Two Fluid Dynamo and Edge-Resonant m=0 Tearing Instability in Reversed Field Pinch

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Abstract. Current-driven tearing instabilities are believed to dominate magnetic relaxation in self-organized high temperature plasmas such as the reversed field pinch (RFP) and spheromak. In the Madison Symmetric Torus (MST) RFP experiments, tearing instabilities are observed in the form of magnetic field, flow velocity and current density fluctuations that follow a temporally cyclic sawtooth behavior. During a sawtooth crash, a surge occurs in the dynamo - a fluctuation-induced mean electromotive force in the generalized Ohm's law that combines the MHD $\mathbf{v} \times \mathbf{B}$ and $\mathbf{j} \times \mathbf{B}$ Hall dynamos. We report new analytic and numerical results on the physics of two-fluid dynamos as well as on the problem of spontaneous (linear) instability of edge-resonant m=0 tearing modes. The key findings are: **(1**) two fluid effects are critically important for dynamo through their influence on the phase between the fluctuations; two-fluid theory yields a non-zero flux surface averaged Hall dynamo, absent in resistive MHD; **(2)** the two fluid version of the NIMROD code confirms analytic results during the linear stage of the instability but exhibits significant broadening of the Hall dynamo profile on the longer time scales of nonlinear evolution; **(3)** improved modeling of force-free RFP equilibrium predicts a wide range of RFP parameters in which m=0 tearing mode is spontaneously unstable, a result that is consistent with recent MST experimental observations.

1. Introduction

Current-driven tearing instabilities are believed to dominate magnetic relaxation in self-organized high temperature plasmas such as the reversed field pinch (RFP) and spheromak. In the Madison Symmetric Torus (MST) RFP experiments, tearing instabilities are observed in the form of magnetic field, flow velocity and current density fluctuations that follow a temporally cyclic sawtooth behavior. During a sawtooth crash, a surge occurs in the dynamo - a fluctuation-induced mean electromotive force in the generalized Ohm's law that combines the MHD $\mathbf{v}\times\mathbf{B}$ and $\mathbf{j}\times\mathbf{B}$ Hall dynamos. The dynamo modifies parallel electric field and plasma current profile. In particular, the mean current density at the magnetic axis drops significantly during plasma relaxation event (magnetic reconnection) accompanied by current increase in the edge. This ultimately leads to current flattening in the core and current sustainment in the plasma edge. The underlying physics of plasma relaxation is of great importance for both laboratory and astrophysical plasmas. Dynamo activity in the RFP has been intensively studied analytically [1,2], by 3D MHD computations [3] and experimentally [4,5]. We report here new analytic and numerical results on the physics of two-fluid dynamos as well as on the problem of spontaneous (linear) instability of edge-resonant m=0 tearing mode. The two fluid quasilinear Hall dynamo theory that was originally derived for a sheared slab [6] is generalized to cylindrical geometry and illuminates the effects of current gradient and field line curvature on the dynamo. We found that 1) two fluid effects are important for dynamo through their influence on the phase between the fluctuations; 2) two-fluid theory yields a non-zero flux surface averaged Hall dynamo that is absent in resistive MHD; 3) the two fluid version of the NIMROD code confirms analytic results during the linear stage of the instability but exhibits significant broadening of the Hall dynamo profile on the longer time scales of nonlinear evolution.

The second group of the results is related to linear stability of the edge resonant m=0 mode in the RFP. The m=0 mode is of a special importance for RFPs because of its impact on mode coupling, ion heating, momentum and energy transport. Robust linear stability of the m=0 mode was verified in the past by many calculations based single parameter equilibrium model for the plasma current (α-model). Recent MST experiments [7] have shown that in some regimes with improved plasma confinement the m=0 mode becomes linearly unstable. This motivates our interest in revisiting the m=0 stability analysis including a broader range of current profiles. A four-parameter model is introduced which permits to vary the position and the width of the current gradient independently. Improved modeling of force-free RFP equilibrium predicts a wide range of RFP parameters in which m=0 tearing mode is spontaneously unstable, a result that is consistent with MST experimental observations.

