Redistribution of Energetic Ions During Reconnection Events in NSTX

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Abstract. The motion of fast ions during a sawtooth event in the spherical torus NSTX is studied with the code GYROXY, which has been extended to simulate the perturbation of the electromagnetic field during a sawtooth crash. The dominant mechanisms of the particle redistribution are elucidated. It is concluded that the resonance interaction of particles with the perturbation and the stochasticity enhanced by the plasma diamagnetism are sufficient to account for the observed strong drops of the neutron yield provided that the pre-crash profile of the safety factor is sufficiently flat in the core.

1. Introduction

Plasma performance and prospects of spherical tori (or spherical tokamaks, ST) as future reactors considerably depend on the behavior of energetic ions – the injected/accelerated particles and the fusion reaction products. Experiments on the National Spherical Torus Experiment (NSTX) show that bursts of MHD events, such as sawtooth oscillations, Internal Reconnection Events (IRE) etc., sometimes result in strong, by a factor of two, drops of the neutron yield and signals on the Neutral Particle Analyser (NPA) [1]. Similar strong reduction of the neutron flux may also occur during quasi-steady-state MHD activity, but then the neutron yield decreases much slower. As in NSTX the neutrons are mainly produced by beam-induced fusion reactions, the mentioned facts indicate that the MHD activity strongly influences the energetic ions.

An example of the strong drops of the neutron yield during MHD events in the NSTX shot #104505 is given in Fig. 1. The earliest two events (at the time moments 0.16 s and 0.19 s, marked with blue dotted lines), most probably, can be identified as sawtooth crashes rather than IRE, which is confirmed by the absence of noticeable current spikes and the presence of $n = 1$ signals in Mirnov spectrograms (n is the toroidal mode number). It was shown in Ref. [2] that the strong neutron flux drops observed during these events cannot be attributed only to a redistribution of the NBI-produced energetic particles and the thermal plasma particles within the plasma, but they result to a large extent from a strong loss of the energetic particles.

The aims of the present work are, first, to analyze mechanisms of the energetic particle redistributions during sawtooth crashes in machines with high β and a large Larmor radius of energetic ions and, second, to explain the strong drops of the neutron emission in NSTX experiments. In contrast to previous analysis [2], which used the guiding-center approximation for the description of the particle motion, in this work we employ the Lorentz code GYROXY [3] able to take the particle gyromotion into account.

2. Mechanisms of the Particle Redistribution During a Sawtooth Crash

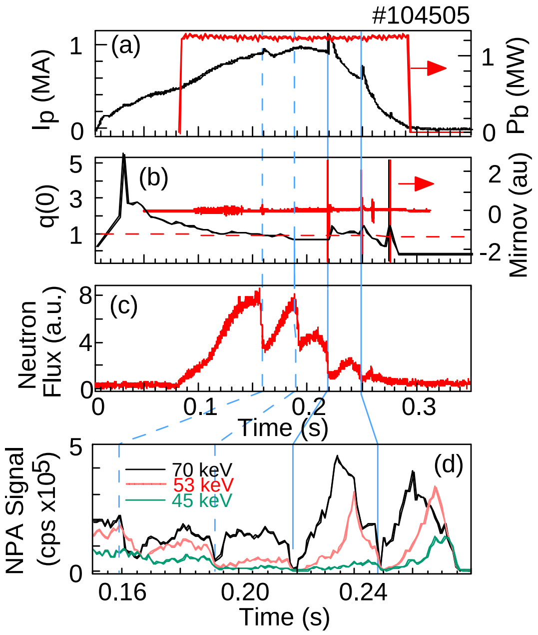

FIG. 1. Influence of sawteeth and reconnection events on the neutron rate and signals of the neutral particle analyzer in the NSTX shot #104505. (Courtesy of S.S. Meadley.)

