Progress in Understanding Multi-Scale Dynamics of Drift Wave Turbulence

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See Posters:TH/2-4, TH/P-19 (B. Coppi)

•Recent Progress on: (see cited works in paper)

1. Coupling of reconnection and resonant-q to inverse cascade in DWT
2. Theory of turbulence spreading
   - spatial transport ↔ spectral transfer
   - ballistic spreading ↔ fast transients
3. Non-diffusive transport of toroidal momentum
   - $k_{||}$ symmetry breaking by v'_E
   - diffusion, 'pinch', torque contributions to flux
4. L-H transition
   - exact analytical solution of minimal model
   - implications for pedestal width

time limitation ⇒ single focus

•Electrostatic Convective Cells, Low-q Resonances and ITBs

- motivation
  - experimental and simulation results
  - multi-scale theory

- requirements of the theory ⇒ electrostatic convective cells

- models
  - secondary vortex cell
  - strong harmonic coupling

  } localized fluctuation profile inhomogeneity

- Scenario: ITB formation

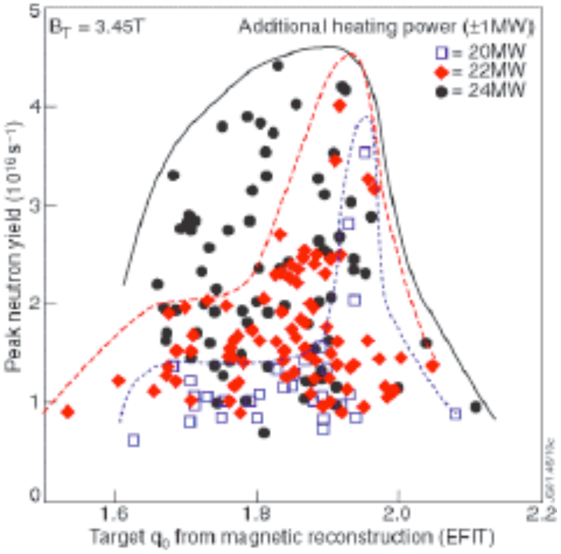
Ackn: K. Burrell, B. A. Carreras, X. Garbet, T.S. Hahm, F. Hinton

•ITB at low-q resonance widely observed, especially in off-axis minimum q (OAM-q) discharges

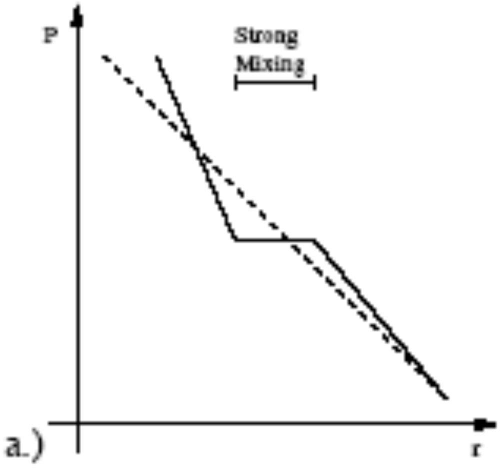
→ message for MFE:
  - q profile improves performance
  - power ↔ q profile trade-offs possible

→ many candidate mechanisms:
  - magnetic islands, localized topology changes
  - energetic particles ↔ electric fields
  - rarefaction of rational surfaces
  - 'zonal flows' ↔ corrugations

→ popular conceptual paradigm:
  - local flattening + adjacent steepening



(Joffrin `03)



