Synergetic effect of TF ripples and MHD modes on fast ion transport

K. Schoepf 1), V. Goloborod'ko 1, 2), S. Reznik 1, 2), V. Yavorskij 1, 2)

1) Institute for Theoretical Physics, University of Innsbruck, Association EURATOM-OEAW, Innsbruck, Austria;
2) Institute for Nuclear Research, Ukrainian Academy of Sciences, Kiev, Ukraine

e-mail contact of main author: Klaus. Schoepf @uibk.ac.at

Abstract. The present paper investigates the combined effect of toroidal field (TF) ripples as well as MHD induced low frequency perturbations on fast ion confinement in tokamak plasmas. Transport coefficients of fusion alphas in presence of TF ripples and TAE modes are calculated using the simplectic method for integration of Hamiltonian systems. It is shown that MHD induced modes may result both in degradation and in improvement of fast ion confinement in a rippled tokamak magnetic field, that is in qualitative agreement with observations of charged fusion products confinement in TFTR.

1. Introduction

Perturbations of the magnetic field associated with toroidal field (TF) ripples and MHD activity in the plasma are known to lead to an enhanced loss of fast ions in tokamaks due to the break down of toroidal momentum conservation [1,2]. Usually, theoretical studies of the effect of these perturbations on fast ion confinement neglect their synergetic impact on the particle transport behaviour. According to Ref. [3], however, TF ripples together with TAE modes may result in a significant synergetic enhancement of fast ion loss in magnetically confined toroidal plasmas. On TFTR the MHD induced modulation of TF ripple loss of charged fusion products has been experimentally demonstrated [2]; both enhancement and reduction of fast ion loss was observed. The objective of the present paper is the modelling of the combined effect of TF ripples and MHD-perturbations on the transport behaviour of fast ions in a tokamak magnetic configuration. We carry out our analysis in the simplest single-harmonic approximation, in which each type of the perturbations is represented by a single harmonic. The combined effect of both perturbations on the stochastic domain in phase space is investigated by analysis of pair correlations of toroidal momentum variations [4] using the simplectic integration method for Hamiltonian systems [5].

2. Approach used

a) Simplectic integration of drift Hamiltonian equations in the presence of TF ripples and MHD perturbations

Our investigation of fast ions orbit instability in a toroidally asymmetric tokamak magnetic field is based on the Hamiltonian formulation of the guiding centre drift motion [6]. Assuming the MHD perturbations to affect only the magnetic field (similar to TF ripples), we start from the normalised drift Hamiltonian of a fast ion in the explicit form [7]

$$H = \left\{ \left[\psi(p_1) + p_2 \right] b(p_1, q_1, q_2) / g \right\}^2 + \lambda b(p_1, q_1, q_2), \tag{1}$$

where ψ is the normalised poloidal flux, $b=B/B_0$ indicates the ratio of the local magnetic field B to its value on the magnetic axis, B_0; $\lambda=\mu B_0/E$ represents the normalised magnetic moment, and $g=\rho_L/(aA)$ normalises the particle gyro-radius $\rho_L = (2E/m)^{1/2}/\omega_B$ to the product of minor plasma radius a and plasma aspect ratio A. Further, the canonical variables are p_1 – the normalised toroidal flux on the trajectory, q_1 – the conjugated poloidal angle, p_2 – the normalised toroidal angular momentum and q_2 – the conjugated toroidal angle. The normalised time is $\tau = tE/(m\omega_B a^2)$. For our modelling the poloidal flux and the magnetic field are chosen to be of the form

$$\begin{aligned} &\psi = \int_0^{p_1} dp_1'/q(p_1'), \quad q = q_0 / \left[1 + 2p_1 (q_0/q_a - 1) \right], \\ &b = \left[1 + \delta_{TF}(p_1, q_1) cos(Nq_2) + \delta_{MHD}(p_1) cos(nq_1 + mq_2) \right] / \left[1 + rcos(q_1)/A \right] \end{aligned} \tag{2}$$

with

$$\begin{aligned} &p_1 = 0.5r^2, \delta_{TF} = \delta_{TF0} exp\left\{ \alpha\sqrt{F} + \beta F \right\}, \\ &F = r^2 + 2r\Delta(r)cosq_1 + \Delta(r)^2, \Delta = \Delta_0 \left[1 - \left(A_c r / A \right)^2 \right], \\ &\delta_{MHD} = \delta_{MHD\max} exp\left\{ -\left[(r - r_c)/\Delta_c \right]^2 \right\}, \\ &p_2 = -\psi + g\chi/b, \ \chi = \sigma\sqrt{1 - \lambda b}, \quad \sigma = \pm 1 \ . \end{aligned} \tag{3}$$

