Dynamical origin of shear flow induced modifications of nonlinear magnetic islands

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Abstract. A generalized Newcomb equation that incorporates inertial contributions from equilibrium shear flows is derived. It is shown from numerical solutions of this equation that the stability parameter Δ' can be significantly influenced by flow as it becomes a sensitive function of the global profiles of the magnetic field and flow velocity. These results have important implications for the nonlinear evolution of magnetic islands in the presence of flows.

1. Introduction

The nonlinear evolution of magnetic islands due to unstable classical or neoclassical tearing modes is a topic of much current interest particularly in the context of confinement limits for long pulse experiments in superconducting tokamaks [1]. The size and life times of these magnetic islands set a limit on the plasma β and are an important concern for future reactor configurations. Much attention is therefore being directed towards the experimental and theoretical elucidation of factors that affect the onset and saturation of such islands. The Rutherford theory of neoclassical tearing modes has provided a particularly useful analytical paradigm for understanding the nonlinear behavior of the islands and has also been the basis for the formulation of various stabilization schemes [2]. Large scale numerical simulation initiatives, such as NIMROD [3], involving direct solution of model MHD equations constitute an alternative and complementary approach to this problem. An important issue that has not yet been satisfactorily resolved and needs detailed exploration relates to the dynamical interaction of the magnetic islands with equilibrium shear flows. Flows are ubiquitous in most tokamak plasmas and can arise from a variety of causes such as unbalanced neutral beam injection, radio frequency heating or as a byproduct of micro-turbulence. Some of the basic issues related to the influence of sheared flows on resistive instabilities have been known for a long time and have been investigated in simplified geometries and model flow profiles in several past studies [4–6]. However their detailed assessment for realistic geometries either through an analytic approach or by means of numerical simulations poses serious mathematical as well as computational challenges. Some recent attempts in this direction e.g. numerical investigations employing a fully toroidal code [7] based on generalized reduced MHD equations [8] have revealed a number of interesting results. It has been found, for example, that differential flow provides a strong stabilizing influence leading to lower saturated island widths for the classical tearing mode and reduced growth rates for the neoclassical tearing mode. The effect of velocity shear is found to depend on the sign of the shear at the mode resonant surface with negative shear providing a stabilizing effect and positive shear acting in a destabilizing fashion [7, 9]. These results have been found for toroidal sheared flows that are restricted in magnitude to be a fraction of the Alfven velocity and for equilibrium plasmas that have a small inverse aspect ratio ($a/R \sim 10$, where a and R are the minor and major radii respectively). While some general qualitative features of these numerical results can be identified from selective switching on and off of various terms of the model equations a detailed analytic understanding of the dynamical origin of many of the flow induced physical effects is still lacking.

In this paper we discuss an important flow induced effect that can significantly alter the general Rutherford model results. In present applications of the Rutherford model for neoclassical tearing modes one generally ignores the influence of flows on the outer layer dynamics and adopts a value for the stability parameter Δ' that is given by the static equilibrium configuration. We derive a generalized version of the outer layer equations in a cylindrical geometry that incorporates both inertial contributions of flow and finite β terms and show through numerical solutions that the value of Δ' can be significantly influenced by the combination of the velocity and magnetic field profiles. Our findings should prove useful in extending the applicability of the Rutherford model and also in the interpretation of numerical investigations carried out on more complex codes like NIMROD. Based on our understanding of these flow induced effects one can hope to develop experimental strategies that can exploit flows for mitigation or better control of island growths in long pulsed tokamak experiments.

