Explosive Growth and Nonlinear Dynamics of the Forced Magentic Island

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Abstract. Novel features of the explosive growth and the nonlinear dynamics of the forced magnetic island subject of the suppression by plasma rotation are studied by nonlinear MHD simulations. The regime of the explosive growth of the externally driven magnetic island was found. This regime occurs after the plasma rotation has been sufficiently reduced due to the interaction with the external field. It is shown that the explosive growth is associated with changes in the structure of the magnetic island and appearance of the localized plasma current around the X-point. It was found that the presence of the localized plasma current sheet. As the result, the long term evolution of the forced magnetic island, such as , e.g., the seed island for the neoclassical tearing modes (NTM), is dominated by the secondary reconnection as the resistivity becomes small.

1 Introduction

It has been well recognized that, in tokamak plasmas, magnetic islands resonant with the low q rational surface deteriorate plasma confinement. The formation of the magnetic islands is a critical issue severely affecting the performance of the reactor [1]. Hence, the suppression and control of the magnetic island is the urgent subject in a tokamak fusion research[2]. There are several principal routes for the origin of the magnetic island. One mechanism is related to the unstable tearing mode (driven by the gradient of the equilibrium current and/or by the pressure gradient); the other mechanism is due to the forced magnetic reconnection from external perturbations such as the residual error field in the magnetic coils or MHD activities. The latter process can be an important source of the seed island for NTM, where the MHD event such as the sawtooth oscillation also acts as the external perturbation for the target mode through the toroidal mode coupling. The initial dynamics of the externally forced magnetic island suppressed by the plasma rotation, in particular, the threshold amplitude of the external perturbation has been studied theoretically [3, 4]. However, less attention has been paid to the subsequent long term behavior of the forced island. These latter processes are important for understanding the overall island effect on the tokamak confinement. In this paper, we study the overall process of nonlinear dynamics of magnetic island due to the growing external perturbation in rotating tokamak plasmas.

2 Model equations

In order to investigate the time evolution of the magnetic island caused by the externally applied magnetic flux perturbation in the rotating plasma, the resistive reduced magnetohydrodynamics (MHD) equations are used in the cylindrical geometry,

$$\frac{\partial}{\partial t}\Psi = \frac{1}{r}[\Psi,\phi] + \frac{B_0}{R_0}\frac{\partial}{\partial\varphi}\phi + \eta J \tag{1}$$

$$\frac{\partial}{\partial t}U = \frac{1}{r}[U,\phi] + \frac{1}{r}[\Psi,J] + \frac{B_0}{R_0}\frac{\partial J}{\partial\varphi} + \mu\nabla_{\perp}(U-U_0)$$

$$J = \nabla_{\perp}^2\Psi, \ U = \nabla_{\perp}^2\phi.$$
(2)

Here, Ψ is the poloidal flux function, ϕ is the flow potential, J is the plasma current, U is the vorticity, φ is the toroidal angle, B_0 is the toroidal magnetic field at the magnetic axis, R_0 is the aspect ratio and set to 10 in this study. These equations include the resistivity η and the viscosity μ . U_0 is the initial background component of the vorticity U. The last term in eq.(2) is added to keep the background flow to the initial profile. In these equations, the parameters are normalized by the plasma minor radius a, the poloidal Alfven time $\tau_{pa} = \frac{\sqrt{\rho a}}{B_{\theta}(a)}$, where the plasma density ρ is set to 1. The resistivity η is normalized such that $\eta = \frac{\tau_{pa}}{\tau_{\eta}}$, where τ_{η} is the plasma skin time. In this study, we treat the q=2 resonant surface, which is tearing stable $\Delta' < 0$. Hence, the single helicity of m/n = 2/1 is assumed for simplicity. The background flow is introduced by the m/n = 0/0 flow potential $\phi_{0/0} = -\frac{2\pi}{\lambda \tau}(1 - r^{\lambda})$. In this study, λ is set to 2. Hence, the plasma surface as the linearly increasing poloidal flux function, $\psi_{2/1}(r = a) = \psi_{2/1}(a) \cdot (t - t_0)$. Here, $\psi_{2/1}(a)$ is the increasing rate of the poloidal flux function at the plasma edge.

