Fishbone Instability Excited by circulating electrons

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Abstract

Fishbone instability excited by the supra-thermal circulating electrons in tokamaks is investigated. It is found for first time that the procession of all the circulating electrons is in ion diamagnetic direction if magnetic share is neglected. The circulating electrons play bigger role on the modes than the barely trapped electrons. The analyses show that the mode frequency is close to the procession frequency of circulating electrons comparable with experiment observations. The correlation of the theory with experiments is discussed.

Ⅰ. Introduction

Internal kink modes driven by the supra-thermal electrons is reported on DⅢ-D tokamak [1]. This instability is most active when electron cyclotron current drive (ECCD) is applied on the high field side of the flux surface. It has a bursting behavior with poloidal/ toroidal mode number =m/n=1/1. In positive magnetic shear plasma this mode becomes fishbone instability. Strong m=1 MHD activities are also observed in HL-1M tokamak during off-axis electron cyclotron resonance heating (ECRH) when the cyclotron location is placed just outside the q=1 flux surface at high field side [2]. Addition of low-hybrid (LH) wave to ECRH significantly enhances the MHD activities. Lower-hybrid related fishbone is observed on FTU tokamak [3]. The fishbone-like structures are located at the q=1 flux surface only during high-power lower hybrid heating. Electron fishbone observed on FTU are strongly excited with LH+ECRH.

Barely trapped electrons may be responsible for the events proposed by Wong et al [1]. The idea is proved in Ref.[4]. However, electrons are heated at high field site in common in all the experiments. This gives a hint that circulating electrons may play bigger role on the modes since a few trapped particles experience high field site.

A set of canonical variables [5] is employed which gives a clear picture of the relativistic particle dynamics. It is found for first time that the procession of all the circulating electrons is in ion diamagnetic direction if magnetic share is neglected seen in Fig.1. Therefore, the supra-thermal circulating electrons have stronger effect than the barely trapped electrons on the modes.

A single dispersion relation is derived including the barely trapped electrons and the circulating electrons. The calculated frequency of the mode is near the supra-thermal electron procession frequency and consistent with experiments [1,2]. The internal kink modes for n=2 are included as well in our model. This may account for the observation on FTU tokamak. In the high power operation the presence of bursts of (1.1) is on the top of (2.1) modes in the device.

II. **Exact guiding center variables**

In tokamak configuration, the relativistic Hamiltonian of a charged particle can be expressed as

$$H = \sqrt{[(P_R - \frac{e}{c}A_R)^2 + (P_Z - \frac{e}{c}A_Z)^2 + (P_\phi - \frac{e}{c}RA_\phi)^2 / R^2]c^2 + m_0^2 c^4} + e\Phi \qquad (1)$$

where A_R, A_Z, and A_ϕ are the vector potential components of the magnetic field, Φ the electrical potential, m_0 is the rest mass, and e the charge. P_R, P_ϕ, P_Z, are the canonical momenta conjugate to R, ϕ, and Z respectively,

$$P_R = m_0 u_R + \frac{e}{c}A_R \qquad (2)$$

$$P_\phi = Rm_0 u_\phi + \frac{\mathrm{e}}{\mathrm{c}}\mathrm{R}A_\phi \qquad (3)$$

$$P_Z = m_0 u_Z + \frac{e}{c}A_Z \qquad (4)$$

where $u = \gamma \upsilon$ and $\gamma = (1 + u^2 / c^2)^{1/2}$ is the relativistic factor.

The magnetic field can be expressed as

$$B = \nabla\phi \times \nabla\Psi + I\nabla\phi \qquad (5)$$

where Ψ is related to the poloidal flux of the magnetic field, I is related to the poloidal current, R is the major radius. Then, in tokamaks we have

$$A_R = 0\,,\ A_Z = -I \ln\frac{R}{R_0}\,,\ A_\phi = -\frac{\Psi}{R} \qquad (6)$$