2. Newcomb equation in the presence of flow

We consider a uniform density compressible plasma in a cylindrical geometry (r, θ, z) that has a uniform equilibrium flow along the z axis and a sheared poloidal flow along the θ direction. To describe the outer layer dynamics we consider the ideal MHD model equations given by,

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\,\mathbf{V}\right] = -\nabla P + \mathbf{J}\times\mathbf{B} \tag{1}$$

$$\mathbf{E} + \mathbf{V}\times\mathbf{B} = 0 \tag{2}$$

$$\nabla\times\mathbf{B} = \mu_0 \mathbf{J} \;\; ; \;\; \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3}$$

$$\nabla\cdot\mathbf{B} = 0 \;\; ; \;\; \nabla\cdot\mathbf{V} = 0 \tag{4}$$

We assume the equilibrium quantities to be of the form,

$$\mathbf{B_0} = B_{0\theta}(r)\hat{\mathbf{e}}_\theta + B_{0z}(r)\hat{\mathbf{e}}_z \; ; \;\; \mathbf{V_0} = V_{0\theta}(r)\hat{\mathbf{e}}_\theta + V_{0z}\hat{\mathbf{e}}_z$$

The ideal equilibrium, obtained from the momentum eqn.(1), is then given by the relation,

$$\frac{\beta}{2}\frac{dP_0}{dr} = \frac{V_{0\theta}^2}{r} - \frac{B_{0\theta}^2}{r} - B_{0\theta}\frac{dB_{0\theta}}{dr} - B_{0z}\frac{dB_{0z}}{dz} \tag{5}$$

where we have normalized the magnetic field by B_{00z}, the velocity field by $V_A\left(= B_{00z}/(\mu_0\rho_0)^{1/2}\right)$, pressure by $P_{00} = \beta B_{00z}^2/2\mu_0$ with P_{00} being the peak value of the pressure, B_{00z} the peak value of the axial magnetic field and $\beta = 2\mu_0 P_{00}/B_{0z}^2$.

We next linearize eqns.(1) and (2) about these equilibrium quantities and neglect the contribution from the time derivative term. This is appropriate for the ideal external region where resistivity can be neglected and the mode growth term, which scales inversely as some power of the resistivity, is also very small. The perturbed quantities then obey the following set of equations,

$$(\mathbf{V_0}\cdot\nabla)\mathbf{V_1}+(\mathbf{V_1}\cdot\nabla)\mathbf{V_0}=-\nabla p_1^*+(\mathbf{B_0}\cdot\nabla)\mathbf{B_1}+(\mathbf{B_1}\cdot\nabla)\mathbf{B_0} \tag{6}$$

$$(\mathbf{B_1}\cdot\nabla)\mathbf{V_0}-(\mathbf{V_0}\cdot\nabla)\mathbf{B_1}+(\mathbf{B_0}\cdot\nabla)\mathbf{V_1}-(\mathbf{V_1}\cdot\nabla)\mathbf{B_0}=0 \tag{7}$$

where $p_1^*=\frac{\beta}{2}P_1+\mathbf{B_0}\cdot\mathbf{B_1}$. We assume the perturbed quantities to have a spatial dependence of the form, $f_1(r,\theta,z)=f_1(r)exp(im\theta+ik_z z)$. We further define,

$$F=\mathbf{k}\cdot\mathbf{B_0}\ \ ;\ \ G=\mathbf{k}\cdot\mathbf{V_0}$$

where $\mathbf{k}=(0,m/r,k_z)$. The radial components of eqns.(6) and (7) give,

$$-FB_{1r}+i\frac{2}{r}\left(V_{0\theta}V_{1\theta}-B_{0\theta}B_{1\theta}\right)+GV_{1r}-i\frac{dp_1^*}{dr}=0 \tag{8}$$

$$V_{1r}=\frac{G}{F}B_{1r} \tag{9}$$

Further, the θ and z components of eqns.(6) and (7) can be used to obtain the following equations,

$$ip_1^*=\frac{B_{1r}}{r}\left(H\frac{\partial F}{\partial r}+\frac{2mB_{0\theta}H}{r^2}-\frac{HF}{r}\right)-\frac{HF}{r}\frac{\partial B_{1r}}{\partial r}-\frac{V_{1r}}{r}\left(H\frac{\partial G}{\partial r}+\frac{2mV_{0\theta}H}{r^2}-\frac{HG}{r}\right)+\frac{HG}{r}\frac{\partial V_{1r}}{\partial r} \tag{10}$$

