Emerging Chaos in Rotation Velocity Profile in Collisional Tokamak Edge Layer

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Since Stringer [1] had shown that any radial location in tokamak plasma is prone to a spontaneous poloidal spin-up with a concomitant rise of a radial electric field, plasma rotation and radial electric field gradually emerged as important factors in stability considerations for toroidal equilibrium and transport. As novel studies indicate, also tokamak turbulence instability due to the geodesic acoustic mode (GAM) or zonal flow have similar mechanisms involving perturbations of the plasma rotation velocities and the radial electric field [2-4]. On the other hand, as plasma collisionality suppresses such instabilities, it is less likely to encounter rotational turbulence, for example, in high collisional plasmas with steep gradients. In the present extension of the neoclassical theory of rotation and electric field in high collisionality tokamak plasmas with steep gradients [5-6], however, it will be shown that in some radial interval the poloidal (and to some extent also toroidal) rotation velocity displays chaotic behaviour.

1. Basic Equations

Using the notations of former publications [5-6], the governing equations for the described regime are, continuity equation for the ions,

$$\partial_t N_i + J^{-1}\partial_\chi (h_\phi h_\psi \Gamma_{\chi,i}) = S_i^{\,N} - J^{-1}\partial_\psi (h_\phi h_\chi \Gamma_{\psi,i}) \tag{1}$$

where $\vec{\Gamma}_i = N_i \vec{U}_i$ and $S_i^{\,N}$ are the ion flux and ion particle source, respectively, and momentum balance equation summed over both species:

$$\partial_i (m_i N_i \vec{U}_i) + \nabla\cdot(m_i N_i \vec{U}_i \vec{U}_i) = -\nabla\cdot\overleftrightarrow{\Pi}_i + \vec{J}\times\vec{B} \tag{2}$$

Here $\overleftrightarrow{\Pi}_i$ is the viscosity tensor including $\overleftrightarrow{\Pi}_{0,i}$, $\overleftrightarrow{\Pi}_{1-2,i}$, $\overleftrightarrow{\Pi}_{3-4,i}$ contributions as defined in Refs. [5-8]. Using a scaling relevant to the tokamak edge, $\varepsilon \sim B_\theta/B_\varphi \sim (qR\nu_J/c_J)^{-1} \sim L_\psi/r \sim r/(qR)$, and by taking the toroidal projection of (2) and averaging over a flux surface, one finally gets the toroidal momentum equation for plasma with circular cross sections

$$\begin{aligned} m_i N_i (1+2q^2)\frac{\partial}{\partial\tau}U_{\theta,i} = {} & q^2 m_i <\tilde{Z}\cos\theta> U_{\theta,i} - \frac{3\eta_{0,i}}{2R^2}\left(U_{\theta,i} + 1.833(e_i B_\phi)^{-1}\frac{\partial T}{\partial r}\right) + 0.54\frac{\eta_{2,i}}{1+Q^2/S^2}q^2 \\ & \times\frac{e_i B_\phi}{T_i}\frac{\partial \ln T_i}{\partial r}\left[\frac{T_i}{e_i B_\theta}\frac{\partial U_\phi}{\partial r} + \frac{1}{2}U_{\phi,i}^{\;2} - U_{\phi,1}\frac{B_\phi}{B_\theta}\left(U_{\theta,i} - \frac{T_i}{e_i B_\phi}\frac{\partial \ln N_i^2 T_i}{\partial r}\right) + 1.90\frac{B_\phi^{\;2}}{B_\theta^{\;2}}\left(U_{\theta,i} - 0.8\frac{T_i}{e_i B_\phi}\frac{\partial \ln N_i^{1.6} T_i}{\partial r}\right)^2\right] - J_r B_\theta \end{aligned} \tag{3}$$

where $\tilde{Z}$ is the poloidal angle dependent part of $Z = S_i^N - J^{-1}\partial_\psi(h_\phi h_\chi \Gamma_{\psi,i})$ which may lead to the Stringer spin-up for a suitable θ dependence of the source and $Q = [4B_\phi U_{\theta,i} - 2.5(T_i / e_i)\partial \ln N_i^2 T_i / \partial r]B^{-1}$ and $S = (2r\chi_{\parallel,i} N_i^{-1})/(q^2 R^2)$, where the parallel heat diffusion coefficient is $\chi_{\parallel,i} = 3.9P_i / m_i \nu_i$. Likewise, one can obtain the neoclassical ambipolarity equation as [5-6]

