Computation of Toroidal-Current Reversal Equilibria for the JT60-U Tokamak

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Abstract. In extreme reversed-shear plasma discharges, the ratio between the poloidal and the toroidal magnetic fields obtained from motional-stark-effect measurements displays significant relative errors inside the core region, with the error bar spanning from small positive values into small negative ones, and therefore magnetic equilibria with toroidal-current reversal cannot be excluded. Following a perturbative approach to solve the Grad-Shafranov equation, a numerical scheme allowing toroidal-current reversed equilibria constrained by experimental data to be computed is proposed and subsequently tested using available measurements from JT60-U plasmas in a typical current-hole scenario.

1. Introduction

Contrary to usual plasma configurations, where the toroidal current density $J_{(\phi)}(R,Z)$ distribution is peaked at the magnetic axis, low or even reversed-shear discharges display hollow $J_{(\phi)}(R,Z)$ profiles and are obtained by externally supplied current drive (neutral beam injection or radio-frequency heating). Strongly reversed-shear scenarios have been regarded with increasing interest, as they favor the build-up of internal transport barriers, thus improving heat and particle confinement [1, 2]. Despite the amount of shear reversal achieved, a poloidal field finite everywhere in the plasma column (except at the axis) has always been believed to be necessary for proper plasma sustainment. However, stable configurations have been observed in several experiments, for which the measured poloidal field is nearly zero throughout a significant region around the magnetic axis (the so-called current-hole) [3, 4, 5]. Moreover, such configurations proved to be resilient, keeping the toroidal current density clamped near zero, even if the supplied current-drive power seems sufficient to decrease $J_{(\phi)}(R,Z)$ towards negative values inside the current hole [5] and thus become a toroidal-current reversal (TCR) equilibrium.

These facts raised a number of questions regarding the possible existence of TCR solutions to the Grad-Shafranov (GS) equation [6, 7]. Indeed, the development in such magnetic configurations of a poloidal-field reversal layer, for which the enclosed toroidal current does vanish, poses several problems to conventional GS equilibrium solvers and precludes their use in TCR scenarios. Part of these problems may be traced back to the widespread assumption that magnetic flux surfaces are always nested, which is not generally the case for TCR configurations [8]. Additionally, the occurrence of poloidal-field reversal layers has been shown to impose some constraints on the zeroth-order (in an inverse aspect-ratio expansion) profiles for $J_{(\phi)}(R,Z)$ and for the plasma pressure $p(R,Z)$ [8]. However, recent developments enabled GS codes to tackle poloidal-field reversal layers and to handle a large variety of internal plasma profiles [8], allowing in this way a suitable modeling of experimental data.

In current-hole regimes, the ratio $B_{(\theta)}/B_{(\phi)}$ between poloidal and toroidal magnetic fields, which is obtained from motional-stark-effect (MSE) measurements, display significant relative errors inside the core region, with the error bar spanning from small positive values into small negative ones. Therefore, such uncertainty does not exclude (at least by itself) GS equilibria with toroidal-current reversal, some of which are herein computed using input profiles and boundary conditions constrained by available experimental data from current-hole discharges in the JT60-U tokamak [4]. Far from pretending to be a rigorous equilibrium reconstruction tool, since it is constrained by MSE and plasma-pressure data along a single chord only, the aim of this approach is to show that TCR equilibria compatible with experimental measurements are able to be computed in extreme reversed-shear scenarios, which may aid to understand the physics behind the reported resilience of tokamak magnetic configurations with a current hole [5].