2. Two-Fluid Dynamo

Standard RFP discharges in the MST are characterized by the periodic sawtooth fluctuations in the plasma velocity $\mathbf{v}^{(1)}$, current density $\mathbf{j}^{(1)}$ and magnetic field $\mathbf{B}^{(1)}$. Corresponding MHD $\langle \mathbf{v}^{(1)}\times\mathbf{B}^{(1)}\rangle$ and Hall $\langle \mathbf{j}^{(1)}\times\mathbf{B}^{(1)}\rangle/ne$ dynamos generate mean electric field and the current accordingly to the generalized Ohm's law $\langle \mathbf{E}\rangle_{||} - \eta\langle \mathbf{j}\rangle_{||} = -(1/c)\langle \mathbf{v}^{(1)}\times\mathbf{B}^{(1)}\rangle_{||} + (1/en^{(0)}c)\langle \mathbf{j}^{(1)}\times\mathbf{B}^{(1)}\rangle_{||}$, where $\langle\ \rangle$ denotes mean (flux surface averaged) value. We model the dynamo by two methods: analytically using two fluid quasilinear tearing mode theory and numerically with the use of the two fluid NIMROD code. Quasilinear theory deals with flux surface averaged quadratic combinations of the linear tearing eigenfunctions of $\mathbf{v}^{(1)}$, $\mathbf{j}^{(1)}$ and $\mathbf{B}^{(1)}$. Standard single fluid resistive MHD yields a non-zero MHD dynamo localized to the rational surface while a naive application of resistive MHD to a Hall dynamo gives zero result due to the 90^0 phase shift between radial B_r and parallel $B_{||}$ components of the perturbed magnetic field. This situation changes radically with the transition to two fluid MHD where B_r and $B_{||}$ are in-phase yielding a large Hall dynamo that dominates on short scales. The "phase effect" is caused by electron-ion decoupling on short scales. In the two fluid case, the cross-field currents of decoupled electrons flow in $\mathbf{E}_\perp\times\mathbf{B}$ direction while in single fluid MHD they are determined by polarization currents which are oriented along $\mathbf{E}_\perp$. This shifts the spatial profile of cross-field current vortexes with respect to the magnetic island and, correspondingly, changes the phase between $B_{||}$ and B_r. (see, Fig. 1.)

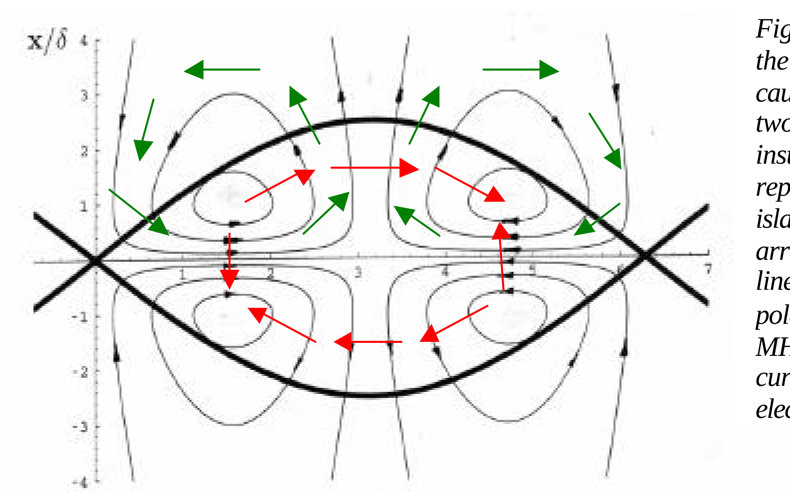


Fig. 1. Qualitative illustration of the phase shift between B_r and $B_{||}$ caused by transition from single to two fluid regime of tearing instability. The thick line represents position of the magnetic island, the thin lines and red arrows, respectively, the stream lines of ***E*** *x* ***B*** *plasma flows and polarization currents in single fluid MHD, the green arrows show current flows of decoupled electrons in two-fluid regime*

Non zero Hall dynamo driven by two fluid tearing instability was first calculated in [6] for a sheared slab model. The range of parameters considered in [6] was related to large values of the stability factor Δ' and the "kinetic Alfven" regime of instability [8]. Detailed analysis of two fluid tearing modes performed in [9] showed that the case of small Δ' and whistler mediated reconnection is more adequate for the parameter space of the MST (see, Fig.2.). We report here analytical results related to this specific regime of Hall dynamo theory in a cylindrical geometry, as well as the results of linear and nonlinear two-fluid NIMROD simulations performed for the slab model.