The sawtooth effect on fast particles in conventional tokamaks has been extensively studied both experimentally and theoretically (see, e.g., the review in Ref. [4] and references therein). The results of theoretical works [4–8] can be briefly summarized as follows. The sawtooth-induced particle redistribution is a result of competition of several processes. First, the electric field associated with the plasma flow tends to move any particle together with the flow. The characteristic time of this motion is τ_{crash}, the sawtooth crash duration. Second, the toroidal precession tends to move a particle so that its radial coordinate, r, is constant and, hence, to prevent the particle redistribution. Third, the longitudinal motion tends to move particles along the displaced flux surfaces. The characteristic time of this motion is the period of the particle trip around a flux surface,

$$\tau_L = \frac{2\pi R_0}{\langle v_\parallel \rangle |q_0^{-1} - 1|}, \tag{1}$$

where $v_\parallel$ is the particle velocity along the magnetic field, q the safety factor, R the distance to the axis symmetry, and the subscript "0" refers to the magnetic axis. The resulting particle motion depends on the relative characteristic times of the three processes, which, in turn, depend on the particle energy and pitch angle. The passing particles with the orbit widths less than r_{mix} are typically characterized by fast longitudinal motion and relatively slow precession; therefore, they are strongly redistributed by a crash. The longitudinal motion of the trapped particles is negligible. Therefore, the intensity of their redistribution depends on their energy: When the energy exceeds the critical value determined by the condition $\tau_{\text{crash}} = \tau_{\text{pr}}$, where τ_{pr} is the precession period, the precession is fast enough to prevent the redistribution [5, 6]. Finally, the precession and bounce periods of the so-called "potato" particles with large orbit widths ($\sim r_{\text{mix}}$) are typically of the same order of magnitude. Therefore, the precession is able to prevent the redistribution (we assume that the bounce period is much less than τ_{crash}), and such particles can be redistributed only due to resonances between their precession and bounce motion [7, 8].

Low aspect ratio and strong plasma diamagnetism in spherical tori modify this pattern. First, in high-β plasmas of modern spherical tori, the particle precession has a strong diamagnetic component (here $\beta = 8\pi p/B^2$, p is the plasma pressure, B is the magnetic field strength). When the magnetic configuration is perturbed by a crash, this component tends to move particles along the surfaces of constant pressure, which are the displaced flux surfaces; therefore, it promotes the particle redistribution instead of preventing it [9]. Second, the relative magnitudes of the mentioned characteristic times in spherical tori are quite different from those in conventional tokamaks. Let us consider 80-keV passing injected deuterons in NSTX. Assuming that $|q^{-1} - 1| \sim 0.1$, $v_\parallel/v = 0.8$, $R_0 \sim 100$ cm, we find: $\tau_L \sim 3 \times 10^{-5}$ s. For a parabolic pressure profile the diamagnetic precession period can be estimated as $\tau_{\rm dia} \sim 2\pi a^2/(v\rho_\perp\beta_0) \sim 10^{-4}$ s [2], where $\rho_\perp = v_\perp/\omega_B$, ω_B is the deuteron cyclotron frequency, $a = 68$ cm is the minor radius of the plasma, $B_0 = 0.3$ T, and β_0 is taken to be 11% for this estimate. The period of the "conventional" precession caused by toroidicity can be estimated as [10]

$$\tau_{\rm pr} = \frac{2\pi R_0^2}{\xi v \rho} \sim 10^{-5}\,{\rm s}, \qquad (2)$$

where $\rho = v/\omega_B$, and ξ is a certain dimensionless parameter determined by the pitch angle and properties of the magnetic configuration, which was assumed to be ~ 1 for this estimate. As $\tau_{\rm crash} \sim 10^{-4}\,$s in most tokamaks (unfortunately, we have no such data concerning spherical tori yet), we observe that the precession, which prevents the particle redistribution, is faster than the rest of factors. Hence, in contrast to conventional tokamaks, the passing 80-keV deuterons in NSTX are not expected to follow the evolving flux surfaces (unless $\xi \ll 1$), and one needs to involve some resonance phenomena to explain their redistribution or loss (the motion of the particles with $\xi \ll 1$ can also be considered as resonant motion with $\omega_\varphi \approx \omega_\vartheta$, where ω_φ and ω_ϑ are the frequencies the poloidal and toroidal motion, respectively). In general, one can conclude that $\tau_{\rm pr}, \tau_L \ll \tau_{\rm dia}, \tau_{\rm crash}$.