2. Method Description

To solve the GS equation

$$-R^2\nabla\cdot(R^{-2}\nabla\psi) = -J_\phi(R,\psi) = R^2\dot{p}(\psi) + \dot{Y}(\psi), \tag{1}$$

a perturbative approach is used [9], where $J_\phi = RJ_{(\phi)}$, $p(\psi)$, and $Y(\psi) = \frac{1}{2}B_\phi^2(\psi)$ are, respectively, the normalized covariant toroidal current density, the plasma pressure, and the squared poloidal current. This has been adapted to cope with the existence of a poloidal-field reversal layer [8], and the solution for the normalized poloidal flux $\psi(r,\theta)$ is sought in laboratory coordinates r and θ (with r the radial distance to the axis normalized to the minor radius a and θ a poloidal angle measured clockwise from the equatorial plane at the high-field side), avoiding the need to define flux coordinates and, consequently, any beforehand assumption about flux-surface topology. In brief, it involves writing an inverse aspect ratio $\varepsilon = a/R_0$ (with R_0 the major radius) power series for each function in Eq. (1), that is

$$R(r,\theta;\varepsilon) = 1 - \varepsilon\, r\cos\theta, \tag{2a}$$

$$\psi(r,\theta;\varepsilon) = \psi_0(r) + \sum_{n=1}^{+\infty}\sum_{k=0}^{k=n}\frac{\varepsilon^n}{n!}\psi_{nk}(r)\cos k\theta, \tag{2b}$$

$$p[\psi(r,\theta;\varepsilon)] = p_0[\psi_0(r)] + \varepsilon\,\dot{p}_0[\psi_0(r)]\big[\psi_{10}(r) + \psi_{11}(r)\cos\theta\big] + \cdots, \tag{2c}$$

$$Y[\psi(r,\theta;\varepsilon)] = Y_0[\psi_0(r)] + \varepsilon\,\dot{Y}_0[\psi_0(r)]\big[\psi_{10}(r) + \psi_{11}(r)\cos\theta\big] + \cdots, \tag{2d}$$

where the dot stands for a flux derivative $d/d\psi$, and subsequently collect for the same powers of ε. Whilst the zeroth-order term $\psi_0(r)$ relates with the zeroth-order profiles $J_\phi^0[\psi_0(r)] = \dot{p}[\psi_0(r)] + \dot{Y}[\psi_0(r)]$ by means of the nonlinear relation

$$r^2\psi_0''(r) + r\psi_0'(r) = r^2 J_\phi^0[\psi_0(r)], \tag{3}$$

the higher-order perturbations $\psi_{nk}(r)$ are computed solving the hierarchy of linear differential equations

$$r^2\psi_{nk}''(r) + r\psi_{nk}'(r) + \big[s(r) - k^2\big]\psi_{nk}(r) = b_{nk}(r), \quad \text{for} \quad s(r) = -r^2\dot{J}_\phi^{(0)}[\psi_0(r)], \tag{4}$$

where the source term $b_{nk}(r)$ couples lower-order perturbations $\psi_{mk}(r)$ and flux derivatives $d^m p/d\psi^m$ and $d^m Y/d\psi^m$ only, with $m < n$, and the boundary conditions $\psi_{nk}^* = \psi_{nk}(r^*)$ at some $r^* > 0$ must be provided for $k \geq 2$.

