Modeling and Interpretation of MHD Active Control Experiments in RFX-mod

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Abstract. A clean magnetic boundary, i.e. a vanishing radial magnetic field, is important for Reversed Field Pinch (RFP) operation. In the past this objective was pursued by setting passive thick conductive shells around the plasma. However the presence of gaps and ports in these shells brings magnetic field errors, which have a negative impact on confinement, and which are hard to control. This led to a large modification of the RFX device (a=0.45 m, R=2 m) into RFX-mod, by adding 192 individually controlled saddle coils which provide a quite flexible control of magnetic boundary. Recent experiments show that this change has produced a significant improvement in plasma MHD behavior.
This paper provides the results of theoretical studies aiming at interpreting these results.

1. Introduction

In fusion devices a "good" conductive wall surrounding the plasma is of key importance for magneto-hydro-dynamic (MHD) stabilization. However, because of the practical difficulty of placing feedback coils inside the stabilizing wall in large devices (like ITER), and the need of real time equilibrium control in steady-state operation, required for a fusion reactor, the conductive wall should permit the penetration of magnetic fields in relatively short time scales (of the order of tens of ms). For these reasons the study of the slow MHD unstable branch of the spectrum, known as the Resistive Wall Modes (RWMs) is of great importance.
The RWM were first detected, partially controlled and recognized as non-resonant current-driven instabilities in a Reversed Field Pinch (RFP) [1]. In tokamak devices these instabilities are instead driven by the pressure gradient at high β's and being resonant in the plasma are strongly influenced by plasma rotation [2]. Recent experiments (in presence of a substantial plasma flow) [3,4] have shown a successful stabilization of the RWMs by active coils.
Also in RFPs a partial stabilization [5] and more recently a complete stabilization [6] of the RWMs was achieved. At difference with tokamaks, where the most unstable RWM has n=1 (n being the Fourier toroidal mode number), in RFPs the spectrum of unstable modes is rich in toroidal numbers and therefore a high number of feedback coils in toroidal direction is needed to avoid side-bands excitation [5,7]. Moreover the fact that RWM are non-resonant in RFPs allows to better decouple the effect of active stabilization of the modes by the external coils, from the presence or not of a substantial plasma flow. This issue is very important for ITER, where the toroidal rotation velocity induced by the Neutral Beam Injectors (NBI) will possibly be too low for stabilization [8] (although this is an active and quite open area of research [9]).
In order to experimentally address these issues, among others, the RFX experiment has been modified (RFX-mod) by replacing a thick, 450 ms vertical field penetration time, with a thinner (50 ms) wall, and by adding a set of 4 (poloidally) X 48 (toroidally) equispaced saddle coils and independent power supplies [10]. This large set of active coils can be used either to control the RWMs and/or to interact with the internal resonant modes, and also to compensate error fields. This system can quite well be considered as a practical realization of the so called "intelligent shell" (but we prefer to call it "virtual shell" (VS)), a concept that

was theoretically studied many years ago [11]. The VS experiments in RFX-mod lead to improvements in plasma duration, reduction of plasma transport with consequent enhancement in ion and electron temperatures [6].
An interesting aspect of the RFPs, if compared with tokamaks, is also that numerical 3D MHD nonlinear simulations, are relatively simpler since cylindrical geometry can be used permitting to considerably simplify the complexity of the problem and to reach interesting physical parameters in computations. For example, the non-obvious a priori (when nonlinear effects are taken into account) RWM stabilization, was theoretically predicted [12] well in advance to the experimental proof.
The main purpose of this paper is to compare the predictions of the nonlinear simulations obtained with a 3D cylindrical MHD code (DEBS), with the new available experimental results in VS and no-VS mode, and also with the old results obtained in RFX before the modification of the shell. In this way we hope to gain a better understanding of the observed improvements and of the underlying physical mechanisms.
The paper is organized as follows: in sec. 2 a brief description of the code is presented; in sec.3 a general comparison of 3 cases relevant to different experimental scenarios is given; in sec.4 the specific problem of the RWM is addressed; summary and conclusions are given in sec.5.