$$m_i N_i \frac{\partial U_{\phi,i}}{\partial t} = \frac{\partial}{\partial r}\left[\eta_{2,i}\left(\frac{\partial U_{\phi,i}}{\partial r} - \frac{0.107q^2}{1+Q^2/S^2}\frac{\partial \ln T_i}{\partial r}\frac{B_\phi}{B_\theta}U_{\theta,i}\right)\right] + J_r B_\theta \tag{4}$$

Above sytem (3-4) represents a two-time-scales problem, namely, it depends on a fast time variable $\tau = t/\varepsilon$, as well as a macroscopic time t. A solution procedure for this problem was suggested in Ref. [9] specializing r to a boundary layer inside the magnetic separatrix by $\xi = (r - r_s)/L_\psi$. A smooth temperature profile model that depends on parameters μ, T_c/T_s, Δ, simulating a steep gradient before the separatrix is taken with this system.

2. Steady State Equations

Now, we look for the equilibrium solutions of Eq.(3) and Eq.(4) for $U_{\varphi,i}$ and $U_{\theta,i}$, using their lowest order expansions and time independent forms. These can be written as

$$\frac{d\,U_{\varphi,i}}{d\xi} = F(U_{\varphi,i}, U_{\theta,i}, \xi) \tag{5}$$

$$\frac{d}{d\xi}\left[\eta_2 \frac{d\,U_{\varphi,i}}{d\xi}\right] = K\,U_{\varphi,i} + \frac{d}{d\xi}\left[G\,U_{\varphi,i}\right] \quad , \text{ where } \quad K \equiv -m_i N_i \nu_{cx} \tag{6}$$

For the neutral beam injection, one could also include a term like $m_i N_i \dot{m}_{\varphi,i}$ to the left hand side of Eq.(6). Here, this term is dropped for simplicity. Substituting Eq.(5) in Eq.(6), we obtain

$$\frac{d\,U_{\theta,i}}{d\xi} = \frac{K\,U_{\varphi,i} + U_{\theta,i}\dfrac{\partial G}{\partial \xi} - (\eta_2' + \eta_2 \dfrac{\partial F}{\partial U_{\varphi,i}})F - \eta_2 \dfrac{\partial F}{\partial \xi}}{\eta_2 \dfrac{\partial F}{\partial U_{\theta,i}} - U_{\theta,i}\dfrac{\partial G}{\partial U_{\theta,i}} - G} \tag{7}$$

where the derivative w.r.t. ξ is denoted by a prime. Functions F and G in Eqs. (5-6) can be written, after normalizing at the separatrix, as

$$F \equiv -\frac{U_{\varphi,i}^2}{2T} + U_{\varphi,i}\left(\frac{U_{\theta,i}}{T} - \frac{d\ln N^2 T}{d\xi}\right) - 1.9T\left(\frac{U_{\theta,i}}{T} - \frac{d\ln N^{1.6}T}{d\xi}\right)^2 + \frac{T^3}{N^2}\frac{U_{\theta,i} + 1.833T' + \Delta}{0.45(d\ln T/d\xi)}\left(1 + \frac{Q^2}{S^2}\right) \tag{8a}$$

$$G = \eta_2 \frac{0.107q^2 (d\ln T/d\xi)}{(1+Q^2/S^2)}. \tag{8b}$$

A possible radial current is denoted here by Δ. The term K, representing charge exchange effects, is taken in the calculations for simplicity as a constant. In Eq.(7) the individual terms indicating partial derivatives of F and G will not be given here as they are lengthy. To this system of two first order nonlinear ordinary differential equations, Eqs.(5,7), an equation for the temperature variation along ξ must also be added. This will be taken now simply as a monotonic function of the minor radius, in which we can adjust the steepness of the profile by a parameter μ:

$$T(\xi) = \frac{1}{2} T_c \left[1 - \tanh\left(a + \frac{\xi}{\mu} \right) \right], \quad \text{where} \quad a = \tanh^{-1}\left(1 - 2\frac{T_s}{T_c} \right). \tag{9}$$