Therefore, the general pattern of the motion of passing particles is determined mainly by the competition of longitudinal motion and precession. As the energy dependence of the characteristic times of these processes is different, $\tau_L \propto \mathcal{E}^{-1/2}$ and $\tau_{\rm pr} \propto \mathcal{E}^{-1}$ ($\mathcal{E}$ is the particle energy), there exists a critical energy for the redistribution of passing particles, $\mathcal{E}_{\rm crit}^{\rm passing}$, determined by the relationship $\tau_L = A\tau_{pr}$, where $A \approx 1/8$ [2, 5]. The particles with the energy below the critical one are strongly redistributed during the crash, whereas for higher energies the crash affects mainly the particles within the resonance island. From Eqs. (1) and (2) we obtain [2]:

$$\mathcal{E}_{\rm crit}^{\rm passing} = \frac{M}{2}\left(\frac{R_0\omega_{B0}}{8\xi}\langle\chi\rangle|q_0^{-1} - 1|\right)^2, \qquad (3)$$

where $\chi = v_\parallel/v$, $\langle\ldots\rangle$ denotes bounce averaging. In particular, $\mathcal{E}_{\rm crit}^{\rm passing} = (\langle\chi\rangle|q_0^{-1} - 1|/\xi)^2 \cdot 50\,{\rm keV}$ in NSTX. Thus, the energy of the ions responsible for the neutron yield, $\mathcal{E} \lesssim 80$ keV, considerably exceeds $\mathcal{E}_{\rm crit}^{\rm passing}$ when $|\xi| \gtrsim 0.2$. One can show that the critical energy of the trapped particles is even lower. This means that these particles can be redistributed only due to resonance effects.

The motion of passing particles during a sawtooth event is determined to a large extent by the resonance $\omega_\varphi/\omega_\vartheta = 1/1$: $\omega_\varphi \approx \omega_\vartheta$ for most circulating particles in the region of sawteeth, and the $m = n = 1$ harmonic dominates in the sawtooth instability [2]. This resonance forms a resonance island near the resonant drift surface, which can cause redistribution of particles or even loss if the island reaches elements of the device hardware. Even some particles outside the island can be lost because of the distortion of their drift surfaces by the perturbation.

In addition, the simulations in Ref. [2, 11] exhibit the presence of an extensive zone of stochastic motion. The numerical calculations taking account of the effect of the large pressure gradients arising during a magnetic reconnection show that finite β increases the region of stochasticity [11]. A possible origin of the stochasticity could be the overlap of nonlinear resonances that appear due to large radial displacements of the particles with small poloidal action under an $m = n = 1$ perturbation [4, 8] (indeed, such displacements increase with the increase of β [9], which should promote stochasticity). However, a detailed analysis based on Refs. [4, 8] shows that the quantity that determines the transition to stochasticity via this mechanism is $(1 - q_0)r_{\rm mix}/(\nu R_0)$, where $\nu = [2\rho/(kR_0)]^{1/3}$, k is the cross section elongation. In previous numerical simulations [4, 8], the transition to global stochasticity took place when this quantity reached ~ 0.5, whereas it is ~ 0.1 for 80-keV deuterons in NSTX. In agreement with this, the stochasticity in the calculation results presented below seems to spread from the island separatrix. Thus, it is likely to originate from the well-known overlap of secondary islands near the island separatrix. As mentioned in Ref. [2], a possible explanation why an the increase of the plasma pressure promotes the stochasticity lies in the fact that the redistribution of the plasma pressure is accompanied by a noticeable variation of the magnetic field strength (i.e., the longitudinal component of the magnetic perturbation is not small). The magnetic reconnection, in particular, creates large pressure gradients on the boundaries of the regions of the reconnected and non-reconnected field. It was noticed in Ref. [2] that the effect of such a perturbation on the particles that cross these boundaries owing to their orbital motion and/or precession will consist in transversal "kicks" of the ∇B drift. As the Fourier spectrum of such perturbations of the particle motion is very rich, one can expect the appearance of numerous secondary resonance islands. As we will see below, numerical simulations of the particle motion without using the guiding-center approximation also reveal an important role of cyclotron resonances in this case.

