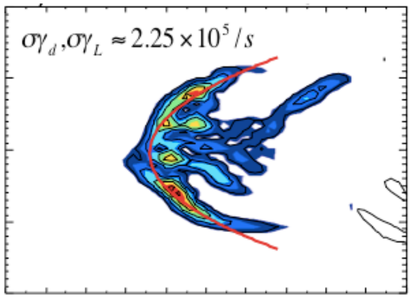
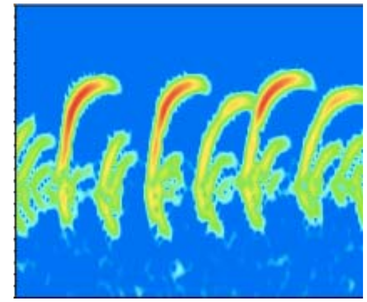
Interpretation of Mode Frequency Sweeping in JET and NSTX





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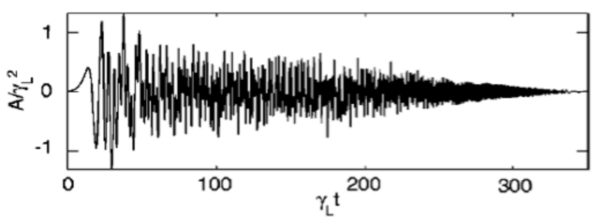
Spontaneous Frequency Sweeping

1. Ubiquitous phenomenon in plasma experiments with kinetic drive
   [e.g. Fishbones, TAE, Hot electron Interchange (TERELLA)]
2. Wave- particle trapping mechanism: universal property applicable to nearly every wave derived from collisionless theory.
3. Trapping mechanism leads to formation of phase space structures: i.e. particle depletion (holes) and accumulation (clumps) regions
4. Frequency sweeping signals give crucial information about the state of plasma and its energetic particle population; this is highly relevant to burning plasmas with substantial energetic alpha particle component
5. Here we report attempts to test the emerging theory of frequency chirping to see if:
   (a) instability drives can be quantitatively calculated,
   (b) if ‘aggressive’ chirp scenarios can be pursued to channel energy
   (c) if phase space structures can be manipulated through heating

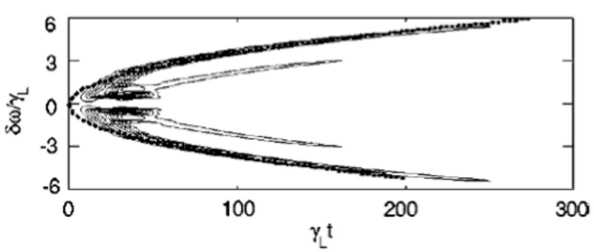
Frequency sweeping from single resonance

- Simulations of bump-on-tail instability (an example of a single resonance) by Petviashvili shows emergence of phase space structures locked to the chirping frequency
- Chirp is induced because of background dissipation, phase space configurations then continually seek lower energy states
- Clumps move to lower energy regions and holes move to higher energy regions

Time evolution of normalized mode Amplitude.



Frequency Sweeping .



$$\left(\begin{array}{l} \omega_b^2 = \dfrac{ek\,|\,E\,|}{m} \\ \text{electrostatic wave} \end{array} \right)$$

$$\omega_b \cong .54\gamma_L$$

$$\delta\omega \cong .44\gamma_L(\gamma_d t)^{1/2}$$

In terms of trapping frequency theoretical description quite similar for nearly any plasma system

Generalization to three dimensions

- Phase Space trapping regions in 1-d generalizes to 3-d
- Instead of resonant point ($\omega = k\mathrm{v}$), there is resonant surface where

$$\omega = \Omega_{p,l,n}(H,\mu,P_\phi) = p\bar{\omega}_\theta + n\bar{\omega}_\phi + l\bar{\omega}_{ci}$$

- Each point on resonant surface traps particles with trapping frequency proportional to $C^{1/2}$ (C is mode amplitude)
- Detailed form of trapping frequency depends on phase space position (see manuscript in conference proceedings)

$$\omega_b = \hat{\omega}_b(H,\mu,P_\phi)C^{1/2}$$

- Trapped particles are entrained in trapping structures and particles' resonances $\Omega_{p,l,n}(t)$, are locked to changing frequency $\omega(t)$.