Here q_0 and q_a are the safety factors at the magnetic axis and at the plasma boundary, $r = (\Phi/\Phi_a)^{1/2}$ is the normalised flux surface radius with Φ being the toroidal flux on the trajectory and Φ_a the value at the plasma boundary, δ_{TF} denotes the TF ripple magnitude [3] for a toroidal coil number of $N=32$ with a minimum value δ_{TF0} ($=5.0\times10^{-7}$) and the parameters $\alpha=8$, $\beta=1$, $A_c=2$; δ_{MHD} represents the MHD perturbation [4] with poloidal and toroidal numbers n and m and a maximum value $\delta_{MHD\max}$($=1.0\times10^{-4}$) localised at $r=r_c$ with the localisation half-width $\Delta_c=0.1$.

Following [8] we rewrite the Hamiltonian equation of motion to the form preserving the Poincaré invariants during long-time integration (more than 10^3 bounce periods). Using the generation function of the third kind, $K(p_0,q)$, determined by

$$p=-\partial K/\partial q, \quad q_0=-\partial K/\partial p_0\,, \tag{4}$$

and expanding $K(p_0,q)$ by means of the time step δt as

$$K=\sum_{\ell=0}^{\infty}\frac{\delta t^{\ell}}{\ell!}K_{\ell}\left(p_0,q\right), \tag{5}$$

the second relation of Eq. (4) can be rewritten into an equation for q [8],

$$q=q_0+\sum_{\ell=1}^{\infty}\frac{\delta t^{\ell}}{\ell!}\frac{\partial K_{\ell}\left(p_0,q\right)}{\partial p_0}\quad , \tag{6}$$

conveniently solved by straightforward iterations. For Eq. (6) we evaluate K up to the third order ($\ell\leq3$) in δt using [8]

$$\begin{aligned} K_1&=H,\\ K_2&=-\frac{\partial K_1}{\partial q}\cdot\frac{\partial H}{\partial p_0},\\ K_3&=-\frac{\partial K_2}{\partial q}\cdot\frac{\partial H}{\partial p_0}+\sum_{i,j=1}^{n}\frac{\partial K_1}{\partial q_i}\frac{\partial K_1}{\partial q_j}\frac{\partial^2 H}{\partial p_{0i}\partial p_{0j}}. \end{aligned} \tag{7}$$

The above treatment can be extended also to the case of time-dependent MHD perturbations.

b) Calculation of transport coefficients

For the calculation of the transport coefficients of fast ions at given energies we follow the numerical approach proposed in [4], i.e.

(i) for any given starting point in phase space $\{r_{max},\lambda\}$ we divide the total time interval of orbit following calculations (about 10^3 bounce periods or $k_t=10^6$ time steps) into n_t subintervals, each containing odd m_t time steps to avoid the effect of regular variations of the toroidal momentum along the trajectory, i.e. $n_t=k_t/m_t$;

(ii) the averaged variation of the particle toroidal momentum within each subinterval is calculated as

$$\overline{p}_{2i} = \frac{1}{m_t}\sum_{j=1}^{m_t}\left(p_2\right)_{(i-1)m_t+j} , \tag{8}$$

where the subscript $(i\text{-}1)m_t+j$ designates the momentary integration interval;

(iii) we determine the diffusion in $\overline{p}_2$ by the coefficient

$$D_{22} = \frac{1}{n_t(n_t-1)}\sum_{i=1}^{n_t}\sum_{j=1}^{i-1}\frac{\left[\left(\overline{p}_2\right)_i - \left(\overline{p}_2\right)_j\right]^2}{\left|t_{(i-1)m_t+0.5(m_t+1)} - t_{(j-1)m_t+0.5(m_t+1)}\right|} ; \tag{9}$$

(iv) to guarantee independence of the initial toroidal angle, D_{22} is averaged over this angle in the range $0 \div 2\pi$;

(v) only trajectories with $n_t>2$ are taken into account.

