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Linear and Nonlinear Aspects of Edge Turbulence and Transport in Tokamaks

R. Singh[1], N. Bisai[1], P. K. Kaw [1],

M. Z. Tokar[2], D. Kalupin[2] and R. Wolf[2]

[1]Institute for Plasma Research, Bhat, Gandhinager-382 428, India

[2]Institut für Plasmaphysik, Forschungszentrum Jülich, Juelich, D-52425, Germany

Contents: *Divided into four parts*

- **Stability of ideal ballooning-Kink (peeling) modes:**
  1. Kink effect reduces the β threshold of ballooning mode.
  2. Near $\beta \sim \beta_c$, 70% of DIIID input power carried by Dissipative modes.

- **Suppression of MHD modes through nonlinear coupling with external field perturbations:**
  1. External field perturbations increase the threshold of BM.
  2. This effect is much more important for ITER conditions than smaller devices.

- **Secondary instabilities of large scale zonal fields and flows in the background of BM turbulence:**
  1. Saturation level of BM in coupled ZF-BM system
  2. Dynamics of zonal flows in relaxation of ELMs is unimportant when $\beta > \beta_c$.

- **Linear theory of non-ideal curvature driven modes:**
  1. For $\beta < \beta_c$, a robust non-ideal curvature driven instability persists due to electron inertia and trapped electrons effects.

Ackn for informative diss.: P. H. Diamond, X. Garbet

1. Linear /Nonlinear Stability of Ballooning- Kink (Peeling) Modes

- **Dispersion relation of BMs with kink and weak collisional effects:**

$$\hat{\omega}(\hat{\omega}+\tau\alpha_i\hat{\omega}_*)+(\alpha_e+\tau\alpha_i)\frac{\beta_e q^2 R}{L_n}$$
$$+\left(\frac{3qR<J_{//0}>^{edge}}{2L_T e n_0 c_A}\frac{\Omega_i}{k_\perp c_A}-1\right)\left[1-i\left(\frac{0.51\nu_e}{\Omega_e}\right)\frac{qRk_\perp^2 c_A}{\Omega_i(\hat{\omega}-\alpha_e\hat{\omega}_*)}\right]=0$$

$\hat{\omega} = \omega\, qR / c_A$, $\hat{\omega}_* = qRk_y\rho_s c_s / L_n c_A$.

- $E_{//} = \eta_{//} <J_{//0}>^{edge}$ ➡ $\partial_r <J_{//0}>^{edge} \sim 3 <J_{//0}>^{edge} / 2L_{Te}$

- **Marginally stable BM with "ω_{*i}" yields:** $k_\perp \delta_R \approx 1$, $\delta_R = [R\rho_s^2/4]^{1/3}$

(Rogers-Drake, PoP-99).

- **Bootstrap current:**

$$J_{//0}^{edg} \sim J_{BS} \approx 2.44(1+\tau_i) e n_0 c_s \varepsilon^{1/2} (\rho_s B / B_\theta L_n)$$

- **Linear growth Ballooning-Kink:**

$$\gamma_k = (c_A / qR)[q^2 R \,|\, d\beta / dr \,|\, (1 + 1.22 \delta_R / L_n \varepsilon^{1/2}) - 1]^{1/2}$$

- **The stability condition with kink effects:**

$$[q^2 R |d\beta/dr \;+\; (1.22/k_\perp L_n \varepsilon^{1/2})(q^2 R|d\beta/dr|)] \;<\; 1$$

- **Peeling term reduces the β threshold and makes the ballooning mode more virulent.**

- **Peeling effect could be significant with that from ballooning one, for:**

$$R/r \approx 3\text{–}4, \quad L_n \approx 2-3\, cm, \quad \rho_s \approx (0.1-0.2)\, cm$$

