Topic: TH-W

Effects of Energetic Beam Ions on Stability Properties of Field-Reversed Configurations*

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Abstract. Stability properties of Field-Reversed Configurations (FRCs) formed by counter-helicity spheromak merging method have been studied numerically using the nonlinear hybrid and MHD simulation code HYM, including the effects of neutral beam injection. It is shown that the beam ions can have a stabilizing or destabilizing effect on the global modes in FRCs, depending on the toroidal mode number n, the mode polarization, and the beam parameters. Linear simulation results agree well with a qualitative analysis based on a generalized energy principle. Nonlinear simulations show that the beam-driven instabilities saturate nonlinearly due to changes in the distribution function of the beam ions. A new stability regime has been found for FRCs with elongation $E \sim 1$, which requires a close-fitting conducting shell and energetic beam ion stabilization. It is shown that the $n = 1$ and $n = 2$ MHD modes can be effectively stabilized by a combination of conducting shell and beam ion effects, and that the residual weakly unstable $n > 2$ modes saturate nonlinearly at low amplitudes. The resulting configuration remains stable with respect to all global MHD modes, as long as the FRC current is sustained.

1. Introduction

Effects of energetic neutral beam ions on the stability properties of field-reversed configurations (FRCs) have been studied numerically using the HYM code. Unlike previous studies [1], which considered the thermal ion kinetic effects on FRC stability, this paper investigates the effects of injection of low-density ($n_b << n_e$) and large injection velocity ($V_b \sim 10V_A$) neutral beams in FRC. FRC offers a unique fusion reactor potential because of its compact and simple geometry, translation properties, and high plasma beta. Injection of energetic ion beams in FRC configurations may help to resolve two major issues in the FRC research by simultaneously providing a stabilizing mechanism and plasma heating and current drive.

The counter-helicity spheromak merging method typically produces FRC configurations with relatively small elongation with $E \sim 1$. It is known that stability properties of oblate FRCs with $E \lesssim 1$ are different from prolate FRCs ($E \gg 1$). In particular, earlier studies have shown that the $n = 1$ tilt mode is an external mode in oblate FRCs, and that this mode can be completely stabilized by a close-fitting conducting shell even in the MHD regime [2]. With conducting-shell stabilization, the $n > 1$ internal co-interchange (kink) modes become the most unstable MHD modes [2]. In contrast to prolate FRCs, the thermal ion FLR stabilization of the low-n modes in oblate FRCs is weak even for low values of the S^* parameter. However, the localization of the low-n kink modes near the magnetic null suggests that neutral beam injection (NBI) may be a very effective stabilizing mechanism for oblate FRCs.
The nonlinear hybrid and MHD simulation code HYM [1] has been used to study stability properties of FRCs with elongation $E \sim 1$ and thermal ion kinetic parameter $S^* \sim 20$, including the effects of neutral beam injection (NBI), and a close-fitting conducting shell. A hybrid version of

the HYM code have been used, which allows a full-orbit kinetic description for both the thermal ions and the beam ions, and a fluid description is used for the electrons. Stability properties of co-interchange (kink) modes with toroidal mode numbers $n = 1 - 4$ have been investigated. Equilibrium calculations show that the beam ions tend to coalesce between the magnetic null and the separatrix near the FRC midplane, in agreement with earlier studies [3,4]. The beam localization increases for stronger beams (i.e., larger beam ion current). Due to the localization, the peak beam ion current density can be comparable to the local plasma current density, even when the fraction of the total current carried by the beam ions is small. As a consequence, the neutral beam ions can significantly modify the stability properties of the internal modes in FRCs.