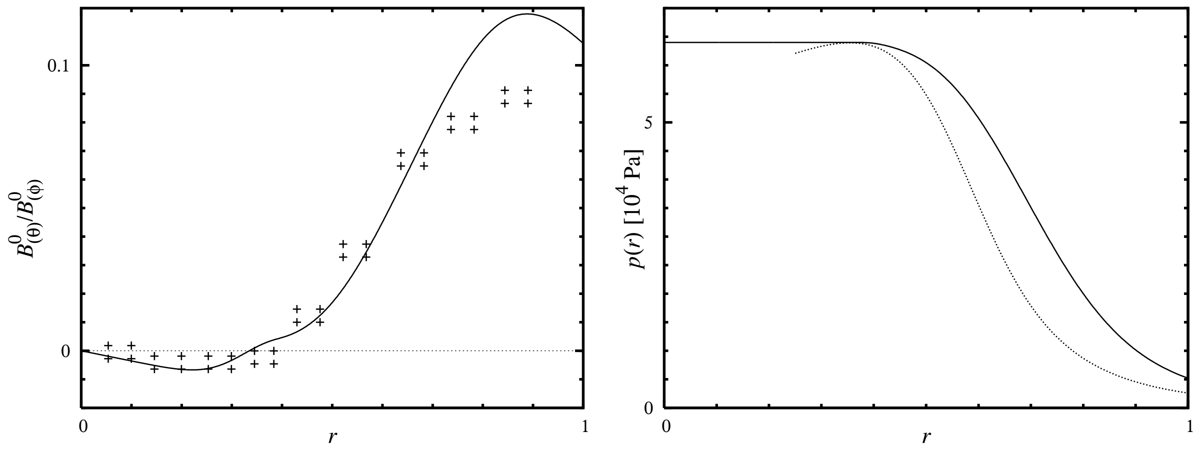

FIG. 1. Zeroth-order ratio $B^0_{(\theta)}(r)/B^0_{(\phi)}(r)$ (solid line, left panel) and plasma pressure $p_0(r)$ (solid line, right panel), with MSE measurements as extracted from Ref. [4] (crosses, left panel) and the function $n_e(r)[T_e(r)+T_i(r)/Z_{eff}]$ as defined in Eq. (7) (dotted line, right panel).

The zeroth-order part of the equilibrium is fully established by setting two of the four profiles $\psi_0(r)$, $p_0[\psi_0(r)]$, $Y_0[\psi_0(r)]$, or $J^0_\phi[\psi_0(r)]$. Unlike regular equilibria, where these can be supplied as free radial functions and their flux derivatives computed using the recursive operator

$$\frac{d^n}{d\psi^n} = \frac{1}{\psi_0'}\frac{d}{dr}\left(\frac{d^{n-1}}{d\psi^{n-1}}\right) \tag{5}$$

as many times as needed [9], in TCR configurations $\psi_0'(r)$ vanishes at the reversal layer $r = r_\mathrm{L}$ and models of the type $J^0_\phi(\psi; a_1, \ldots, a_i)$ and $p(\psi; b_1, \ldots, b_j)$, depending on a few parameters $a_1, \ldots, a_i$ and $b_1, \ldots, b_j$ which are chosen to get a suitable solution to Eq. (3), must be provided [8]. To this end, let

$$J^0_\phi(\psi_0) = \begin{cases} -\frac{7}{10} - (90\psi_0)^3 & \Leftarrow r \le r_\mathrm{L} \\ \frac{375\Delta + 1000\Delta^2}{1+65\Delta-200\Delta^2+600\Delta^3} + f(\frac{1}{2}, 10^{-3}, \Delta)\sum_{p=0}^{4} a_p\Delta^p & \Leftarrow r > r_\mathrm{L} \end{cases} \tag{6a}$$

$$p_0(\psi_0) = \begin{cases} 40 & \Leftarrow r \le r_\mathrm{L} \\ 40\exp(-6\Delta) + f(1, 10^{-3}, \Delta)\sum_{p=0}^{5} b_p\Delta^p & \Leftarrow r > r_\mathrm{L} \end{cases} \tag{6b}$$

with the switch-like function $f(\alpha, \Delta_0, \Delta) = \frac{1}{2}\{1 + \tanh[\alpha^{-1}(1 - \Delta/\Delta_0)]\}$ and $\Delta = \psi_0(r) - \psi_0(r_\mathrm{L})$. The sets of parameters $a_0, \ldots, a_4$ and $b_0, \ldots, b_5$ are chosen in order to ensure the continuity of $J^0_\phi(\psi_0)$ and $p_0(\psi_0)$, along with their flux derivatives up to the fourth and fifth orders respectively, over the reversal layer. Inserting the proposed model for $J^0_\phi(\psi_0)$ into Eq. (3), one may solve the latter for $\psi_0(r)$ and afterwards compare the zeroth-order ratio $B^0_{(\theta)}(r)/B^0_{(\phi)}(r) = \psi_0'(r)/\sqrt{2Y_0(r)}$ with available MSE data and the zeroth-order plasma pressure $p_0[\psi_0(r)]$ with the function $n_\mathrm{e}(r)[T_\mathrm{e}(r) + T_\mathrm{i}(r)/Z_\mathrm{eff}]$ where