3. Numerical Model

To clarify the physical mechanisms that cause the particle loss, we carried out numerical calculations with the code GYROXY [3]. The code simulates the ion motion in a toroidal magnetic configuration, using the Lorentz equations (i.e., without employing the guiding-center approximation).

The code was supplemented by a semi-analytical model of the electromagnetic field evolution during a Kadomtsev-type reconnection. In contrast to a previous model [5], this model takes into account the perturbation of the magnetic field strength due to the plasma diamagnetism. Neglecting the variations of the Shafranov shift, we take the magnetic field during the reconnection in the form

$$\mathbf{B} = \Xi(\psi, \alpha)\nabla\psi \times \nabla\alpha + \nabla\varphi \times \nabla\psi_*(\psi, \alpha), \tag{4}$$

where $(\psi, \vartheta, \varphi)$ are flux coordinates in the initial axisymmetric state, ψ is the toroidal magnetic flux, φ is the toroidal angle, ϑ is the poloidal angle chosen so that $\varphi - q\vartheta$ is constant on the magnetic field lines, $\alpha = \vartheta - \varphi$, $\psi_* = \psi_p - \psi$ is the magnetic flux through the helical surface with $m = n = 1$, ψ_p is the poloidal magnetic flux, the functions $\psi_*(\psi, \alpha)$ and $\Xi(\psi, \alpha)$ describe the evolution of the flux surfaces and the effect of the plasma pressure redistribution on the toroidal magnetic field, respectively.

We take Ξ in the form

$$\Xi = \Xi_0 \left\{ 1 - \frac{1}{2}[\beta(\psi, \alpha) - \beta_0(\psi)] \right\}, \tag{5}$$

where the subscripts "0" refer to the initial state. One can see that this expression correctly represents the effect of the plasma pressure variations on the toroidal magnetic field in axisymmetric plasmas. As mentioned above, β is characterized by discontinuities at the magnetic separatrix (the boundary of the growing magnetic island). The reason for this is that the pressure of the mixed plasma within the magnetic island is not equal to the pressure of the plasma in the regions that has not passed through the reconnection yet. In reality, this discontinuity is smoothened by the transversal diffusion, the finite time of the longitudinal diffusion, and other factors. Nevertheless, one can expect that the characteristic transversal size of the pressure inhomogeneity is much smaller than the Larmor radius of the particles of interest, so that they perceive it as a discontinuity. We took this factor into account in our calculations. To decrease the integration error, we smoothened the function $\beta(\psi, \alpha)$ near the magnetic separatrix. However, this did not seem to affect the results.

For the function ψ_*, we use a model expression, which corresponds to the evolution of the flux surfaces during a Kadomtsev-type [12] sawtooth crash. This model, which will be described in detail elsewhere, is similar to that described in Ref. [5]. However, since the particles of interest are not expected to follow the evolving flux surfaces, in the new model the requirement that the flux is frozen into plasma is not observed very strictly, which gives a possibility to accelerate the calculations.

As shown in Sec. 2, the characteristic times of the particle motion in the electromagnetic field of a sawtooth are much shorter than the expected crash duration. This enables us to use Poincaré maps in order to understand the main features of the particle motion and evaluate the loss region. Therefore, we used GYROXY to generate Poincaré maps of the particle motion under a static magnetic perturbation, varying the parameters of the particle studied, the equilibrium magnetic configuration, and the perturbation amplitude. Note that the fact that the crash duration is large means at the same time that the electric component of the perturbation is less important than the magnetic component. We calculated the particle trajectories, used them to obtain the trajectories of the guiding centers (to first order in the Larmor radius), and found the crossings of the guiding-center trajectories with a given vertical plane. It should be emphasized that we did *not* use the guiding-center approximation when simulating the particle motion. The guiding centers are used on our Poincaré maps instead of the real positions of the particles only because excluding the gyromotion makes the presentation more clear.