2. Linear Stability Results

Previous theoretical studies have investigated the effects of energetic beam ions on FRC equilibrium and stability properties[3-6]. Equilibrium studies have shown that the beam ions tend to coalesce near the magnetic null, and the beam coalescence increases with the beam current [3,4]. The low-frequency stability properties of a hybrid system in which field reversal is created both by plasma currents, as in the FRC configuration, and by a low-density energetic component of large-orbit ions, have been studied by means of a generalized energy principle [5], and also by using three-dimensional (3D) numerical simulations [3,6]. It has been shown for the low-n co-interchange (kink) modes that the energetic beam ion contribution is stabilizing provided the condition $n|\Omega| > \omega_\beta$ is satisfied [5], where Ω is the ion toroidal rotation frequency, ω_β is the betatron frequency, and wave frequency is assumed to be small $\omega \ll \Omega$.

Numerical simulations of the tilt instability including the effects of the energetic beam ions have been performed for prolate configurations [3]. In some of the cases considered, stabilization of the tilt instability was obtained for normalized beam ion density n_b/n_e <2%, and large beam ion velocities $V_b \geq 10V_A$, where V_A is the characteristic Alfven velocity. It was also found that beam coalescence reduces the effectiveness of the stabilization [3]. Destabilization and subsequent nonlinear saturation of the $n = 4$ kink mode in prolate FRCs for the case of a cold ion beam has also been observed in 3D particle simulations [6].

This paper presents the results of hybrid simulations, which have been performed for oblate FRCs and the following beam ion parameters: toroidal velocity of $V_b \sim 4 - 6V_A$, and relative peak density of $n_b/n_e = 1.5 - 3\%$. An exponential rigid-rotor distribution function is assumed for the beam ions, corresponding to $f_b(\varepsilon, p_\phi) = A\exp[-(\varepsilon - \Omega_0 p_\phi)/T_b)]$, where A is a normalization constant, $p_\phi = m_i R v_\phi - e\psi$ is the canonical toroidal angular momentum, ε is the beam ion energy, T_b is the beam ion temperature (assumed constant), and Ω_0 =const. The beam ion distribution function corresponds to a local, shifted Maxwellian distribution with constant toroidal angular rotation frequency Ω_0. The ion density and current density are calculated from the distribution of the simulation particles, which follow the ions trajectories using the Lorentz-force equations. The nonlinear delta-f particle simulation method is employed in order to reduce the numerical noise level. The HYM simulations have been performed with a cylindrical grid size of $50 \times 32 \times 60$ in (r, ϕ, z) using 2×10^6 simulation particles. The region outside the separatrix (open field lines) is modeled as a low-density, high-resistivity plasma.

Self-consistent equilibria have been calculated including the contribution of the beam ions [7].

It has been shown that the beam ions tend to coalesce between the magnetic null and the separatrix near the FRC midplane. Strong beams are highly localized, whereas weaker beams (i.e. with relatively small beam ion current) result in a broader beam density profile. Due to localization, the peak beam current density can be comparable to the local plasma current density, even when the fraction of the total current carried by the beam ion is small. Relatively intense ion beam can significantly modify the equilibrium profiles. In particular, the inclusion of beam ion effects results in the increase in the separatrix radius, a reduction in the elongation, and more peaked current profiles.

The growth rate of the most unstable mode for each toroidal mode number with and without beam ion effects are shown in Fig. 1, where growth rates are normalized to the value of the $n = 1$ mode growth rate. For an oblate FRC configuration with thermal ion parameter $S^* = 18$ and elongation $E \sim 1$, the stability parameter $S^*/E \sim 18$ is well above the empirical stability boundary at $S^*/E = 3 - 4$. The $n = 1$ tilt mode and other $n > 1$ MHD modes are expected to be strongly unstable in this regime. Indeed, Fig. 1 shows that in the absence of beam ion effects, the $n = 1$ mode is the most unstable mode, with growth rate $\gamma = 0.83\gamma_{mhd}$, where $\gamma_{mhd} = 1.2V_A/Z_s$ is the growth rate of the $n = 1$ mode in the MHD regime. The growth rate of the $n = 2$ mode is comparable to that of the tilt mode, whereas the growth rates of the $n = 3$ and $n = 4$ modes are reduced, probably due to stronger thermal ion FLR stabilization at higher n.