We introduce a generating function[5,] for changing to the guiding center variables,

$$F_1 = -\frac{m_0 \Omega_0 R_0^2}{2} \exp(\frac{X}{m_0 \Omega_0 R_0})(\ln\frac{R}{R_0} - \frac{X}{m_0 \Omega_0 R_0})^2 tg\alpha - ZX \qquad (7)$$

where

$$X = m_0 \Omega_0 R_0 \ln\frac{R_C}{R_0} \qquad (8)$$

and Ω_c is the toroidal gyro-frequency taken absolute value for electron, ρ the Larmor radius, α the gyro-phase, subscripts o and c refer to the values at the magnetic axis and the guiding center respectively. X and α are the new coordinates conjugate to the momenta

$$P_X = Z + \rho \sin\alpha + \frac{\rho^2}{4R_C} \sin 2\alpha \quad (9)$$

$$P_\alpha = \frac{1}{2} m_0 \Omega_C \rho^2 \quad (10)$$

where P_X is actually the guiding center of Z coordinate, Z_c. That the moment is turned to be coordinate often occurs during area-conserved canonical transformation [6]. The other two canonical variables P_ϕ and ϕ do not change in the new coordinates. The old coordinates are connected with new ones through four identical equations,

$$P_R = m_0 \Omega_c \rho e^{\frac{\rho}{R_c}\cos\alpha} \sin\alpha \quad (11)$$

$$P_Z = -X \quad (12)$$

$$R = R_C \exp(-\frac{\rho \cos\alpha}{R_C}) \quad (13)$$

$$Z = P_X - \rho \sin\alpha - \frac{\rho^2}{4R_C} \sin 2\alpha \quad (14)$$

The Jacobian in the area-conserved transformation is unity [6], that is,

$$d\tau = J dP_\alpha dP_x dP_\phi d\alpha dX d\phi \quad (15)$$

$$J = 1 \quad (16)$$

The exact Hamiltonian for the relativistic particles is

$$H = \sqrt{\{2m_0\Omega_c P_\alpha[(\frac{R_c}{R})^2 \sin^2\alpha + \cos^2\alpha] + \frac{1}{R^2}[P_\phi + e\Psi]^2\}c^2 + m_0^2 c^4} + e\Phi \quad (17)$$

It is suitable for particle simulation. The equations of motion and Vlasov's equation could be derived from the Hamiltonian for the relativistic particles.

Ⅲ. Dispersion relation

The plasma consists of two components. One is a relatively cold MHD part. The other is a hot particle component treated with gyro-kinetic description.

For the gyro-kinetics the Hamiltonian in Eq.(17) could be averaged with the gyro-phase;

$$H = \sqrt{(2m_0\Omega_c P_\alpha + m_0^2 u_\phi^2)c^2 + m_0^2 c^4} + e\Phi \quad (18)$$

We form a new invariant [6],

$$\Pi = \frac{1}{2\pi}\oint P_x dX \tag{19}$$

For the trapped particles in the large aspect ratio configuration, that is, $\varepsilon << 1$, we get

$$\Pi_t = \frac{8qR_0 m_0 (\varepsilon\Omega_0 P_\alpha / m_0)^{0.5}}{\pi}[E(k_1) - (1-k_1{}^2)K(k_1)] \tag{20}$$

which is the toroidal magnetic flux enclosed by drift surface. The bounce frequency and the procession frequency are obtained [4，7],

$$\omega_{bt} = \frac{\pi(\varepsilon\Omega_0 P_\alpha / m_0)^{0.5}}{2\gamma q R_0 K(k_1)} \tag{21}$$

$$\omega_{\zeta t} = \frac{2\Omega_0 P_\alpha}{\gamma\Omega_p m_0 R_0^2}[\frac{E(k_1)}{K(k_1)} - \frac{1}{2}] + \frac{4\Omega_0 P_\alpha \hat{s}}{\gamma\Omega_p m_0 R_0^2}[\frac{E(k_1)}{K(k_1)} - (1-k_1{}^2)] \tag{22}$$

where Ω_p is the poloidal gyro-frequency, $\varepsilon = r / R_0$, $k_1^2 = \dfrac{m_0 u_{\phi 0}^2}{4\varepsilon\Omega_0 P_\alpha}$, $\gamma = (1 + \dfrac{2\Omega_C P_\alpha / m_0 + u_{\phi 0}^2}{c^2})^{\frac{1}{2}}$, and $\hat{s}$ is the magnetic shear. $E(k_1)$ and $K(k_1)$ are complete elliptic function.