2. The numerical code

DEBS is a cylindrical 3D numerical code (see [12] for more details) solving the Faraday induction equation for the magnetic field coupled to the Euler equation describing the plasma motion. This PDE system is solved in a cylinder using Fourier decomposition in the poloidal (θ) angle and in z coordinate (assuming that the unknowns are periodic along this direction), and a finite difference scheme in radius. A semi-implicit advancement in time, which allows reasonably high time steps, is also implemented.
The code, as is customary for this type of problems, assumes initially (at t=0) given magnetic field and velocity profiles, and also the radial profiles for the two main physical quantities controlling dissipation: resistivity and viscosity. The two dissipation scales define two important normalized control parameter for the simulations, i.e. the Lundquist number S (defined as the ratio between resistive diffusion and Alfvèn time scales) and the Prandtl number P (defined as the ratio between viscosity to resistivity).
The simulations presented here have $S=10^5$ and P=10. Furthermore we assume equal monotonic increasing radial profiles for resistivity and viscosity (almost flat till r/a=0.8 and then rapidly increasing towards r/a=1).
An interesting feature of DEBS is the possibility of specifying different boundary conditions for the magnetic field, including resistive walls (up to two) and external feedback control [12].
The code employs normalized units: the radius is normalized to the minor plasma radius, the time to the resistive diffusion time and the velocity to the Alfvèn velocity.

3. Comparison of different scenarios

As already mentioned in the introduction we simulate and compare 3 cases:

-the case of the old RFX (with thick passive shell) : named OLD
-the case of the RFX-mod (with the thin passive shell): named NEW
-the case of RFX-mod in VS mode: named ID (ID for Ideal shell)

These 3 cases differ only for the boundary conditions applied to the numerical simulations. As regard the macroscopic control parameter Θ, defined as the ratio between the poloidal field at the wall and the average toroidal field, it is fixed in simulations to 1.6 (experimentally it ranges from 1.4 to 1.6).

In both cases OLD and NEW a resistive shell is present. The shell proximity b/a, being respectively , 1.18 and 1.1 (as in the experiment), while the shell penetration time is 8 times longer for the OLD case if compared to the NEW (again as in the experiment). In the ID case an ideal boundary is present just at the plasma edge, b/a=1. This is slightly different from the experimental VS mode where the virtual ideal wall is produced at the sensor radius (b/a=1.1).

The main experimental observation is that the old RFX and new RFX-mod (when the VS operation is turned off) are not significantly different, in terms of magnetic perturbation amplitudes, dominant spectral components etc., being however the new device slightly worse[13].

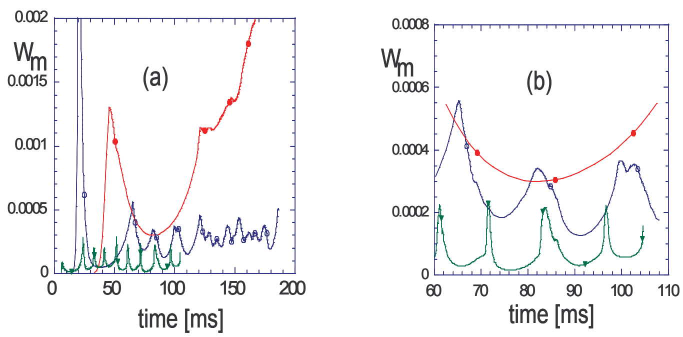

Figure3. 1: Total magnetic energy vs. time.

In Fig.3.1(a,b) (Fig.3.1(b) shows an expanded time window of Fig.3.1(a)) the nonlinear simulation results in terms of the total (radial+poloidal+toroidal) perturbed magnetic energy (in normalized unity) is shown for the 3 cases: red line corresponds to the NEW, blue line to the OLD and green line to the ID cases respectively. The general observation is that Fig.3.1 shows a good agreement between code and the reported experimental results [6,13] about perturbation amplitudes in the 3 cases analyzed. In particular the comparison of the two cases OLD and NEW, show that, as in experiment, the OLD case has a lower level of magnetic fluctuations, while the ID case (corresponding to the VS operation) shows the smallest fluctuations amplitude. A summary in terms of total and radial m=1 energy/field is given in Table 1. The time average fluctuation levels in the expanded time window of Fig.3.1b are considered.