3 Linear critical value for the onset of the rapid island growth

One of the important problems of the magnetic island evolution by the external perturbation in the rotating plasma is the onset of the rapid growth of the magnetic island width.

Several theoretical works were carried out to understand the critical value ζ_c beyond which the magnetic island shows the rapid growth. In these works, the balance between the externally applied magnetic flux and the dissipation at the resonant surface is assumed treating this problem in the steady or quasi steady state. Hence, in order to check these theoretical results, the external perturbation is applied slowly in the simulation study. Such a situation is consistent with the quasi-static error field. In the case of the MHD event, like as the sawtooth or ELM, however, the magnetic flux is applied to the target mode at the different time scale through the toroidal mode coupling.

Figure 1 shows the time evolution of the magnetic island width and the background flow at the initial resonant surface q = 2 for the different resistivity η .



FIG. 1: Time evolution of the magnetic island width (solid curve) and the background poloidal flow (broken curve) at the resonant surface for $\eta = 5 \times 10^{-5}, \ 10^{-5}, \ 5 \times 10^{-6}.$

As shown in Fig.1, the background flow at the initial q = 2 surface decreases as the magnetic island width increases. There is a clear correlation between the occurrence of the

explosive growth regime and the reduction of plasma rotation at the resonant magnetic surface. This feature is consistent with the previous studies[3].

Figures 2(a) and (b) show the dependence of ψ_{crit}^{ext} on the resistivity η and that on the viscosity ν , respectively. According to the previous study[3], the parameter regime used in this study is categorized to the nonlinear inertial regime.

this study categorized is As shown in Fig.2(a), the exponent α , where it is assumed that $\zeta_{crit} \propto \eta^{\alpha}$, changes from -0.179 to -0.359 by changing the increasing rate of the external perturbation from $\frac{d}{dt}\Psi = 10^{-6}$ to 10^{-7} . On the other hand, in the parameter regime used in this study, the critical value ψ_{crit}^{ext} does not show clear dependence both on the viscosity and the increasing rate of the external perturbation, as shown in Fig.2(b). In the parameter regime used in this study, the onset of the rapid magnetic island growth is triggered by the reduction of the background flow, which is consistent with the former theoretical and numerical simulation results, while the dependence of the critical external perturbation on the resistivity η changes by



FIG. 2: (a) Dependence of the trigger timing on the resistivity η . (b) Dependence of the trigger timing on the viscosity μ . The trigger timing is proportional to the critical external perturbation. The closed circle corresponds to the case of $\dot{\psi}^{ext} = 10^{-6}$ and the closed square to $\dot{\psi}^{ext} = 10^{-7}$.

the increasing rate of the external perturbation and that on the viscosity ν is weak.

4 Nonlinear growth of the forced magnetic island

In the previous section, we investigated the effects of the resistivity η and viscosity ν on the critical value ψ_{crit}^{ext} . As shown in Figs.2, the η dependence is clearly pronounced while, for our parameters, we do not observe substantial dependence on ν . In this section we study the effects of the varying resistivity on the long term evolution of the magnetic island, in particularly, on the nonlinear stage. Figure 3 shows the time evolution of the magnetic island width for the different η in the range $\eta = 5 \times 10^{-5} \sim 10^{-7}$. Figure 4 shows the time evolution of the magnetic island width and the poloidal angle of the O and X-points for $\eta = 10^{-6}$. In the high resistivity regime, $\eta \ge 5 \times 10^{-6}$, one can clearly see three phases in the time evolution of the magnetic island width; the flow-suppressed growth phase (phase A), the nonlinear rapid growth phase (phase B) and the Rutherfordlike growth phase (phase C). In the flow-suppressed phase, which is from t = 0 to $t \simeq 1600$ for the $\eta = 10^{-5}$ case, the magnetic island has a finite phase difference with respect to the externally applied perturbation, $\psi^{ext}(a)$. In this phase, the island grows gradually while remaining at relatively low amplitude. After ψ^{ext} exceeds the critical value ψ^{ext}_{crit} , the magnetic island enters the fast growth phase which is from $t \simeq 1600$ to $t \simeq 2400$ for the $\eta = 10^{-5}$ case. In this phase, the magnetic island also moves in the poloidal direction and locks to the externally applied perturbation. These features are also well seen in the evolution of the X and O points as shown in Fig.4. After that, the magnetic island enters the slow growing phase, which is from $t \ge 2400$ for the $\eta = 10^{-5}$ case. In this phase, the flow potential $\phi_{2/1}$ is very close to the flux function [7]. These features of the magnetic island evolution are consistent with the previous theoretical and simulation results [3]. These phases are well pronounced in the dependence of the magnetic island width in time.