$$i\frac{2}{r}\left(V_{0\theta}V_{1\theta}-B_{0\theta}B_{1\theta}\right)=\frac{2m(ip_1^*)}{r^2}\frac{(V_{0\theta}G-B_{0\theta}F)}{(F^2-G^2)}+\left[\frac{2(FB_{1r}+GV_{1r})}{(F^2-G^2)}\left(\frac{B_{0\theta}^2}{r^2}-\frac{V_{0\theta}^2}{r^2}\right)-\frac{4V_{0\theta}B_{0\theta}}{r^2}\frac{(GB_{1r}+FV_{1r})}{(F^2-G^2)}-\left(\frac{\partial V_{0\theta}^2}{\partial r}-\frac{\partial B_{0\theta}^2}{\partial r}\right)\frac{\left(\frac{FB_{1r}}{r}-\frac{GV_{1r}}{r}\right)}{(F^2-G^2)}\left(B_{0\theta}\frac{\partial V_{0\theta}}{\partial r}-V_{0\theta}\frac{\partial B_{0\theta}}{\partial r}\right)(GB_{1r}-FV_{1r})\right] \tag{11}$$

where $H=r^3/(k_z^2r^2+m^2)$. Using eqn.(10) and eqn.(11) to substitute for p_1^* and $(V_{0\theta}V_{1\theta}-B_{0\theta}B_{1\theta})$ in eqn.(8) and after some rearrangement of terms one gets,

$$\begin{aligned}
&F\frac{d}{dr}\left(H\frac{d\psi}{dr}\right)-\frac{2mH}{r^2\alpha}\frac{G}{F}\left(V_{0\theta}-\frac{G}{F}B_{0\theta}\right)\frac{d\psi}{dr}-\psi\frac{d}{dr}\left(H\frac{dF}{dr}\right)-F\psi\left[r+\frac{2m}{r^2F^2\alpha}\left(H\frac{dF}{dr}+\frac{2mB_{0\theta}H}{r^2}-\frac{HF}{r}\right)\left(B_{0\theta}-\frac{G}{F}V_{0\theta}\right)-\frac{2}{rF^2\alpha}\left(B_{0\theta}^2+V_{0\theta}^2\right)+\frac{4}{rF^2\alpha}\frac{G}{F}V_{0\theta}B_{0\theta}+\frac{1}{F^2\alpha}\left(\frac{dV_{0\theta}^2}{dr}-\frac{dB_{0\theta}^2}{dr}\right)+\frac{r}{F}\frac{d}{dr}\left(\frac{2mB_{0\theta}H}{r^3}\right)-\frac{r}{F}\frac{d}{dr}\left(\frac{HF}{r^2}\right)-\frac{H}{rF}\frac{dF}{dr}-r\frac{G}{F}\left(B_{0\theta}\frac{dV_{0\theta}}{dr}-V_{0\theta}\frac{dB_{0\theta}}{dr}\right)\right]\\
&=G\frac{d}{dr}\left(H\frac{dW}{dr}\right)-\frac{2mH}{r^2\alpha}\left(V_{0\theta}-\frac{G}{F}B_{0\theta}\right)\frac{dW}{dr}-W\frac{d}{dr}\left(H\frac{dG}{dr}\right)-GW\left[r+\frac{2m}{r^2GF\alpha}\left(H\frac{dG}{dr}+\frac{2mV_{0\theta}H}{r^2}-\frac{HG}{r}\right)\left(B_{0\theta}-\frac{G}{F}V_{0\theta}\right)+\frac{2}{rF^2\alpha}\left(B_{0\theta}^2+V_{0\theta}^2\right)-\frac{4}{rGF\alpha}V_{0\theta}B_{0\theta}+\frac{1}{F^2\alpha}\left(\frac{dV_{0\theta}^2}{dr}-\frac{dB_{0\theta}^2}{dr}\right)+\frac{r}{G}\frac{d}{dr}\left(\frac{2mV_{0\theta}H}{r^3}\right)-\frac{r}{G}\frac{d}{dr}\left(\frac{HG}{r^2}\right)-\frac{H}{rG}\frac{dG}{dr}-r\frac{F}{G}\left(B_{0\theta}\frac{dV_{0\theta}}{dr}-V_{0\theta}\frac{dB_{0\theta}}{dr}\right)\right]
\end{aligned} \tag{12}$$

where $\alpha = 1 - (G^2/F^2)$ and we have simplified the notation somewhat by using ψ for normalized B_{1r} and W for normalized V_{1r}. Further, using eqn.(5) and eqn.(9) in the above equation (12) and after affecting some simplifications one can get the following equation in the single variable ψ.