Table 1

	W_{tot}/W_{old}	b_{tot}/b_{old}	$W_{r\,m=1}$ / $W_{r\,old}$	$b_{r\,1}$ / $b_{r\,1\,old}$
OLD	**1**	**1**	**1**	**1**
NEW	**1.18**	**1.06**	**1.57**	**1.25**
ID	**0.34**	**0.56**	**0.26**	**0.5**

It is shown that the degree of enhancement (or reduction) with respect to the OLD case depends on which components of the fluctuations are considered. For example in [6,13] an estimate of about a factor 2-3 decrease in the internal radial m=1 (m being the poloidal mode number) amplitudes between the VS and non VS operation in RFX-mod using a Newcomb solver reconstruction of the internal fields has been reported. Here we find a factor 2.5 (0.5/1.25) (see Table 1 last column) , in good agreement with the Newcomb method based estimate. The results obtained here are, at least qualitatively, in agreement with previous simulations of a RFP plasma in presence of a resistive shell [14].

Looking at Fig.3.1 other observations can be done. First, it is possible to see that after 100 ms the NEW case shows a large increase of the magnetic energy. This is due to the excitation of the m=1, n=4 RWM (see the discussion of Sec.4). Another point is that regular oscillations, with slightly different time scales (slower in OLD case), are present in the OLD and ID cases. These oscillations are resembling those observed experimentally in RFX . In particular a strong sawtooth activity has been reported in RFX-mod with VS control [6]. In the old RFX, although sometime present, these oscillations were not so clear.

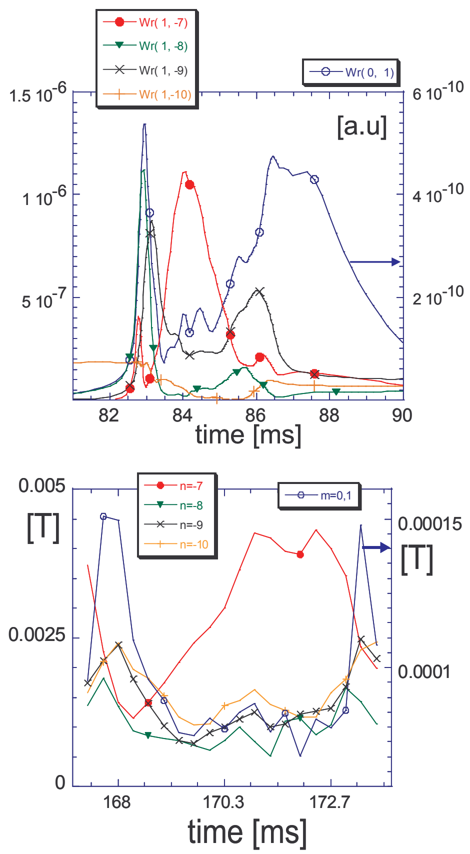

Fig.3.2 Sawtooth activity in simulations (volume average radial energy) (top) and in experiment (edge radial field) (bottom).

An example of a sawtooth in numerical simulations (ID case) is shown in Fig.3.2. The behavior of few m=1 modes and of the dominant m=0 (n=1) (blue line) is shown. After an initial short phase in which many internal resonant m=1 are excited simultaneously, an n=-7 mode is emerging, while the other modes (including the m=0) are decreasing in amplitude. This is similar to what observed experimentally (Fig.3.2 bottom).
It is not easy, even in numerical simulations, to identify the cause of the oscillation, but it seems clear from our results that an ideal wall (or a thick wall) is really a key ingredient. Relaxation oscillations at relatively higher Θ (=1.9) have been reported for 3D ideal wall simulations [15].
Another subject which is of great interest for RFPs is the interaction of an external perturbation with the internal resonant modes. This subject is important both to better understand the dynamics of the tearing modes and also in the framework of excitation/control of the so called "Single Helicity" (SH) state. In [16] the feedback modified version of the DEBS code was employed to actively induce a SH state. The main result of [16] is reproduced in Fig.3.3, where the 1/-7 is driven to be the dominant mode, while the others (with the exception in this case of the n=2 RWM) "naturally" decrease exponentially to low energy.
Similar experiments have been carried out in RFX-mod where also a dominant n=-7 was actively induced. The main difference with the numerical case is that experimentally the secondary modes do not decrease so strongly after the growth of the n=-7.