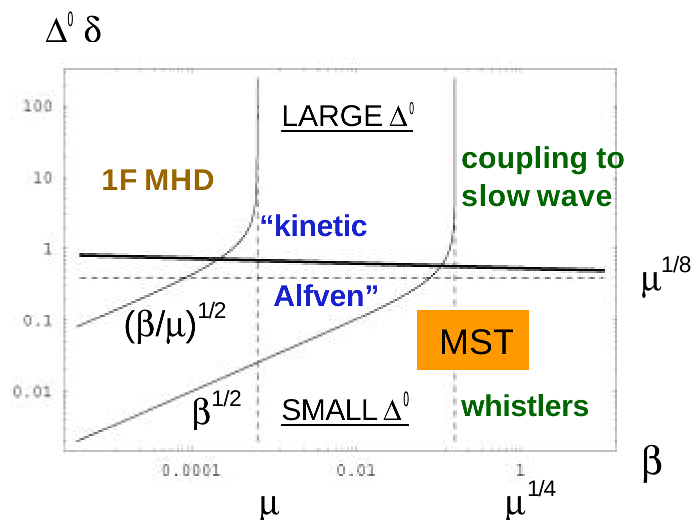


Fig.2. Classification of the different regimes of two-fluid tearing instability in terms of D' and plasma b (m = m_e/m_i, $d^2 = 1/(g\, t_A S)$, g and S are the electron skin depth, growth rate and Lunquist number). The MST case, D'd ~ 0.2, b~ (5-10)% belongs to the range of small D' and transitional regime between "kinetic Alfven" and whistler mediated reconnection.

2.1 . Hall dynamo theory

We consider the cylindrical model of the RFP assuming force free equilibrium magnetic configuration. In contrast to tokamaks, reduced MHD approximation is not applicable for the stability study in the RFPs because perturbations of the guide field $B_{||}^{(1)}$ are comparable with the radial component $B_r^{(1)}$. Using helical harmonics expansion yields

$$B^{(1)}_{r,||} = B_{r,||}(r)\exp(\gamma t + im\theta - kz)$$

After flux surface averaging (integration over z and θ), the parallel component of the Hall dynamo driven by single tearing mode takes a form

$$\epsilon^{(H)}_{||} \propto <\mathbf{b}^{(0)}[\mathrm{Re}\ \mathbf{j}^{(1)} \times \mathrm{Re}\ \mathbf{B}^{(1)}]> \rightarrow \frac{1}{2}\frac{\partial}{\partial r}\left(|B_r||B_{||}|\cos\phi\right), \phi = \phi_r - \phi_{||}$$

Radial profile of $\varepsilon_{||}^{(H)}$ can be found in quasilinear approximation with the use of linear eigenfunctions of the system. In tearing mode theory, the eigenfunctions are calculated by matching of inner (resistive) and outer (ideal) solutions. Within the scope of single fluid MHD, the interrelation between outer (ideal) solutions for $B_{||}$ and B_r is as follows

$$B_{||} = \frac{i}{k_0^2}\left(\frac{k_{||}}{r}\frac{\partial}{\partial r}(rB_r) - \frac{4\pi j^{(0)} k_{\perp}}{cB}B_r\right)$$

This equation predicts a constant (not dependent on r) phase shift $\phi = \pi/2$ between $B_{||}$ and B_r and, therefore, $\varepsilon_{||}^{(H)} = 0$ in the outer region. Resistive tearing layer equations [11] show that the $B_{||}$ and B_r perturbations are out of phase not only in the outer region but in the resistive layer too. Thus, within the scope of single fluid resistive MHD, $\varepsilon_{||}^{(H)} \equiv 0$ everywhere in a cylinder. Note, that the different situation takes place in the case of the MHD dynamo

$$\epsilon_{||}^{(MHD)} = -\frac{1}{c} < \mathrm{b}^{(0)}[\mathrm{Re}\ \mathrm{v}^{(1)} \times \mathrm{Re}\ \mathbf{B}^{(1)}] > \ = \ \frac{i}{4k_\perp c}\frac{\partial}{\partial r}(v_r B_r^* - v_r^* B_r)$$