Above T_c, T_s are values of the temperature at the core and the separatrix, respectively. Similarly, the density profile is assumed to be given by a relationship like $N=T^{1/\gamma}$, where parameter γ can be taken, approximately as $\gamma = 1.6$. Thus, we reduce our system of equations to an autonomous system of first order differential equations:

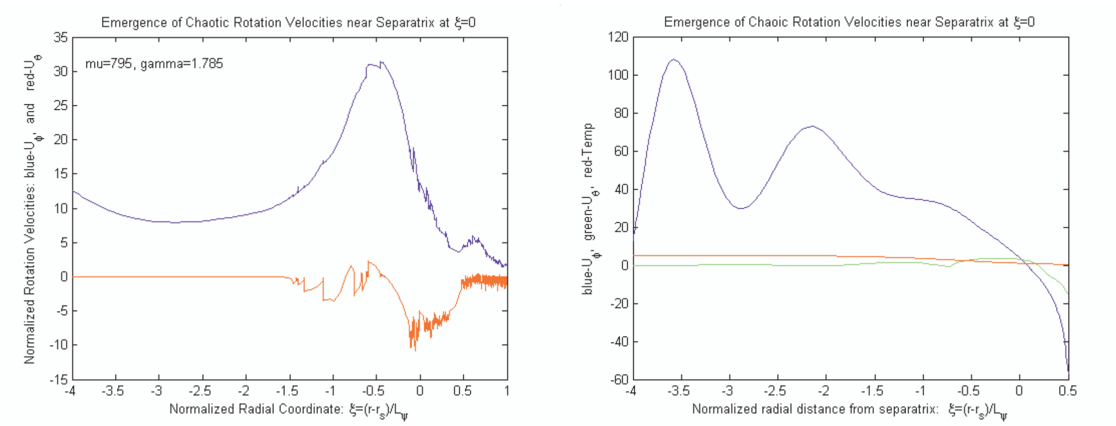
$$\frac{dU_{\varphi,i}}{d\xi} = f_1(U_{\varphi,i}, U_{\theta,i}, T; M), \quad \frac{dU_{\theta,i}}{d\xi} = f_2(U_{\varphi,i}, U_{\theta,i}, T; M), \quad \frac{dT}{d\xi} = f_3(T; M) \tag{10}$$

where M is a vector of control parameters (μ, γ, Tc/Ts, K, $\dot{m}$, Δ,..). The particular model chosen for T in Eq.(9) allows us to express T' and T'', which are implicit in Eqs. (10), also as functions of T, viz.,

$$T' = -(2/\mu)T[1-(T_s/T_c)T], \qquad T'' = (2/\mu)^2 T[1-(T_s/T_c)T][1-2(T_s/T_c)T] \quad .$$

3. Numerical Solutions

While integrating system of Eqs.(10) near the magnetic separatrix, solutions at times are observed to display chaotic oscillations. One also observes that this effect becomes more pronounced when the steepness of the temperature profile is increased, namely, when the control parameter μ is decreased. Such effects are usually due to the bifurcations of the solutions near the fixed points of the vector field $\mathbf{f}_i$ depending on the parameter values.



(a) **(b)**

Figure 1. Depending on the parameters steady state solutions can be highly oscillatory (a), or not (b).

A systematic numerical study for various parameter values indicate that the profiles of $U_{\varphi,i}$ and $U_{\theta,i}$ in a radial interval inside and near the separatrix corresponding to the threshold of the temperature rise become multivalued displaying tendency to entanglement. As this behaviour persists for a broad range of the parameter values, it seems to be generic within the model assumptions, namely, for a collisional edge layer with a steep temperature gradient.
For various boundary value tripples [$U_\varphi(0)$, $U_\theta(0)$, $T(0)$], the dynamical system given by Eqs.(10) can be integrated numerically to yield U_φ, , U_θ and T at an arbitrary location ξ.

