

- Theory predicts possible observation of fishbone-like MHD modes near (below) the GAM frequency.
Relevance for burning plasmas

- The bounce averaged dynamics of both trapped as well as barely circulating electrons depends on energy (not mass).
- Their effect on low frequency MHD modes can be used to simulate/analyze the analogous effect of charged fusion products in the small dimensionless orbit limit (unlike fast ions in present EXP).
- The combined use of ECRH and LH provide extremely flexible tools to investigate various nonlinear behaviors, of which FTU experimental results provide a clear example.
- Possibility of validating models for Integrated Modeling of fishbone NL dynamics and induced transports.
Nonlinear evolution equations for the fishbone cycle (i)

- Nonlinear suprathermal response due to wave-particle resonances is computed from nonlinear GKE:
  \[
  \frac{\partial}{\partial t} \delta \xi_0 = \frac{\delta \xi_{r0}}{r_s}
  \]

  \[
  \frac{\partial}{\partial t} H_{NL} = -\frac{2}{r} \omega_c \omega^2 \frac{\partial}{\partial r} \left[ \left( 1 - \frac{(q-1)\bar{v}_\parallel}{\omega qR_0} \right) \text{Im} \left( \frac{\bar{\omega}_d}{\bar{\omega}_d - \omega} \right) \left( \frac{QF_0}{\omega} \right) r^2 r_s^2 |\delta \xi_0|^2 \right]
  \]

- Fast electron density relaxes according to diffusion processes due to fishbone fluctuations:
  \[
  \frac{\partial}{\partial t} n_h = \dot{N}_h - \frac{2}{r} \omega_c \omega^2 \frac{\partial}{\partial r} \left[ r^2 r_s^2 |\delta \xi_0|^2 f_{eff,h} \left( \frac{Q_{res} n_h}{\omega} \right) \right]
  \]

Nonlinear evolution equations for the fishbone cycle (ii)

- Mode frequency chirps downward according to

\[ \delta \hat{W}_f + \text{Re} \delta \hat{W}_k + \text{Re} \delta \hat{W}_{k,NL} = (S/\sqrt{2}) \Lambda^{3/2} \simeq 0 \]

\[
\frac{\partial}{\partial t} \text{Re} \delta \hat{W}_{k,NL} \simeq -2 \frac{\pi^2}{B_0^2} m \omega_c^3 \frac{R_0}{r_s^2} \int_0^{r_s} \frac{r^2}{q} d\xi d\lambda \sum_{\nu_{\parallel}/\nu_{\perp} = \pm 1} \tilde{\omega}_d^2 \text{Im} \left( \frac{Q}{\bar{\omega}_d - \omega} \right) \frac{1}{\varepsilon} \frac{\partial}{\partial \varepsilon} \left\{ \tau_b \tilde{\omega}_d \varepsilon^3 \frac{\partial}{\partial r} \left[ r^2 r_s^2 Q F_0 \left( 1 - \frac{(q - 1) \bar{v}_{\parallel}}{\omega q R_0} \right) |\delta \xi_0|^2 \right] \right\}
\]

- Characteristic time scale for frequency chirping

\[ \propto |\delta \xi_0|^{-2} \]

- Nonlinear evolution equations for fishbone amplitude and resonant particles

\[ \frac{d}{dt} |\delta \xi_0|^2 = 2 \Gamma \left[ \int_0^{r_s} \left( \frac{r}{r_s} \right) \left( \frac{\partial \beta_{h,\text{res}}}{\partial r} \right) dr - \beta_{h,c} \right] |\delta \xi_0|^2 \]

\[ \frac{\partial}{\partial t} \left[ |\delta \xi_0|^2 \left( \frac{\partial}{\partial t} - \nu_{\text{ext}} \right) \frac{\partial}{\partial r} \beta_{h,\text{res}} \right] = 2 C \omega^2 r_s^2 |\delta \xi_0|^4 \frac{\partial^2}{\partial r^2} \left( r^2 \frac{\partial}{\partial r} \beta_{h,\text{res}} \right) \]

- Fishbone amplitude and resonant electron transport time scale

\[ \simeq |\delta \xi_0|^{-1} \]
Discussions and Conclusions

- Fishbone – like mode excitations by suprathermal electrons is possible in extremely different conditions: ECRH, LH, combinations.
- Analytic theory is successful in explaining these behaviors: theory also predicts optimal conditions for (electron/ion) fishbones at frequencies comparable with GAM/BAE.
- Relevance of these studies for burning plasmas stability and transport of both fast electrons and fusion products.
- ECRH+LH (as in FTU) provide very flexible tools for these studies and a testbed for verifying prediction of nonlinear theories.
- Presented a simple (yet relevant) nonlinear first-principle based model for the fishbone cycle and fast particle transport.
- NEXT: integrated modeling of the fishbone cycle and induced transport