Taking, for example, the relationship between v_r and B_r in the outer ideal region, yields small (due to smallness of γ) but non-zero MHD dynamo in this area

$$\gamma B_r = ik_{||}Bv_r \quad \rightarrow \quad \epsilon_{||}^{(MHD)} = \frac{\gamma}{2k_\perp c}\frac{\partial}{\partial r}\left(\frac{|B_r|^2}{k_{||}B}\right)$$

Transition from single to two fluid MHD does not change the plasma momentum equation but brings an additional $\nabla \times \mathbf{j} \times \mathbf{B}$ Hall term into the induction equation. This term provides magnetic field frozen condition into the electron component. This modifies standard shear Alfven (SA), compressional Alfven (CA) and magnetoacoustic (MA) modes on short scales and gives rise to "kinetic Alfven" and whistler mediated regimes of tearing instability. As it was shown in [9], at small values of Δ', the tearing instability is driven mainly by the electrons and, correspondingly, the ion motion can be ignored in the electron layer. Then, the cylindrical components of the induction equation simplified in the vicinity of the resonant magnetic surface $k_{||} = 0$ can be written as follows

$$B_r - \delta^2\nabla^2 B_r = \frac{k_{||}k_\perp B}{\gamma}\tilde{B}_{||}, \qquad \tilde{B}_{||} = \left(B_{||} - \frac{j^{(0)}}{ik_\perp B}B_r\right)$$

$$\tilde{B}_{||} + \tilde{B}_{||}\left(\frac{1}{\beta} + \frac{k_{||}^2}{\gamma^2} + \frac{id_i k_\perp}{\gamma}\frac{2B_\theta^2}{rB}\right) - \delta^2\nabla^2\tilde{B}_{||} = \frac{d_i}{\gamma}\left(\frac{Bk_{||}}{k_\perp}\frac{\partial^2 B_r}{\partial r^2} - B_r\frac{\partial j^{(0)}}{\partial r}\right)$$

where dimensionless γ is $\gamma\tau_A$, electron skin depth $\delta^2 = 1/(\gamma S)$, ion skin depth $d_i = c/(a\,\omega_{pi})$.
By equating the inertia term in cross-field $\mathbf{b} \times \mathbf{e}_r$ component of plasma momentum equation to zero, we decouple the CA branch from the equations. We introduced the function $\tilde{B}_{||}$ that describes deviation of $B_{||}$ from its asymptotical outer solution $B_{||}^{(outer)}$. The term proportional to B_θ^2 in the second equation describes the effect of curvature that does not exist in the slab model. Without this term, the solution for $\tilde{B}_{||}$ is in phase with B_r providing non zero Hall dynamo in the tearing layer. Outside the layer, $\tilde{B}_{||}$ tends to zero that changes the phase between $B_{||}$ and B_r to $\phi = \pi/2$. Although amplitude of $B_{||}$ is finite everywhere, the Hall dynamo vanishes in the outer area due to the "phase effect". The curvature term and the term proportional to the gradient of the equilibrium current effect on the profile of $\varepsilon_{||}^{(H)}$ but do not change the general picture. The above equations give a quantitative description of the phase behavior while the qualitative illustration is presented in Fig. 1.

2.2. Numerical simulations

Numerical studies of Hall dynamo driven by two-fluid tearing instability are performed in a sheared slab geometry using the two-fluid version of NIMROD. In the linear stage of tearing instability, the Hall dynamo is localized on a short scale very near the resonance surface and is much larger than the MHD dynamo effect. On larger scales of order of ion gyroradius, the Hall dynamo diminishes and is comparable to the MHD dynamo. Nonlinear single mode computations are also performed with $L > r_S$ and in the other parameter regimes. In the nonlinear regime, the Hall dynamo broadens to the same scale as the MHD dynamo and contributions from the Hall and MHD terms to the dynamo are comparable (see, Fig. 3)