$$n_\mathrm{e}(r) = 1 + \frac{1.65 + 0.45r}{1 + 47r^8}, \quad T_\mathrm{e}(r) = \frac{5.15 + 2.74r}{1 + 7.51r^5}, \quad \text{and} \quad T_\mathrm{i}(r) = \frac{7.55 + 2.67r}{1 + 15.3r^7} \tag{7}$$

are rational functions that best fit measured electron density (m^{-3}) and electron and ion temperatures (KeV) [4], which are valid for $r \ge 0.25$ only, and $Z_\mathrm{eff} = 1$ (Fig. 1).

At first glance, the proposed zeroth-order profiles seem to afford only a rather crude approximation to experimental data. Nonetheless, one must bear in mind that whenever

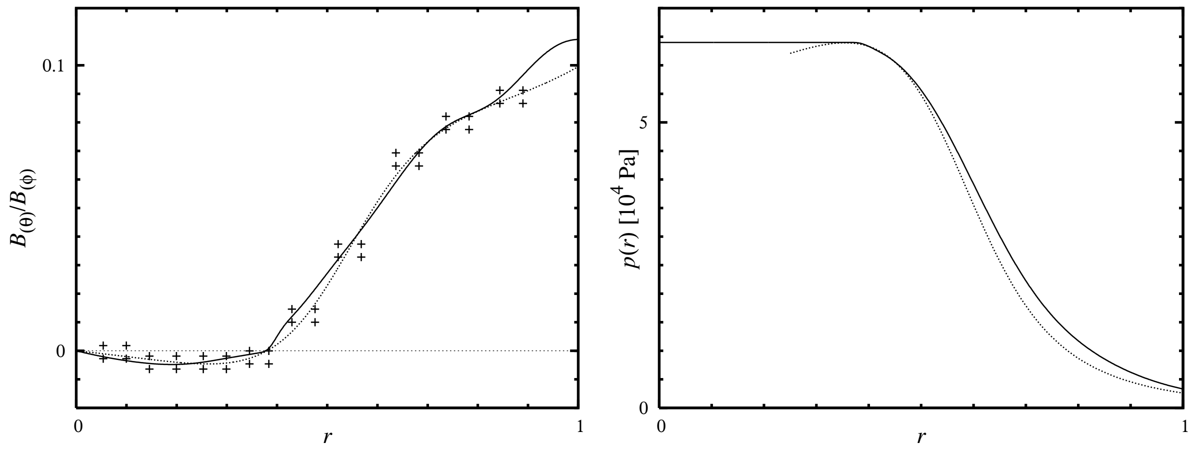

FIG. 2. Third-order ratio $B_{(\theta)}(r,\pi)/B_{(\phi)}(r,\pi)$ (solid line, left panel) and plasma pressure $p(r,\pi)$ (solid line, right panel), with the best-fit function $h(r)$ as defined in Eq. (9) (dotted line, left panel). Other items as in Fig. 1.

an higher-order perturbation is computed and added to the considered solution $\psi(r,\theta)$ the profiles $B_{(\theta)}(r,\pi)/B_{(\phi)}(r,\pi)$ and $p[\psi(r,\pi)]$, along the chord $\theta=\pi$, will change accordingly. Therefore, the aim is to somehow direct these changes in a way that minimizes the difference between numerically computed and measured quantities. To achieve this, one starts noting that if everything else is kept fixed, equilibria accurate up to third-order terms in ε are determined by the three values ψ^*_{22}, ψ^*_{32}, and ψ^*_{33}, which are assigned to the boundary conditions of the terms $\psi_{22}(r)$, $\psi_{32}(r)$, and $\psi_{33}(r)$ at $r^*=1$ when solving Eq. (4). Once a given equilibrium is computed, a three-valued cost function is defined as