Solutions depend on the parameters describing steepness of gradients, composition, and the likely external disturbances on the system. Among such external disturbances, a likely weak periodic excitation of temperature over the radial coordinate would be a likely trigger for another homoclinic orbit bifurcation in our 3-D, saddle-node system. Indeed, the stability of temperature profile itself is sensitive to many physical factors, as shown in Refs.[10, 11] by Bachmann et al.. Namely, in radiative edge plasmas, temperature bifurcations and chaos can be driven by a given time-modulated impurity density. In the light of those studies, it is not unrealistic to add also a weak periodic temperature disturbance superimposed on the monotonous model-T profile with a controllable steepness as assumed here. In that case, further complicated bifurcations in the system would be the outcome. One can further look into the effects of random temporal and spatial perturbations in radial temperature and density profiles in above equations, as they are likely to exist in tokamaks. Perturbation of the regular profiles by such random components can further lead to chaotic behavior of the rotation speeds, as above coupled equations would act like stochasticity amplifier or even as a stochasticity generator. However, a time dependent analysis looking into the stability of the full partial differential system, Eq.(3) and Eq.(4), in the slow-time scale is not attempted here. Although, due to the coupling of governing equations, however, transient U_θ would play a role in driving U_φ .

As some 2-D projections of the numerical solutions for poloidal and toroidal rotation velocities were observed to display multivaluedness, as seen in Fig.2, studies were extended to the bifurcation behaviour in the 3-D phase space due the existing parameters.

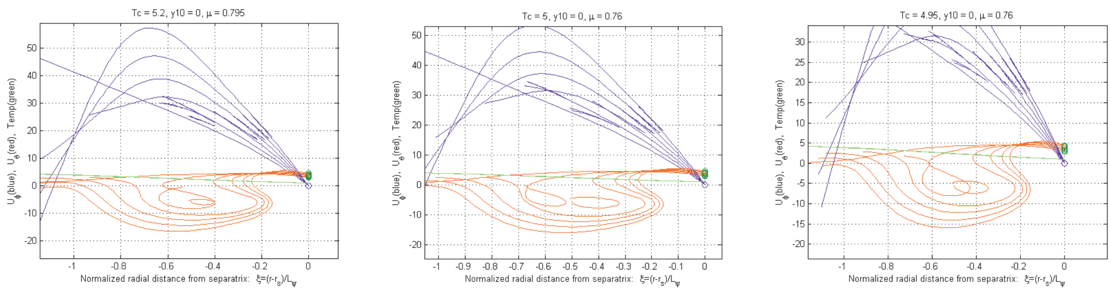


Figure 2-a,b,c: Examples of entanglement of poloidal (lower curves) and toroidal (upper curves) rotation velocity profiles on the core side of the separatrix for various plasma parameters.

Primary parameter to be considered here is μ, which controls the steepness of the assumed temperature profile given in Eq.(9) . Variation of the parameter μ between 0.795 and 4 while keeping other parameters fixed reveals a topological change of the trajectories occupying the phase space, as seen in Figs.3-8. Namely, when μ is gradually reduced, causing the steepness to increase, near μ=2 there arises a dense set of spiraling heteroclinic trajectories on the core side of the separatrix whose osculating planes almost overlap. Each of these orbits leaves an