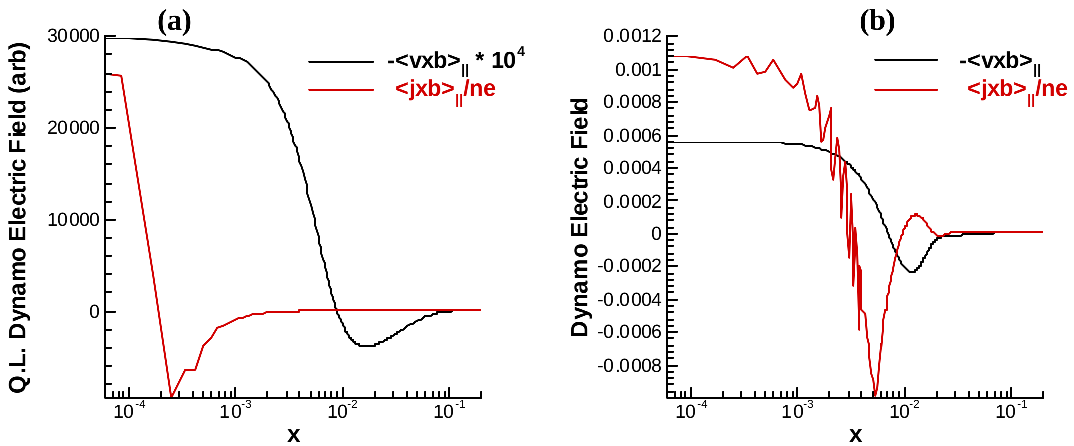


Fig.3. MHD and Hall dynamos during linear (a) and nonlinear (b) stages of tearing instability. Quasilinear computations (a) show strong and localized Hall dynamo, consistent with the analytical prediction. In nonlinear saturation (b), the Hall dynamo expands to the equilibrium scale L, and the two dynamos become comparable in magnitude.

Numerically computed growth rates are in good agreement with the analytical dispersion relation, provided that the tearing layer is sufficiently small with respect to the equilibrium scale. Viscous dissipation was anticipated for the nonlinear computations, and a parameter scan shows very little impact on linear growth rates with some broadening of the **V** profile of the eigenfunction. Examining the dynamo contributions locally in y shows that both Hall and MHD dynamos act in phase with respect to y in the quasilinear result; though, the x-scales and magnitudes are very different. In the nonlinear state, the local Hall electric field acts primarily near the separatrix (see, Fig. 4.) and remains large in magnitude relative to the MHD contribution while the broad profile is expected for current density redistribution caused by nonlinear MHD dynamo shown in Fig.5.

Fig.4. Contours of local fluctuation-induced Hall electric field.

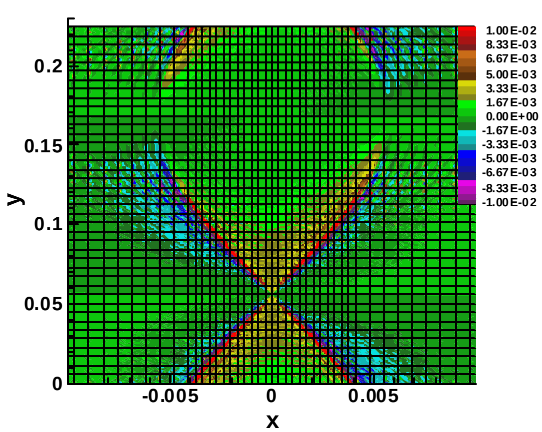
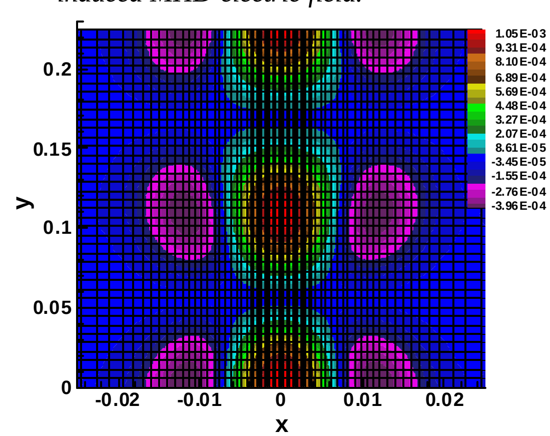


Fig. 5. Contours of local fluctuation-induced MHD electric field.