$$H\frac{d^2\psi}{dr^2} + \left(\frac{dH}{dr} + h_f\right)\frac{d\psi}{dr} - \left[\frac{g}{F^2} + \frac{g_f}{F^2} + \frac{1}{F}\frac{d}{dr}\left(H\frac{dF}{dr}\right)\right]\psi = 0 \tag{13}$$

where,

$$g = \frac{(\alpha m^2 - 1)rF^2}{\alpha(k_z^2r^2 + m^2)} + \frac{k_z^2r^2}{\alpha(k_z^2r^2 + m^2)}\left(\alpha rF^2 + F\frac{2(k_zr - mB_{0\theta})}{k_z^2r^2 + m^2} + \beta\frac{dP_0}{dr}\right)$$

$$h_f = \frac{2H}{\alpha}\frac{G}{F}\left(\frac{G}{F}\frac{1}{F}\frac{dF}{dr} - \frac{1}{F}\frac{dG}{dr}\right)$$

$$\begin{aligned}
g_f \;=\; & \frac{2H}{\alpha}\frac{G}{F}\frac{dF}{dr}\left(\frac{G}{F}\frac{dF}{dr} - \frac{dG}{dr}\right) + \frac{4}{r\alpha^2}\frac{G}{F}V_{0\theta}B_{0\theta} + \frac{GH}{r\alpha}\left(\frac{\partial G}{\partial r} + \frac{2mV_{0\theta}}{r^2}\right) + \frac{2V_{0\theta}}{\alpha}\frac{dV_{0\theta}}{dr} \\
& -\frac{G}{\alpha}\frac{d}{dr}\left(\frac{2mHV_{0\theta}}{r^2}\right) + \left(\frac{m^2}{k_z^2r^2 + m^2} - \frac{2}{\alpha}\right)\frac{2V_{0\theta}^2}{r\alpha} + \frac{Gr}{\alpha}\frac{d}{dr}\left(\frac{HG}{r^2}\right) \\
& + \left(\frac{4}{\alpha^2}\frac{k_z^2r^2}{k_z^2r^2 + m^2}\frac{G}{F}\frac{B_{0\theta}}{r} - \frac{2mH}{r^2\alpha^2}\left(\frac{dG}{dr} - \frac{G}{r}\right)\right)\left(V_{0\theta} - \frac{G}{F}B_{0\theta}\right) \\
& - \left(\frac{2mH}{r^2\alpha^2}\frac{G}{F}\left(\frac{dG}{dr} + \frac{2mV_{0\theta}}{r^2} - \frac{G}{r}\right)\right)\left(B_{0\theta} - \frac{G}{F}V_{0\theta}\right)
\end{aligned}$$

For $G = 0$ and $\beta = 0$ eqn.(13) reduces to the standard outer layer equation that has been analyzed in the paper by Furth, Rutherford and Selberg [10]. More recently, Nishimura *et al* [11] have extended the results of [10] to include finite β effects and have shown that finite β can have a stabilizing effect on Δ'. The effect of equilibrium sheared flows on Δ' has been examined in the past by Chen and Morrison [5] but only in a simple slab geometry. For G finite and in the limit of a slab geometry ($r \to \infty$, $d/dr \to d/dx$) our eqn.(13) reduces to the set of equations that have been discussed by Chen and Morrison [5]. Note that in the slab limit the finite β contribution disappears. Thus eqn.(13) represents a more generalized description of the outer layer dynamics that takes into account finite β contributions, cylindrical curvature effects as well as sheared flow effects.