3. Results of the Calculations

For our calculations we took an NSTX configuration with $B_0 = 0.36$ T. In order to study the dependence of the particle motion on the parameters of the configuration, we varied the poloidal magnetic field in the core, changing q_0 and the mixing radius. Since the pressure of the poloidal field in the core is much less than that of the toroidal field and the variation of q is always less than 30%, the force balance is not broken by such modifications very strongly. Most calculations presented here are obtained for $q_0 = 0.9$ and $r_{\mathrm{mix}} \approx 0.7a$. Figure 2 shows the footprints left by the guiding centers of 80-keV deuterons with pitch angles $\chi \sim 0.8$ ($\lambda = 0.36$, where $\lambda = \mu B_0/\mathcal{E}$, μ is the magnetic moment of the particle) in the vertical plane for the case when the perturbation is relatively weak (the beginning of the magnetic reconnection, when the fraction of the reconnected helical flux, Δ_ψ, is only 0.05). The diamagnetic component of the perturbation (i.e., the perturbation of the toroidal magnetic field) is suppressed in this calculation. The visual width of the particle orbits is a consequence of the fact that we use the first-order guiding centers, which still undergo gyromotion although with much smaller amplitudes than the particles themselves. One can see the island of the resonance $\omega_\varphi/\omega_\vartheta = 1/1$ (the particle shown in blue) at the

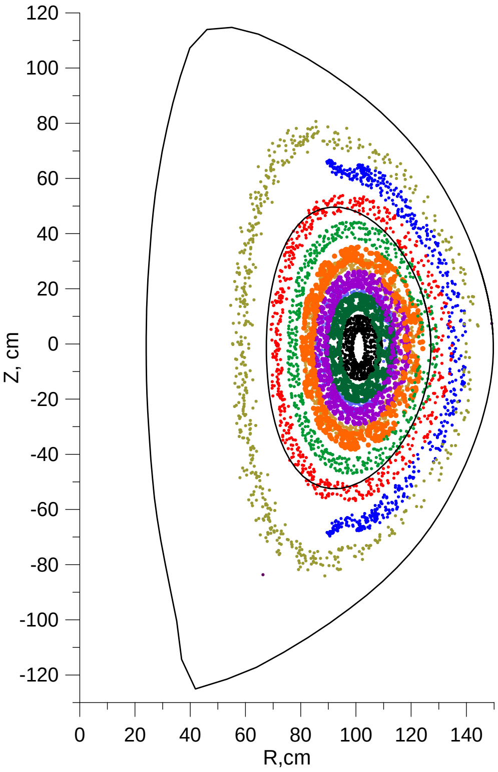

FIG. 2. Poincaré maps of particles with $\lambda = 0.0975$ *for* $\Delta_\psi = 0.05$, $r_{mix} \approx 0.7a$. *Different colors correspond to particles with different initial positions.*

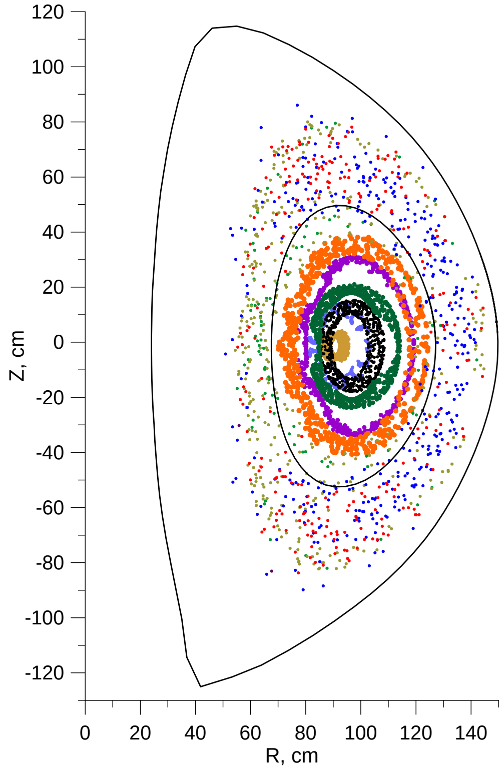

FIG. 3. The same as Fig. 2 but for $\Delta_\psi = 0.95$.