The effect of pitch-angle scattering on the pitch-angle cosine ξ is modelled by use of a standard Monte-Carlo approach [9]:

$$\begin{aligned}
&\xi_i' = \xi_i - 0.5\cdot\nu_\perp\left(r_i\right)\cdot\chi_i\cdot\Delta t + \sqrt{0.5\cdot\nu_\perp\left(r_i\right)\cdot\lambda_i\cdot b_i}\cdot\Delta w ,\\
&\lambda_i = \left(1-\xi_i^2\right)/b_i , \quad b = B\left(q_1,p_1,q_2,p_2\right)/B_0 ,\\
&\Delta w = \begin{cases} 0 , & 0 \le \chi \le 1/3 \\ -\sqrt{3\cdot\Delta t} , & 1/3 < \chi < 2/3 , \\ \sqrt{3\cdot\Delta t} , & 2/3 \le \chi \le 1 \end{cases}
\end{aligned} \tag{10}$$

where χ is a normally distributed random number in the interval [0,1], Δt represents the time step taken for pitch-angle cosine random perturbation ($\Delta t \ge dt$ =time step for trajectory calculations) and $\nu_\perp$ is the pitch-angle scattering rate.

c) 3D Fokker-Planck model in COM space

The simulation of the combined effect of TF ripples and MHD-perturbations on the confinement of fast ions is based here on the steady-state 3D Fokker-Planck equation in the constant of motion (COM) space [10],

$$\frac{1}{\sqrt{g_{\mathbf{c}}}}\sum_{i,j=1}^{3}\frac{\partial}{\partial c_i}\sqrt{g_{\mathbf{c}}}\left[d^i - D^{ij}\frac{\partial}{\partial c_j}\right]f - \bar{S}(\mathbf{c}) = 0, \quad \mathbf{c} = \{E, \lambda, r_{\max}\}. \tag{11}$$

In Eq. (11) the respective contributions of TF ripples and MHD-perturbations are accounted for by the "radial" diffusion coefficient

$$D^{33} = \left(\partial p_2 / \partial r_{\max}\right)^2 D_{22}. \tag{12}$$

The perturbative contributions to the coefficients D^{ij} other than D^{33} are small and hence neglected here as were the minor variations induced in the convection coefficients d^i.

3. Simulation results

In the collisionless limit the calculated distributions of the diffusion coefficient in the toroidal momentum are plotted in Figs.1a-c. Figure 1a displays the typical resonant structure of unstable motion caused by TF ripples only, which are known to enhance the collisional ripple induced diffusion of fast ions. Figs.1b, c demonstrate the distortions of this structure by additional weak helical perturbations (δ_{MHD}=0.01%) localised in the plasma core at the

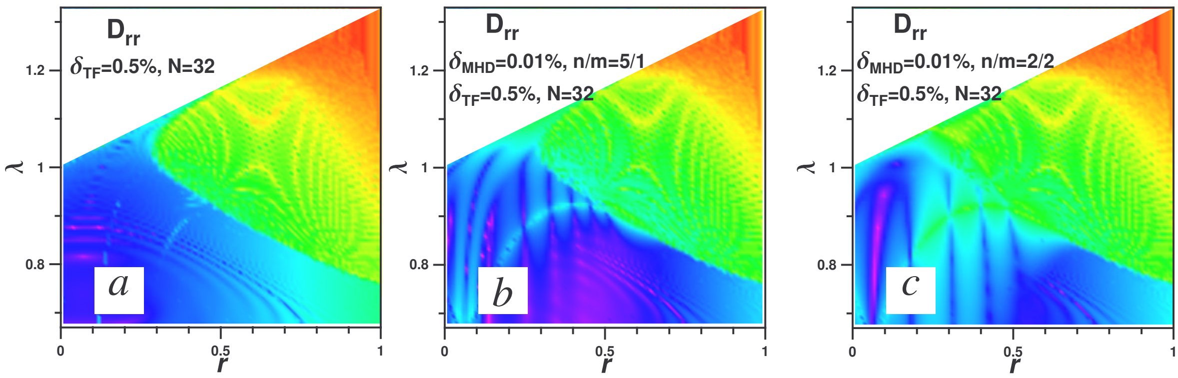

Fig. 1: Fine resonant structure induced by TF ripples (yellow) and its destruction by low n and low m harmonics of additional helical perturbations of the tokamak magnetic field. Blue colour corresponds to stable particle motion and orange to the stochastic domain.

radius r_c=0.3. It is seen that helical harmonics have an effect on both trapped ($1+r/A > \lambda > 1-r/A$) and circulating particles ($\lambda<1-r/A$).

Taking into account Coulomb collisions, the co-action of TF ripples and core localised MHD perturbations in a MHD quiescent TF-rippled plasma is shown to result potentially in both, either an enhancement or a weakening of the radial transport of fast ions.