3. Numerical evaluation of Δ'

To investigate the influence of flows on the outer layer dynamics we have solved eqn.(13) numerically to determine Δ' for a model set of profiles of $B_{0\theta}$ and velocity $V_{0\theta}$. An advanced shooting method, first developed by Nishimura *et al* [11] for the finite β problem, has been adopted for this purpose. The algorithm involves numerical integration of the equation away from the singular layer towards the boundaries. The following analytic expressions representing asymptotic solutions for ψ near the resonant surface have been used to launch the numerical solutions.

$$\psi = A_l|s|^{h+1} - B_l|s|^{-h} \quad ; \; for \; x < x_s \tag{14}$$

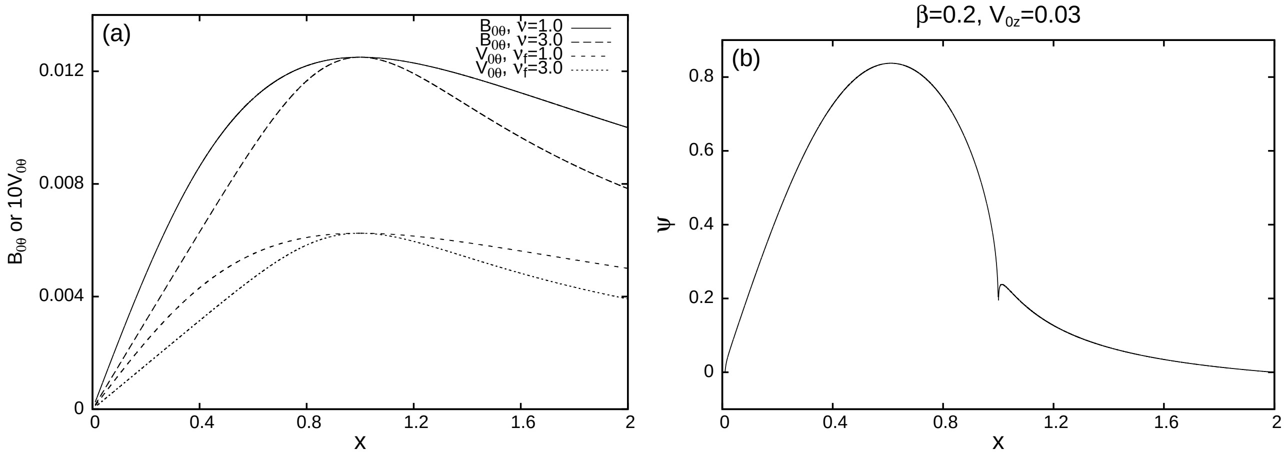

Figure 1. (a) Profiles of $B_{0\theta}$ and $V_{0\theta}$ (b) Eigenfunction ψ for (m=2,n=1) mode

$$\psi = A_r|s|^{h+1} + B_r|s|^{-h} \quad ; \; for \; x > x_s \tag{15}$$

where $x = r/r_s$, r_s is the location of the singular layer and $s = x - x_s$.

$$h = -\frac{1}{2} + \frac{1}{2}\sqrt{1 - 4D_s}$$

The Mercier coefficient D_s is given as,

$$\begin{aligned} D_s \;=\; & -\frac{q_s^2}{{q_s'}^2 \alpha x_s}\left[\beta\frac{dP_0}{dx} + \frac{2x}{Hk_z^2r_s^2}V_{0\theta}\frac{dV_{0\theta}}{dx} + \frac{2}{Hk_z^2r_s^2}\left(\frac{m^2}{k_z^2r_s^2x^2+m^2} - \frac{2}{\alpha}\right)V_{0\theta}^2\right. \\ & + + \left(\frac{4}{\alpha}\frac{G}{F}\frac{B_{0\theta}}{x} - \frac{2m}{\alpha x k_z^2 r_s}\frac{dG}{dx}\right)\left(V_{0\theta} - \frac{G}{F}B_{0\theta}\right) - \frac{2m}{\alpha x k_z^2 r_s}\frac{G}{F}\left(\frac{dG}{dx} + \frac{2mV_{0\theta}}{r_s x^2}\right) \\ & \times \left.\left(B_{0\theta} - \frac{G}{F}V_{0\theta}\right) + \frac{4}{\alpha H k_z^2 r_s^2}\frac{G}{F}V_{0\theta}B_{0\theta}\right]_{x=x_s} \end{aligned} \tag{16}$$

We iterate the constants A and B until the solution satisfies the appropriate boundary conditions [11]. The value of Δ' is then obtained as,

$$\Delta' = \frac{A_r}{B_r} - \frac{A_l}{B_l}$$

In the absence of flow our numerical values of Δ' agree with those of Nishimura *et al* [11] for a choice of their model profile.