• Dissipative mode near marginal point

$$\beta \approx \beta_c \qquad 0.51\nu_e qRk_\perp^2 c_A / \Omega_e \Omega_i (\hat{\omega} - \alpha_e \hat{\omega}_*) < 1$$

$$\boxed{\gamma_k \approx [(2\alpha_e + \tau_i \alpha_i)\frac{c_s^2}{L_n R}]^{1/3} \nu_e^{1/3} (\frac{k_\perp c}{\omega_{pe}})^{2/3}}$$

- **For pedestal of DIII-D:** $B_0 = 1.6T$, $I_p = 1.1MA$, $R_0 = 1.72m$, $a = 0.585m$, $\varepsilon = a/R_0 = 0.34$, $\kappa = 1.8$. **Other parameters in the pedestal:** $T_e \approx 250eV$, $n_0 \approx 1\times10^{19} m^{-3}$, **neutral beam power =** $5.1MW$, $\nu_{ei} \approx 1.09\times10^{5} \sec^{-1}$, $q_{95} = 3.7$, $L_n \sim 0.028\,m$, $L_T \sim 0.02m$, **and** $Z_{effie} = 3.0$, **and total power** $Q \approx 6.0MW$ **(assuming loop voltage** $\sim 0.9Volt$**).**

- **Dissipative instability driven thermal transport:**

  1. **Mixing Length:** $\chi_{i\perp}^{A} \approx \gamma_k / k_\perp^2 = 1.86\, m^2/\sec$.

  2. **Simple power balance without radiation losses:**

  $Q = 2 n_0 \chi_{i\perp}^{A} |\partial T_i / \partial r| S \;\approx\; 4.2MW$ **~** 70% **of input power in DIIID.**

2. Suppression of MHD modes through nonlinear coupling with external field perturbations

- A standard technique of parametric instability in four-wave interaction process with low frequency pump - **Kaw *et al* JGR (1976).**

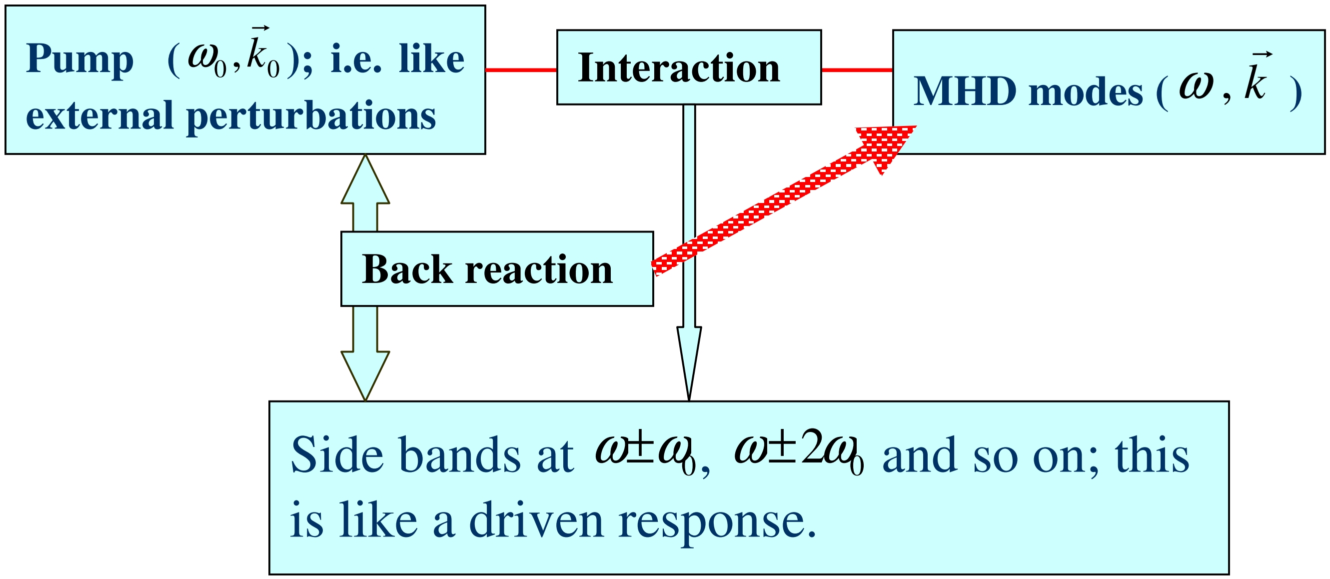
- The external magnetic field perturbations are represented as oscillations of the vector potential