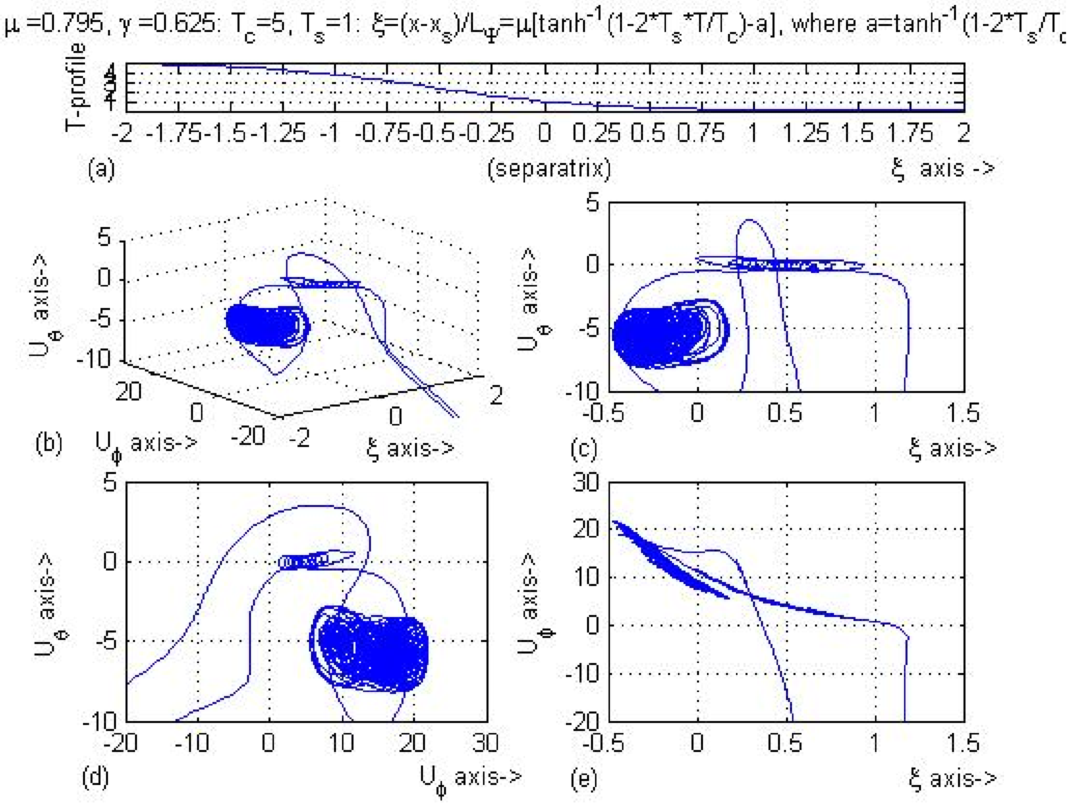


Figure 3. Some representative orbits for a rather steep temperature profile with μ=0.795. On the core side of the separatrix there exists a system of isolated heteroclinic orbits with very close osculating surfaces.