Analysis of the linear mode structure shows that the most unstable $n = 2$ and $n = 3$ modes are axially-polarized co-interchange modes similar to the $n = 1$ tilt mode, i.e. for these modes the perturbed poloidal velocity has a large axial component, v_z, and a relatively small radial velocity component. [Note, these modes are also called “odd modes”, based on the $v_r(z)$ symmetry.] By contrast, the most unstable $n = 4$ kink mode has a different polarization, with a large radial velocity component (which is symmetric in z). This type of mode structure is referred to here as a “radially polarized” mode or an “even” mode.

Figure 1 shows that the growth rates of the most unstable $n = 1 - 4$ modes are changed significantly for given beam ion parameters. It is also evident that the effects of the beam ions depend on the toroidal mode number n in a complex way. In particular, the growth rate of the $n = 1$ tilt mode is increased, whereas the growth rate of the most unstable $n = 2$ mode is reduced, and it remains an axially-polarized mode. Moreover, the $n = 3$ mode becomes strongly unstable, but the polarization changes to that of a radially-polarized mode. The growth rate of the most unstable $n = 4$ mode is reduced, and it remains a radially-polarized mode.

Therefore, our simulations which include realistic FRC geometry show that linear stability properties are much more complex than previously envisioned [5] in the bycycle-tire limit or infinite-length limit. In particular, the expression for the beam-plasma interaction term, $I \sim 1/[(n\Omega - \omega)^2 - \omega_z^2] \approx 1/[n^2\Omega^2 - \omega_z^2]$, was derived for axially-polarized modes with rigid-tilt structure, where Ω is the beam ion toroidal rotation frequency, ω_z is axial betatron frequency of the beam ions, and the wave frequency is assumed small $\omega \ll \Omega$. The beam ion contribution is stabilizing provided $I > 0$, and it is destabilizing otherwise. For simulations shown in Fig. 1, the average beam ion toroidal frequency and axial betatron frequency are $\bar{\Omega} = 0.4\omega_{ci}$ and $\bar{\omega}_z = 0.63\omega_{ci}$, respectively. Accordingly, the beam ions are expected to be destabilizing for the $n = 1$ mode ($I < 0$), whereas for other modes the beam-plasma interaction

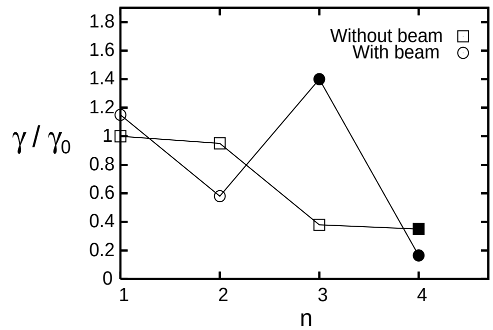

Figure 1: *Normalized growth rates of most unstable $n = 1 - 4$ modes in FRC with and without the beam ion effects. Open and solid symbols correspond to axial and radial mode polarization, respectively.*

term is positive ($I > 0$ for $n > 1$), and the beam effects should be stabilizing. This is clearly not the case for the $n = 3$ mode in Fig. 1.

The more general derivation of the beam-plasma iteraction term demonstrates that the expression for I should include other terms associated with secondary resonances of the form $n\Omega - \omega = l\omega_z + m\omega_r$, where l and m are integers, and ω_z and ω_r are axial and radial betatron frequencies, respectively. Moreover, the form of the interaction term depends on the polarization of the mode. For elongated FRCs ($E > 1$) the analysis is simplified due to the separation of the radial and axial time scales $\omega_z/\omega_r \sim 1/(2E) << 1$, and the beam-plasma interaction term can be expressed for axially-polarized (odd) modes as

$$I_{odd} = \frac{A_1}{n^2\Omega^2 - \omega_z^2} + \frac{A_3}{n^2\Omega^2 - 9\omega_z^2} + \dots \tag{1}$$

For even modes (with radial polarization), we obtain

$$I_{even} = \frac{A_2}{n^2\Omega^2 - 4\omega_z^2} + \frac{A_4}{n^2\Omega^2 - 16\omega_z^2} + \dots\ , \tag{2}$$

where the coefficients A_l are positive and depend on the exact poloidal mode structure.