4. Results and Discussion

We now present our numerical results of Δ' values obtained for the (m=2, n=1) tearing mode. We have used the following equilibrium profile for the normalized poloidal magnetic field,

$$B_{0\theta}(x) = \frac{r_s}{Rq_0}\frac{x}{(1+x^{2\nu})^{1/\nu}} \quad ; \; q(x) = q_0(1+x^{2\nu})^{1/\nu}$$

where R the major radius is taken to be a constant quantity and ν is an index that controls the flatness of the magnetic profile. To account for finite β effects we have chosen the normalized pressure profile to be,

$$P_0(x) = 1 - \left(\frac{x}{x_b}\right)^2$$

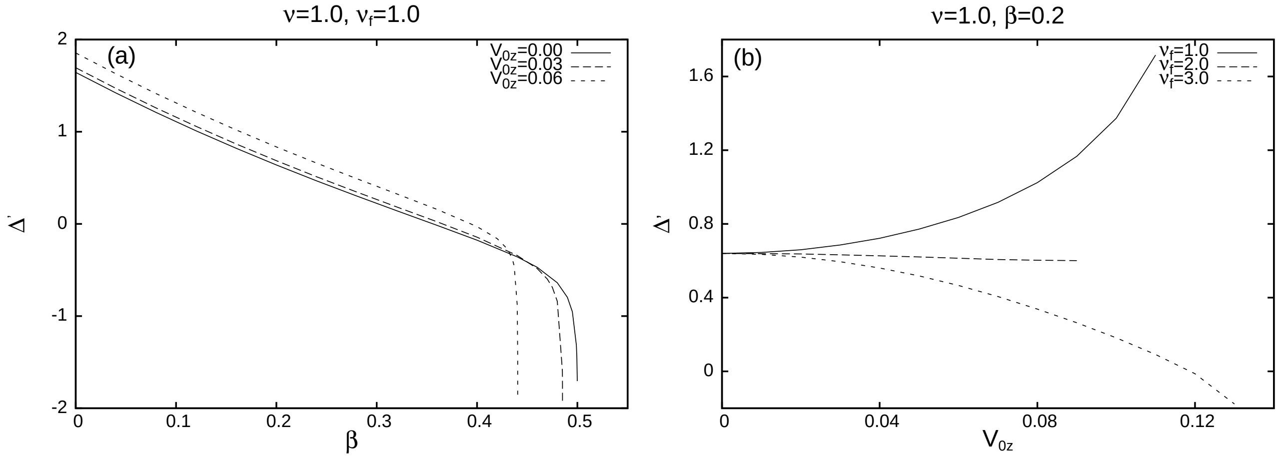

Figure 2. (a) Change of Δ' with β for different V_{0z} (b) Change of Δ' with V_{0z} for different $V_{0\theta}$ profiles

The equilibrium pressure balance is ensured by providing a small variation in B_{0z}. The plasma boundary is chosen to be at $x_b = 2$ and by construct $x_s = 1$. Note that since $q(x_s) = m/n = 2$, for the $(2,1)$ tearing mode, the index ν and the quantity q_0 are related as $q_0 = 2^{1-(1/\nu)}$. For the velocity profile we have chosen the following model form,

$$V_{0\theta}(x) = \frac{r_s V_{0z}}{R q_{v0}} \frac{x}{(1+x^{2\nu_f})^{1/\nu_f}} \quad ; \quad q_v(x) = q_{v0}(1+x^{2\nu_f})^{1/\nu_f} \quad ;$$