$$\Omega_{p,l,n}(H_0 + \delta E(t), \mu_0 + \delta\mu(t), P_{\phi 0} + \delta P_{\phi 0}(t)) = \omega_0 + \delta\omega(t)$$

- Thus as frequency changes, trapped particles move to new regions of phase space but value of trapped particle distribution does not change

$$F_{trap}\left(H,\mu,P_\phi\right) = F_0\left(H - \delta H(t), \mu - \delta\mu(t), P_\phi - \delta P_\phi(t)\right)$$

where $F_0\left(H,\mu,P_\phi\right)$ is the unperturbed equilibrium distribution

Solution for Mode Saturation and Sweeping Rate

Saturated electric field amplitude

$$|C|^{1/2} = \frac{16}{3\pi^2} < \frac{\tilde{\gamma}}{\hat{\omega}_b(\Gamma)} > \text{ where } < \frac{\tilde{\gamma}}{\hat{\omega}_b^s(\Gamma)} > \equiv \int d\Gamma \sum_{p,l} \delta(\omega_0 - \Omega_{p,l,n}) \frac{\tilde{\gamma}_{p,l,n}}{\hat{\omega}_b^s(\Gamma)}, \text{ note } < \frac{\tilde{\gamma}}{\hat{\omega}_b^0(\Gamma)} >= \gamma_L$$

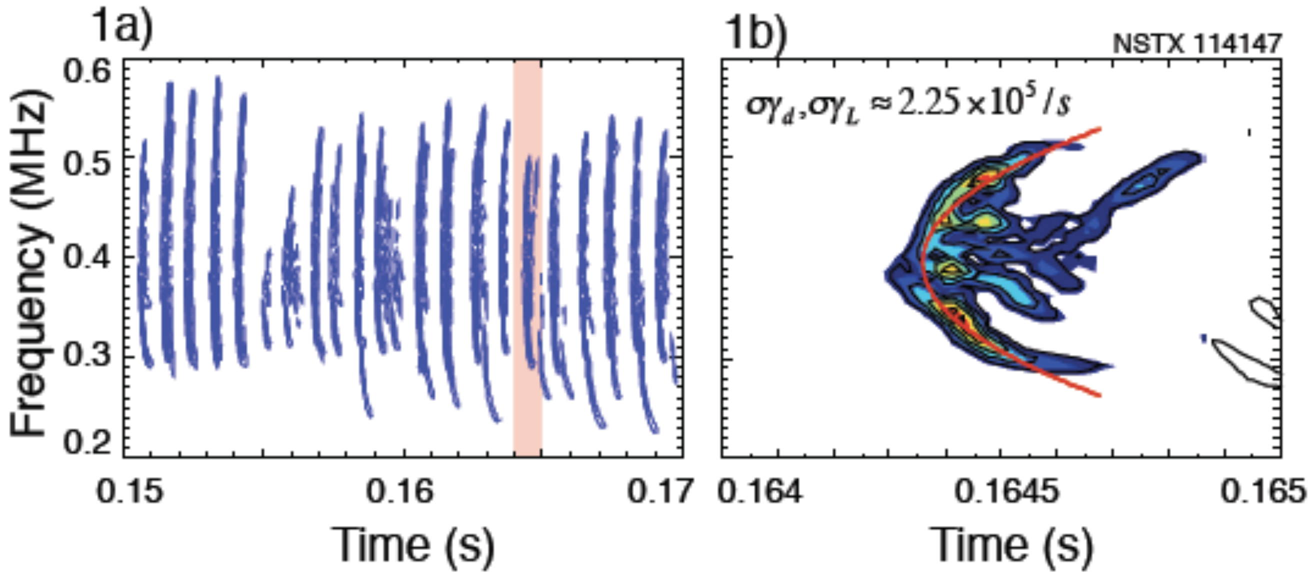
Note: predicted saturation is a weighted average over growth rate contributions from each resonance regions

Predicted sweeping rate

$$\omega - \omega_0 = \frac{\pm 16\sqrt{2}\gamma_L \left(\sigma\gamma_d t\right)^{1/2}}{3\pi^2\sqrt{3}}; \ \sigma = \frac{< \frac{\tilde{\gamma}}{\hat{\omega}_b} >^3}{< \frac{\tilde{\gamma}}{\hat{\omega}_b^0} >^2 < \frac{\tilde{\gamma}}{\hat{\omega}_b^3} >}$$

The numerical factor σ contains contributions from entire resonant surface

Alfven Wave Chirping in NSTX



- predominantly chirp both up and down during early NBI on NSTX.
- Rapidity increasing with applied NBI
- n=5 toroidal number determined
- Wave propagates opposite beam injection and plasma rotation direction
- Blow up of single frequency chirp fits $t^{1/2}$ and linear growth rate (to within factor σ) is inferred.