For the average beam ion frequencies $\bar{\Omega} = 0.4\omega_{ci}$ and $\bar{\omega}_z = 0.63\omega_{ci}$, the sign of the first term in the expression for I_{odd} is negative for $n = 1$, and positive for $n = 2$. Accordingly, the beam ions should have a destabilizing effect on the $n = 1$ tilt mode, but reduce the growth rate of the axially-polarized $n = 2$ mode. Figure 1 shows that these predictions for the $n = 1$ and $n = 2$ modes agree with the numerical results. Similarly, the first term in the expression for I_{even} is positive for $n = 4$, which is consistent with the reduction of the growth rate of the radially-polarized $n = 4$ mode evident in Fig. 1. The strong instability of the radially-polarized $n = 3$ mode seen in Fig. 1, can also be explained using Eq. (2). For $n = 3$, the first term in I_{even} is large (and negative) due to the resonance at $3\bar{\Omega} \approx 2\bar{\omega}_z$, which indicates a strong destabilization of the radially-polarized $n = 3$ mode by the interaction with the beam ions.

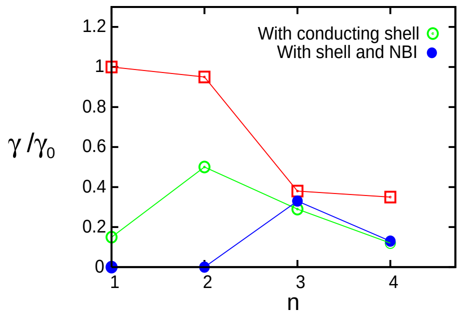

Figure 2: *Normalized growth rates of the* $n = 1 - 4$ *modes obtained from linearized hybrid simulations including thermal ion kinetic effects (red), the effects a of conducting shell (green), and the combined effects of a conducting shell and NBI stabilization (blue) for an FRC with elongation* $E = 1.1$.

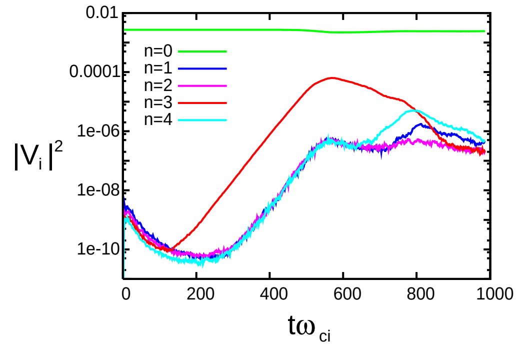

Figure 3: *Plots of the time evolution of the* $n = 0 - 4$ *Fourier harmonics of the ion kinetic energy obtained from 3D nonlinear hybrid simulations including the effects of the energetic beam ions and the close-fitting conducting shell.*

Expressions (1) and (2) can serve as a convenient tool allowing to predict qualitatively effects of the energetic beam ions on the linear stability of various MHD modes in FRCs based on the equilibrium properties of the beam. In particular, it can be seen that the direct stabilization of the $n = 1$ mode by the beam ion effects is difficult to achive for any FRC elongation. Namely, for the $n = 1$ tilt mode, the condition $I_{odd} > 0$ can be satisfied either for very weak beams, i.e., for small beam ion current density (which will have small effect on stability properties), or for large beam ion temperature, comparable to the beam ion directed energy ($T_b \sim m_i V_0^2/2$). This is related to the tendency of the strong and cold beams to coalesce near the midplane, and therefore satisfy $\Omega < \omega_z$.