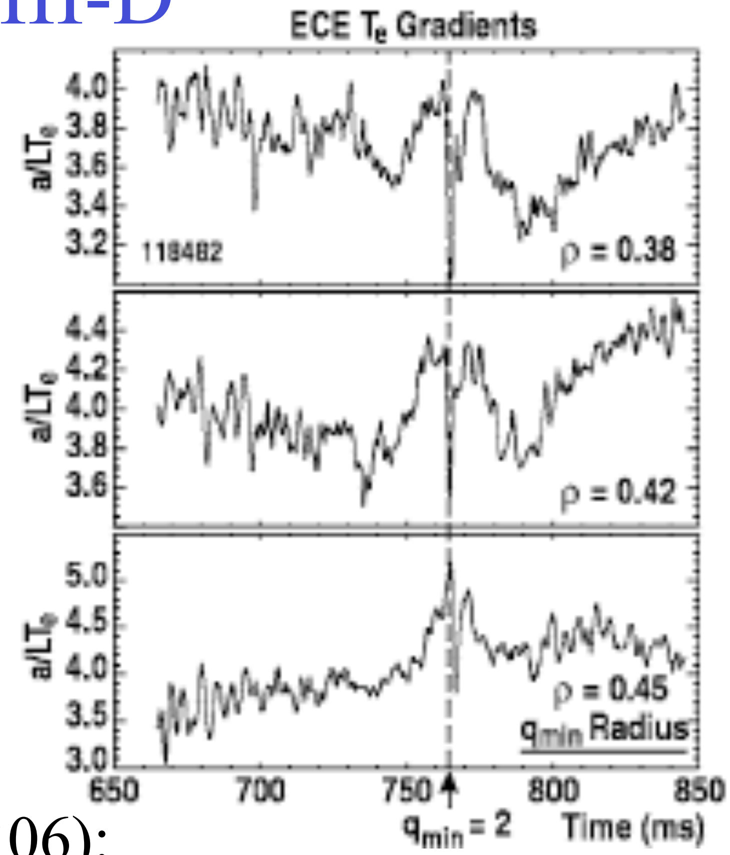
•Recent Experiments on DIII-D

(Austin, Burrell, et. al.)

→no magnetic signal detected at ITB formation

→∇T_e variation:

  – steepens before q_{min}=2
  – *flattens* at q_{min}=2
  – steepens after q_{min}=2



(Austin, et. al. `06)

→GYRO simulations (Waltz, Candy `06):

  –exhibit profile ‘corrugations’ at q-resonance
  –indicate “zonal flows” correlated with corrugation structure
  –*suggest* zonal flows as ITB trigger

→sufficient $\langle v_E \rangle'$ *required* for ITB formation

•Critical Issues

→*The* question:
Why are shear flows linked to resonant-q surfaces?

→ Answer must address coexistence of:

1) region of profile flattening *at* resonant surface
   - ⇒ region of localized mixing, transport
2) barrier formation *nearby* resonance
   - ⇒ neighboring region of strong shear flow

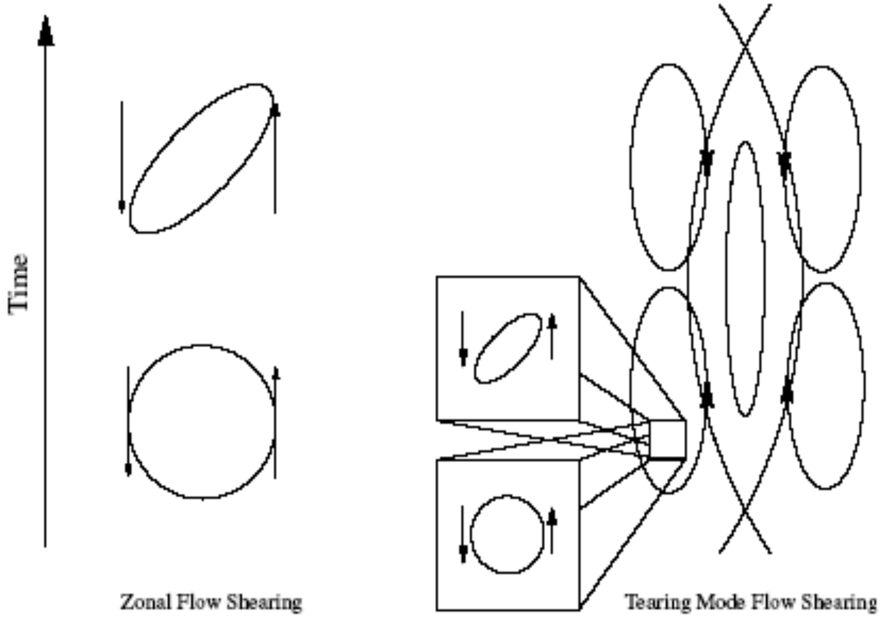
→ Broader theoretical theme: “Multi-scale Problem”