Matching Predicted Growth Rates

- Fast ion distribution from TRANSI
- CAE frequency at r/a=0.5 fits frequency condition

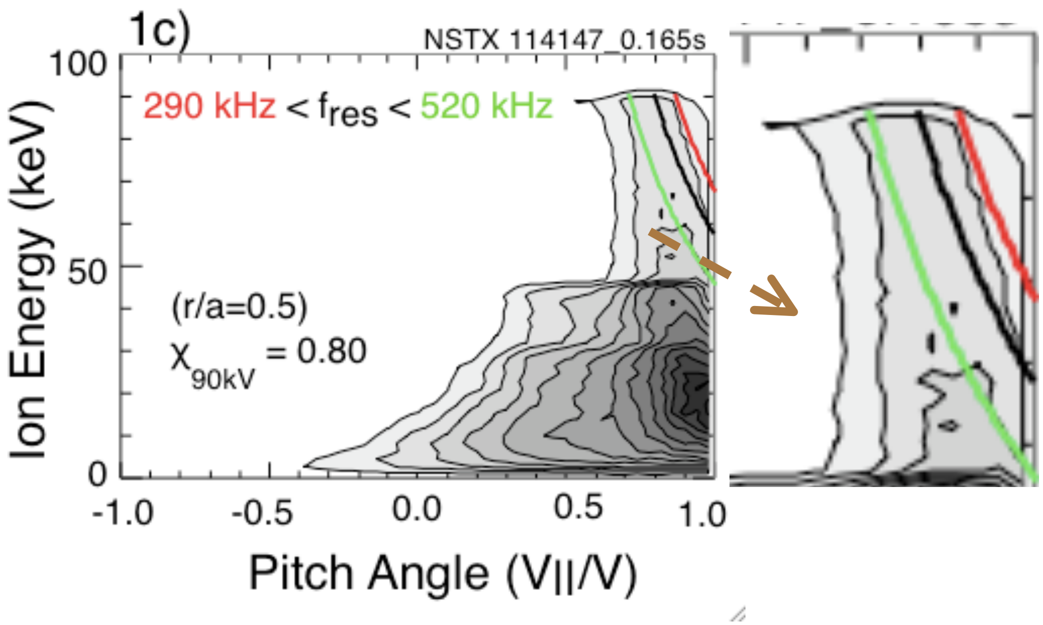
$$\omega_{CAE}^2 = k^2 v_A^2 \cong k_\perp^2 v_A^2$$

(but not unique, GAE at smaller radii fits; internal measurements needed)

- Mode frequency of 420 kHz (in plas frame) together with Doppler rotation gives 394 kHz observed in lab.
- using Doppler resonance condition,

$$\omega \equiv k v_A = -|k_\parallel| v_\parallel + \omega_{ci}$$

chirp range matches transit frequency of particles



Comparison of growth rate predictions

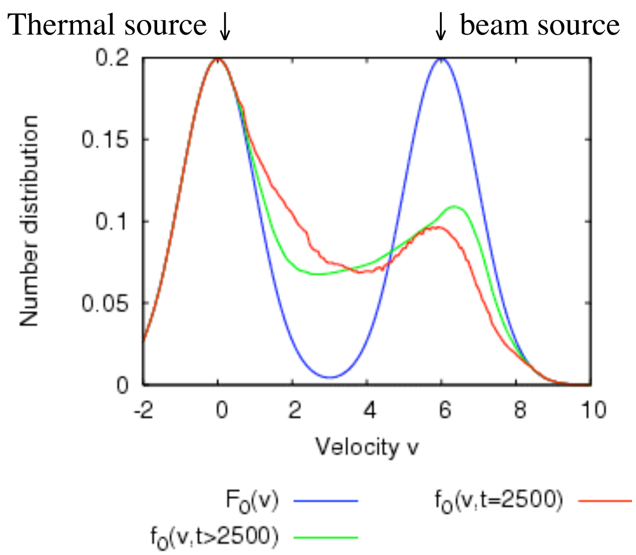
$$\frac{\gamma_L(linear\ theory)}{\omega} = 0.040 \text{ (from slowing downdistribution)}$$