One can see that in the flow suppressed case the island is shifted in phase from the

external perturbation. In this stage, the X- O point phase difference is $\pi/2$ which corresponds to the standard X-point configuration on the separatrix. In the second stage, the X-O point phase difference changes pointing out the formation of the current sheet and Y-type structure [5]. One can see from Fig 4 that this stage terminates when the X-O point phase difference becomes stochastic oscillating function. We relate this with the instability of the current sheet leading to the formation of secondary islands[5]. As it follows from Fig. 4, this transition stage is followed by the slow growth (Rutherford) stage in which the X-O point phase difference becomes regular again and equal to $\pi/2$. The transition stage is clearly pronounced in the low resistivity regime, $\eta \leq 10^{-6}$. In the low resitivity regimes, the growth rate of magnetic island width is reduced as η becomes smaller and the magnetic island width shows oscillation.



FIG. 3: Time evolution of the magnetic island FIG. 4: Time evolution of the Owidth for the different resistivity, $\eta = 5 \times 10^{-5} \simeq \frac{\text{point}, X\text{-point and the magnetic island}}{\text{width for } \eta = 10^{-6}}$.

Figure 5 shows the temporal growth rate $\gamma_W = \frac{dW}{dt}$ as a function of the resistivity η . As shown in Fig.5, by assuming $\gamma \propto \eta^{\alpha}$, $\alpha \simeq 0.55$, which is close to 1/2, in phase A. This means that the magnetic island width increases via the resistive diffusion while the externally applied torque is balanced by the plasma flow inertia. In the fast grow stage, our simulations show weak dependence of γ_W on η and $\alpha \simeq 0.14$ in phase B. This is reminiscent of the phase instability regime [6] that is probably important for the phase flip and the magnetic island locking to the external perturbation.

Figures 6 show the contour plots of the helical flux function, i.e. magnetic island, at each time. It is worth noting that in the flow-suppressed growth phase (phase A), at t=500 and 1800, the magnetic island is deformed by the background flow. In this phase, the magnetic island is asym-



FIG. 5: Dependence of the growth rate of the magnetic island width during the phase I and II on the resitivity η .

metric to the resonant surface. Such deformation is caused by the out of phase component of the applied torque. During the nonlinear rapid growth phase (phase B), at t=2200 and 2400, the magnetic island deformation is further enhanced. In this phase, the magnetic island shows asymmetry both in the radial and poloidal directions. This island deformation stretches out the initial X-type reconnection region into the Y-type reconnection region. Such a process was also recently reported in [8]. As shown in Fig.6(e), at t=2600,



the second islands are formed at the Y-type reconnection region.

FIG. 6: Contour plots of the helical flux, i.e. the magnetic island, at (a)t=500, (b)t=1800, (c)t=2200, (d)t=2400, (e)t=2600.