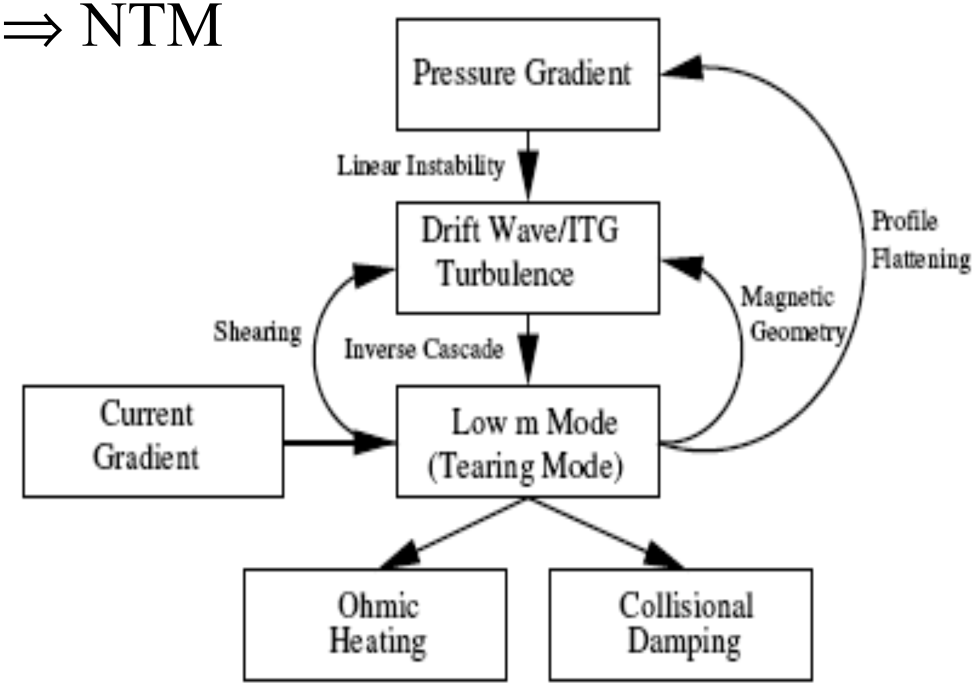
‘inverse cascade’ in
drift wave turbulence ⟷ resonant q

•Multi-Scale Problem: General Structure

→ Interaction self-consistent description of large scale ↔ small scales

$$\text{i.e.} \begin{cases} \text{Zonal flow} + \text{DWT} \Rightarrow \text{confinement} \\ \text{Tearing mode} + \text{DWT} \Rightarrow \text{NTM} \end{cases}$$





→Key Physics (McDevitt, Diamond `06):

→Tearing couples to generic *inverse cascade* in DWT via ions, as well as turbulent dissipation, transport via electrons

→Multi-scale interaction ↔ *'negative viscosity'* phenomena

•Multi-Scale Problem: Limiting Case

→ natural question: What happens to tearing + DWT problem in limit $\Delta' << 0$?

→ equivalent: What type of structure can form at resonant surface in MHD-stable limit?

→ Answer, for simple model:

Secondary vortex cell!

- electrostatic convective cell (Dawson, Sagdeev)
- finite-m analogue of zonal flow
- driven by modulational instability
- seems relevant to ITB in OAM-q ...