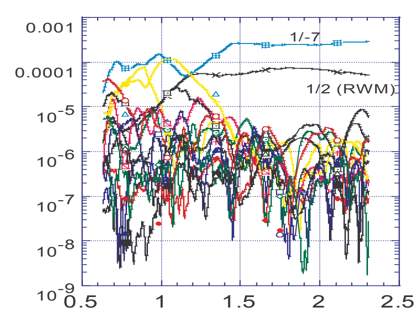

Fig.3.3 Mode energy vs. time (in normalized units).

4.Code predictions for the RWM branch

A very interesting point to address is the comparison between the code predictions and the experiments regarding the RWM spectrum. The prediction of the possibility of actively stabilize the RWM was already obtained theoretically [12] and observed experimentally [5,6].

It was also demonstrated [12] that RWMs behave according to the linear theory and that the coupling with other branches is weak or absent. In Fig.4.1 an example is shown of the linear growth of different m=1 RWMs. An exponential curve, which fits very well the data, is also superimposed for all modes. It is seen that, in terms of energy content, the dominant mode is n=4. In Fig.4.2 a comparison is presented between the results of linear simulations at different F values (colored bars), obtained using the equilibrium profiles that can be inferred from experimental data in RFX-mod (without control) and the nonlinear simulations (horizontal dashed bars).

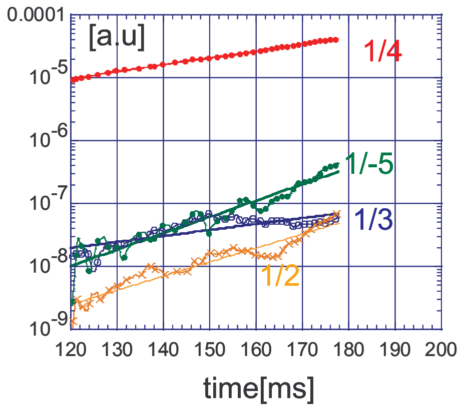

Fig.4.1 RWMs growth in the NEW case.

The figure shows normalized (to the wall time assumed to be 50 ms) growth rates versus toroidal mode number (again for m=1 modes). The nonlinear simulations predict slightly larger growth rates. We observe that the linear growth (shown by the colored bars in Fig.4.2) agrees well with experimental data obtained in RFX-mod (bottom frame of Fig.4.2, thick lines refer to experimental data). Moreover, in experiments, at F values around -0.15, the dominant RWM (in terms of energy content) seems to be the n=-6 (and not the n=4). It is clear however that in real experiment the field errors can also play a key role in the RWM growth.

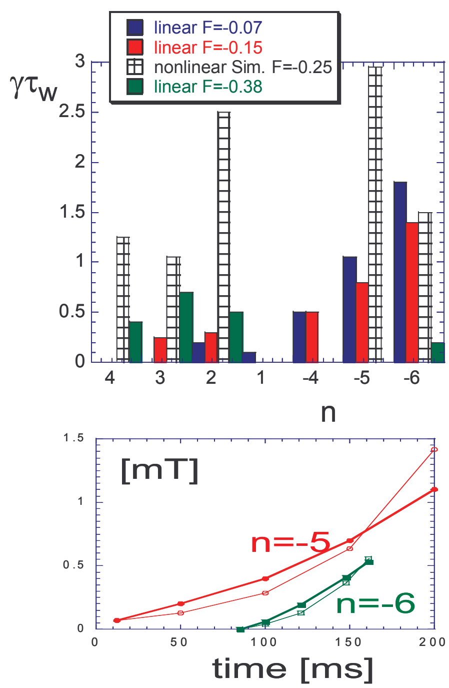

Fig.4.2 Growth rates (top) for RWMs. Dashed bars are obtained from 3D simulations. Comparison with experimental time traces (bottom).