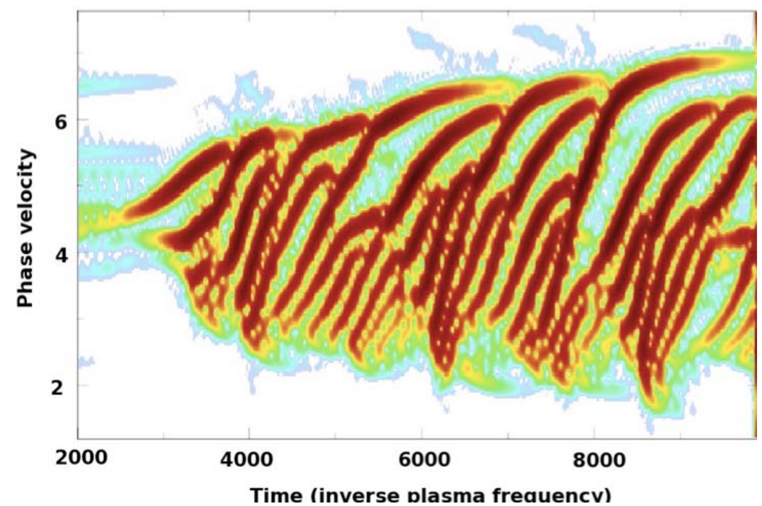
$$\frac{\gamma_L(sweep)}{\omega} = 0.053$$

(using σ=1.6 found from slowing down distribution)