•Critical Issues, revisited

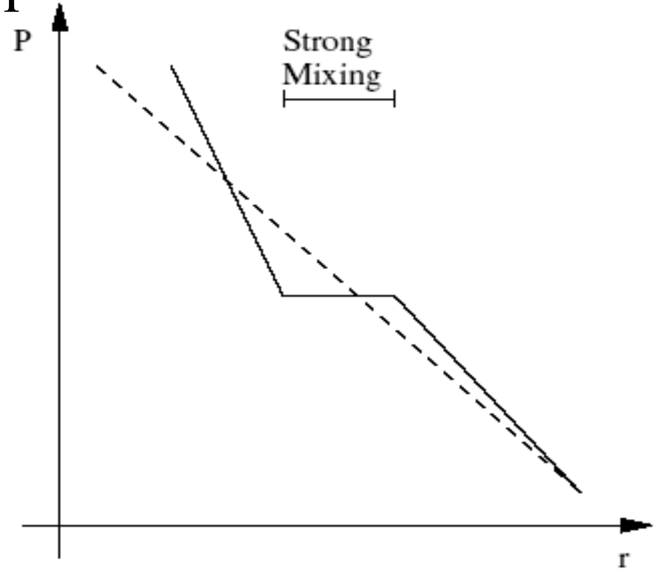
→ Q: How to link shear flows and resonant-q?

A: Via electrostatic convective cells!

→ Possible Mechanisms:

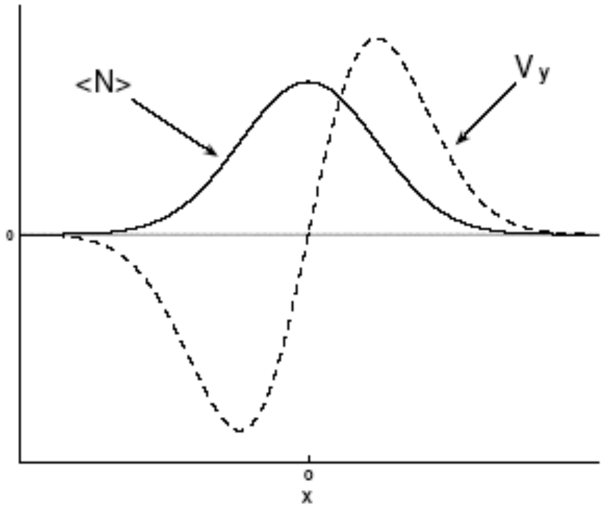
a) secondary vortex cell (McD & D, `06)

- radial scale ↔ regulated by magnetic shear
- $v_r \neq 0$ → profiles mixed near resonance
- turbulence sheared by flow
- especially relevant → *weak magnetic shear*



b) strong harmonic coupling (Carreras, Diamond et. al. `92)

- nonlinear interaction of many co-located harmonics at low-q surface
- local peak in turbulence intensity, transport
- dipolar shear layer
- relevant in weak *and* normal magnetic shear



•Secondary Vortex Cell - Theory I

→ 'minimal model'

– large scales:

$$\frac{\partial}{\partial t}\psi = v_A\frac{\partial}{\partial z}\phi + \eta\nabla_\perp^2\psi$$

Reynolds stress

$$\left(\frac{\partial}{\partial t} + \gamma_d\right)\nabla_\perp^2\phi = v_A\frac{\partial}{\partial z}\nabla_\perp^2\psi + \nu_c\nabla_\perp^2\nabla_\perp^2\phi - \frac{c}{B_0}\left\langle(\hat{\mathbf{z}}\times\nabla\tilde{\phi})\cdot\nabla\nabla_\perp^2\tilde{\phi}\right\rangle$$

– for electrostatic DWT:

$$\frac{\partial\langle N\rangle}{\partial t} = \frac{\partial}{\partial k_x}\left(D_k\frac{\partial\langle N\rangle}{\partial k_x}\right) + \frac{\partial}{\partial x}\left(D_x\frac{\partial\langle N\rangle}{\partial x}\right) + \gamma_k\langle N\rangle - \Delta\omega_k\langle N\rangle^2$$

$$D_k = k_y^2\sum_q R(k,q)\,q_x^4\,|\phi_q|^2 \qquad D_x = \sum_q R(k,q)\,q_y^2\,|\phi_q|^2$$