For the circulating particles,

$$\Pi_c = \frac{\Omega_0 m_0 r^2}{2} + \frac{2qR_0 m_0 u_{\phi 0}\sigma}{\pi}E(k) \tag{23}$$

$$\omega_{bc} = \frac{\pi u_{\phi 0}\sigma}{2\gamma q R_0 K(k)} \tag{24}$$

$$\begin{aligned}\omega_{\zeta c} &= q\omega_{bc}\sigma + \frac{u_{\phi 0}^2}{2\gamma\Omega_p r R_0}[\frac{E(k)}{K(k)} - (1-\frac{k^2}{2})] + \frac{u_{\phi 0}^2 \hat{s}}{\gamma\Omega_p R_0^2}\frac{E(k)}{K(k)} \\ &= q\omega_{bc} + \frac{u_{\phi 0}^2}{2\gamma\Omega_p r R_0}G\end{aligned} \tag{25}$$

where G is the normalized procession of the circulating particle seen in Fig.1, σ represents direction of the circulating particle and $k^2 = k_1^{-2}$. It is found for first time that the procession of all the circulating electrons is in ion diamagnetic direction if magnetic share is neglected.

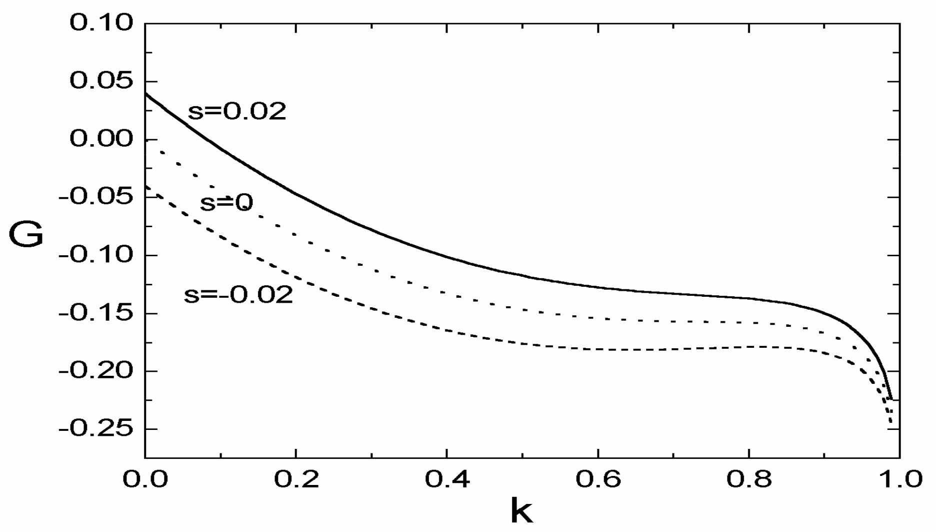

Figure 1 Normalized procession velocity of the circulating electrons versus k. **s** is the magnetic shear.

New momenta $\Pi, P_{\alpha,} P_\zeta$ are conjugate to η, α, ζ in which $\dfrac{d\eta}{dt} = \omega_b$, $\dfrac{d\zeta}{dt} = \omega_\zeta$, $P_\zeta = -\dfrac{e}{c}\Psi_0 = \Psi^\bullet$ which is actually the position variable [7].

The bounce-averaged gyrokinetic equation for the non-relativistic scenario is easy to be derived,

$$\frac{\partial f_1}{\partial t} + \dot{\zeta}\frac{\partial f_1}{\partial \zeta} = -\frac{i}{T}(\omega - \omega^\bullet)F_m \delta H_1 \tag{26}$$

where $\delta H_1 = -\omega_\zeta \Omega_p R_0 \xi$ is the perturbed Hamiltonian by the radial MHD displacement,

$$\xi = \xi_0 e^{i(n\zeta - \omega t)}, \tag{27}$$

indicating that the change of guiding center position induces change of particle kinetic energy and $\omega^\bullet$ is the electron diamagnetic drift frequency. The bounce-averaged equation Eq.(26) is derived in a complete set of canonical variables for both the trapped particles and circulating particles. The effects are canceled for the particles flying in different directions in Eq.(25). Only processions are left. .