3. Spontaneous tearing instability of m=0 modes

Typical MST regimes of operation exhibit cyclic sawtooth oscillations associated with core (m=1, n=6,7,…) and edge (m=0, n=1) resonant tearing modes. In typical discharges, it is believed that the core tearing modes are spontaneously unstable, while linearly stable m=0 modes are nonlinearly driven by coupling to core resonant modes. This scenario of forced m=0 magnetic reconnection is based in part on robust linear stability properties demonstrated in the past by various Δ' calculations which predict stability. Recent MST experiments [7] have shown that in some regimes with improved plasma confinement the m=0 mode becomes linearly (spontaneously) unstable. Efforts to resolve this observation with theory inspired improved modeling of the equilibrium current profile. A four-parameter cylindrical model is introduced that allows independent variation in the radial position and width of the current gradient. These calculations determine a wide class of unstable equilibriums showing a strong sensitivity of stability properties to small variations of current profiles. We report on ideal MHD Δ' analysis as well as the results obtained from a cylindrical resistive eigenvalue code.

Force free RFP equilibrium is characterized by the parallel current profile $j = \lambda(r) B$ where $\lambda(r) = \mu_0 f(r)$ and

$$f(r,d,w,\alpha) = \frac{\left[\exp\left(\frac{r^\alpha - d^\alpha}{w}\right)+1\right]^{-1} - \left[\exp\left(\frac{1-d^\alpha}{w}\right)+1\right]^{-1}}{\left[\exp\left(-\frac{d^\alpha}{w}\right)+1\right]^{-1} - \left[\exp\left(\frac{1-d^\alpha}{w}\right)+1\right]^{-1}}$$

Four factors μ_0, w, d and α allow us to control independently the amplitude, width and position of the current gradient. At $w >> 1$, this model reduces to the standard α model

$$\lambda(r) \to \mu_0(1 - r^\alpha)$$

The values of the stability factor Δ' are found by solving the Newcomb equation

$$\frac{d^2 b_r}{dr^2} + \frac{f^*}{r}\frac{db_r}{dr} - g^* b_r = 0 \ ,$$

and matching left and right solutions at the resonant surface r_s ($q(r_s) = 0$) with the boundary conditions $b_r(0) = b_r(1)$. The results are presented in Fig. 6. for a few profiles $\lambda(r)$ and $\mu_0 = 3.35$, $\alpha = 3$.

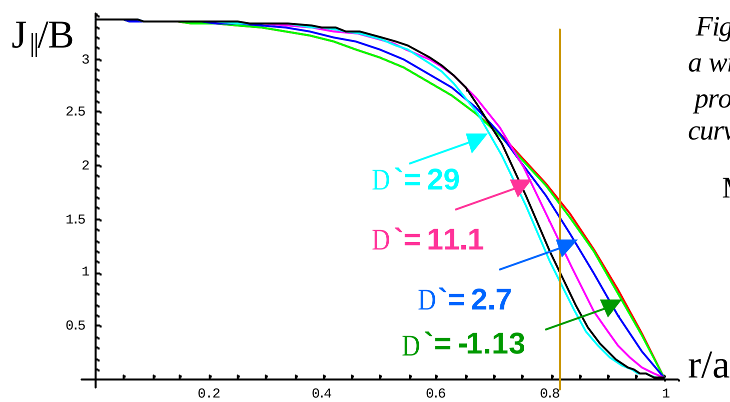


Fig. 6. Four parameter model describes a wide class of unstable ($\Delta'> 0$) current profiles while the a - model profile (green curve) is mostly stable.

Magnitude of Δ' as a function of RFP parameters F and Θ :

F=-0.06, Q=1.6, D`= 29
F=- 0.24, Q=1.7, D`= 11.1
F=- 0.23, Q=1.6, D`= 2.7
F=- 0.22, Q=1.6, D`= -1.13

Accordingly to the four parameter model there is a strong sensitivity of the stability properties to small variations of the current profile. The model reveals a wide class of

unstable configurations with large positive Δ' whose $\lambda(r)$ profiles are just slightly different from the stable α-model curve. Calculations of the reversal F and pinch parameter Θ for these configurations show that the profiles with large Δ' correspond to weakly reversed configurations. This is not consistent with the MST results where spontaneous m=0 instability is observed at F = – 0.5, while at F = -0.2 the mode is stable. In order to address this issue we analyzed the stability of these configurations not in terms of Δ', but by comparing the growth rates obtained from resistive cylindrical eigenvalue code by
V. Svidzinski. In Fig.7., the numerical results are compared with the analytic FKR expression [11] and more precise equation by G. Bertin [12] that takes into account the gradient of the equilibrium current.