→Key effects:

$$\left.\begin{array}{l}\text{–modulation}\\ \text{shearing}\end{array}\right\}\ \text{ala' zonal flow feedback, with } m\neq 0$$

– localization by magnetic shear

•Secondary Vortex Cell - Theory II

→ two scale analysis + wave kinetics for DWT ⇒

$$\frac{v_A^2 q_y^2}{\eta L_s^2}\frac{d^2\phi_q}{dq_x^2} = \left[\left(\nu_c + \nu_T\left(q_x\right)\right)q_x^2 + \gamma_d + \frac{\partial}{\partial t}\right] q_x^2\phi_q$$

$$\nu_T = c_s^2 \sum_k R\left(k,q\right)\frac{\rho_s^2 k_y^2}{\left(1+\rho_s^2 k_\perp^2\right)^2} k_x \frac{\partial\left\langle N\right\rangle}{\partial k_x}\,; \quad R\left(\mathbf{k},\mathbf{q}\right) = \frac{\gamma_k}{\left(\Omega - \mathbf{v}_{gr}\cdot\mathbf{q}\right)^2 + \gamma_k^2}$$

→elements:

–drive → ‘negative viscosity’ from modulational instability

$$\nu_T\left(q_x\right) < 0 \qquad \text{for} \qquad \partial\left\langle N\right\rangle/\partial k_x < 0$$

→ $\nu_T\left(q_x\right)$ decreases for large q_x

–damping → friction, collisional viscosity

–localization → field line bending ↔ magnetic shear, m-dependence

⇒profile *‘corrugation’, flat spots wider for weak shear*

⇒consistent with GYRO simulations

•Secondary Vortex Cell - Theory III

→ 'predator-prey' structure, ala' Reynolds stress driven shear flow

→ Eigenvalue ⇒ fluctuation intensity threshold to excite cell

$$N \approx \Gamma + \left(\frac{3\pi}{2}\frac{1}{1-\epsilon}\right)^{2/3} \frac{\nu_c^{2/3}}{\eta^{1/3}} \left(\frac{v_A q_y}{L_s}\right)^{2/3} \frac{1}{\gamma_k}$$

frictional damping

shear and viscous damping

$$\epsilon \equiv \frac{3}{4}\alpha\left(\ln\left(\frac{16}{\alpha}\right) - \frac{N-2\Gamma}{N-\Gamma}\right), \quad \alpha \equiv \frac{1}{4}\frac{N}{(N-\Gamma)^2}\hat{\nu}_c$$

→Dependencies of critical intensity:

- –decreases for weaker shear → OAM-q?!
- –increases with ν_c → synergy with magnetic shear
  i.e. field line bending localizes cell ⇒
  thinner cell sensitive to viscosity
- –increases with m → low order rationals preferred

•Secondary Vortex Cell - Theory IV

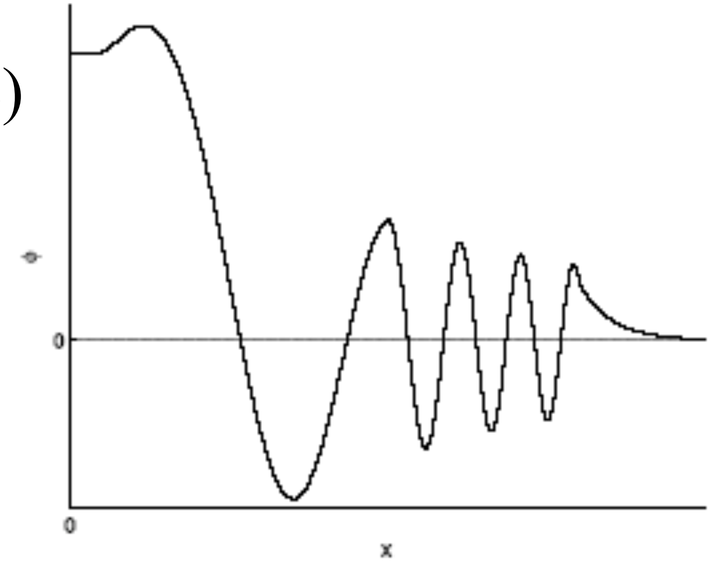
→Cell structure:

– potential profile (asymptotic)

$$\phi(x) \sim \frac{1}{x^{3/4}} \exp\left(i\frac{2}{3}\left(\frac{x}{\Delta x}\right)^{3/2} \right)$$

–radial cell scale

$$\Delta x \equiv \left(|\nu_T(0)|\,\eta\right)^{1/6} \left(L_s/\left(v_A q_y\right)\right)^{1/3}$$



–cell flow shear profile

$\left|v_y'\right| \sim x^{1/4}$ ⇒ flow shear stronger off resonance

⇒transport suppression more effective *off* resonant surface

∴ secondary vortex satisfies dual requirements of:

–profile flattening *at* resonance

–shearing *near* resonance

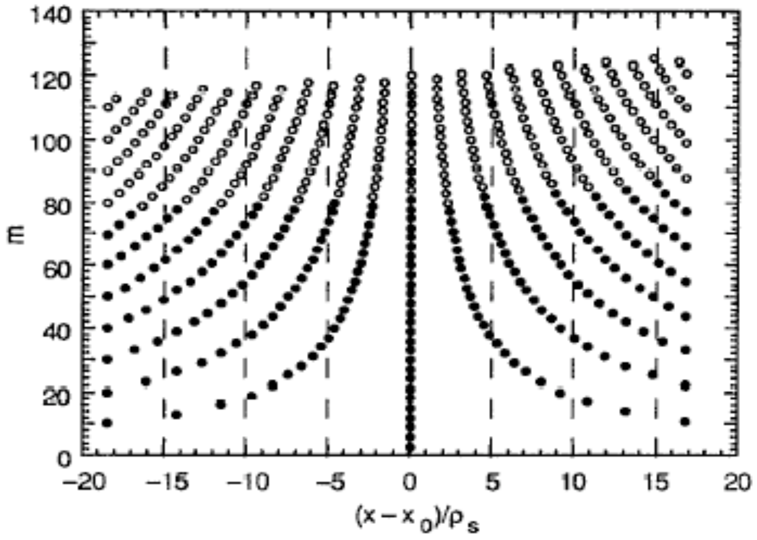
•Resonant Harmonic Coupling - Theory I

→ Low-q resonance ⇒ many co-located resonant harmonics

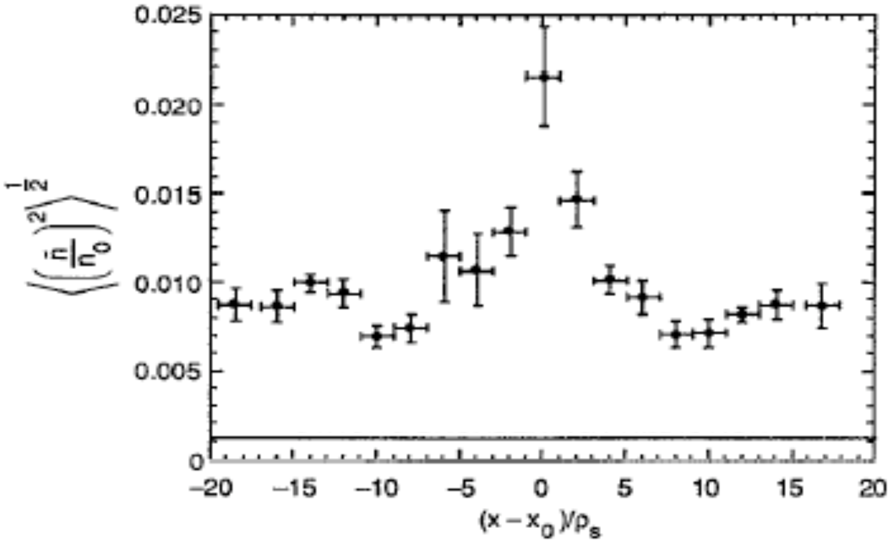
→ Implications (Carreras et. al. `92)

  - local maximum in fluctuation intensity
  - single helicity interaction stronger than usual multiple helicity, “turbulent” couplings

⇒ *intensity profile fine structure develops*



Resonant mode distribution



Intensity Profile peaked on resonance

•Resonant Harmonic Coupling - Theory II

→ strong single helicity interaction at resonant-q modifies coupling to dissipation damping

⇒novel structure, dynamics!?