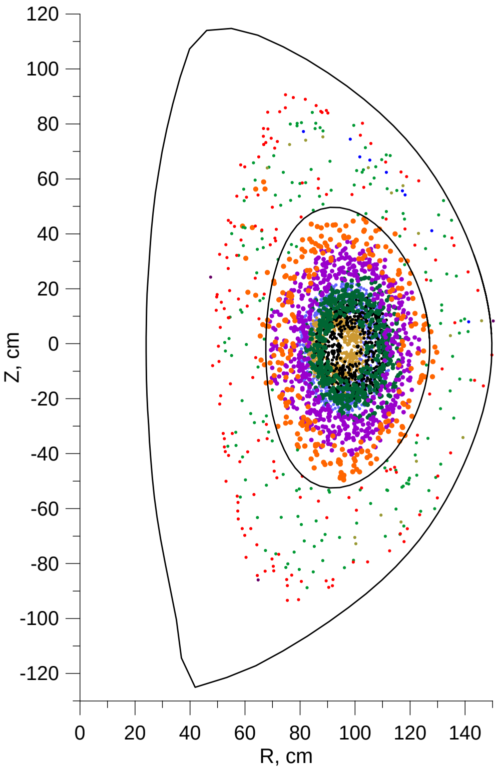

FIG. 4. The same as Fig. 3 but with a diamagnetic perturbation, $\Delta\beta = 0.3$.

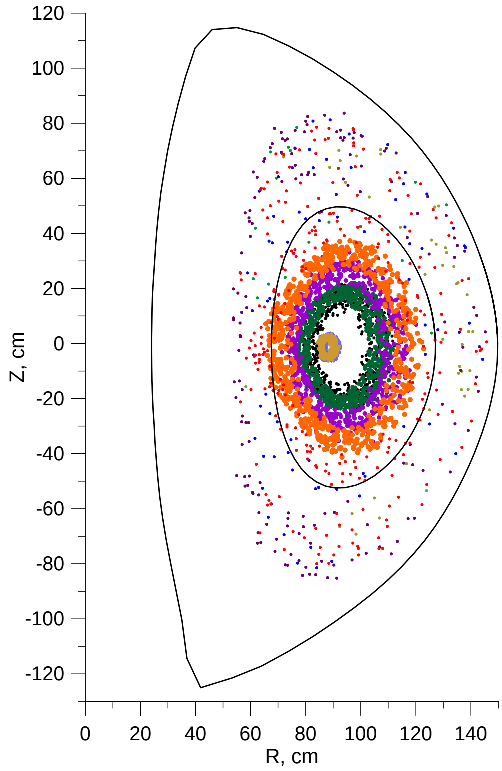

FIG. 5. The same as Fig. 4 but for $\lambda = 0.0975$.

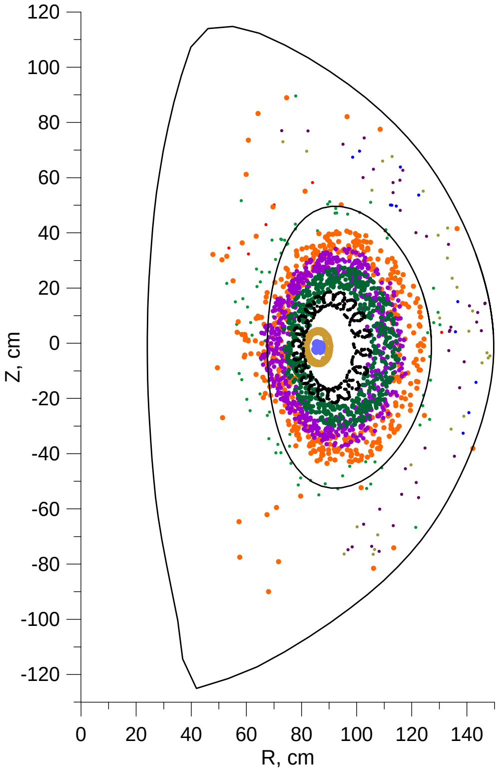

FIG. 6. The same as Fig. 5 but for $r_{\rm mix} = 0.8a$*,* $\Delta_\psi = 0.999$*.*