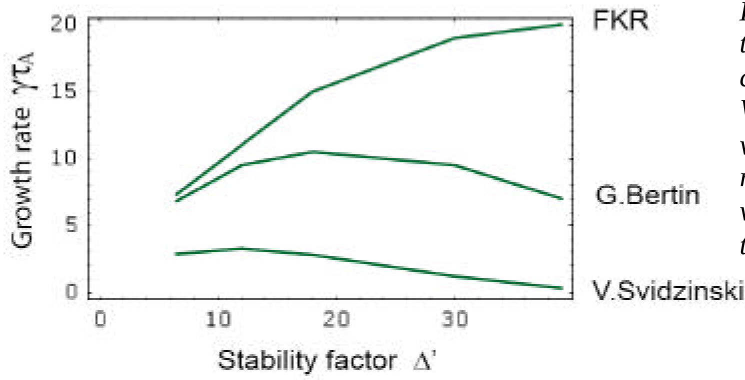


Fig. 7. Numerical calculations of the growth rate for different current profiles (b=0, S=10^4*). Weakly reversed configurations with large* Δ' *have smaller growth rates than deeply reversed states with small* Δ' *that contradicts to the FKR analytic expression.*

Strong suppression of the growth rates at large Δ' may also result from finite β effects found in numerical calculations (see, Fig.8.)

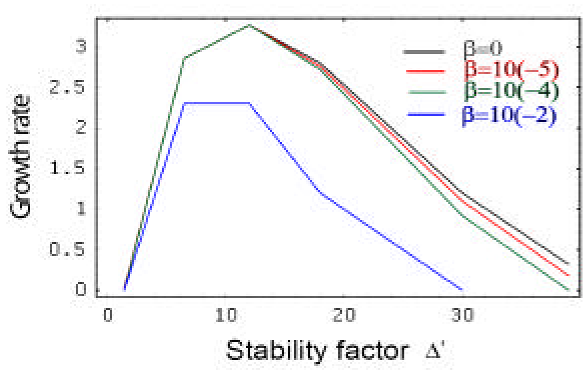


Fig. 8. Numerical calculations of the growth rate for different current profiles and values of b (S=10^4 *). The m=0 tearing instability driven by large* $\Delta' > 30$ *are suppressed by finite b > 1%.*

4. Summary

Three important effects for Hall dynamo theory are obtained within the scope of the cylindrical RFP model: (a) spatial variation of the phase between B_r and $B_{||}$ eliminates Hall dynamo in the ideal outer region, but makes this effect dominating in the tearing layer;
b) contribution from the field curvature can be ignored at small β; (c) gradient of equilibrium current is not important for quadrupole structure of out-of plane component at $\Delta' >> 1$.
Implicit the leapfrog numerical algorithm accurately reproduces two-fluid tearing instability in slab geometry with a large guide field, at large time-step. When the mode is at small amplitude, the computations reproduce the quasilinear prediction for Hall and MHD dynamo effects. With nonlinear saturation, the net Hall dynamo effect broadens and decreases in

magnitude, becoming comparable to the net MHD dynamo effect; though, locally the amplitude of the Hall electric field remains far greater. Multi-helicity two-fluid computations are needed to understand relaxation through nonlinear interactions among fluctuations.

Robust linear stability demonstrated in the past for m=0 tearing mode was based on the α-model. Improved four-parameter model predicts wide class of unstable current profiles and shows strong sensitivity of Δ' to small variation of the current profile. Dependence of the FKR growth rates on the reversal parameter F contradicts to the MST observations. Improved analytic model with $dj/dr \neq 0$ shows better agreement between the theory and the experiment. Effect of finite β makes m=0 tearing instability faster growing in deep reversal configurations with small Δ', than in weak reversal configuration with large Δ'.

Results obtained are applicable to the problem of generation of parallel flows, momentum transport driven by tearing instabilities and magnetic reconnection in space plasmas.

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