⇒unexpected effects of shearing

→ effect occurs *both* for normal and weak magnetic shear

→ Key question: How ‘low’ is ‘low-q’?

- compare SH and MH interaction ⇒
- criterion:

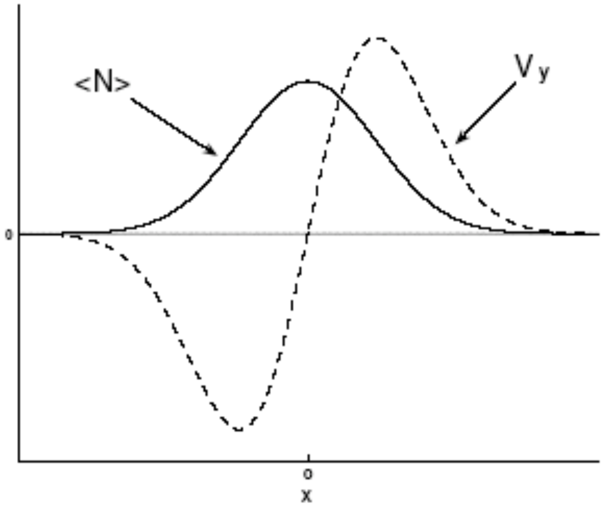
$$m < m_{crit}\left(L_s/L_n, \rho/a, \bar{k}_\theta \rho\right)$$

•Resonant Harmonic Coupling - Theory III

→previous analysis did *not* address Reynolds stress-driven flow shear

→flow ~ intensity profile gradient

$$\frac{\partial \langle v_\theta \rangle}{\partial t} \sim -\frac{\partial}{\partial r} \langle \tilde{v}_r \tilde{v}_\theta \rangle + \cdots$$



⇒ideal for dipolar shear layer *near* q_{res}

→extension to include self-generated shear flow straightforward ala' Diamond, et. al. `94

→Key result: fluctuation intensity required to drive flow

$$N_{\text{crit}} \approx \left(\frac{1}{\rho_s^2 \alpha_h}\right)\left(\frac{1}{\alpha_h L_s^2}\right)\left(\frac{\gamma_d}{\nu}\right)$$

•ITB Formation I

→ sheared flow of electrostatic convective cell as ‘*trigger*’ for ITB formation

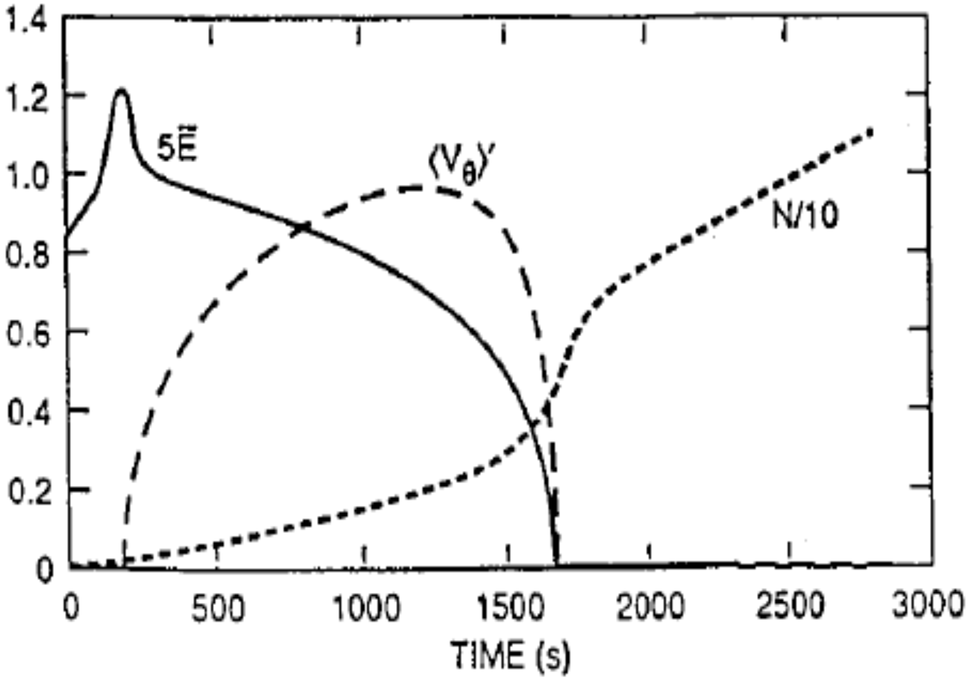
→ two stage process: (Diamond, et. al. `94)