where $q_v = rV_{0z}/(RV_{0\theta})$, and ν_f is an index that controls the flatness of the flow velocity profile. For convenience we choose $q_{v0} = m/2^{(1/\nu_f)}$ which makes the magnetic and velocity profiles to have a similar behavior near the singular layer. Fig. 1 (a) shows two sets of profiles of $B_{0\theta}$ and $V_{0\theta}$ plotted for values of $(\nu = 1.0, 3.0)$ and $(\nu_f = 1.0, 3.0)$ respectively and Fig. 1 (b) depicts a typical eigenfunction for the $(m = 2, n = 1)$ mode. In Fig. 2 (a) we have plotted the variation of Δ' with β for various values of the flow velocity V_{0z}. The flatness profile indices ν and ν_f are held constant at the value of unity. The solid curve (no flow case) corresponds to the previous result of Nishimura *et al* [11] and shows the stabilizing effect of finite β on Δ'. Due to a factor of 2 difference in the definition of β between our normalization and that adopted in [11] the x axis scale is expanded by a factor of 2 in our case. When finite flow velocity is turned on (at the same values of ν and ν_f) we notice two differences from the no flow result. At low β finite flow has a slightly destabilizing effect but the threshold β at which the curve begins to sharply drop to negative values is decreased as seen from the two other curves in the figure. Thus one can access higher β values more easily in the presence of flows. This trend however is strongly influenced by the shape of the velocity profile. This is shown in Fig. 2 (b) where the variation of Δ' with V_{0z} is shown at a fixed value of $\beta = 0.2$, $\nu = 1$ and for different values of ν_f. As ν_f increases we see that there is a change in the behavior of Δ' beyond a threshold value of ν_f and flow begins to have a destabilizing effect. This sensitivity to the profile parameter is also seen for the magnetic field. In Fig. 3(a) we show how Δ' changes with the magnetic field flatness parameter ν for a zero β plasma. As can be seen there is a dramatic rise in the value of Δ' as ν increases i.e. as the magnetic field profile gets more peaked. At a given value of ν if ν_f is raised then Δ' decreases somewhat indicating that raising the peakedness of the flow profile has a stabilizing influence. The stabilizing influence is more pronounced at higher values of ν. Fig. 3 (b) shows the effect of finite β on Δ' with velocity profiles of different ν_f and for a fixed value of $\nu = 1$. The figure shows that β has a stabilizing influence on Δ' at a given value of $\nu = 1$ in agreement with

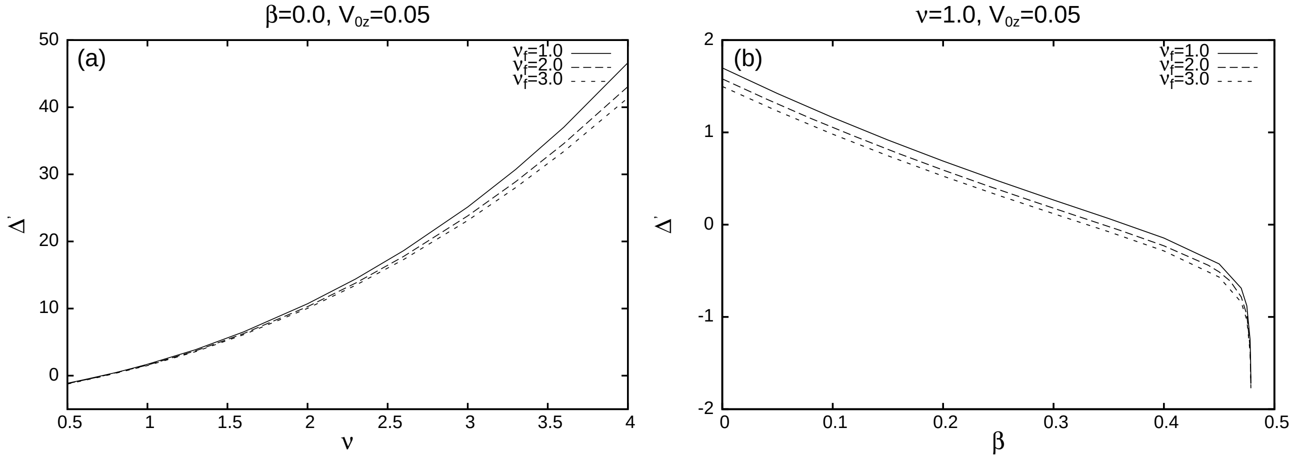

Figure 3. (a) Change of Δ' with ν for different $V_{0\theta}$ profiles (b) Change of Δ' with β for different $V_{0\theta}$ profiles