agreement fair, many uncertainties

Benign amelioration of instability drive from frequency sweeping

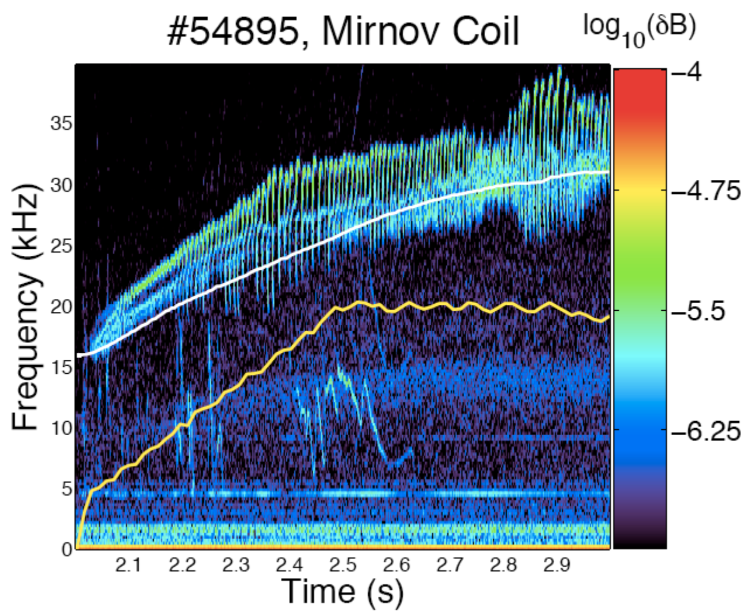


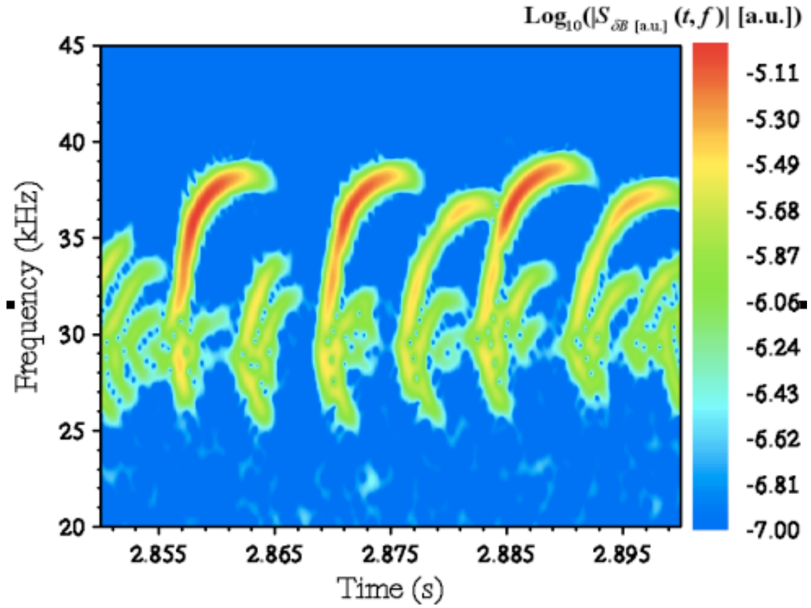


Numerical simulation of double humped distribution, with source, sink, and background plasma dissipation. System evolves to a marginal stable state, maintained by continual chirping. Power being transferred independent of stored energetic particle distribution even though stored energetic particle energy considerably lower due to frequency sweeping (compare green curve to blue curve)

This is just how we would like to deal with alpha particles, i.e. keep stored alpha particle energy low without affecting power to plasma [effectively, energy channeling (Fisch)]

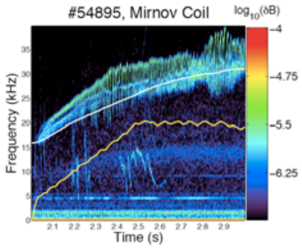
Frequency Chirping of n=0 mode in JET



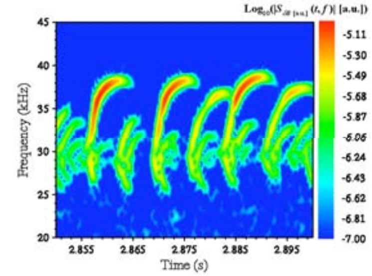


Chirp of n=0 mode in JET (first found by R. Heeter), is due to excitation of geodesic acoustic mode(note, frequency tracks $T_e^{1/2}$ white curve with high field side ICRF power (brown curve) being applied. GAM frequency, $\omega_{GAM} = (T_e / MR)^{1/2}(2 + 1/q^2)^{1/2} \quad (T_e >> T_i)$ matches bounce frequency of ~250kev trapped protons, which have inverted energy distribution. n = 0 mode causes diffusion of energy, but does not cause particle loss, since angular momentum is conserved as well as magnetic moment ($\omega << \omega_{ci}$)

Can chirping GAM be 'magic bullet' to ameliorate alpha particle build-up in burning plasma without loss of power transfer to plasma (a mechanism for alpha channeling)?



Energy channeling



- Linear GAM theory presently not fully understood, but chirping theory and observation allows estimate of channeled power
- observe sweep of 5kHz in 1 ms leads to $\gamma_L \cong .09\omega$ and $\omega_b \approx 0.5\gamma_L$
- Internal mode amplitude then obtained from trapping formula for electrostatic mode: $\Rightarrow e\phi_0 \approx \left(\gamma_L / \omega\right)^2 \mu B r / R$
- Sweeping signal continuously present but with little overlap
- We take $P_{\mathrm{chnnl}} = 2\gamma_L \; W_{md}$ (as $\gamma_d \approx \gamma_L$) with mode wave energy W_{md} temporally steady
- Then,with W_{md} expressed in terms of electrostatic potential $\Rightarrow$

$$W_{md} = 2 n_p m_p \frac{c^2 |< \nabla\phi |^2 >}{2B^2} V_{md} \approx n_p m_p \frac{c^2 \phi_0^2}{B^2 \Delta_m^2} V_{md}$$