$$\tilde{A}_{0//} = A_0(r)\exp(-i\omega_0 t + ik_0 y)$$

- Amplitude $\tilde{A}_0$ much larger than that of MHD fluctuations

- Neglect the potential and pressure perturbations on the short scale of pump wave ($\omega_0, \vec{k}_0$)

- **Physics:**



- **Taking nonlinear terms up to order $|A_0|^2$, then the dispersion relation of Ballooning – Kink modes:**

$$\varepsilon_k \approx \frac{\beta_e}{2} k_\perp^2 [(\hat{z}\times\vec{k})\cdot\vec{k}_0]^2 \left(1 - \frac{i2k_\perp^2}{\beta_e \hat{\chi}_e \omega}\right) |A_0|^2 \left[\left(\frac{\omega^2}{\omega^2 - \omega_0^2}\right) + \frac{2}{\beta_e} k_{//}^2 k_\perp^2 \left(1 - \frac{k_{0\perp}^2}{k_\perp^2}\right) \left\{ \left(\frac{2}{\varepsilon_1} + \frac{2}{\varepsilon_2}\right) + \left(\frac{k_{0//}}{k_{//}} + \frac{\omega_0}{\omega}\right)\left(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2}\right) \right\} \right]$$

- For $\beta_e q_a^2 R / L_n >> (L_n / R)(\rho_s / \delta_R)^2$ that's relevant for the H-mode pedestal region and $\omega_0 < \omega$, $k_0 < k_\perp$.

- **Growth rate:**

$$\gamma_k = (c_A / qR)[q^2 R \,|\, d\beta / dr \,|\, (1 + 1.22\delta_R / L_n \varepsilon^{1/2}) - 1 - (qR / \delta_R)^2 (\delta B_r / B)^2]^{1/2}$$

- **Reduction in growth can be viewed as additional field line bending effects arising via nonlinear coupling of external, low m, perturbations with MHD modes.**

- **This effect is similar to the magnetic Reynolds stresses effect in stabilizing the streamers and zonal flows in drift wave turbulence through random magnetic shearing.**

- **Stabilization due to external perturbations are important, if**

$$|\delta B_r / B| = b_{cr} \geq \delta_R / qR \quad \alpha \quad (RB)^{-2/3}$$

- **For DIII-D conditions** $B=1.6T$**,** $R=1.72m$**,** $q_{95}=3.7$**,** $T_{ped}=250eV$**, the critical level of** b_{cr} **is:** $b_{cr} \geq 0.1\%$

- **Note the considered effect works synergistically with the increase of transport by large** m **external magnetic field perturbations between ELMs.**

4. Secondary instabilities of large scale magnetic fields in the background of short scales ideal ballooning mode turbulence

- **A simple self-consistent theoretical model of multi- scale interaction of ELMs governed by ballooning mode with zonal magnetic fields and zonal flows.**
- **We estimate the nonlinear saturation level of MHD mode (ballooning).**

a. Large scale zonal field instability

- **There is a sufficient non-axisymmetric in the spectrum, which separates long scale zonal magnetic fields $(\Omega, q_\perp)$ and small scale MHD mode $(\omega, k_\perp)$.**
- **Equations for slow zonal waves are averaged over fast time and space scales,**