5 Torque analysis

In this section we consider the contribution of various terms to the total momentum balance. Based on the equation of motion of plasma fluid,

$$\rho(\frac{\partial}{\partial t}\vec{V} + \vec{V} \cdot \nabla \vec{V}) = -\nabla P + \vec{J} \times \vec{B} + \mu \nabla^2 \vec{V}$$
(3)

we introduce the following components of the torque in the poloidal direction: electromagnetic torque : $T_{\theta}^{EM}(r,\theta) = r\vec{e_{\theta}} \cdot \vec{J} \times \vec{B}$, nonlinear inertial torque $T_{\theta}^{I} = r\vec{e_{\theta}} \cdot (\vec{V} \cdot \nabla)\vec{V}$, and viscous torque: $T_{\theta}^{V} = r\mu\vec{e_{\theta}} \cdot \nabla^{2}\vec{V}$. The total torque is defined as a sum of the above components: $T_{\theta}^{T} = T_{\theta}^{EM}(r,\theta) + T_{\theta}^{V}(r,\theta) - T_{\theta}^{I}(r,\theta)$. Here, $\rho = 1$ is assumed. The total torque is balanced by the non-stationary part of the inertial torque: $r\vec{e}_{\theta} \cdot \partial \vec{V} / \partial t$. Figures 7 show the contour plots of the magnetic island and the total torque. Red color shows positive values, where the torque enforce the fluid element to move in the positive poloidal angle direction, blue color shows negative values, where the torque enforce the fluid element to move in the negative direction. As shown in fig. 7(a), around the O-point, the total torque becomes positive and, around the X-point, it becomes negative. This means that plasma is forced in the opposite directions around X- and O-points. In the flow-suppressed phase (phase A), the magnetic island has a finite phase difference with respect to the external perturbation. In the nonlinear rapid growth phase (phase B), the magnetic island shifts in the poloidal direction so that the displacements of the O- and X- points are different. This causes the Y type point near the separatrix. In the transition phase (phase C), there are positive and negative regions of the total torque within the magnetic island separatrix. This non-monotonic torque profile within the magnetic island separatrix causes the complex magnetic island motion. In the Rutherford-like phase (phase D), the positive and negative region of the total torque at the same flux surface within the magnetic island separatrix are balanced so



FIG. 7: Contour plots of the magnetic island and the total torque at (a)t=1000, (b)t=1800, (c)t=2600 and (d)t=3400. The red color shows positive and the blue one negative. The positive torque acts the fluid element to move in the large θ direction and the negavive one acts the fluid element to move in the small θ direction.

that the flux surface average torque within the magnetic island separatrix becomes small. This is consistent with the stationary magnetic island locked to the external perturbation, where the background flow around the magnetic island reduces almost to zero.

The complex profile of the torque indicates the complex plasma motion inside the island separtix [7], the feature that has been mostly neglected in the standard theory of the forced magnetic reconnection. Our simulations are done in the regime of low viscosity so that the contribution of the viscosity to the total is small, as it is shown in Fig.8. Figures 8 show the radial profile of the flux averaged torques. The JxB force and nonlinear inertia are the contributors to the total torque. As it is indicated in Fig.8, their relative contributions are different at various stages of the island evolution. At initial stage, the nonlinear inertia is small while it increases in the fast growth and transition stages indicating the importance of plasma flows at these stages. Note that the nonlinear inertia force is equivalent to the Reynolds stress contributon.



FIG. 8: radial profile of the flux averaged torques. The initial flux surface without the magnetic island is used for averaging.

6 Localized current sheet and secondary instability

One of the most important features of our simulations is formation of secondary islands in the transition phase (phase C). The secondary islands appears as a result of the instability of the current sheet that has been formed on the island separatrix. The formation of the current sheet can be seen from the different motion of the O- and X-points in the poloidal direction. Figure 9 shows the time evolution of the phase difference of the O- and X-points in the poloidal in the poloidal direction.