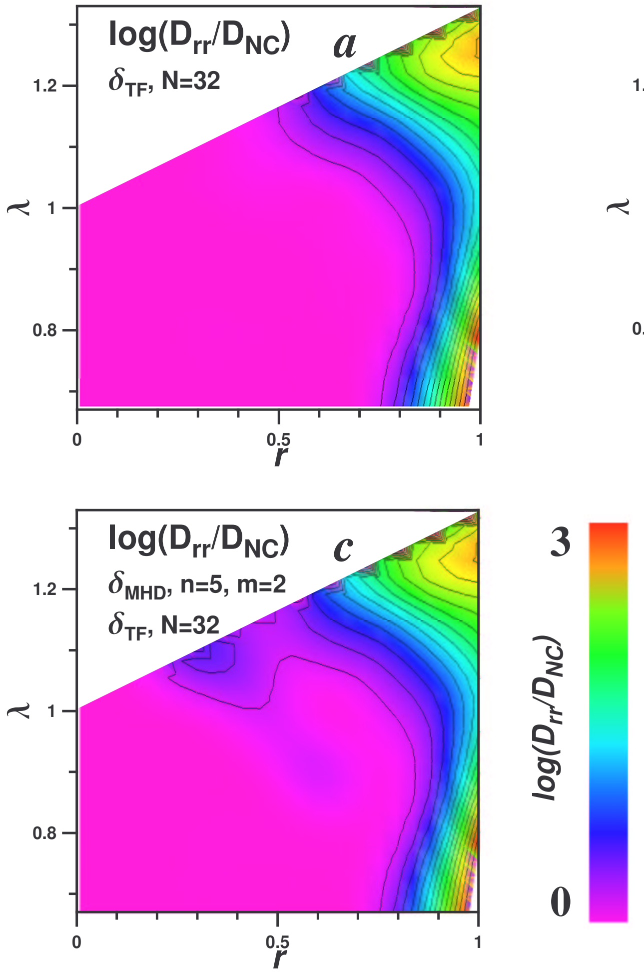

Fig. 2. Ratio of the ripple induced radial diffusion coefficient to the neoclassical one, D_{NC}, in (a) a MHD quiescent plasma and (b,c) in the presence of MHD modes taking δ_{TF}=0.5% and δ_{MHD}=0.05%.

This is seen in Fig. 2 illustrating the ratio of the radial (D_{rr}) and the neoclassical ($D_{\rm NC}$) diffusion coefficient for the cases of a MHD quiescent TF rippled plasma (Fig. 2a) and of co-acting TF ripples (δ_{TF}=0.5%) and MHD modes (δ_{MHD}=0.05%) as displayed in Figs. 2b,c for different mode numbers. Fig. 3 clearly demonstrates the effect of TF ripple and MHD-mode synergy on the collisional radial diffusion of 1.7 MeV alphas [7]. To measure the pure synergistic contribution, neoclassical diffusion has been subtracted for the comparison of co-acting and separate TF ripple and MHD-mode diffusion. As seen in Fig. 3, the synergy of the n=5, m=16

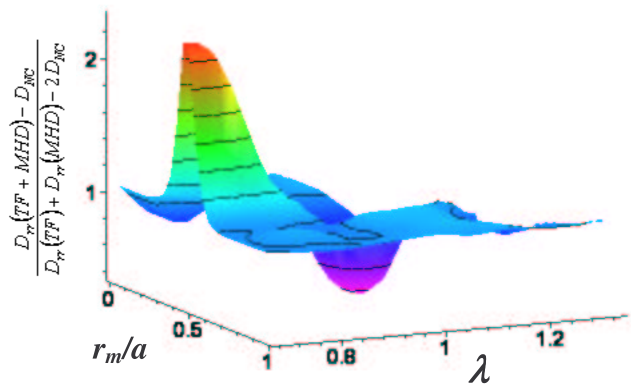

Fig. 3 Synergistic effect of the combined action of magnetic field ripples and MHD perturbation (m=16, n=5) on the radial diffusion rate of fast alphas with E=1.7 MeV.

MHD mode with ripples can lead to a 2.5-fold enhancement of the radial diffusion of marginally circulating alphas with λ~0.7, r_{max}~0.4, whereas for well trapped alphas the radial diffusion is reduced by about 70% at λ~1, r_{max}~0.5. Thus, due to the synergism considered the radial diffusion of fast ions in tokamak plasmas may be both enlarged as well as reduced depending on the position in phase space.