- **Vorticity equation:**

$$\partial_t \nabla_\perp^{\ 2} \hat{\phi}_q = -[(\partial_{yy} - \partial_{xx}) < \partial_x \hat{\phi}_k \partial_y \hat{\phi}_k > - \partial_{xy} < \partial_y \hat{\phi}_k)^2 - (\partial_x \hat{\phi}_k)^2 >] + (\beta_e / 2)[(\partial_{yy} - \partial_{xx}) < \partial_x \hat{A}_{\parallel k} \partial_y \hat{A}_{\parallel k} > - \partial_{xy} < (\partial_y \hat{A}_{\parallel k})^2 - (\partial_x \hat{A}_{\parallel k})^2 >]$$

- **Parallel electron momentum equation:**

$$(1 - (\lambda_s^2 / \rho_s^2) \nabla_\perp^2) \partial_t \hat{A}_{\parallel q} - \hat{\eta} \nabla_\perp^{\ 2} \hat{A}_{\parallel q} + (2/\beta_e) \nabla_\parallel \hat{\phi}_q = \partial_x < \partial_y \hat{\phi}_k \hat{A}_{\parallel k} > - \partial_y < \partial_x \hat{\phi}_k \hat{A}_{\parallel k} > - (\lambda_s^2 / \rho_s^2) \partial_x < \partial_y \hat{\phi}_k \nabla_\perp^2 \hat{A}_{\parallel k} > + (\lambda_s^2 / \rho_s^2) \partial_y < \partial_x \hat{\phi}_k \nabla_\perp^2 \hat{A}_{\parallel k} >$$

- **Further simplified: (a) For zonal fields** $q_x c / \omega_{pe} < 1$ **(b) via quasilinear relation between** $\hat{\phi}_k$ **and** $\hat{A}_{\parallel k}$ **:**

$$\hat{A}_{\parallel k} \approx 2 \hat{k}_\parallel (\hat{\omega}_{rk} - i \gamma_k) / \beta_e \, | \hat{\omega}_k |^2 \, \hat{\phi}_k .$$

- **Equations of zonal flows, $\hat{\phi}_q$ and zonal field, $\hat{A}_{\parallel k}$:**

$$\partial_t \hat{\phi}_q = \sum (1 - 2\hat{k}_{\parallel}^2 / \beta_e |\omega_k|^2) \hat{k}_x \hat{k}_y |\hat{\phi}_k|^2$$

$$\partial_t \hat{A}_{\parallel q} + \hat{\eta} \hat{q}_x^2 \hat{A}_{\parallel q} = -i \hat{q}_x \sum (2\hat{\gamma}_k \hat{k}_{\parallel} \hat{k}_y / \beta_e |\omega_k|^2) |\hat{\phi}_k|^2$$

- **Response of zonal fields and zonal flows on turbulence can be calculated from standard wave kinetic equation**

$$\partial_t N_k + \partial_{\vec{k}} \omega_{nl} \cdot \partial_{\vec{x}} N_k - \partial_{\vec{x}} \omega_{nl} \cdot \partial_{\vec{k}} N_k = 2\gamma_{nl} N_k - \Delta\omega_k N_k^2$$

- **In limit $q_x < k_\perp$, the action density (N_k) of background turbulence:** $N_k = E_k / \omega_k$, E_k **Energy of the underlying turbulence.**

$$N_k = \Lambda_k |\hat{\phi}_k|^2,$$

$$\Lambda_k = [k_\perp^2 + (2\hat{k}_\parallel \hat{k}_\perp / \beta_e |\omega_k|)^2 + (\alpha_i \hat{k}_y / \tau_i |\omega_k|)^2 / \alpha_k \hat{k}_y]$$

- **Here, action density N_k, $\hat{\omega}_{rk}$, and $\hat{\gamma}_k$ are varying slowly at zonal wave scales**

$$\boxed{\hat{\omega}_k = \hat{\omega}_{rk}^{(0)} - \hat{k}_y \partial_x \hat{\phi}_q}, \quad \boxed{\hat{\gamma}_k \approx \hat{\gamma}_k^{(0)} + \hat{\gamma}_k^{(1)} - (\hat{k}_y \beta_e / 2)(\partial \hat{\gamma}_k^{(0)} / \partial \hat{k}_{\parallel} + \partial \hat{\gamma}_k^{(1)} / \partial \hat{k}_{\parallel}) \partial_x \hat{A}_{\parallel q}}$$