$$\chi^2(\psi^*_{22},\psi^*_{32},\psi^*_{33}) = \frac{\int_0^1 [B_{(\theta)}(r,\pi)/B_{(\phi)}(r,\pi) - h(r)]^2\, dr}{\int_0^1 h^2(r)\, dr}, \tag{8}$$

where $B_{(\theta)}(r,\theta) = R^{-1}\partial\psi(r,\theta)/\partial r$, $B_{(\phi)}(r,\theta) = R^{-1}\sqrt{2Y(r,\theta)}$, and

$$h(r) = -\frac{r}{100}\,\frac{19-49r}{10-37r+48r^2-18r^3} \tag{9}$$

is a radial function that best fits the measured MSE data. Then, standard nonlinear minimization algorithms may be employed to find boundary conditions yielding local minima for χ.

3. Results

After implementing the procedure described above using the Nelder-Mead Simplex search algorithm [10], a minimum for the cost function defined in Eq. (8) was found at $\psi^*_{22}=23$, $\psi^*_{32}=-255$, and $\psi^*_{33}=130$. The ratio $B_{(\theta)}/B_{(\phi)}$ and the plasma-pressure profiles for the corresponding equilibrium are depicted in Fig. 2, where they are seen to provide a suitable approximation to measured data along the chord $\theta=\pi$. In Fig. 3, the contours of constant poloidal flux $\psi(r,\theta)$ are displayed, with the island system which unfolds around the reversal layer being clearly visible. When compared against other available equilibrium reconstructions (e.g. Fig. 1 in Ref. [4]), the flux surfaces seem a bit distorted, with accentuated triangularity and misplaced external x-points. However, this should be expected because a) the only constraint was made along a single chord, b) no data other

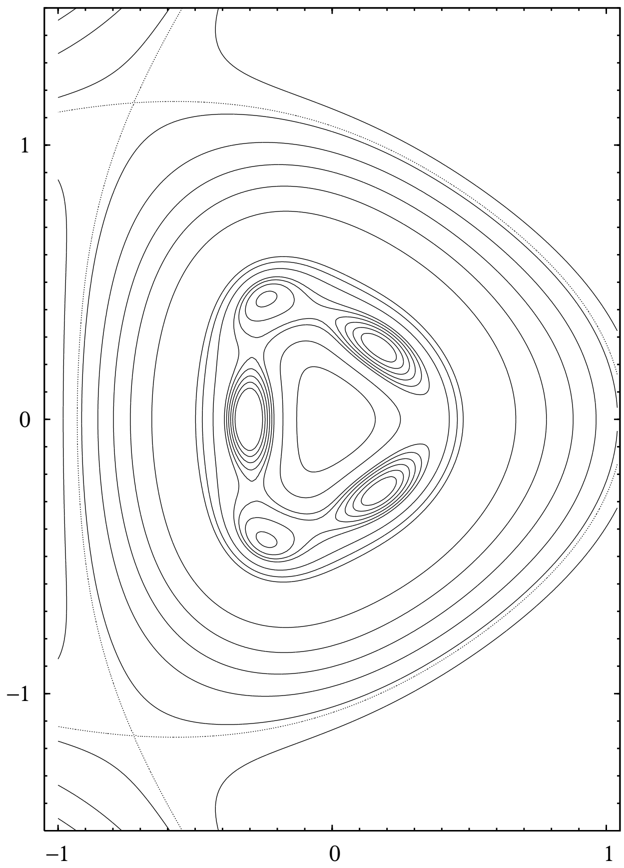

FIG. 3. Contour levels of the poloidal-flux function $\psi(r, \theta)$ (solid lines) and the external separatrix (dotted line).