In Fig. 4.3(a,b) the profile of the 1/4 and 1/-6 radial field components from simulations and reconstructed from the experiment using the Newcomb's solver and a time dependent equilibrium reconstruction are shown . The 1/4 profile evolves almost self-similarly in time both in simulations and experiment, with the on-axis and edge values increasing simultaneously at the given rate. This is not completely true for the n=-6 in simulations, where the on-axis and edge values are more decoupled. The Newcomb's solver instead finds, also in this case, a self-similar evolution and a broader profile if compared with simulations.

Therefore it can be concluded that there are differences, although not too dramatic, in the predictions of the two approaches. It is clear that at least part of the differences is due to the different underlying equilibria in the two cases. In the Newcomb case the equilibria are fitted to the experimental data (with an arbitrary choice on the class of the fitting functions), while in the 3D case they are self-consistently determined by the nonlinear evolution.

The comparison presented here seems to suggest that detailed predictions about mode amplitudes and radial structures based on Newcomb methods may not be completely satisfactory. This observation holds very likely not only for RWMs but particularly for internally resonant modes, for which the nonlinear coupling is much stronger. On the other hand also 3D simulations have a drawback, since the Lundquist number is too low (if compared with experiments) and the viscosity is chosen somehow arbitrarily.

However, despite this caveats, as shown in Sec.3, the estimate of global reduction/enhancement of the average fluctuation amplitude based on the Newcomb's solver, agrees quite well with the nonlinear simulations.

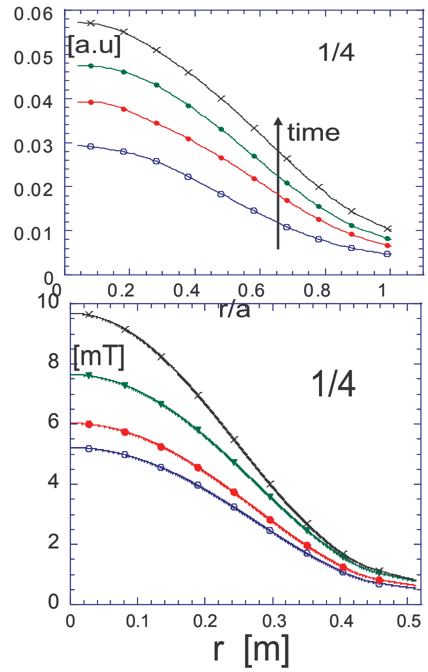

Fig.4.3(a)Radial magnetic field profile in time (simulation top and experiment (#17278) bottom).

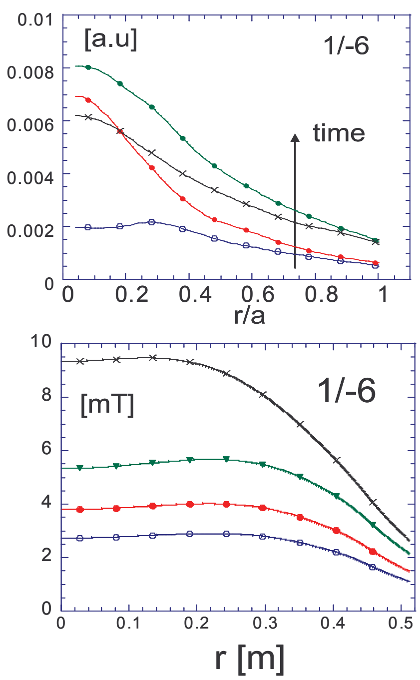

Fig.4.3(b)Radial magnetic field profile in time (simulation top and experiment (#17278) bottom)

5. Summary and Conclusions

3D nonlinear cylindrical simulations have been presented in this paper in the attempt of gaining some insight about the dynamical behavior of a RFP device under 3 different boundary conditions: from an ideally conducting shell (with zero radial magnetic field at the plasma boundary) , to a thick resistive shell allowing some slow magnetic field penetration, and finally, to a much thinner wall with faster field penetration. The 3 cases are relevant for the 3 operational scenarios already achieved in the RFX device. In particular the resistive wall cases (labeled as OLD and NEW) refer to the operation with no active control, in the old RFX and in the new RFX-mod devices, while the ideal wall simulation (labeled as ID) resembles the active mode control operation named VS, where a cage of external coils can mimic the presence of an ideally conducting wall surrounding the plasma region.