1. $\langle \tilde{v}_r \tilde{v}_\theta \rangle$ drives flow ⇒

$$\begin{cases} \nabla P, \nabla v_\phi \text{ steepen} \\ \text{fluctuations reduced} \end{cases}$$

2. fluctuations quenched ⇒

$$\begin{cases} \text{flow damps away} \\ \langle v_E \rangle' \text{ from } \nabla P, \nabla v_\phi \end{cases}$$



→ end state is ITB

•ITB Formation II

→ *trigger threshold*: fluctuation intensity to drive flow against damping

→ power balance and convert transport model to power threshold, i.e.

$$P_{\rm in} \sim R r_b v_{th} T_i \eta_i \epsilon_T^{-1/2} \tau^2 N_{crit}\,, \text{ where}$$

$$\left.\begin{array}{ll} N_{\rm crit} \approx \Gamma + \left(\dfrac{\frac{3\pi}{2}+\delta}{1-\epsilon}\right)^{2/3} \dfrac{\nu_c^{2/3}}{\eta^{1/3}} \left(\dfrac{v_A q_y}{L_s}\right)^{2/3} \dfrac{1}{\gamma_k} & \begin{array}{c}\text{vortex}\\ \text{cell}\end{array} \\ N_{\rm crit} \approx \left(\dfrac{1}{\rho_s^2 \alpha_h}\right) \left(\dfrac{1}{\alpha_h L_s^2}\right) \left(\dfrac{\gamma_d}{\nu}\right) & \begin{array}{l}\text{harmonic}\\ \text{coupling}\end{array} \end{array}\right\} \begin{array}{l}\text{detailed}\\ \text{quantitative}\\ \text{study}\\ \text{needed to}\\ \text{discriminate}\end{array}$$

→ Important: Flow damping, not γ_L vs $\gamma_{E\times B}$, sets threshold for trigger

→ Experiments ⇒ Proximity to “usual” transition threshold required

•Conclusions

→ electrostatic convective cells viable as trigger for OAM-q ITB transition

→ cell paradigm physics satisfies requirements:

  i. flattening/“corrugation” *at* low-q resonance

  ii. sheared flow *near* low-q resonance

→ theory of candidate mechanisms examined:

  – secondary vortex cell

  – resonant harmonic interaction

  – NOT MUTUALLY EXCLUSIVE

→ *detailed radial structure of intensity profile of DWT* is critical element of underlying physics

•Future Work

→ Simple:

- improve model → structure of collisionless electrostatic cell?
  → Landau damping, etc. → radial scale?
- predict when: transient confinement improvement *or* ITB transition

  → flow threshold vs. ∇P transport bifurcation threshold?

→ Not-so-simple:

- improve theoretical understanding of intensity profiles, as in theory of turbulence spreading (Hahm; this session)
  - → extend to encompass q-resonance distribution!?
- explore broader implications of electrostatic cells and q(r) structure

  i.e. origin of observed “choppy profile”? etc.