than MSE and plasma pressure where used, and c) up-down symmetry was assumed, all these intended to keep the formalism as simple as possible. Adding more information as, for instance, the shape of the last closed flux surface, is likely to improve computed equilibria. Moreover, the internal x-point located at low-field side appears to be related with the flat pressure profile prescribed to the core zone, and further steps should be taken to assess if a non-flat pressure distribution can transform it into an o-point. This would contribute to prevent the inner negative current channel from being pushed outwards, where it can reconnect with the positive-current plasma [11]. Finally, the distribution of the toroidal current density along the equatorial plane is plotted in Fig. 4. This plot shows that a moderate negative current density of the order of $-250\,\mathrm{kA\,m^{-2}}$ flowing near the axis ($r \leq 0.1$) accounts for a small ratio $B_{(\theta)}/B_{(\phi)} \approx 3 \times 10^{-3}$.

4. Conclusions

For the first time, TCR equilibria were computed using available experimental data from tokamak plasma discharges with extreme reversed shear. Although the proposed scheme should not be taken has a rigorous equilibria reconstruction tool, its purpose is twofold: In the first place, it shows that TCR equilibria are possible, within measurement errors, in current-hole scenarios for which the toroidal current has been assumed to be nearly zero.

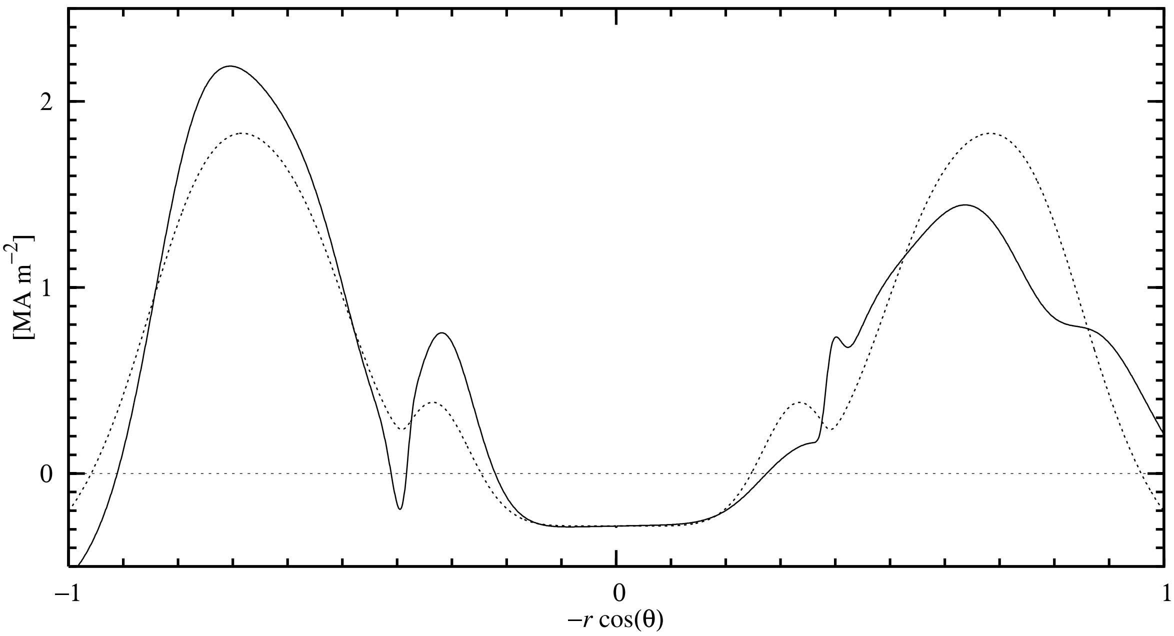

FIG. 4. Toroidal current density distribution along the equatorial plane ($\theta = 0$ and $\theta = \pi$) for the zeroth-order (dotted line) and the third-order (solid line) equilibrium.

Secondly, it illustrates how such equilibria can be computed and related with experimental data, opening the way for more elaborate approaches to be devised.

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