The general conclusion of this study is that the 3D results are in good agreement with experimental observations under several respects.

A general reduction of the fluctuation amplitude is observed in experiments with VS, with strong consequences on the transport and enhancement of ion and electron temperatures [6]. On the basis of the analysis presented here the total average radial field amplitude in the plasma region can be estimated to be a factor 2.5 smaller than without active control. This value is in line with the estimate using a simpler

Newcomb's solver [6,13,17]. The simulations also show that the RFX-mod device (with no control) is slightly worse than the old RFX with thick passive shell. Although a detailed comparison is not possible, since RFX had much less measurement coils, this result is consistent with transport rate estimates in the two devices [13]. It should also be noted that error fields were likely larger in RFX, possibly partially compensating the modest enhancement of the fluctuation level predicted by the model in RFX-mod without control.
Another interesting similarity between numerical results and experiments is the observation of a clear sawtooth activity of the n=-7 mode, in anti-phase to other internal modes and to the m=0 mode [6]. A clear identification of the physical mechanism responsible for this activity has not been possible, but this study clearly indicates that only ideal wall (or quasi-ideal) boundary conditions are able to trigger this phenomenon. Taking into account this result, the observation of very frequent sawtooth activity in VS mode in RFX-mod indirectly confirms that the VS operation is effectively and efficiently working as to replace the ideal wall.
Regarding RWMs the main conclusion is that qualitatively the 3D simulations agree with the experiments. In particular the spectrum of unstable modes is the same. Some differences can be found if a more quantitative comparison is carried out. In particular 3D simulations seem to indicate slightly different dominant modes and larger growth rates.
However for both these aspects, a careful analysis should be done in experiments, to exclude the presence of error fields, which can excite specific modes and also affect the growth rates. In simulations instead a scanning in the F parameter could help for a more quantitative comparison.
To conclude: 3D MHD simulations compare quite well with experimental data in RFPs and can help to better understand the basic physical mechanisms operating in these devices.

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References

[1] B. Alper et. al. Plasma Phys. Controlled Fusion **31**, 205 (1989).
[2] A. Bondeson and D.J. Ward, Phys. Rev. Lett. **72**, 2709 (1994).
[3] E.J. Strait et. al., Phys. Plasmas **7**, 3681 (2000).
[4] M. Okabayashi et al., Nucl. Fusion **45**, 1715 (2005)
[5] P. Brunsell et. al., Phys. Rev. Lett. **93**, 225001 (2004).
[6] R. Paccagnella et. al., Phys. Rev. Lett. **95**, 075001 (2006).
[7] R. Paccagnella, D. Gregoratto, A. Bondeson, Nucl. Fusion **42**, 1102 (2002).
[8] Y.Q. Liu, A. Bondeson, Y. Gribov and A. Polevoi, Nucl. Fusion **44**, 232 (2004).
[9] G. M. Staebler and H.E. St John, Nucl. Fusion **46**, L6-L8 (2006).
[10] P. Sonato et. al., Fusion Eng. Des. **66**, 161 (2003).
[11] C.M. Bishop, Plasma Phys. Controlled Fusion **31**, 1179 (1989).
[12] R. Paccagnella, D. Schnack, M. S. Chu, Phys. Plasmas **9**, 234 (2002).
[13] S. Ortolani et. al. Plasma Phys. Controlled Fusion to be published in November 2006.
[14] D.D.Schnack, S. Ortolani, Nucl. Fusion **30** 277 (1990).
[15] S. Cappello, D. Biskamp, Nucl. Fusion **36** 571 (1996).
[16] R. Paccagnella, Proceedings of the Theory of Fusion Plasma, Joint Varenna-Lausanne International Workshop, Italy, 2002 (Ed. SIF Bologna, Italy) Vol. ISPP-20 p.73.
[17] S. Martini, et. al. "Overview of RFX-mod Results with Active MHD Control" This IAEA Conference paper EX/7-3.