inthe In the initial stages, as shown in Fig.9, the phase difference between X and O points remains constant (and $=\Pi/2$) which indicates the standard X-point configuration. We interpret the deviation of the $\Delta \theta (O-X)$ phase difference from $\Pi/2$ as the appearance of the Y-type reconnection. Note that the appearance of the Y-type structures coincides with the beginning of the explosive growth phase. At lower resistivity, the Y-type current sheet occurs at later times. However, since at low resistivity island growth rate is smaller, the critical island width when the Y current sheet occurs is smaller. The maximum value of $\Delta \theta (O-x)$ (hence maximal deformation of the island) is larger for low resistivity. The current sheet persist for some time but becomes unstable later. At this time secondary islands form. We observe oscillations of $\Delta \theta (O - X)$ phase difference. Eventually, oscillations disappear, $\Delta \theta (O-X)$ settles to $\pi/2$ and island enters the Rutherford like growth. In this



FIG. 9: Time evolution of the phase difference $\Delta \theta$ between the O- and X-points for the resistivity $\eta = 10^{-5}, 5 \times 10^{-6}$ and 10^{-6} .

stage the island is quasi symmetric. The duration of the oscillations corresponds to the transition phase (phase C). The transition phase becomes longer as the resistivity becomes decreases. Figures 10 show the contour plots of the helical flux function, i.e. the magnetic island, and the plasma current for the different resistivity. Figs.10(a) and (b) are both at t = 2200, when it is just before the nonlinear rapid growth phase (phase B) for $\eta = 10^{-6}$ and in the flow-suppressed growth phase (phase A) for $\eta = 10^{-7}$. As shown in figs.10, there exists the localized plasma current near the original X-point. In both cases, the same increase rate of the externally applied perturbation is used. Hence, the magnetic flux inputted into the system is the same for both cases. The amplitude of the localized plasma current, however, becomes larger for lower resistivity. As the result, it takes longer time to dissipate the localized current.

7 Summary

In this study, the time evolution of the magnetic island driven by the externally applied magnetic perturbation in the rotating plasma is investigated. Overall, the time evolution of the driven magnetic island in the rotating plasma can be divided into several phases. In the flow-suppressed phase, when the magnetic island width increases slowly while the plasma rotation slows down (Phase A). When the plasma flow velocity becomes sufficiently small, we observe a transition to the fast growth regime (Phase B). In the slow growth phase the island has a finite phase shift with respect to the external perturbation. The locations of the O and X points (and distance between them) changes in the beginning of the fast growth phase. The important feature of the fast growth phase is appearance of the current sheet as it is indicated by the instability of the current sheet when the secondary magnetic islands appear near the former X points (Phase

C). This phase manifests itself in the stochastic oscillations of the X-and O points as well as in the oscillations of the magnetic island width. This phase serves as a transition stage between fast growth regime and Rutherford type growth regime (Phase D).

In a low collisionality regimes the duration of the phase B reduces and the transition phase C start early. In the low η regime, the second island formation dominates the time evolution of the magnetic island.

We have studied the 2dimensional structure of the torque profile. It is shown that the JxB torque leads to a complex plasma flow structures inside the magnetic island. In the low viscosity regime, as studied in this work, the vis-



FIG. 10: Contour plots of the magnetic island (black solid curve) and the plasma current (colored contour) for the resistivity $\eta = 10^{-6}$ and 10^{-7} .

cous force is small, so that the $j \times B$ torque is mostly balanced by the inertia.

Forced magnetic reconnection is a central process for the excitation of the seed island for neoclassical tearing modes. The seed island can be formed by other MHD activity such as sawteeth and ELMs. The formation of the current sheet and subsequent development of secondary instabilities introduces new dynamics in the seed island formation. This will affect the final amplitude and time scale for the seed island. The secondary instability of the current sheet will also affect NTM via direct modification of the bootstrap current drive in the region around the magnetic separatrix. It was shown that competition of the parallel and perpendicular heat conductivities in the secaratrix region may be an important mechanism for NTM threshold[10]. An irregular (stochastic) region near the magnetic separatrix formed by the secondary instability will strongly affect the heat flow in this region thus modifying the transport threshold for NTM formation.

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