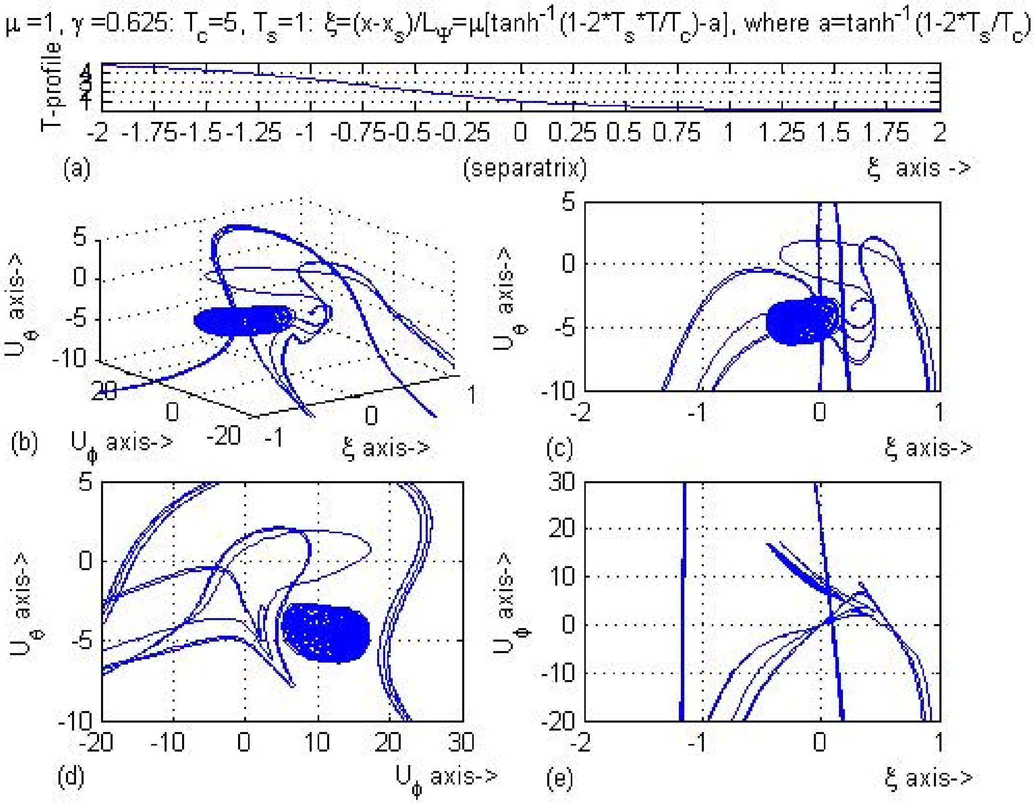


Figure 4. One can clearly see in (d) that the heteroclinic set of trajectories comprise an isolated system. Almost overlapping of the spiraling heteroclinic orbits can be seen from their projection onto the U_φ and ξ plane as depicted in Fig. 4(e).

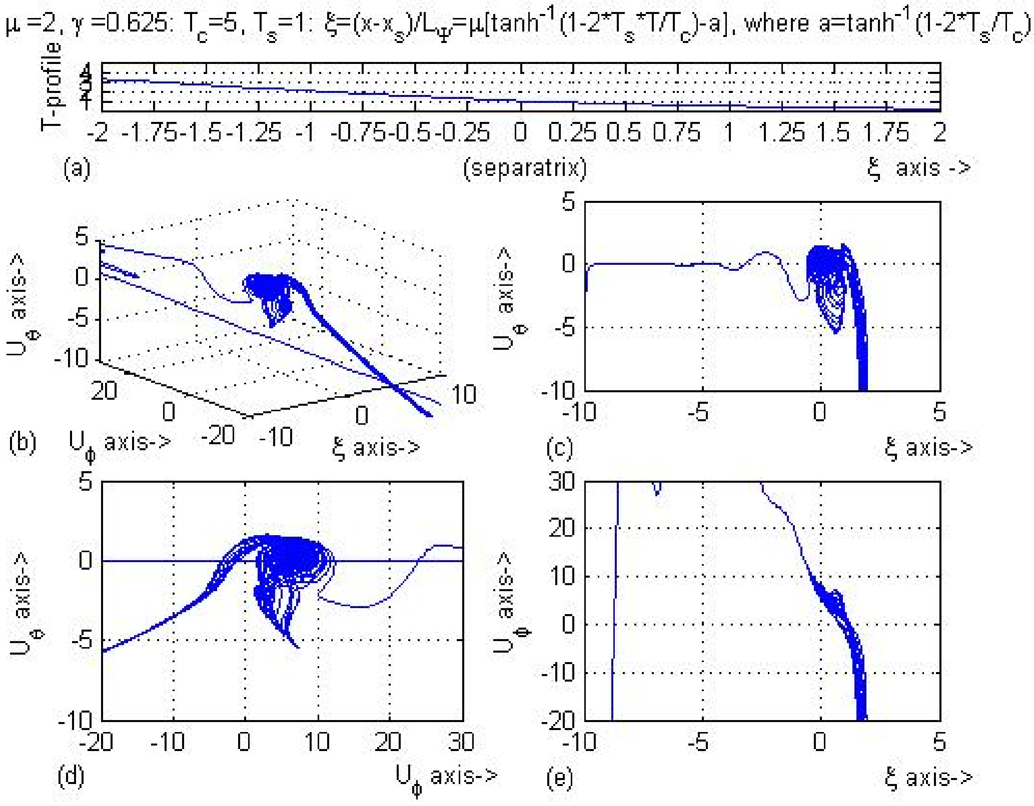


Figure 5. When temperature profile gets less steep (μ=2), the isolated heteroclinic system disappears and orbits coming from distant locations of phase space end up in densely located foci near the separatrix as seen in (b)-(e).

unstable singular point and reach a stable focus or a point attractor. Behaviour of these heteroclinic orbits can be studied by calculating orientation and position of their local osculating planes via their normals $\vec{n}_i = d\vec{r}_i \times d\vec{r}_{i+1} / |d\vec{r}_i \times d\vec{r}_{i+1}|$, and distances to the origin $d_i = \vec{n}_i \cdot \vec{r}_i$. These calculations reveal that the orbital binormals (or normals of the osculating planes) of the heteroclinic orbits induced by steep temperature profiles are almost parallel to the ξ–axis.