- **Note ϕ_q, $A_{\parallel q}$ at long scales are generated in MHD turb.**

- **Now, the response from WK equation at slow scale is:**

$$\boxed{\overline{\Omega}_q \delta N_k = -i \hat{q}_x^2 k_y \partial_{k_x} \overline{N}_k \hat{\phi}_q + \beta_e \hat{q}_x \hat{k}_y (\partial \gamma_k^{(0)} / \partial \hat{k}_{\parallel} + \partial \gamma_k^{(1)} / \partial \hat{k}_{\parallel}) \overline{N}_k \hat{A}_{\parallel q}}$$

$$\boxed{\overline{\Omega}_q = \Omega_q - \hat{q}_x V_{gx} + 2i\hat{\gamma}_k}$$

- **Growth rates of zonal fields / Flows:** $\beta_c = \beta q^2 R / L_{pi}$

$$\gamma_q^{\phi} \approx -\nu_i + k_{\parallel} c_A (q_x \rho_s)^2 [1 - \frac{1}{(\beta_c - 1)}] (\frac{k_y L_{pi}}{2})^2 (\frac{R}{L_{pi}}) \frac{\beta_c}{(\beta_c - 1)^{1/2}} |\frac{e\phi}{T_e}|^2 \Theta(\beta_c - 1)$$

Neoclassical damping

$$\gamma_q^{A_{\parallel}} \approx -\frac{\nu_e}{\beta_e}\frac{m_e}{m_i}(q_x\rho_s)^2 + k_{\parallel} C_A (q_x \rho_s)^2 (\frac{k_y L_{pi}}{2})^2 (\frac{R}{L_{pi}}) [\frac{2\beta_c}{(\beta_c - 1)^{3/2}} \Theta(\beta_c - 1) - \frac{\beta_c}{f(\nu_e)}] |\frac{e\phi}{T_e}|^2$$

Resistive damping, $f(\nu_e) \approx [(0.51\nu_e / \Omega_e)(qRk_{\perp}^2 c_A / \Omega_i)]^{1/3}$

- **Important to note – when $\beta_c \geq 1$, ballooning mode is unstable, the magnetic Reynolds stress effects completely suppresses the zonal flow growth.**

- **Dynamics of zonal flows in ELMs relaxation is unimportant when $\beta_c \geq 1$.**

b. Saturation level of ballooning mode

- **For estimation of ballooning mode saturation: "Predator-prey" for $<\Phi_k> = |e\phi_k / T_e|^2$, (Diamond *et al.*, PPCF- 2005).**

$$\partial_t <\Phi_k> = \gamma_k <\Phi_k> - \alpha_0 \hat{A}_{\parallel q} <\Phi_k> - \Delta\omega <\Phi_k>^2$$

$$\partial_t \hat{A}_{\parallel q} = \alpha_0 \hat{A}_{\parallel q} <\Phi_k> - \eta_0 \hat{A}_{\parallel q};$$

Where:

$$\eta_0 = (\nu_e m_e / \beta_e m_i)(q_x \rho_s)^2; \quad \alpha_0 = k_\parallel C_A (q_x \rho_s)^2 (k_y L_{pi})^2 (R/4L_{pi})(2\beta_c / (\beta_c - 1))^{3/2}$$

- **In steady state, balancing the growth of zonal fields and damping due to resistivity sets the turbulent level, and that yields** $<\Phi_k>=\eta_0/\alpha_0$

$$<\Phi_k>=\left(\frac{2}{q\beta_e^2}\right)\left(\frac{\nu_e}{k_\perp^2 C_A R}\frac{m_e}{m_i}\right)(\beta_c-1)^{3/2}$$