- With these elements we obtain estimate of channeled power transfer

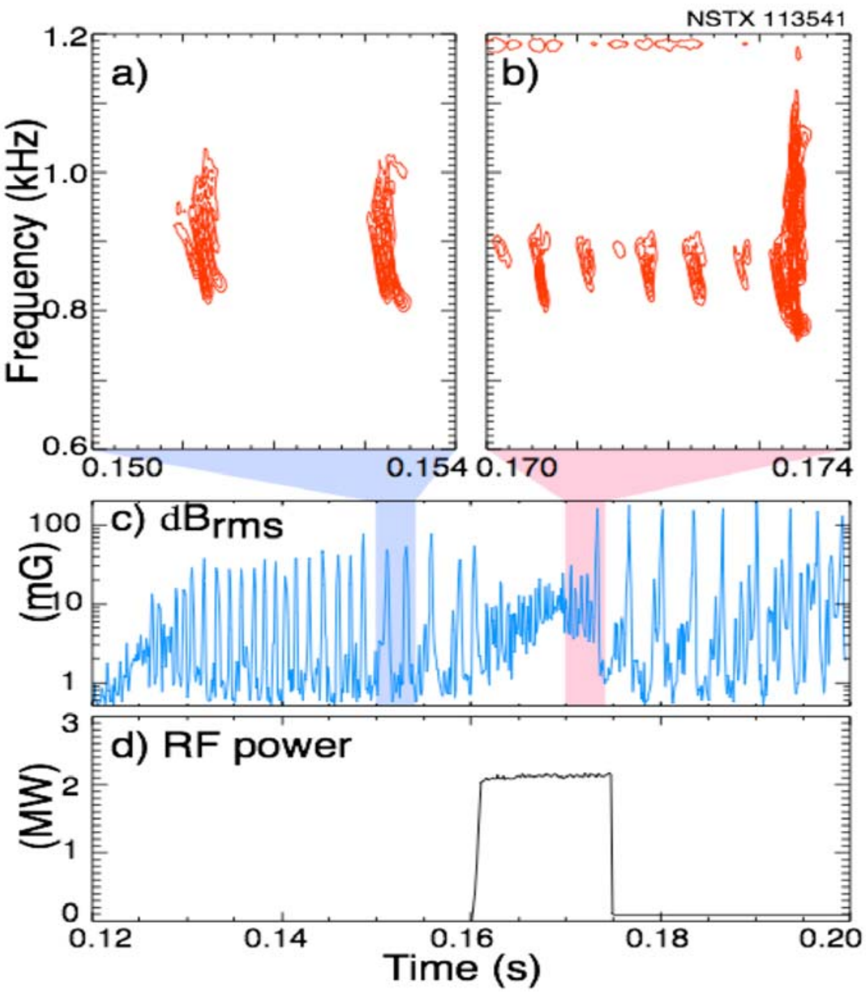
Channeled Power Transfer, P_{chnnl}

$$P_{chnnl} = 0.75\,MW\,\frac{a_r^2}{r_m\Delta_m}\,\frac{2.5r_m}{a}\,\frac{3.5a}{R}\,\frac{n}{1.7\times10^{19}\,m^{-3}}\left(\frac{R}{3(m)}\right)^3$$

- This result is disappointingly low. In JET most of above factors about unity, except $a_r^2 / r_m\Delta_m \approx 0.01$, where $a_r \equiv$ Larmor radius of resonant particle, $\Delta_m \equiv$ mode width for mode width ~ banana width of resonant particle
- Applied ICRF power - 5 Mw. Thus channeled power fraction appears low. This estimate gives ~ 0.15% of ICRF heating channeled to plasma heating by the chirping waves.
- Theory needs confirmation by more direct observations to avoid extrapolations

Effect of Stochasticity

- Frequency chirping can be altered by effects of orbits stochasticity
- Terella dipole experiment exhibited termination of frequency sweeping
- Attempt to replicate this experiment for observed sweeping in NSTX of fishbone, TAE and CAE modes using high harmonic fast wave heating (HHFW)
- results often confusing with many issues still in need of resolution
  a. fishbone chirp never effected by rf
  b. TAE sometimes stabilized or not effected
  c. CAE often gives expected effect as chirping range reduced as seen in accompanying figures



Frequent negative results may indicate that:

- Insufficient rf power applied (estimated that applied power at borderline of needed power)
- Basic equilibrium configuration is being altered by rf
- Alternate sweeping mechanisms may explain chirping (e.g. Zonca et. al.)

Summary

1. Theoretical model has been developed that describes chirping as due to formation of long lived coherent phase space structures, that is applicable to nearly any wave a plasma can support
2. Mode chirping near onset of instability seen in many experiments for many different waves can be an important diagnostic tool
3. Application of this model to NSTX high frequency Alfven wave gives growth rates compatible with what is inferred from direct theoretical calculation
4. Significant energy channeling exhibited in toy simulation model
5. However, our calculations indicate that the chirping $n = 0$ geodesic acoustic mode in JET, does not lead to significant energy channeling
6. Introduction of stochasticity with HHFW heating produces some success but there are also many unresolved issues