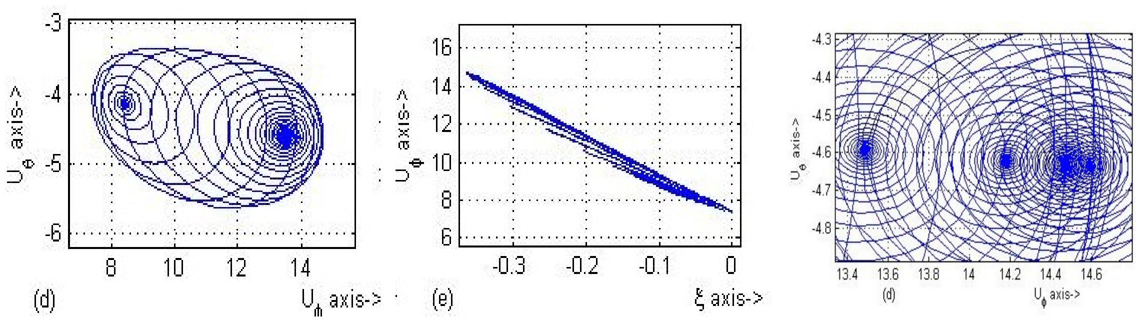


Fig 6. Left picture shows the projection of a particular hetroclinic orbit from Fig. 4-(d). The same orbit is seen in the middle to lie almost on the same plane. An enlarged view of the projection of a few chosen stable foci causing entanglement of the heteroclinic orbits is seen on the right.

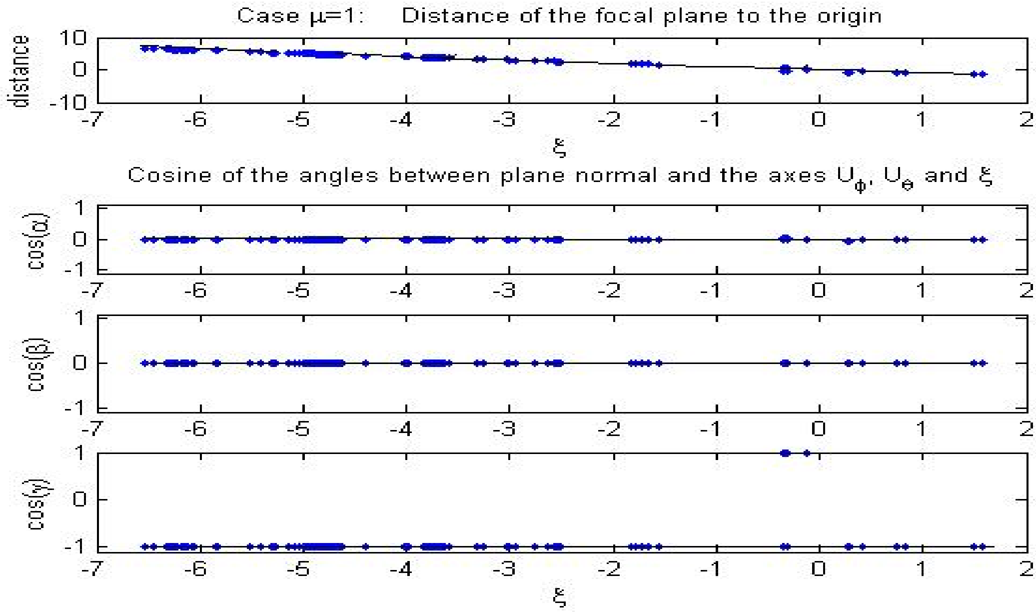


Figure 7. Properties of the focal osculating planes: An exemplary calculation for μ=1 shows that normals of the focal osculating planes are parallel to the ξ-axis and their distance to the origin vary continuously.