Figs. 4 and 5 present Fokker-Planck simulation results of the effect of combined TF ripple and MHD-perturbations on the confinement of DT fusion alphas in a deuterium

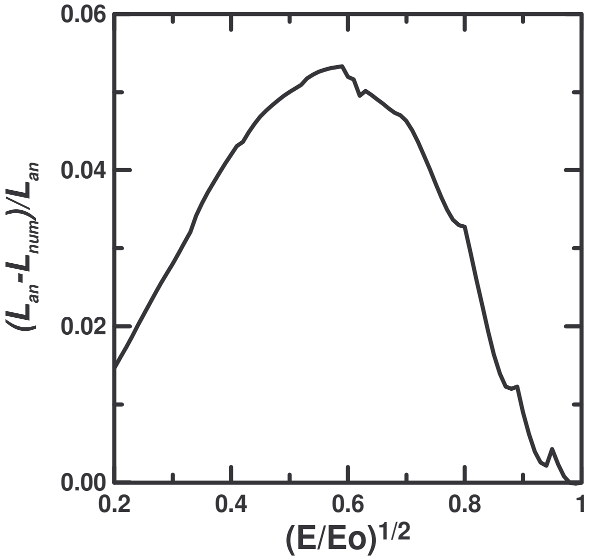

Fig. 4. Comparison of the pure neoclassical alpha loss fraction, once calculated with an analytical radial diffusion coefficient [11] and once with numerical radial diffusion rates derived in this paper.

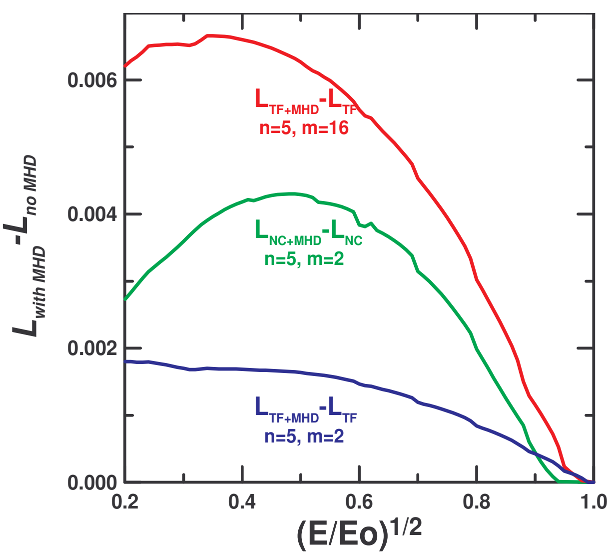

Fig. 5. MHD induced enhancement of alpha loss. The largest effects are seen in the low and intermediate energy range.

tokamak plasma with circular cross section; the results refer to a plasma with monotonic current ($q_a/q_0 = 3/1$) and flat density profile $n = n_0[0.8(1-r^2)^{0.4} + 0.2]$ taking $n_0 = 5 \cdot 10^{13}\, cm^{-3}$, the aspect ratio A=3 and the toroidal magnetic field B=3.5T. Fig. 4 demonstrates a rather small (<6%) deviation of the axisymmetric neoclassical alpha loss fraction calculated with an analytical radial diffusion coefficient from that obtained with the numerical radial diffusion coefficient resulting from the approach described in Sec. 2. Finally, Fig. 5 illustrates the enhancement of alpha loss induced by MHD perturbations. Neoclassical and ripple induced losses of fusion alphas are seen to be enlarged by MHD modes mostly in the intermediate energy range. Note that, in the presence of TF ripples, the MHD induced contribution to the alpha loss is more pronounced for higher toroidal mode numbers.

2. Summary

Using the simplectic method for integration of Hamiltonian systems, an approach is presented for evaluation of fusion alpha confinement in tokamak plasmas with TF ripples and MHD modes. It enables the numerical investigation of fast ion transport with strong inhomogeneities in phase space as induced by resonant interaction of ion motion with magnetic field perturbations. In this context MHD modes also were characterised as perturbing periodically the magnetic field. Coulomb collisions were accounted for in this approach as well. The modelling demonstrates an essential synergistic effect of the two specific perturbations, TF ripples and MHD-modes, on the radial diffusion of fusion alphas. While, in the absence of TF ripples, MHD modes induce radial ion transport additional to the neoclassical diffusion, those modes can both increase as well as reduce the intensity of radial diffusion in the presence of TF ripples. Note that the study of the co-action effect of TF ripples and MHD modes presented here helps to interpret the observations of charged fusion products losses in TFTR [1], where in the presence of MHD activity both degradation as well as improvement of fast ion confinement was detected.

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