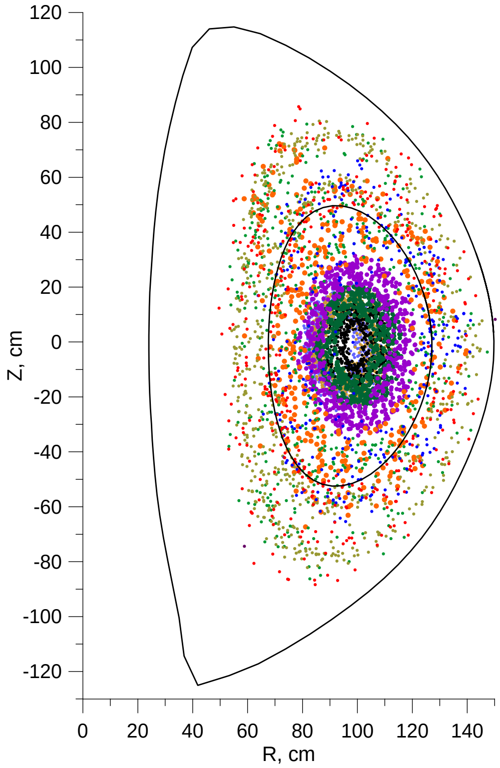

FIG. 7. The same as Fig. 4 but for $r_{\rm mix} = 0.6a$*.*

left-hand side of the cross section. When the perturbation is stronger, corresponding to a final stage of the reconnection ($\Delta_\psi = 0.95$), the island is wider (see Fig. 3). The particles orbits in the island seems stochastic, although it is difficult to distinguish exactly between stochastic and regular motion because of the remaining gyromotion.

Figure 4 shows the motion under the same perturbation as in Fig. 3, but with strong diamagnetic perturbation corresponding to $\Delta\beta = 30\%$, where $\Delta\beta$ is the difference $\beta(r = 0) - \beta(r = r_{\rm mix})$ before the crash. The region occupied by the island in Fig. 3 has turned into a zone of stochastic motion. The loss time of the particles in the stochastic zone is rather short (less than the expected crash duration, $\sim 10^{-4}$ s), which means that they would be lost during the crash. The motion of the particles with smaller perpendicular velocities ($\lambda = 0.0975$) is almost the same, in spite of weaker diamagnetic drift (Fig. 5). As mentioned above, one of the possible explanations of the stochasticity is the kicks of ∇B-drift at the magnetic separatrix [2]. However, our calculations indicate that the cyclotron resonances play an important role in the formation of the stochasticity. A graphical example is shown in Fig. 6, where the motion in a configuration with $r_{\rm mix} \approx 0.8a$ is shown. One can see a trajectory with clear cyclotron resonance (in black) just on the boundary of the stochastic zone. Simulations with a smaller mixing radius ($r_{\rm mix}/a \approx 0.6$) show qualitatively the same picture. However, a detailed analysis of the trajectories shows the particles survive within the plasma for longer time periods exceeding the crash duration.

4. Summary and Conclusions

The code GYROXY has been extended to include a model of the electromagnetic field evolution during a Kadomtsev-type sawtooth crash. The model, in particular, can take account of the diamagnetic perturbation of B due to the pressure redistribution. This gives a possibility to model the effect of sawteeth on fast ion, taking into account the finite gyroradius of fast ions. It was found that the dominant mechanisms that can cause fast ion losses during sawtooth events are the island of the resonance $\omega_\varphi/\omega_\vartheta = 1/1$ and the concomitant stochasticity. The stochasticity is enhanced by the diamagnetism in high-β shots. Cyclotron resonance are found to play an important role in the development of the stochasticity. The results of the calculations show that the strong drops of the neutron yield during sawteeth in NSTX (by a factor of 2) can be explained if the mixing radius of the sawteeth is as large as $\sim 0.7a$, which is possible when the safety factor profile is flat in the plasma core (which is usually the case in NSTX) and q_0 is sufficiently low.

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