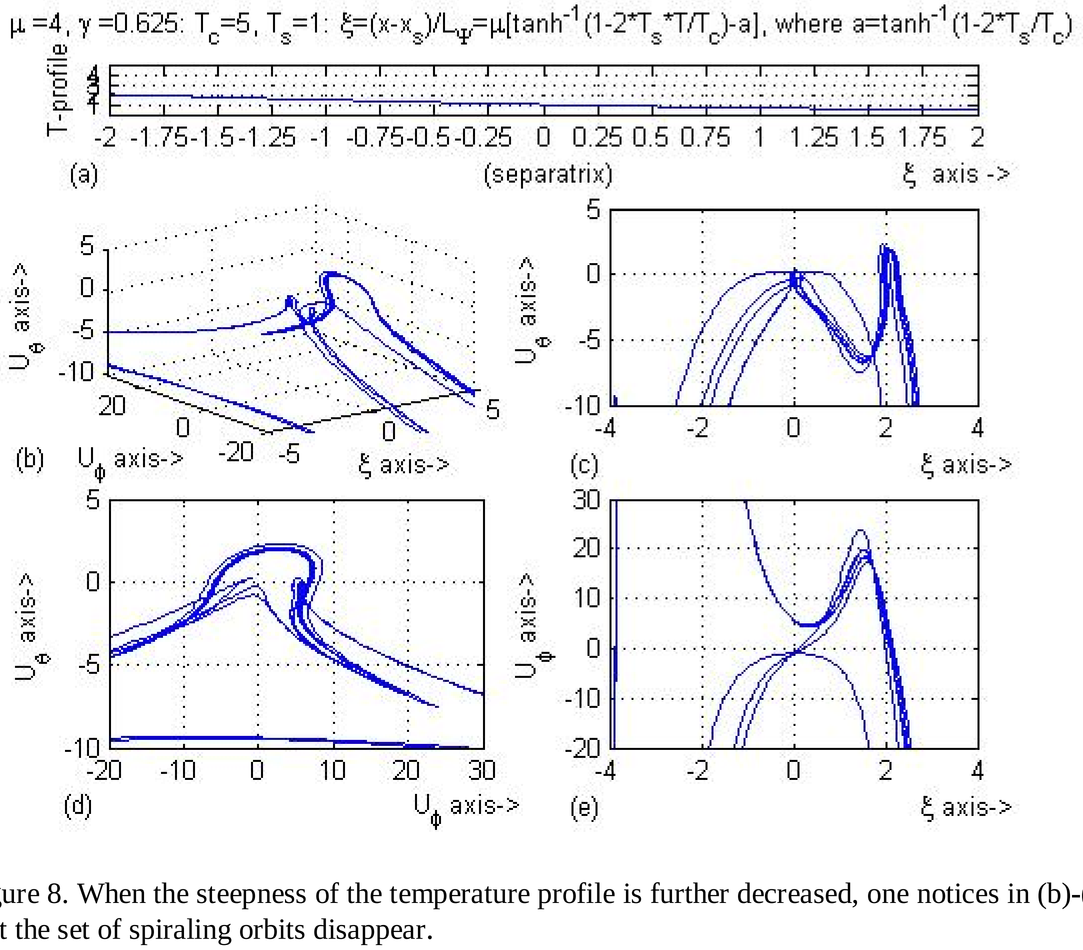


Figure 8. When the steepness of the temperature profile is further decreased, one notices in (b)-(e), that the set of spiraling orbits disappear.

4. Conclusions

Here we have shown that a steep temperature gradient in a highly collisional plasma under discussion can induce a topological change, or a global bifurcation in the phase portrait of rotation velocities. Steepness of the model temperature profile was varied by means of a controlling parameter, μ. At a critical μ, it was shown, that a dense set of singular points appeared in a particular section of the phase space. This section extends from the magnetic separatrix to some short distance on the core side. The heteroclinic trajectories among these singular points were found to form a flat layer. Indeed, the osculator planes of orbits reaching point attractors are found to become parallel to each other. As the line formed by the dense point attractors is almost tangential to these planes, orbits reaching point attractors are rather sheared or entangled w.r.t. the other orbits in their surrounding, namely w.r.t. those moving on the immediately neighboring planes. The shear providing the cause for local entanglement of the trajectories, plasma stream is thus locally in a state of emerging chaos [12]. Since in the present study of stationary field of velocities, particle trajectories and stream lines formed by velocity vectors coincide, chaos of velocity field implies stochastic behaviour of passive particles. Hence, this situation gives rise to what is called Lagrangian turbulence. A highly colisional tokamak boundary layer plasma is thus shown to be able, similar to a weakly collisional plasma, to provide a mechanism for generating turbulence.

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