Using the solution of Eq.(26) and Eq.(27) we can calculate the kinetic energy,

$$\delta\hat{W}_k = 2^{7/2}\pi^2 \sum_\sigma \int_0^1 y dy \int_0^{\lambda_0} d\lambda \int_0^\infty dE E^{5/2} \frac{\omega^\bullet}{n\omega_\zeta - \omega} \cdot \frac{k_b r_s^2 R_0^2 \hat{\omega}^2}{q^2 T^3} F_m \tag{28}$$

where λ_0 is the procession reversal point which varies with magnetic shear seen in Eq. (25) and mentioned in Ref. [8], $\lambda = \dfrac{\Omega_0 P_\alpha}{E}$, q is the safety factor, k_b is related to the particle bounce period [9], $\omega_\zeta = \frac{E}{T}\hat{\omega}$, $\omega^\bullet > \omega_\zeta$, r_s is the manor radius of the resonant surface,

$$\omega^* = \frac{nT}{\Omega_p R_0 m_0 L_n}, \qquad L_n = [-B_p R_0 \frac{\partial}{\partial \Psi} \ln(f_m)]^{-1} \ . \tag{29}$$

Since Ω_p is negative for electrons, $\omega^\bullet$ is positive only for the case of spatial density gradient reversal. E integration is straight. If we define poloidal beta of the hot electrons,

$$\beta_h = -\frac{2}{\varepsilon_s^2}\int_0^1 ydy \frac{2r_s n_e T}{L_n B^2} f_r \tag{30}$$

where

$$f_{rt} = \frac{2}{3}\sqrt{2\varepsilon}\int_{\frac{1}{1+\varepsilon}}^{\lambda_0} \lambda^{\frac{1}{2}} d\lambda \cdot \{[E(k_1) - \frac{1}{2}K(k_1)] + 2\hat{s}[E(k_1) - (1-k_1{}^2)K(k_1)]\} \tag{31}$$

for the trapped particles, while for the circulating particle,

$$f_{rc} = \frac{2}{3}\int_0^{\frac{1}{1+\varepsilon}} [1-\lambda(1-\varepsilon)]^{\frac{1}{2}} d\lambda \cdot \{[E(k) - (1-\frac{1}{2}k^2)K(k)] + 2\hat{s}E(k)\} \tag{32}$$

f_r roughly is proportional to the kinetic energy either from circulating particles or from barely trapped particles which destabilize the internal kink modes. We have calculated the ratios of the circulating particles to the barely trapped particles for different devices. For DIII-D $\varepsilon = 0.1136$ $\frac{f_{rc}}{f_{rt}} = 17$. For HL-1M $\varepsilon = 0.05$, $\frac{f_{rc}}{f_{rt}} = 40$. We have seen that the supra-thermal circulating electrons have stranger effect than the barely trapped electrons on the modes.

We assume that $\hat{\omega}$ is insensitive to y and define $\Omega = \frac{\omega}{n\hat{\omega}}$, then we get

$$\delta\hat{W}_k = \beta_h[1 + \frac{2}{3}\Omega + \frac{4}{3}\Omega^2 + \frac{4}{3}\Omega^{5/2}Z(\Omega^{1/2})] \tag{33}$$

The dispersion relation is obtained,

$$-i\Omega \cdot \frac{n\hat{\omega}}{\varpi_A} + \delta\hat{W}_f + \beta_h[1 + \frac{2}{3}\Omega + \frac{4}{3}\Omega^2 + \frac{4}{3}\Omega^{5/2}Z(\Omega^{1/2})] = 0 \tag{34}$$

where $\varpi_A = V_A/(3^{1/2}R_0\hat{s})$ and $Z(\Omega^{1/2})$ is the plasma dispersion function. The dispersion relation is slightly different from ones of the ion fishbone [10, 11].