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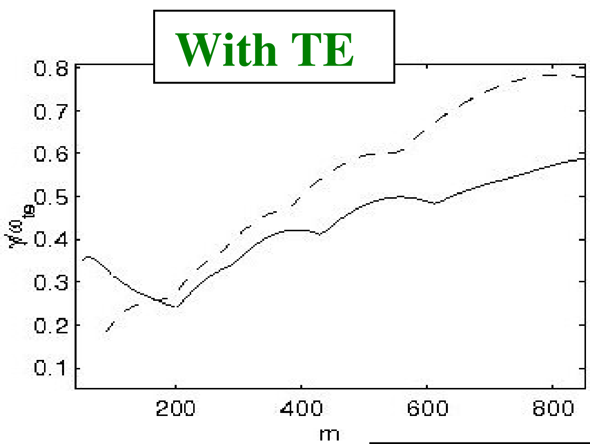
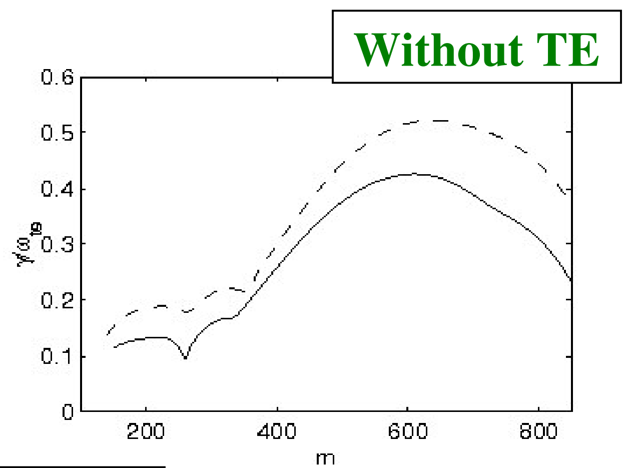
5. Collisionless ballooning instability in tokamak edge

- **Trapped particles affect on non-ideal curvature driven edge instabilities; for** $\beta<\beta_c$**, and** $qR<\lambda_{mfp}$ **case.**
- **Fluid equations for ions, untrapped and trapped electrons**

$$[c(\beta)(\omega+i0.51\nu_{ei})]^{-1}d^2\tilde{\psi}/d\theta^2+[\omega-\omega_c/2]\tilde{\psi}$$
$$+\{(\omega_{*n}-\omega)(1-\sqrt{\varepsilon})/F(\theta)\}[\{(\omega-\omega_c/2+i\nu_{eff}(1+\sqrt{\varepsilon})\}/\{\omega-\omega_c/2+i\nu_{eff}\}]\tilde{\psi}=0$$

- **An Eigen value problem is solved numerically and analytical.**





ES-dashed and EM-solid

Conclusions

- **A detailed understanding of particle and energy transports in the presence of external magnetic perturbations is still an open challenging topic to investigate.**

- **We have proposed a theoretical model for mitigation of ELMs due to stochastic field lines as observed in DIII-D.**

- **Linear analysis shows that the kink effect significantly modifies the threshold of ballooning mode.**

- **Reaction of external magnetic field perturbations can drastically modify the threshold of the ballooning-kink mode if** $|b_{cr}| > \delta_R / qR > 0.1\%$ **in D-IIID.**

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- **Simple self-consistent theoretical model of multi-scale interaction of the ideal ballooning-Kink modes interacting with zonal magnetic fields and zonal flows.**

- **Secondary instability of zonal field is used to estimate saturation level BM. The dynamics of zonal flows in relaxation of BM is unimportant when** $\beta > \beta_c$ **.**

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- **Linear instabilities of non-ideal curvature driven modes, including the effects of trapped electrons, and electron inertia is investigated.**

- **For** $\beta < \beta_c$ **, electron inertia and trapped effects can derive a robust non-ideal curvature driven instability.**

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THANK YOU