Complexity of the interaction of the beam ions with the global modes in the FRC described by the expressions (1) and (2) implies that for any FRC configuration and the beam parameters, there will be several MHD modes destabilized by the energetic beam ions. More optimistic result is obtained in the linearized simulations including the effects of the close-fitting conducting shell (Section 3). Nonlinear hybrid simulations including energetic beam ion effects have demonstrated that the beam-driven instabilities can saturate nonlinearly due to self-consistent changes of the beam ion distribution function.

3. Stabilization of Oblate FRCs by a Combination of Conducting Shell and Beam Ion Effects

A new stability regime has been discovered for FRC configurations with elongation $E \sim 1$ which requires both a close-fitting conducting shell and energetic beam ions for stabilization. It has been shown in this regime that the $n = 1$ tilt mode and the $n = 2$ mode can be linearly stabilized (Fig.2), whereas the residual weak instabilities of the $n > 2$ modes saturate nonlinearly at small amplitude (Fig.3). Figure 2 shows the normalized growth rates of the most unstable mode for each toroidal mode number for three sets of simulations (where $\gamma_0 = V_A/Z_s$, and Z_s is the separatrix half-length). In the absence of a conducting shell and beam ion effects, the $n = 1$ tilt mode is the most unstable mode, with growth rate comparable to the MHD growth rate. Analysis of the linear mode structure shows that the $n = 1$ tilt mode is an external mode, which has a large perturbation amplitude at the separatrix, whereas the higher-n modes, $n > 1$, are more localized and have smaller radial extent. The growth rate of the $n = 1$ tilt mode is reduced almost by an order-of-magnitude when a close-fitting conducting shell is used for stabilization (Fig. 2, green curve), and the growth rates of the $n > 1$ radially polarized co-interchange modes are also strongly reduced due to conducting-shell effects. With conducting-shell stabilization, the $n = 2$ axially polarized kink mode becomes the most unstable mode, and all unstable low-n modes are localized near the magnetic null.

Figure 2 also shows the simulation results with the combined effects of the conducting shell and energetic beam ion stabilization (blue curve). It is evident that the beam ions have a strong stabilizing effect on the $n = 1$ and $n = 2$ modes, which are stabilized completely. For the same set of beam ion parameters, the growth rates of the $n = 3$ and $n = 4$ modes remain approximately the same, with the $n = 3$ mode being more unstable than the $n = 4$ mode. Numerical simulations with larger beam ion density, $n_b/n_e = 0.05$, show a reduction of the growth rates of these modes by a factor 1.5, but not complete stabilization.

A set of 3D nonlinear simulations have also been performed to study the nonlinear evolution of the FRC in the presence of a conducting shell and energetic beam ions for the same set of beam ion parameters. Figure 3 shows the time evolution of the $n = 0 - 4$ Fourier components of the ion kinetic energy from these simulations. It is evident that the $n = 3$ instability saturates nonlinearly at $t \approx 600(1/\omega_{ci}) = 30t_A$ at low amplitude (Fig. 3), and therefore it is not a dangerous mode. Simulation runs which model a "sustained FRC" (i.e., without decay of the equilibrium current) show that after the $n = 3$ and the $n = 4$ modes saturate, the resulting configuration remains stable with respect to all global MHD modes. In contrast, in simulations of the decaying FRC (not shown), the $n = 1$ mode eventually becomes unstable due to reduction of the FRC separatrix radius and the reduction of the stabilizing effect of the conducting shell. Note that this is a new stability regime (i.e., a combination of small elongation, conducting shell, and beam ion effects in a sustained FRC), which has not yet been studied before. The stabilizing effects of the neutral-beam-induced bulk ion rotation have not been included in the present numerical study, but these effects will likely contribute to the stabilization of the low-n MHD modes.

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