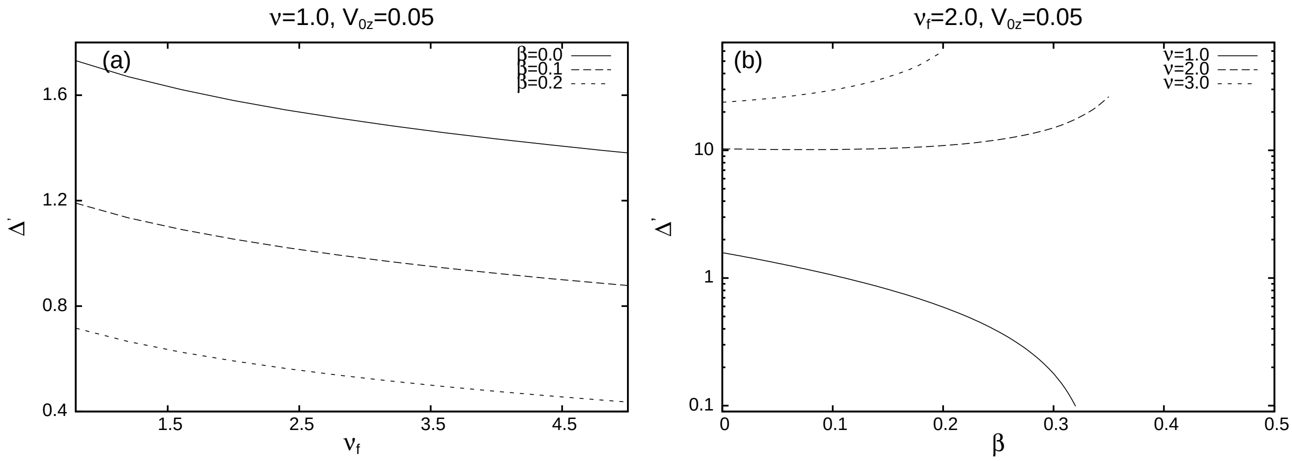

Figure 4. (a) Change of Δ' with ν_f for different β (b) Change of Δ' with β for different $B_{0\theta}$ profiles

the results of [11]. Increasing ν_f at this value of ν and for a given value of β provides a further stabilizing influence but the incremental effect is small. This is further supported by Fig. 4(a) where the curve of Δ' *vs* ν_f for a given β is seen to be quite quite flat. The above picture changes however when the value of ν is increased to higher values. As Fig. 4(b) demonstrates, for a value of ν exceeding a critical value increasing β can have a destabilizing effect (e.g. the curves for $\nu = 2, 3$ with $\nu_f = 2$). This is very similar to the turnover behavior that is observed in Fig. 2 (b) for Δ' *vs* V_{0z} curve where when ν_f is changed keeping β and ν constant.

These numerical results suggest that the combination of the magnetic and velocity profile variations along with finite β effects can profoundly influence the magnitude of Δ' and consequently the stability of the tearing mode. This global dependence of Δ' needs to be appropriately accounted for when estimating stability thresholds or saturation widths of magnetic islands in the nonlinear Rutherford theory. Our model outer layer equation (13) provides a means for estimating Δ' in the presence of sheared flows particularly for large aspect ratio machines. When toroidal effects become important it is necessary to generalize the equation to account for the additional geometric effects. Our present calculations were done with simple model profiles and in a limited parametric space to highlight the sensitivity of Δ' to equilibrium profile parameters. A more direct utility of our equation would be to estimate Δ' using realistic equilibrium profiles obtained from MHD equilibrium codes. We are presently carrying out such a calculation using profiles from TOQ in order to get a better understanding of the stability results obtained from the NEAR code [7].

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