Ⅳ. Numerical solution

Solution of the dispersion relation is obtained by using standard nonlinear complex solver. For the cases $\delta\hat{W}_f = 0$ and $\delta\hat{W}_f = \frac{n\hat{\omega}}{\varpi_A}$ we get Ω_i versus β_h in Fig. 2 where Ω_i is related to the growth rate by $\gamma = n\hat{\omega}_c\Omega_i / 2\pi$. The critical β_h for $\delta\hat{W}_f = 0$ is

$$\beta_{hcrit} = 1.5 \frac{n\hat{\omega}}{\overline{\omega}_A} \tag{35}$$

which is comparable with precious works.[9,10]. For n=1, $T_e = 36$ Kev ($\upsilon_{\phi 0}^2 = 2T_e$), B_T =1.77 tesla, r_s =0.2m, $\hat{s}$ =0 and G=0.13 seen in Fig.1 we obtain procession frequency of the circulating electrons from Eq.(25) $f_c = 10.496 \times 10^3 s^{-1}$ which is consistent with experiment [1,2]. For MHD-stable case, $\delta\hat{W}_f = 2\frac{n\hat{\omega}}{\overline{\omega}_A}$, the growth rates are obviously reduced, but the mode frequency is increased (seen Fig. 3). For the n=2 mode the threshold is increased according to Eq.(35). Only high power can drive the mode. This may account for the phenomena observed on FTU. During high power operation there is presence of burst of (1,1) on top of the (2,1) modes [3].

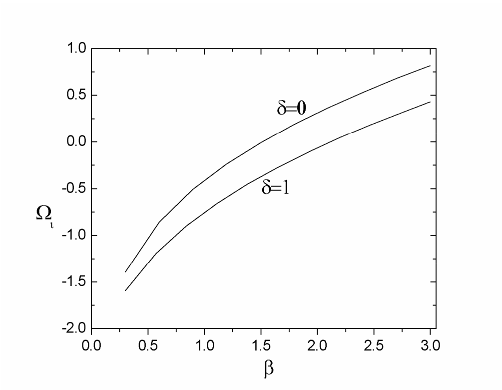

Fig.2 Ω_i versus β, $\gamma = n\hat{\omega}_c\Omega_i / 2\pi$ and $\beta = \beta_h \frac{\overline{\omega}_A}{n\hat{\omega}}$. $\delta = 0$ for $\delta\hat{W}_f = 0$ case. $\delta = 1$ for $\delta\hat{W}_f = \frac{n\hat{\omega}}{\overline{\omega}_A}$ case.

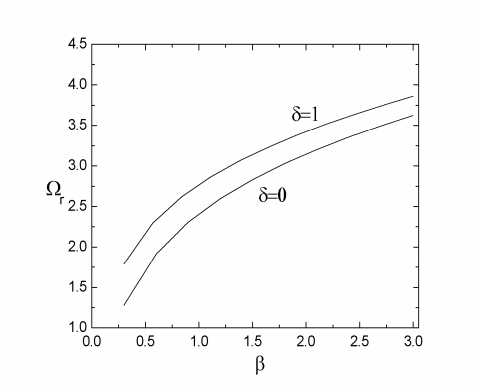

Fig. 3 Ω_r versus β, $f = n\hat{\varpi}_c\Omega_r / 2\pi$ and $\beta = \beta_h \frac{\overline{\omega}_A}{n\hat{\omega}}$ $\delta = 0$ for $\delta\hat{W}_f = 0$ case. $\delta = 1$ for $\delta\hat{W}_f = \frac{n\hat{\omega}}{\overline{\omega}_A}$ case

Ⅴ. **Summary**

A set of canonical variables is employed which gives a clear picture of the relativistic particle dynamics. It is found for first time that the procession of all the circulating electrons is in ion diamagnetic direction if magnetic shear is neglected seen in Fig.1. The electron fishbone instability is investigated. Circulating electrons play bigger role on the modes than the barely trapped electrons. The analyses show that the mode frequency is close to the procession frequency of circulating electrons comparable with experiment observations [1,2]. Spatial gradient reversal is necessary for the instability. The high toroidal wave number is considered that has higher threshold. The theory may apply for FTU. During high power operation there is presence of burst of (1,1) on the top of (2.1) modes.

Negative magnetic shear increases f_r of the hot electrons and β_h, therefore, is destabilizing. This may account for the crucial feature, that is, the presence of a slightly inverted q profile in the center during fishbone period on FTU. Shear evolves with time [1] that may account for occurrences of the fishbones.

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